AERO 630 : HW #1

INTRODUCTION TO RANDOM DYNAMICAL SYSTEMS

1. A rod of length "l" is broken at two points chosen at random. What is the probability that a triangle can be formed from the segments obtained?

2. N points are scattered at random and independent of one another inside a sphere of radius "R".
   (a) What is the probability that the distance from the center of the sphere to the nearest point will not be less than "r"?
   (b) What is the limit of the probability found in part (a) if $R \to \infty$ and $N/R^3 \to 4\pi\lambda/3$?

   NOTE: this problem is taken from astronomy: in the neighbourhood of the sun, $\lambda \approx 0.0063$ if R is measured in parsecs.

3. The probability that a molecule which has collided with another at time $t=0$ and undergone no further collisions with other molecules upto time "t" will have a collision with another molecule in the interval "t" to "t+Δt" is $\lambda \Delta T + o(\Delta T)$. Determine the probability that the time of free motion (i.e., the time between successive collisions) is greater than "t".

4. The random variable ξ has $F(x)$ as its distribution function ($p(x)$ is the density function). Find the distribution function (density function) for each of the random variables:

(a) $\eta = a\xi + b$

(b) $\eta = \xi^{-1}$ ($P\{\xi = 0\} = 0$).

(c) $\eta = \tan \xi$

(d) $\eta = f(\xi)$ where f is a continuous strictly monotonic function.

5. A point is chosen at random on the segment of the η-axis between the points $(0,0)$ and $(0,R)$. Through this point we draw the chord to the circle "$x^2+y^2 = R^2$" that is perpendicular to the η-axis. Determine the distribution of the length of this chord.

6. The random variable ξ has a continuous distribution function $F(x)$. How is the random variable $\eta = F(\xi)$ distributed?