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Assignment -2 MAN-001

Q1 (a) $\begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3-\lambda & 1 & 1 \\ 2 & 4-\lambda & 2 \\ 1 & 1 & 3-\lambda \end{bmatrix} = 0$

$(3-\lambda)[(4-\lambda)(3-\lambda)-2] - [2(3-\lambda)-2] + [2-4+\lambda]$

$(3-\lambda)[\lambda^2-7\lambda+10] - [4-2\lambda] + [-2+\lambda]$

$3\lambda^2-21\lambda+30-\lambda^3+7\lambda^2-10\lambda-4+2\lambda-2+\lambda = 0$

$-\lambda^3+10\lambda^2-28\lambda+24=0$

$-\lambda^3+2\lambda^2+8\lambda^2-16\lambda-12\lambda+24=0$

$[\lambda-2][-\lambda^2+8\lambda-12]$

$[\lambda-2][\lambda-2][\lambda-6]$

$\lambda = 2, 2, 6.$

$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$x_1 + x_2 + x_3 = 0$ $\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$\lambda = 6$

$\begin{bmatrix} -3 & 1 & 1 \\ 2 & -2 & 2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = $ $\left.\begin{matrix} x_1 + x_3 = x_2 \\ x_1 + x_2 = 3x_3 \\ x_2 + x_3 = 3x_1 \end{matrix}\right\} \rightarrow$ $x_1 - x_2 = x_2 - 3x_1$

$2x_1 = x_2$

$x_1 = x_3$

$\begin{bmatrix} x_1 \\ 2x_1 \\ x_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1-\lambda & 1 & 0 \\ 0 & 1-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{bmatrix}$ $(1-\lambda)^3 = 0$

$\lambda = 1$

$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$x_2 = 0$

$\begin{bmatrix} x_1 \\ 0 \\ x_3 \end{bmatrix} = \begin{bmatrix} a_1 \\ 0 \\ a_3 \end{bmatrix}$

(c) $\begin{bmatrix} 1-\lambda & -3 & 3 \\ 3 & -5-\lambda & 3 \\ 6 & -6 & 4-\lambda \end{bmatrix}$

$(1-\lambda)[(-5-\lambda)(4-\lambda)+18] + 3[12-3\lambda-18] + 3[-18+30+6\lambda]$

$= -\lambda^3 + 12\lambda + 16.$

$(\lambda-2)(\lambda^2+2\lambda-8) = (\lambda+2)(\lambda+2)(\lambda-4)$

$\lambda = -2, -2, 4.$

$\cancel{\begin{bmatrix} -1 & -3 & 3 \\ 3 & -7 & 3 \\ 6 & -6 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}$

$\cancel{-x_1 - 3x_2 + 3x_3 = 0}$

$\cancel{3x_1 - 7x_2 + 3x_3 = 0}$

$\cancel{6x_1 - 6x_2 + 2x_3 = 0}$

$\cancel{-2x_1 - 6x_2 + 6x_3 = 0}$

$\cancel{-6x_1 - 6x_2 + 2x_3 = 0}$

$\cancel{8x_3 = 8x_1}$

$\cancel{x_3 = x_1}$

$\cancel{3x_2 = 2x_1}$

$\cancel{x_2 = \frac{2x_1}{3}}$

$\cancel{\begin{bmatrix} x_1 \\ \frac{2}{3}x_1 \\ x_1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}}$

$\begin{bmatrix} 3 & -3 & 3 \\ 3 & -3 & 3 \\ 6 & -6 & 6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_1 - x_2 + x_3 = 0.$

$\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$\begin{bmatrix} -3 & -3 & 3 \\ 3 & -9 & 3 \\ 6 & -6 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$

$-x_1 - x_2 + x_3 = 0$

$x_1 - 3x_2 + x_3 = 0$

$x_1 = x_2$

$\begin{bmatrix} x_1 \\ x_1 \\ 2x_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

$x_3 = 2x_1$

(d) $\begin{bmatrix} 7-\lambda & 2 & -2 \\ -6 & -1-\lambda & 2 \\ 6 & 2 & -1-\lambda \end{bmatrix}$

