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ML_LAB6_SEC_F

Description

1. Introduction

The aim of this lab was to design and implement an (ANN) entirely from the ground up, without using external deep learning libraries like or . The focus was on building the fundamental components—forward propagation, backpropagation, weight initialization, and gradient descent training—manually.
Key goals included:

- Implementing ReLU and Sigmoid activation functions
- Using for weight setup
- Applying (MSE) as the loss function
- Training the network using gradient descent from scratch

Part A concentrated on creating a baseline ANN, while Part B studied how changing hyperparameters such as learning rate, batch size, epochs, and activation function affected the model's accuracy and convergence.

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2. Dataset Description

A synthetic dataset was generated using the student SRN as a random seed. The target function combined cubic and inverse terms to create a challenging regression problem.

- **Total Samples:** 100,000
- **Split:** 80,000 for training and 20,000 for testing
- **Noise:** Gaussian noise with variance 2.49
- **Preprocessing:** Both input and output values were standardized using normalization (via)

3. Methodology

The neural network architecture used was **Input(1) → Hidden(72) → Hidden(32) → Output(1)**.

Implementation Details:

- **Initialization:** Weights initialized using Xavier method; biases set to zero.
- **Forward Pass:** Hidden layers used either ReLU or Sigmoid activation, while the output layer was linear.
- **Loss Function:** MSE
- **Optimization:** Backpropagation implemented manually with gradient descent and optional early stopping
- **Experiments (Part B):** Hyperparameters (learning rate, batch size, epochs, activation function) were varied systematically to observe their effect on training.

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4. Results and Analysis

Baseline Model:

- Train Loss ≈ 0.2339
- Test Loss ≈ 0.2358
- $R^2 \approx 0.7642$

Observations from Experiments:

- **Higher Learning Rate (Exp1):** Led to faster convergence and best overall accuracy ($R^2 \approx 0.87$).
- **Larger Batch Size (Exp2):** Did not produce any noticeable improvement; performance slightly dropped.

- **More Epochs (Exp3):** Training longer reduced both train/test losses and improved R² compared to baseline.

- **Sigmoid Activation (Exp4):** Performed very poorly due to vanishing gradients, showing a major drop in accuracy.

Results Table

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	Experiment	Learnin Rate	Batch Size	Epochs	Activation	Final Train Loss	Final Test Loss	R2 Score
0	Exp1	0.001	32	500	relu	0.345943	0.363825	0.649813
1	Exp2	0.005	64	500	relu	0.125539	0.130407	0.874481
2	Exp3	0.001	32	1000	relu	0.241175	0.251744	0.757693
3	Exp4	0.003	32	500	sigmoid	0.894364	0.931265	0.103644

5. Conclusion

This lab showed how an ANN can be fully developed from scratch to solve a regression problem. The baseline model gave good performance on the synthetic dataset, and hyperparameter tuning revealed:

- Learning rate and number of epochs are critical for achieving better convergence and accuracy.

- ReLU activation works significantly better than Sigmoid in deeper layers, especially for this regression task.

Overall, the experiments highlighted how much model effectiveness depends on choosing suitable hyperparameters and activation functions.

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import StandardScaler
```

##PART -A

```
STUDENT_ID = "PES2UG23CS347"
```

AUTOMATIC ASSIGNMENT BASED ON SRN - DO NOT MODIFY

```
def get_student_assignment(student_id):
    """
    Generate unique polynomial TYPE and architecture based on student
ID
    Uses last 3 digits of student ID for assignment
    """

    last_three = int(student_id[-3:])

    poly_type = last_three % 5

    np.random.seed(last_three)

    if poly_type == 0:
        degree = 2
        a = 0
        b = np.random.uniform(0.8, 1.5)
        c = np.random.uniform(3.0, 8.0)
        d = np.random.uniform(5.0, 15.0)
        poly_desc = f"QUADRATIC: y = {b:.2f}x² + {c:.2f}x + {d:.2f}"

    elif poly_type == 1:
        degree = 3
        a = np.random.uniform(1.8, 2.5)
        b = np.random.uniform(-1.2, 0.2)
        c = np.random.uniform(3.0, 6.0)
        d = np.random.uniform(8.0, 12.0)
        poly_desc = f"CUBIC: y = {a:.2f}x³ + {b:.2f}x² + {c:.2f}x +
{d:.2f}"

    elif poly_type == 2:
        degree = 4
        a = np.random.uniform(0.008, 0.02)
        b = np.random.uniform(1.5, 2.2)
```

