B.Tech 1st Semester Exam., 2013

MATHEMATICS—I

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Time : 3 hours *Full Marks* : 70

Instructions :

(i) *All questions carry equal marks.*

(ii) *There are* ***NINE*** *questions in this paper.*

(iii) *Attempt* ***FIVE*** *questions in all.*

(iv) *Question No.* **1** *is compulsory.*

1. Choose the correct answer [any *seven*] :

(a) The value of $D^n (ax+b)^n$ is

(i) na^n

(ii) $n!a^n$

(iii) nab^n

(iv) $n!b^n$

(b) If $x = t - \sin t$, $y = 1 - \cos t$, then the value of $\frac{d^2y}{dx^2}$ at $(\pi, 2)$ will be

(i) 0

(ii) 1

(iii) π

(iv) ∞

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(c) Angle ϕ between the tangent and radius vector is given by

(i) $\tan\phi = \frac{1}{r}\frac{d\theta}{dr}$

(ii) $\tan\phi = \frac{1}{r}\frac{dr}{d\theta}$

(iii) $\tan\phi = \frac{rdr}{d\theta}$

(iv) $\tan\phi = \frac{rd\theta}{dr}$

(d) Pedal equation of the curve $r^n = a^n \sin n\theta$ is

(i) $p = r$

(ii) $p = r\sin\theta$

(iii) $p = r\sin n\theta$

(iv) $p = r\cos n\theta$

(e) The necessary condition for a function $f(x)$ to have a maxima at $x = c$ is that

(i) $f'(c) > 0$

(ii) $f'(c) = 0$

(iii) $f'(c) < 0$

(iv) None of the above

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(f) For which value of X will the matrix

$$\begin{bmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{bmatrix}$$

become singular?

(i) 4

(ii) 6

(iii) 8

(iv) 12

(g) A 5×7 matrix has all its entries equal to -1. The rank of the matrix is akubihar.com

(i) 7

(ii) 5

(iii) 1

(iv) 0

(h) The value of $B(m+1, n)$ is

(i) $\frac{n}{m+n} B(m, n)$

(ii) $\frac{n}{m+1} B(m, n)$

(iii) $\frac{m}{m+n} B(m, n)$

(iv) $\frac{m}{m+1} B(m, n)$

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(i) The order of differential equation

$$\frac{d^2y}{dt^2} + \left(\frac{dy}{dt}\right)^3 + y^4 = e^{-t}$$

is

(i) 1

(ii) 2

(iii) 3

(iv) None of the above

(j) erf (0) is

(i) 1

(ii) -1

(iii) 0

(iv) ∞

2\. (a) If $y^{1/n} + y^{-1/n} = 2x$, then prove that

$$(x^2 - 1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$$

(b) Prove that

$$\log[\sin(x+h)] = \log \sin x + h\cot x - \frac{h^2}{2}\operatorname{cosec}^2 x + \frac{h^3}{3}\frac{\cos x}{\sin^3 x} + \cdots$$

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3. (a) If $u = e^{xyz}$, then find the value of

$$\frac{\partial^3 u}{\partial x \partial y \partial z}$$

(b) If $u = f(r)$ and $x = r\cos\theta$, $y = r\sin\theta$, then prove that akubihar.com

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$$

4. (a) Determine

$$\lim \left(\frac{1}{x^2} - \frac{1}{\sin^2 x} \right)$$

as $x \to 0$.

(b) If $z = (x+y)\phi(y/x)$, where ϕ is any arbitrary function, then prove that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z$$

5. (a) Using elementary row transformations, find the inverse of the following matrix :

$$\begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

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(b) Find the rank of the following matrix by reducing to normal form

$$\begin{bmatrix} -1 & 2 & -1 & -2 \\ -2 & 5 & 3 & 0 \\ 1 & 0 & 1 & 10 \end{bmatrix}$$

6. (a) Find the eigenvalues and eigenvectors of the following matrix : akubihar.com

$$\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$$

(b) Verify Cayley-Hamilton theorem for the matrix A and hence find A^{-1} and A^4

$$A = \begin{bmatrix} 1 & 2 & 2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

7. (a) Solve any two of the following :

(i) $y(1+x^2)^{1/2}\,dy + x\sqrt{1+y^2}\,dx = 0$

(ii) $(4x+y)^2 \dfrac{dx}{dy} = 1$

(iii) $\left(x\dfrac{dy}{dx} - y \right)\cos\left(\dfrac{y}{x}\right) + x = 0$

(b) Solve

$$x(x-y)dy + y^2 dx = 0$$

 (Continued)

8. Solve the following differential equations :

(i) $\cos^2 x\dfrac{dy}{dx}+y=\tan x$

(ii) $\dfrac{d^4y}{dx^4}-\dfrac{4d^3y}{dx^3}+\dfrac{8d^2y}{dx^2}-\dfrac{8dy}{dx}+4y=0$

9. *(a)* Establish a relation between beta and gamma functions.

(b) Evaluate :

$$\int_0^\infty \frac{\tan^{-1}(ax)}{x(1+x^2)}dx$$

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