$(7-\lambda)[(1+\lambda)^2 - 4] - 2[6+6\lambda-12] - 2[-12+6+6\lambda]$

$(7-\lambda)[\lambda^2 - 3 + 2\lambda] + 12 - 12\lambda$ ~~...~~ $+12-12\lambda$

$7\cancel{\lambda^2} - 21 + 14\lambda - \lambda^3 + 3\lambda - 2\lambda^2$ ~~+ 4...~~ $= 0$

$-\lambda^3 + 5\lambda^2 + 7\lambda + 27 = 0$

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$7\lambda^2 - 21 + 14\lambda - \lambda^3 + 3\lambda - 2\lambda^2 + 12 - 12\lambda + 12 - 12\lambda = 0$

$+\lambda^3 + 5\lambda^2 + \cancel{E} + \cancel{7}\lambda + 3 = 0$

$\lambda^3 - 5\lambda^2 + 7\lambda - 3 = 0$

$(\lambda - 1)(\lambda - 1)(\lambda - 3) = 0$

$\lambda = 1, 1, 3.$

$$\begin{bmatrix} 6 & 2 & -2 \\ -6 & -2 & 2 \\ 6 & 2 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad 3x_1 + x_2 = x_3$$

$$\begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 & -2 \\ -6 & -4 & 2 \\ 6 & 2 & -4 \end{bmatrix}$$

$2x_1 + x_2 = x_3$

$3x_1 + 2x_2 = x_3$

$3x_1 + x_2 = 2x_3$

$x_1 + x_2 = 0$

$x_1 = -x_2$

$2x_1 = 2x_3 \quad (x_1 = x_3)$

$$\begin{bmatrix} x_1 \\ -x_1 \\ x_1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

Q2 (a) $AX = \lambda X$

$IX = \lambda A^{-1} X$

$\frac{1}{\lambda} X = A^{-1} X$

$\boxed{A^{-1} \to \frac{1}{\lambda}}$

$AX = \lambda X \quad (A - KI)X = (\lambda I - KI)X$

$\therefore$ eigen value of $A - KI = (\lambda I - KI)$

(b) Characteristic polynomial eqn of $A \to |A - \lambda I| = 0$

$\to |A - \lambda I|^T = 0.$

$|A^T - \lambda I^T| = 0 \Rightarrow |A^T - \lambda I| \to$ characteristic polynomial of A^T

$\therefore$ Both show same eigen value. Eigen vectors are only same for $A = A^T$ or when A is symmetric.

(c) $Ax = \lambda x$

P is non singular. hence P^{-1} exists.

Pre-multiplying P^{-1} on both sides.

$$P^{-1}Ax = P^{-1}\lambda x$$

Set $x = Py$.

we get $P^{-1}APy = \lambda P^{-1}Py$

~~$P^{-1}APy = \lambda y$~~

$(P^{-1}AP)y = \lambda y$

$\therefore$ λ is also an ~~eg~~ eigen value for $P^{-1}AP$.

<u>Hence proved</u>

Q3. (b) Let $Ax = \lambda x$.

$x^*A = (A^*x)^* = (-Ax)^* = (-\lambda x)^* = -\lambda^* x^*$

$x^*Ax = -\lambda^* x^*x$

$x^*\lambda x = -\lambda^* x^*x$

$(\lambda + \lambda^*)x^*x = 0$

$\lambda = -\lambda^*$

$\therefore$ ~~should be real~~, λ should 0 or purely imaginary.

(a) Let $Ax = \lambda x$

~~$x^*A = A$~~ $(x^*A) = (A^*x)^* = (Ax)^* = \lambda^* x^*$

$x^*Ax = \lambda^* x^* x$

$x^*\lambda x = \lambda^* x^* x.$

$\lambda = \lambda^*$

$\therefore$ λ should be purely real.

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(c) Let A be a symmetric matrix with distinct eigenvalues λ & μ and corresponding eigen vector x & y.

Then $AX = \lambda X$

& $AY = \mu Y$

$(AY)^T = (\mu Y)^T$

$Y^T A^T = \mu Y^T$

$Y^T A = \mu Y^T$

$Y^T A X = \mu Y^T X$

$\lambda Y^T X = \mu Y^T X$

$(\lambda - \mu) Y^T X = 0$

$\lambda \neq 0$

$\therefore \underline{Y^T X = 0}$

$\rightarrow$ when both vectors are orthogonal.