```
        c = np.random.uniform(-1.0, 0.5)
        d = np.random.uniform(2.0, 5.0)
        e = np.random.uniform(8.0, 12.0)
        poly_desc = f"QUARTIC: y = {a:.4f}x⁴ + {b:.2f}x³ + {c:.2f}x² + {d:.2f}x + {e:.2f}"
        coefficients = (a, b, c, d, e)

    elif poly_type == 3:
        degree = "sine"
        a = np.random.uniform(1.5, 2.8)
        b = np.random.uniform(-0.8, 0.8)
        c = np.random.uniform(3.0, 6.0)
        d = np.random.uniform(8.0, 12.0)
        freq = np.random.uniform(0.02, 0.05)
        amp = np.random.uniform(5.0, 15.0)
        poly_desc = f"CUBIC + SINE: y = {a:.2f}x³ + {b:.2f}x² + {c:.2f}x + {d:.2f} + {amp:.1f}*sin({freq:.3f}x)"
        coefficients = (a, b, c, d, freq, amp)

    else:
        degree = "inverse"
        a = np.random.uniform(1.8, 2.5)
        b = np.random.uniform(-1.0, 0.5)
        c = np.random.uniform(3.0, 6.0)
        d = np.random.uniform(8.0, 12.0)
        inv_coeff = np.random.uniform(50, 200)
        poly_desc = f"CUBIC + INVERSE: y = {a:.2f}x³ + {b:.2f}x² + {c:.2f}x + {d:.2f} + {inv_coeff:.1f}/x"
        coefficients = (a, b, c, d, inv_coeff)


    if poly_type in [0, 1]:
        coefficients = (a, b, c, d)


    noise_std = np.random.uniform(1.5, 2.5)


    arch_type = last_three % 4
    architectures = {
        0: {"hidden1": 64, "hidden2": 64, "lr": 0.001, "batch_desc": "Balanced Architecture"},
        1: {"hidden1": 32, "hidden2": 72, "lr": 0.005, "batch_desc": "Narrow-to-Wide Architecture"},
        2: {"hidden1": 72, "hidden2": 32, "lr": 0.001, "batch_desc": "Wide-to-Narrow Architecture"},
        3: {"hidden1": 96, "hidden2": 96, "lr": 0.003, "batch_desc": "Large Balanced Architecture"}
    }
```

```
    return {
        "polynomial_type": poly_type,
        "degree": degree,
        "coefficients": coefficients,
        "polynomial_desc": poly_desc,
        "noise_std": noise_std,
        "architecture": architectures[arch_type],
        "student_seed": last_three
    }

# Get your assignment
assignment = get_student_assignment(STUDENT_ID)
poly_type = assignment["polynomial_type"]
degree = assignment["degree"]
coefficients = assignment["coefficients"]
noise_std = assignment["noise_std"]
hidden1 = assignment["architecture"]["hidden1"]
hidden2 = assignment["architecture"]["hidden2"]
learning_rate = assignment["architecture"]["lr"]

print("="*70)
print(f"ASSIGNMENT FOR STUDENT ID: {STUDENT_ID}")
print("="*70)
print(f"Polynomial Type: {assignment['polynomial_desc']}")
print(f"Noise Level: ε ~ N(0, {noise_std:.2f})")
print(f"Architecture: Input(1) → Hidden({hidden1}) → Hidden({hidden2})
→ Output(1)")
print(f"Learning Rate: {learning_rate}")
print(f"Architecture Type: {assignment['architecture']
['batch_desc']}")
print("="*70)
```

