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Code : 121201

2012 (A)

MATHEMATICS—III

Time : 3 hours

Full Marks : 70

Instructions :

(i) All questions carry equal marks.

*(ii) There are **TEN** questions in this paper.*

*(iii) Attempt any **FIVE** questions.*

1. Discuss singular points and solve the differential equation $x(x-1)y'' + (3x-1)y' + y = 0$ in series.

2. (a) Prove :

$$xJ_n' = -nJ_n + xJ_{n-1}$$

(b) (i) Prove :

$$J_{\frac{1}{2}}(x) = \sqrt{\left(\frac{2}{\pi x}\right)} \sin x$$

(ii) Prove :

$$J_{-n}(x) = (-1)^n J_n(x)$$

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3. (a) Prove :

$$xJ_n' = nJ_n - xJ_{n+1}$$

Legendre polynomials.

4. (a) Form a partial differential equation from

$$x^2 + y^2 + (z-c)^2 = a^2$$

(b) Obtain the solution of the wave equation

$$\frac{\partial^2 y}{\partial t^2} = e^2 \frac{\partial^2 y}{\partial x^2}$$

using the method of separation of variables.

5. (a) Find the d'Alembert's solution of the wave equation

$$\frac{\partial^2 y}{\partial t^2} = e^2 \frac{\partial^2 y}{\partial x^2}$$

(b) Find the deflection of a vibrating string of unit length having fixed ends with initial velocity zero and initial deflection $f(x) = k(\sin x - \sin 2x)$.

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6. (a) Show that the polar form of Cauchy-Riemann equation are

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}, \quad \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$$

Deduce that

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$$

(b) Define analytic function and show that the sufficient condition for a function $f(z) = u + iv$ to be analytic at all the points in a region R is (i) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ and (ii) $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ are continuous functions of x and y in region R.

7. (a) State Laurent's theorem and obtain the Taylor's or Laurent series which represents the function

$$f(z) = \frac{1}{(1+z^2)(z+2)}$$

when

(i) $1 < |z| < 2$

(ii) $|z| > 2$

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(b) Evaluate the integral using the residue theorem

$$\int_C \frac{4-3z}{z(z-1)(z-2)}\,dz$$

where C is the circle $|z| = \frac{3}{2}$.

8. (a) State Bayes' theorem. An urn I contains 3 white and 4 red balls and an urn II contains 5 white and 6 red balls. One ball is drawn at random from one of the urns and is found to be white. Find the probability that it was drawn from urn I.

(b) A husband and wife appears in an interview for two vacancies in the same post. The probability of husband's selection is 1/7 and that of wife's selection is 1/5. What is the probability that—

(i) both of them will be selected;

(ii) only one of them will be selected;

(iii) none of them will be selected?

9. (a) Prove that the mean of binomial distribution is np.

(b) Three numbers are chosen at random from the given numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. Find the probability that the chosen number will be in AP.

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10. (a) An incomplete frequency distribution is given as below :

Variable	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency :	12	30	?	65	?	25	18

Given that the total frequency is 229 and mean is 46, find the missing frequencies.

(b) In experiments on pea breeding, the following frequencies of seeds were obtained :

Round and Yellow	*Wrinkled and yellow*	*Round and green*	*Wrinkled and green*	*Total*
315	101	108	32	556

Theory predicts that the frequencies should be in proportions 9 : 3 : 3 : 1. Examine the correspondence between theory and experiment.

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