(d) $AA^T = I$

$AX = \lambda X$

$\bar{X}^T \bar{A}^T = \bar{\lambda} \bar{X}^T$

$\bar{X}^T \bar{A}^T A X = \bar{\lambda} \bar{X}^T \lambda X$

$\bar{X}^T X = |\lambda|^2 \bar{X}^T X$

$\therefore |\lambda|^2 = 1$

$\therefore$ modulus of eigen values of a unitary matrix $= 1$.

(e) $A = -A^T$

$|A| = |-A^T| = (-1)^n |A^T|$

$n \rightarrow$ odd $\quad |A| = -|A| \qquad 2|A| = 0 = |A| \rightarrow$ hence skew symmetric matrix of odd order have 0 determinant

(f) $A^2 = A$, $AX = \lambda X$

$A^2 X = \lambda A X \Rightarrow AX = \lambda^2 X \qquad AX = \lambda^2 X \therefore \lambda X = \lambda^2 X$

$\lambda = \lambda^2$

$\lambda^2 - \lambda = 0$

$\lambda(\lambda - 1) = 0$

$\boxed{\lambda = 0, 1}$

9. $A^k = 0$,

$Ax = \lambda x$.

$AAx = \lambda Ax$

$A^2 x = \lambda^2 x$.

$\therefore A^k x = \lambda^k x$.

$\lambda^k = 0$

$\lambda = 0$

since $x \neq 0$

Q4. $(AB)x = \lambda x$

$BABx = \lambda Bx$

Let $Bx = Y$

$\therefore BAY = \lambda Y$.

hence λ is the eigen value of BA too.

$(I - BA)$ invertible $\Rightarrow |I - BA| \neq 0$

$\Rightarrow |B - BAB| \neq 0$.

$B(I - AB) = (I - BA)B$.

$(I - BA)^{-1} B (I - AB) = B$.

$|(I - BA)^{-1}| \, |B| \, |I - AB| = |B|$

$|(I - BA)^{-1}| \, |I - AB| = 1$.

$\because I - BA \neq 0$

so $|I - AB| \neq 0 \rightarrow$ invertible.

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Q5 Let A be real symmetric matrix & B be real skew symmetric
then $\bar{A} = A$ $\bar{B} = B$ & $A^T = A$ $B^T = -B$

Now, $H = A + iB$

$\bar{H} = \overline{A + iB}$

$\bar{H} = \bar{A} - i\bar{B} = A - iB$

$\bar{H}^T = (A - iB)^T = A^T - (iB)^T = A - i(-B)$

$= A + iB = H$

$\therefore H^\theta = H$, means $A + iB \rightarrow$ hermitian matrix

Q6. $A = \begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix}$ $\rightarrow$ To prove $(I-A)(I+A)^{-1} \rightarrow$ unitary matrix.

$$X = (I-A)(I+A)^{-1} = \begin{bmatrix} 1 & -1-2i \\ 1-2i & 1 \end{bmatrix} \begin{bmatrix} 1 & 1+2i \\ -1+2i & 1 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 1 & -1-2i \\ 1-2i & 1 \end{bmatrix} \cdot \frac{1}{1^2 - (2i)^2 + 1^2} \begin{bmatrix} 1 & -1-2i \\ 1-2i & 1 \end{bmatrix}$$

$$= \frac{1}{6} \begin{bmatrix} -4 & -2-4i \\ 2-4i & -4 \end{bmatrix}$$

$$X^\theta = \frac{1}{6} \begin{bmatrix} -4 & 2+4i \\ -2+4i & -4 \end{bmatrix}$$

$$XX^\theta = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Hence $(I-A)(I+A)^{-1} \Rightarrow$ unitary matrix.