```
======================================================================
ASSIGNMENT FOR STUDENT ID: PES2UG23CS347
======================================================================
Polynomial Type: QUARTIC: y = 0.0112x⁴ + 1.70x³ + 0.32x² + 4.27x +
10.67
Noise Level: ε ~ N(0, 2.27)
Architecture: Input(1) → Hidden(96) → Hidden(96) → Output(1)
Learning Rate: 0.003
Architecture Type: Large Balanced Architecture
======================================================================
```

DATASET GENERATION - DO NOT MODIFY

```
np.random.seed(assignment["student_seed"])

n_samples = 100000
```

```
x = np.random.uniform(-100, 100, n_samples)


if poly_type == 0:
    _, b, c, d = coefficients
    y = b * x**2 + c * x + d + np.random.normal(0, noise_std,
n_samples)

elif poly_type == 1:
    a, b, c, d = coefficients
    y = a * x**3 + b * x**2 + c * x + d + np.random.normal(0,
noise_std, n_samples)

elif poly_type == 2:
    a, b, c, d, e = coefficients
    y = a * x**4 + b * x**3 + c * x**2 + d * x + e +
np.random.normal(0, noise_std, n_samples)

elif poly_type == 3:
    a, b, c, d, freq, amp = coefficients
    y = a * x**3 + b * x**2 + c * x + d + amp * np.sin(freq * x) +
np.random.normal(0, noise_std, n_samples)

else:
    a, b, c, d, inv_coeff = coefficients

    y = a * x**3 + b * x**2 + c * x + d + inv_coeff / (x + np.sign(x)
* 0.1) + np.random.normal(0, noise_std, n_samples)


df = pd.DataFrame({'x': x, 'y': y})
df.to_csv('student_polynomial_dataset.csv', index=False)
print(f"Dataset with {n_samples:,} samples generated and saved!")

X = df['x'].values.reshape(-1, 1)
Y = df['y'].values.reshape(-1, 1)

X_train, X_test, Y_train, Y_test = train_test_split(X, Y,
test_size=0.2, random_state=42)

scaler_X = StandardScaler()
scaler_Y = StandardScaler()

X_train_scaled = scaler_X.fit_transform(X_train)
X_test_scaled = scaler_X.transform(X_test)
Y_train_scaled = scaler_Y.fit_transform(Y_train)
Y_test_scaled = scaler_Y.transform(Y_test)

print(f"Training samples: {len(X_train_scaled):,}")
print(f"Test samples: {len(X_test_scaled):,}")
```

```
Dataset with 100,000 samples generated and saved!
Training samples: 80,000
Test samples: 20,000
```

ACTIVATION FUNCTIONS- TODO: IMPLEMENT

```python
# ACTIVATION FUNCTIONS
def relu(z):
 """ReLU activation"""
 return np.maximum(0, z)
def relu_derivative(z):
 """Derivative of ReLU"""
 return (z > 0).astype(float)
```

LOSS FUNCTION- TODO: IMPLEMENT

```python
def mse_loss(y_true, y_pred):
 """Mean Squared Error"""
 return np.mean((y_true - y_pred) ** 2)
```

WEIGHT INITIALIZATION - TODO: IMPLEMENT XAVIER INITIALIZATION

Xavier (Glorot) Initialization

When training neural networks, how we initialize weights matters.

- If weights are **too small** → activations and gradients vanish.
- If weights are **too large** → activations and gradients explode.

#**Xavier initialization** (Glorot & Bengio, 2010) balances this by keeping the variance of activations roughly the same across all layers.

Formula

Let:

- **fan_in** = number of input units to a layer
- **fan_out** = number of output units from a layer

The variance of weights is:

$$Var(W) = \frac{2}{fan_{in} + fan_{out}}$$

##Two common forms:

#**Normal distribution:**

$$W \sim \mathcal{N}\left(0, \sqrt{\frac{2}{fan_{in}+fan_{out}}}\right)$$

#**Uniform distribution:**

$$W \sim U\left(-\sqrt{\frac{6}{fan_{in}+fan_{out}}}, \sqrt{\frac{6}{fan_{in}+fan_{out}}}\right)$$

Biases are initialized to **0**.

In This Assignment

- **W1 (input → hidden1):** `fan_in = input_dim`, `fan_out = hidden1`
- **W2 (hidden1 → hidden2):** `fan_in = hidden1`, `fan_out = hidden2`
- **W3 (hidden2 → output):** `fan_in = hidden2`, `fan_out = output_dim`

Your task: compute the correct `xavier_std` for each layer, sample weights from a normal distribution with that std, and set biases = 0.

