8. (a) Define error function and prove that—

(i) $\operatorname{erf}(-x) = -\operatorname{erf}(x)$

(ii) $\operatorname{erf}(\infty) = 1$

(b) Evaluate

$$\int_0^\infty \frac{\log(1+\alpha x)}{1+x^2}\,dx$$

9. (a) Test for the convergence of the improper integral

$$\int_{-a}^{a} \frac{dx}{\sqrt{a^2 - x^2}}$$

(b) Prove that—

(i) $\Gamma(1/2) = \sqrt{\pi}$

(ii) $\int_0^\infty \frac{e^{-\sqrt{x}}}{x^{1/4}}\,dx = \frac{3}{3}\sqrt{\pi}$

* * *

Code : 211101

2012

MATHEMATICS—I

Time : 3 hours *Full Marks : 70*

Instructions :

(i) All questions carry equal marks.

*(ii) There are **NINE** questions in this paper.*

*(iii) Attempt **FIVE** questions in all.*

*(iv) Question No. **1** is compulsory.*

1. Answer any *five* questions :

(a) Explain elementary transformations on a matrix.

(b) Define characteristic equation of a matrix and find the eigenvalues of $\begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$.

(c) State Leibnitz theorem.

(d) Write Euler's theorem on homogeneous function.

(e) Solve :

$$\cos x\,dy = y(\sin x - y)\,dx$$

(f) State comparison test for convergence of improper integral.

(g) Define beta and gamma functions.

2. (a) Determine the values of b such that the rank of A is 3, where

$$A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ b & 2 & 2 & 2 \\ 9 & 9 & b & 3 \end{bmatrix}$$

(b) State and prove Cayley-Hamilton theorem.

3 (a) If $y = \sin(m \sin^{-1} x)$, then prove that

$$(1-x^2)\, y_{n+2} - (2n+1)\, x y_{n+1} - (n^2 - m^2)\, y_n = 0$$

(b) Using Maclaurin series, show that

$$e^{x \cos x} = 1 + x + \frac{x^2}{2} - \frac{x^3}{3} + \cdots$$

4. (a) Explain curvature and obtain a formula to find radius of curvature of a given curve.

(b) Determine the point on $y = 4x - x^2$, where the curvature is maximum.

5. (a) Define harmonic function. If

$$v = (x^2 + y^2 + z^2)^{-1/2}$$

then prove that $\dfrac{\partial^2 v}{\partial x^2} + \dfrac{\partial^2 v}{\partial y^2} + \dfrac{\partial^2 v}{\partial z^2} = 0.$

(b) Discuss the maxima and minima of

$$f(x, y) = x^3 y^2 (1 - x - y)$$

6. (a) Solve any *two* of the following

(i) $(1 + e^{x/y})\, dx + e^{x/y}(1 - x/y)\, dy = 0$

(ii) $\left(\dfrac{e^{-2\sqrt{x}}}{\sqrt{x}} - \dfrac{y}{\sqrt{x}}\right)\dfrac{dx}{dy} = 1$

(iii) $\dfrac{dz}{dx} + \left(\dfrac{z}{x}\right)\log z = \dfrac{z}{x}(\log z)^2$

(b) State and prove the necessary and sufficient condition for differential equation to be exact.

7. (a) Solve : $\frac{1}{8} \times \frac{1}{9}$ (1-

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$$

$\frac{6-5}{6}$

(b) Solve :

$$x^3 \frac{d^3y}{dx^3} + 3x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + 8y = 65\cos(\log x)$$

$x^3 + 3x^2 d$