Q7. (a) $D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ $P = \begin{bmatrix} -2 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

$$[P|I] = \left[\begin{array}{ccc|ccc} -2 & -1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-\frac{R_1}{2},\ R_2 - R_1} \left[\begin{array}{ccc|ccc} 1 & 1/2 & -1/2 & -1/2 & 0 & 0 \\ 0 & -1/2 & 1/2 & 1/2 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{\substack{R_1 + R_2 \\ R_2(-2) \\ R_3 - R_2}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & -2 & 0 \\ 0 & 0 & 2 & 1 & 2 & 1 \end{array}\right] \xrightarrow[R_2 + R_3]{R_3/2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1/2 & -1 & 1/2 \\ 0 & 0 & 1 & 1/2 & 1 & 1/2 \end{array}\right]$$

$$P^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ -1/2 & -1 & 1/2 \\ 1/2 & 1 & 1/2 \end{bmatrix}$$

$$A = PDP^{-1} = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 2 & 0 \\ 1 & 2 & 3 \end{bmatrix}$$

(b) (a) $\begin{bmatrix} 4 & -2 & 0 \\ -2 & 2 & -2 \\ 0 & -2 & 4 \end{bmatrix} \rightarrow \begin{vmatrix} 4-\lambda & -2 & 0 \\ -2 & 2-\lambda & -2 \\ 0 & -2 & 4-\lambda \end{vmatrix} = 0$

$\lambda(\lambda-6)(4-\lambda) = 0$

$\lambda = 0, 4, 6$

$\lambda = 0 \quad \begin{bmatrix} 4 & -2 & 0 \\ -2 & 2 & -2 \\ 0 & -2 & 4 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$4u_1 - 2u_2 = 0$

$-u_1 + u_2 - u_3 = 0$

$-u_2 + 2u_3 = 0$

$2u_1 = u_2 = 2u_3$

$[1, 2, 1]^T$

$\lambda = 4 = \begin{bmatrix} 0 & -2 & 0 \\ -2 & -2 & -2 \\ 0 & -2 & 0 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\quad u_2 = 0$

$u_1 + u_3 = 0$

$[-1, 0, 1]^T$

$\lambda = 6 = \begin{bmatrix} -2 & -2 & 0 \\ -2 & -4 & -2 \\ 0 & -2 & -2 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\quad u_1 + u_2 = 0$

$u_2 + u_3 = 0$

$[1, -1, 1]^T$

$P = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}$

Q6 (a) $A = \begin{bmatrix} -4 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & -2 \end{bmatrix} \rightarrow |A - \lambda I| = 0$

$(-4-\lambda)(-3-\lambda)(-2-\lambda) = 0$

$\lambda = -2, -3, -4$

for $\lambda = -2 \quad x_2 = x_3, \; 2x_1 = x_2 \quad [1, 2, 2]^T$

$\lambda = -3 \rightarrow x_3 = 0, \; x_2 = x_1 \quad [1\ 1\ 0]^T$

$\lambda = -4 \rightarrow x_3 = x_2 = 0 \quad [1\ 0\ 0]^T$

$P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \rightarrow P^{-1} = \begin{bmatrix} 1 & -1 & 1/2 \\ 0 & 1 & -1 \\ 0 & 0 & 1/2 \end{bmatrix}$

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$$D = \begin{bmatrix} 4 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

$A = PDP^{-1}$

$2A = P(2D)P^{-1}$

$e^{2A} = P(e^{2D})P^{-1}$

$$e^{2A} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} e^{-8} & 0 & 0 \\ 0 & e^{-6} & 0 \\ 0 & 0 & e^{-4} \end{bmatrix} \begin{bmatrix} 1 & -1 & 1/2 \\ 0 & 1 & -1 \\ 0 & 0 & 1/2 \end{bmatrix}$$

$$e^{2A} = \begin{bmatrix} e^{-8} & e^{-6} - e^{-8} & e^{-8} - e^{-6} + e^{-4}/2 \\ 0 & e^{-6} & e^{-4} - e^{-6} \\ 0 & 0 & e^{-4} \end{bmatrix}$$

Similarly
$$A^{50} = \begin{bmatrix} 4^{50} & 3^{50} - 4^{50} & -3^{50} + \frac{2^{50}}{2} \\ 0 & 3^{50} & 2^{50} - 3^{50} \\ 0 & 0 & 2^{50} \end{bmatrix}$$

b) Eigen Value → $-2, 4, 4$.