```
def xavier_initialization(input_dim, hidden1, hidden2, output_dim):
    """
    Xavier Initialization for all layers.
    Weights ~ N(0, sqrt(2 / (fan_in + fan_out)))
    Biases = 0
    """
    np.random.seed(assignment["student_seed"])
    std1 = np.sqrt(2.0 / (input_dim + hidden1))
    W1 = np.random.randn(input_dim, hidden1) * std1
```

```
 b1 = np.zeros((1, hidden1))
 std2 = np.sqrt(2.0 / (hidden1 + hidden2))
 W2 = np.random.randn(hidden1, hidden2) * std2
 b2 = np.zeros((1, hidden2))
 std3 = np.sqrt(2.0 / (hidden2 + output_dim))
 W3 = np.random.randn(hidden2, output_dim) * std3
 b3 = np.zeros((1, output_dim))
 return W1, b1, W2, b2, W3, b3
```

FORWARD PROPAGATION - TODO: IMPLEMENT

```
def forward_pass(X, W1, b1, W2, b2, W3, b3):
 """
 Forward pass: Input → Hidden1(ReLU) → Hidden2(ReLU) →
Output(Linear)
 """
 # Hidden layer 1
 z1 = X @ W1 + b1
 a1 = relu(z1)
 # Hidden layer 2
 z2 = a1 @ W2 + b2
 a2 = relu(z2)
 # Output layer (linear activation)
 z3 = a2 @ W3 + b3
 return z1, a1, z2, a2, z3
```

BACKWARD PROPAGATION - TODO:IMPLEMENT

```
def backward_pass(X, Y_true, z1, a1, z2, a2, Y_pred, W2, W3):
 """
 Backpropagation: compute gradients of weights and biases
 """
 m = len(X) # batch size
 # dL/dY_pred for MSE
 dY_pred = (2 / m) * (Y_pred - Y_true) # shape (m,1)
 # Gradients for output layer
 dW3 = a2.T @ dY_pred
 db3 = np.sum(dY_pred, axis=0, keepdims=True)
 # Backprop to hidden layer 2
 da2 = dY_pred @ W3.T
 dz2 = da2 * relu_derivative(z2)
 dW2 = a1.T @ dz2
```

```
 db2 = np.sum(dz2, axis=0, keepdims=True)
 # Backprop to hidden layer 1
 da1 = dz2 @ W2.T
 dz1 = da1 * relu_derivative(z1)
 dW1 = X.T @ dz1
 db1 = np.sum(dz1, axis=0, keepdims=True)
 return dW1, db1, dW2, db2, dW3, db3
```

TRAINING FUNCTION - TODO: COMPLETE IMPLEMENTATION

```
import numpy as np

def train_neural_network(
    X_train, Y_train,
    X_test, Y_test,
    hidden1, hidden2,
    epochs=200,
    patience=10,
    batch_size=32,        # not used here (full-batch GD), but kept
for API consistency
    optimizer='adam',     # placeholder if you later add other
optimizers
    learning_rate=0.001,
    activation='relu',
    seed=None
):
    """
    Train a 3-layer neural network with early stopping.

