- 8. (a) Dofine error function and prove that
 - filerti-xi -cri(x):



ibi Evaluate

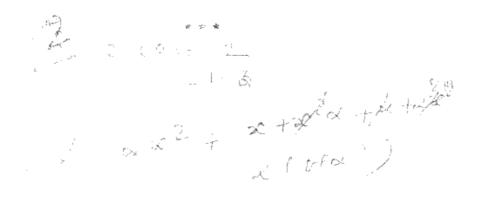
$$\int_0^{\infty} \frac{\log(1+\alpha x)}{1+\alpha^2} dx$$

Lat lest for the convergence of the improper untegral

$$\int_{-\infty}^{\infty} \frac{dx}{\sqrt{x^2 + x^2}}$$

do Prove that -

$$|(21)| = \frac{8\pi \rho^{-\sqrt{4}}}{8\pi^{-\sqrt{4}}} (2x + \frac{3}{3}\sqrt{7})$$



Code 21:101

Code: 211101

2012

MATHEMATICS-1

- Time: 3-hours

- Full Marks : 70 ----

Instructions:

- All questions carry equal marks.
- 497 There are NINE questions in this paper.
- (iii) Attempt FIVE questions in all.
- (iv) Question No. 1 is compalisory.
- 1. Answer any five questions .
 - (a) Explain elementary transformations on a matrix.
 - Define characteristic equation of a matrix and find the eigenvalues of $\begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$.
 - (cl- State Leibnitz theorem.
 - Write Euler's theorem on homogeneous function.
 - Solve:

- State comparison test for convergence of improper integral.
- Define beta and gamma functions,
- Determine the values of b such that the rank of A is 3, where

$$A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ b & 2 & 2 & 2 \\ 9 & 9 & b & 3 \end{bmatrix}$$

- State and prove Cayley-Hamilton theorem
- (a) If $y = \sin(m\sin^{-1}x)$, then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 - m^2)y_n = 0$
 - (b) Using Maclaurin series, show that $e^{x \cos x} = 1 + x + \frac{x^2}{2} - \frac{x^3}{2} + \cdots$
- Explain curvature and obtain a formula to find radius of curvature of a given curve.
 - Determine the point on $y = 4x x^2$, where the curvature is maximum.

- Define harmonic functions: If $v = (x^2 + u^2 + x^2)^{-1/2}$ then prove that $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial u^2} + \frac{\partial^2 v}{\partial z^2} = 0$.
 - Discuss the maxima and minima of $f(x, y) = x^3y^2(1 - x - y)$
- Solve any two of the following (i) $(1 + e^{x/y}) dx + e^{x/y} (1 - x/y) dy = 0$ (ii) $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right) \frac{dx}{dy} = 1$ (iii) $\frac{dz}{dx} + \left(\frac{z}{x}\right) \log z = \frac{z}{x} (\log z)^2$
 - State and prove the necessary and sufficient condition for differential equation to be exact.
- 7. (a) Solve :
- $x^{3} \frac{d^{3}y}{dx^{3}} + 3x^{2} \frac{d^{2}y}{dx^{2}} + x \frac{dy}{dx} + 8y = 65\cos(\log x) + \frac{4}{5} \frac{5}{4} \frac{1}{5}$ $3c^{3} + 33c^{2} + \frac{1}{5} \frac{1}$