$$P = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad D = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} \quad \text{so} \quad P^{-1} = \frac{1}{6}\begin{bmatrix} 6 & -4 & -3 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\Rightarrow e^{2A} = Pe^{2D}P^{-1} = \frac{1}{6}\begin{bmatrix} 6\alpha & 4(\beta-\alpha) & 3(\beta-\alpha) \\ 0 & 6\beta & 0 \\ 0 & 0 & 6\beta \end{bmatrix} \quad \alpha = e^{-4},\ \beta = e^{8}$$

$$A^{50} = PD^{50}P^{-1} = \frac{1}{6}\begin{bmatrix} 6\alpha & 4(\beta-\alpha) & 3(\beta-\alpha) \\ 0 & 6\beta & 0 \\ 0 & 0 & 6\beta \end{bmatrix}, \quad \alpha = 2^{50},\ \beta = 4^{50}$$

Q1 Using Cayley - Hamilton Theorem

① $$\begin{bmatrix} 1 & 3 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 3 \end{bmatrix}$$

Eqn $= \lambda^3 - 7\lambda^2 - 7\lambda + 1 = 0$

~~So by the~~ Applying Theorem

$A^3 - 7A^2 - 7A + I = 0$

$A^{-1} = -A^2 + 7A + 7I$

$$\boxed{A^{-1} = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}} \quad A^2 = \begin{bmatrix} 7 & 24 & 24 \\ 8 & 28 & 27 \\ 8 & 27 & 28 \end{bmatrix}$$

② $$\begin{bmatrix} 7 & 6 & 2 \\ -1 & 2 & 4 \\ 3 & 6 & 8 \end{bmatrix}$$

Eqn $= A^3 - 17A^2 + 62A - 40I = 0$

$A^{-1} = \frac{1}{40}[A^2 - 17A + 62I]$

$$A^2 = \begin{bmatrix} 41 & 66 & 54 \\ 3 & 22 & 70 \\ 34 & 78 & 94 \end{bmatrix} \quad \therefore A^{-1} = \begin{bmatrix} -0.20 & -0.50 & 0.50 \\ 0.50 & 1.25 & -0.75 \\ -0.30 & -0.60 & 0.50 \end{bmatrix}$$

Q9 $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ $A^3 - 11A^2 + 6A - I = 0$

$A^{-1} = A^2 - 11A + 6I$

On solving, we get,

$$A^{-1} = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

Q10 equation $= A^3 - 3A^2 + 5A - 11 = 0$

$$A^3 = \begin{bmatrix} 3 & 2 & 6 \\ 11 & 3 & 1 \\ 4 & 24 & 4 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -1 & 4 & 2 \\ 2 & -1 & 2 \\ 3 & 8 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 1 & 1 \\ 4 & 0 & 1 \end{bmatrix}$$

$\therefore$ On putting the above values in the equation we get.

$$A^{-1} = \frac{1}{11}\begin{bmatrix} 1 & -2 & 2 \\ 5 & 1 & -1 \\ -4 & 8 & 3 \end{bmatrix}$$

$A^4 = 3A^3 - 5A^2 + 11A.$

$$\therefore A^4 = \begin{bmatrix} 25 & 8 & 8 \\ 12 & 25 & 4 \\ 16 & 32 & 33 \end{bmatrix}$$

Q11 $A = \begin{bmatrix} 4 & \alpha & -1 \\ 2 & 5 & \beta \\ 1 & 1 & \gamma \end{bmatrix}$ Given $\lambda = 3, 3, \delta$ $(\delta \neq 3)$.

$A \rightarrow$ diagonalizable.

find $\alpha, \beta, \gamma, \delta$.

$\delta + 6 = 9 + \gamma + 9$

$\therefore \delta = \gamma + 3$

for diagonalizable AM = GM.

GM = 2.

~~rank~~ = 3 - 2 = 1

$\therefore$ ~~$8\theta = 2\alpha = 0$~~

~~$\alpha = 10$~~

~~$10\beta + \epsilon = 0$~~

~~β~~

$\lambda = 3.$

$$A - \lambda I = \begin{bmatrix} 1 & \alpha & -1 \\ 2 & 2 & \beta \\ 1 & 1 & \gamma - 3 \end{bmatrix}$$

$\alpha = 1$

$\beta = -2.$

$\gamma = 2.$

$\delta = \gamma + 3 = 5.$

Q13. $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{15} \\ a_{21} & a_{22} & \cdots & a_{25} \\ \vdots & & & \\ a_{51} & \cdots & & a_{55} \end{bmatrix}$ $x = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

Given, sum of all rows = 1.