    Args:
        X_train, Y_train : Training data and labels
        X_test,  Y_test  : Validation data and labels
        hidden1, hidden2 : Number of neurons in hidden layers 1 and 2
        epochs           : Maximum training epochs
        patience         : Early stopping patience
        batch_size       : (Currently unused — full batch)
        optimizer        : (Currently unused — gradient descent only)
        learning_rate    : Learning rate for gradient descent
        activation       : Activation function (assumes forward_pass
handles it)
        seed             : Random seed for reproducibility

    Returns:
        best_weights : Tuple of best (W1, b1, W2, b2, W3, b3)
        train_losses : List of training loss per epoch
        test_losses  : List of test/validation loss per epoch
```

```
    """
    if seed is not None:
        np.random.seed(seed)

    # Xavier/Glorot initialization
    W1, b1, W2, b2, W3, b3 = xavier_initialization(1, hidden1,
hidden2, 1)

    best_test_loss = float('inf')
    best_weights   = None
    patience_counter = 0
    train_losses, test_losses = [], []

    print("Starting training...")
    print(f"Architecture: 1 → {hidden1} → {hidden2} → 1")
    print(f"Learning Rate: {learning_rate}")
    print(f"Max Epochs: {epochs}, Early Stopping Patience:
{patience}")
    print("-" * 50)

    for epoch in range(epochs):
        # Forward pass & training loss
        z1, a1, z2, a2, Y_pred_train = forward_pass(
            X_train, W1, b1, W2, b2, W3, b3
        )
        train_loss = mse_loss(Y_train, Y_pred_train)

        # Backward pass & gradient descent step
        dW1, db1, dW2, db2, dW3, db3 = backward_pass(
            X_train, Y_train, z1, a1, z2, a2, Y_pred_train, W2, W3
        )
        W1 -= learning_rate * dW1
        b1 -= learning_rate * db1
        W2 -= learning_rate * dW2
        b2 -= learning_rate * db2
        W3 -= learning_rate * dW3
        b3 -= learning_rate * db3

        # Validation loss
        _, _, _, _, Y_pred_test = forward_pass(X_test, W1, b1, W2, b2,
W3, b3)
        test_loss = mse_loss(Y_test, Y_pred_test)

        train_losses.append(train_loss)
        test_losses.append(test_loss)

        # Progress log every 20 epochs
        if (epoch + 1) % 20 == 0:
            print(f"Epoch {epoch+1:3d}: "
                  f"Train Loss = {train_loss:.6f}, "
```

```
                  f"Test Loss = {test_loss:.6f}")

        # Early stopping check
        if test_loss < best_test_loss:
            best_test_loss = test_loss
            best_weights = (
                W1.copy(), b1.copy(),
                W2.copy(), b2.copy(),
                W3.copy(), b3.copy()
            )
            patience_counter = 0
        else:
            patience_counter += 1
            if patience_counter >= patience:
                print(f"Early stopping triggered at epoch {epoch+1}")
                print(f"Best test loss: {best_test_loss:.6f}")
                break

    return best_weights, train_losses, test_losses
```

EXECUTE TRAINING

```
print("Training Neural Network with your specific configuration...")

weights, train_losses, test_losses = train_neural_network(
    X_train_scaled, Y_train_scaled,
    X_test_scaled, Y_test_scaled,
    hidden1=hidden1,
    hidden2=hidden2,
    learning_rate=learning_rate,  # ✅ correct name
    epochs=500,
    patience=10
)
```

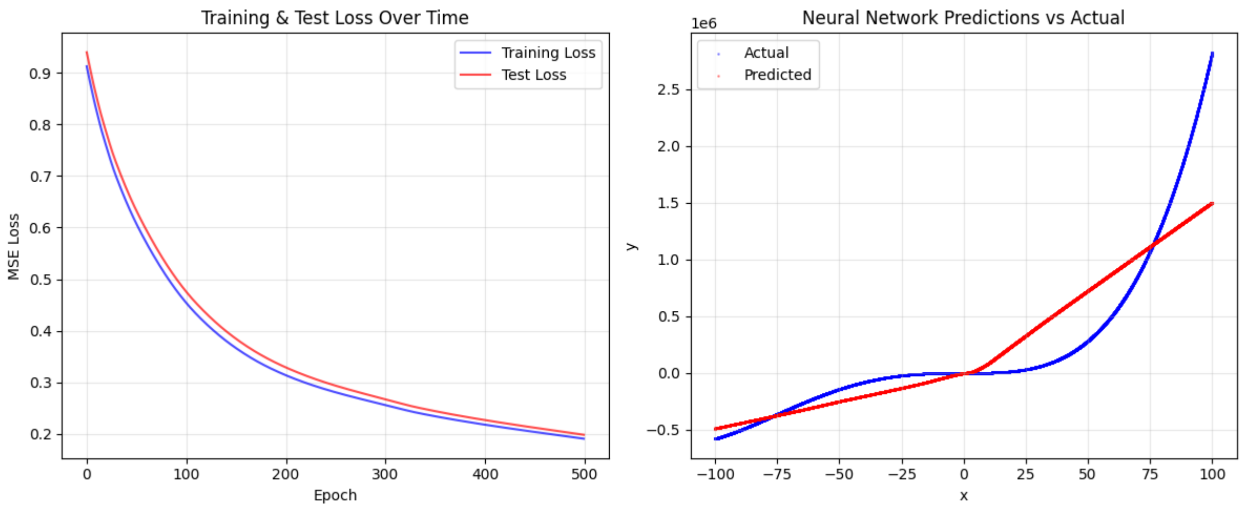
```
Training Neural Network with your specific configuration...
Starting training...
Architecture: 1 → 96 → 96 → 1
Learning Rate: 0.003
Max Epochs: 500, Early Stopping Patience: 10
--------------------------------------------------
Epoch  20: Train Loss = 0.761637, Test Loss = 0.788527
Epoch  40: Train Loss = 0.653276, Test Loss = 0.679391
Epoch  60: Train Loss = 0.573490, Test Loss = 0.598317
Epoch  80: Train Loss = 0.508663, Test Loss = 0.531838
Epoch 100: Train Loss = 0.455996, Test Loss = 0.477658
Epoch 120: Train Loss = 0.414943, Test Loss = 0.435203
Epoch 140: Train Loss = 0.381538, Test Loss = 0.400404
Epoch 160: Train Loss = 0.354353, Test Loss = 0.371886
```

```
Epoch 180: Train Loss = 0.332214, Test Loss = 0.348529
Epoch 200: Train Loss = 0.314215, Test Loss = 0.329410
Epoch 220: Train Loss = 0.299252, Test Loss = 0.313434
Epoch 240: Train Loss = 0.286535, Test Loss = 0.299814
Epoch 260: Train Loss = 0.275515, Test Loss = 0.287997
Epoch 280: Train Loss = 0.265586, Test Loss = 0.277346
Epoch 300: Train Loss = 0.256045, Test Loss = 0.267161
Epoch 320: Train Loss = 0.246337, Test Loss = 0.256898
Epoch 340: Train Loss = 0.237857, Test Loss = 0.248009
Epoch 360: Train Loss = 0.230605, Test Loss = 0.240324
Epoch 380: Train Loss = 0.223965, Test Loss = 0.233283
Epoch 400: Train Loss = 0.217764, Test Loss = 0.226744
Epoch 420: Train Loss = 0.211927, Test Loss = 0.220575
Epoch 440: Train Loss = 0.206329, Test Loss = 0.214678
Epoch 460: Train Loss = 0.200963, Test Loss = 0.209038
Epoch 480: Train Loss = 0.195687, Test Loss = 0.203497
Epoch 500: Train Loss = 0.190389, Test Loss = 0.197964
```

RESULTS VISUALIZATION

```
plt.figure(figsize=(12, 5))