$\therefore AX = X$

$A^{-1}AX = A^{-1}X$

$X = A^{-1}X.$

$\therefore AX = A^{-1}X.$

$(A - A^{-1})X = 0$

$\boxed{A = A^{-1}}$

$\therefore$ since sum of all entries = 5 in A.

sum of all entries of A^{-1} = 5.

Q14. A is a nilpotent matrix.

It's eigen values = 0, 0, ..

$(I + A)$ has eigen values $\lambda + 1, \ldots$ = 1, 1, 1...

Product of all eigen values = 1.

$\therefore |I + A| = 1 \neq 0$

$\therefore I + A$ is invertible

Q18 (a) ~~Sum of eigen value of = Trace value.~~
~~4 + 6 + 8 = 3 + 3 + 5.~~

~~A → diagonalizable~~

Q12 (a) $|A-\lambda I| = \begin{vmatrix} 2-\lambda & 3 \\ 3 & -1-\lambda \end{vmatrix} = 0 \Rightarrow \lambda = -2, 8.$

$\lambda = -2 \rightarrow 9x_1 + 3x_2 = 0 \quad x = (-1, 3)^T$

$\lambda = 8) \quad -x_1 + 3x_2 = 0 \rightarrow \quad x = (3, 1)^T$

$\begin{bmatrix} -1 & 3 \\ 3 & 1 \end{bmatrix} \rightarrow$ for orthogonal $P = \begin{pmatrix} -1/\sqrt{10} & 3/\sqrt{10} \\ 3/\sqrt{10} & 1/\sqrt{10} \end{pmatrix}$

(b) $|A-\lambda I| = \begin{vmatrix} 11-\lambda & -8 & 4 \\ -8 & -1-\lambda & -2 \\ 4 & -2 & -4-\lambda \end{vmatrix} = 0 \Rightarrow \lambda = -5, 16, -5$

$\lambda = -5) \rightarrow 4x_1 - 2x_2 + x_3 = 0 \quad \rightarrow \quad x = \left(\frac{-5}{\sqrt{105}} \quad \frac{-8}{\sqrt{105}} \quad \frac{4}{\sqrt{105}} \right)^T$

$x = (0 \quad 1/\sqrt{5} \quad 2/\sqrt{5})$

$\lambda = 16 \rightarrow 5x_1 + 8x_2 = 4x_3 \; ; \; 8x_1 + 17x_2 = -2x_3$

$4x_1 = 2x_2 + 20x_3 \quad x = \left(\frac{4}{\sqrt{21}} \quad \frac{-2}{\sqrt{21}} \quad \frac{1}{\sqrt{21}} \right)$

$\therefore P = \begin{bmatrix} 0 & -5/\sqrt{105} & 4/\sqrt{21} \\ 1/\sqrt{5} & -8/\sqrt{105} & -2/\sqrt{21} \\ 2/\sqrt{5} & 4/\sqrt{105} & 1/\sqrt{21} \end{bmatrix}$

(c) $|A-\lambda I| = \begin{vmatrix} 1-\lambda & 1-i \\ 1+i & 2-\lambda \end{vmatrix} = 0 \rightarrow \lambda = 0, 3.$

$\lambda = 0) \quad x = \left[\frac{-2}{\sqrt{6}} \quad \frac{1+i}{\sqrt{6}} \right]^T$

$\frac{x_1}{2} = \frac{-x_2}{1+i}$

$\lambda = 3) \quad \frac{x_1}{1} = \frac{x_2}{1+i} \rightarrow x = \left[\frac{1}{\sqrt{3}} \quad \frac{1+i}{\sqrt{3}} \right]^T$

$\therefore P = \begin{bmatrix} -2/\sqrt{6} & 1/\sqrt{3} \\ \frac{1+i}{\sqrt{6}} & 1+i/\sqrt{3} \end{bmatrix}$