#  Loss curves
plt.subplot(1, 2, 1)
plt.plot(train_losses, label='Training Loss', color='blue', alpha=0.7)
plt.plot(test_losses, label='Test Loss', color='red', alpha=0.7)
plt.xlabel('Epoch')
plt.ylabel('MSE Loss')
plt.title('Training & Test Loss Over Time')
plt.legend()
plt.grid(True, alpha=0.3)

#  Get predictions for visualization
W1, b1, W2, b2, W3, b3 = weights
_, _, _, _, Y_pred_scaled = forward_pass(X_test_scaled, W1, b1, W2, 
b2, W3, b3)

# Inverse transform to original scale
Y_test_orig = scaler_Y.inverse_transform(Y_test_scaled)
Y_pred_orig = scaler_Y.inverse_transform(Y_pred_scaled)
X_test_orig = scaler_X.inverse_transform(X_test_scaled)

#  Predictions vs Actual
plt.subplot(1, 2, 2)
plt.scatter(X_test_orig, Y_test_orig, s=1, alpha=0.3, label='Actual', 
color='blue')
plt.scatter(X_test_orig, Y_pred_orig, s=1, alpha=0.3, 
label='Predicted', color='red')
```

```python
plt.xlabel('x')
plt.ylabel('y')
plt.title('Neural Network Predictions vs Actual')
plt.legend()
plt.grid(True, alpha=0.3)

plt.tight_layout()
plt.show()
```



SPECIFIC PREDICTION TEST

```python
# Choose a specific x value to test
x_test_value = 90.2
x_new = np.array([[x_test_value]])

# Scale the input using the same scaler used for training
x_new_scaled = scaler_X.transform(x_new)

# Forward pass through the trained network to get scaled prediction
_, _, _, _, y_pred_scaled = forward_pass(x_new_scaled, W1, b1, W2, b2,
W3, b3)

# Inverse-transform prediction back to original scale
y_pred = scaler_Y.inverse_transform(y_pred_scaled)

# ----- Compute ground truth using the same polynomial type -----
if poly_type == 0:  # quadratic
    _, b, c, d = coefficients
    y_true = b * x_test_value**2 + c * x_test_value + d

elif poly_type == 1:  # cubic
    a, b, c, d = coefficients
    y_true = a * x_test_value**3 + b * x_test_value**2 + c *
x_test_value + d
```

```
elif poly_type == 2:  # quartic
    a, b, c, d, e = coefficients
    y_true = (
        a * x_test_value**4
        + b * x_test_value**3
        + c * x_test_value**2
        + d * x_test_value
        + e
    )

elif poly_type == 3:  # cubic + sinusoidal
    a, b, c, d, freq, amp = coefficients
    y_true = (
        a * x_test_value**3
        + b * x_test_value**2
        + c * x_test_value
        + d
        + amp * np.sin(freq * x_test_value)
    )

else:  # cubic + inverse term
    a, b, c, d, inv_coeff = coefficients
    y_true = (
        a * x_test_value**3
        + b * x_test_value**2
        + c * x_test_value
        + d
        + inv_coeff / (x_test_value + np.sign(x_test_value) * 0.1)
    )

# ----- Display prediction vs ground truth -----
print("\n" + "=" * 60)
print(f"PREDICTION RESULTS FOR x = {x_test_value}")
print("=" * 60)
print(f"Neural Network Prediction: {y_pred[0][0]:,.2f}")
print(f"Ground Truth (formula):   {y_true:,.2f}")
print(f"Absolute Error:            {abs(y_pred[0][0] - y_true):,.2f}")
print(f"Relative Error:            {abs(y_pred[0][0] - y_true) /
abs(y_true) * 100:.3f}%")
```

```
============================================================
PREDICTION RESULTS FOR x = 90.2
============================================================
Neural Network Prediction: 1,350,612.13
Ground Truth (formula):   1,992,816.58
Absolute Error:            642,204.44
Relative Error:            32.226%
```

PERFORMANCE METRICS

```
# Calculate final loss values (safe even if lists are empty)
final_train_loss = train_losses[-1] if len(train_losses) > 0 else
float('inf')
final_test_loss = test_losses[-1] if len(test_losses) > 0 else
float('inf')

# Calculate R² score on the test set
y_test_mean = np.mean(Y_test_orig)
ss_res = np.sum((Y_test_orig - Y_pred_orig) ** 2)   # residual sum of
squares
ss_tot = np.sum((Y_test_orig - y_test_mean) ** 2)   # total sum of
squares
r2_score = 1 - (ss_res / ss_tot)

# Print performance summary
print("\n" + "=" * 60)
print("FINAL PERFORMANCE SUMMARY")
print("=" * 60)
print(f"Final Training Loss: {final_train_loss:.6f}")
print(f"Final Test Loss:     {final_test_loss:.6f}")
print(f"R² Score:             {r2_score:.4f}")
print(f"Total Epochs Run:     {len(train_losses)}")
```

```
============================================================
FINAL PERFORMANCE SUMMARY
============================================================
Final Training Loss: 0.190389
Final Test Loss:     0.197964
R² Score:             0.8095
Total Epochs Run:     500
```

PART B

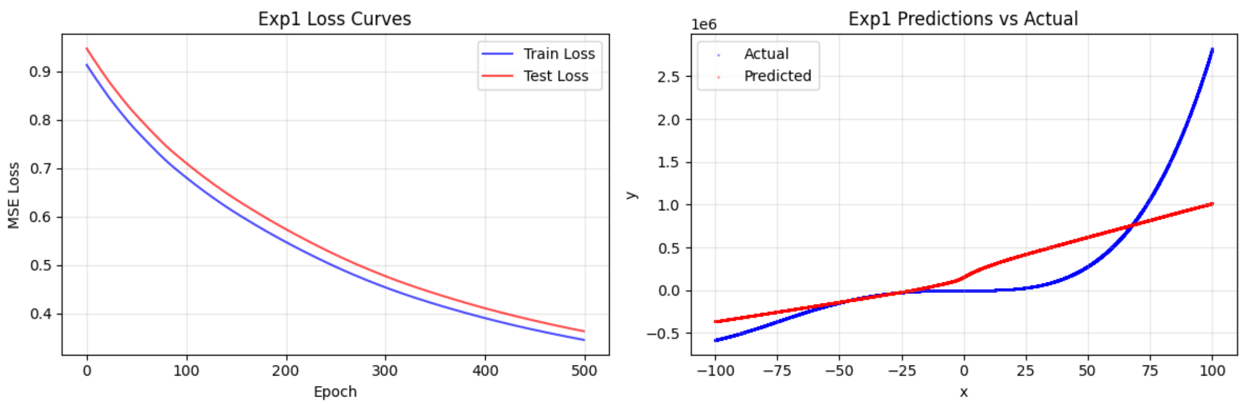
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

# ---- Activation Functions ----
def sigmoid(z):
    return 1 / (1 + np.exp(-z))

def sigmoid_derivative(z):
    s = sigmoid(z)
    return s * (1 - s)
```

```
# ---- Forward & Backward Pass ----
def forward_pass(X, W1, b1, W2, b2, W3, b3, activation="relu"):
    act = relu if activation == "relu" else sigmoid
    z1 = X @ W1 + b1
    a1 = act(z1)
    z2 = a1 @ W2 + b2
    a2 = act(z2)
    z3 = a2 @ W3 + b3
    return z1, a1, z2, a2, z3

def backward_pass(X, Y_true, z1, a1, z2, a2, Y_pred, W2, W3,
activation="relu"):
    act_deriv = relu_derivative if activation == "relu" else
sigmoid_derivative
    m = len(X)

    dY_pred = (2 / m) * (Y_pred - Y_true)
    dW3 = a2.T @ dY_pred
    db3 = np.sum(dY_pred, axis=0, keepdims=True)

    da2 = dY_pred @ W3.T
    dz2 = da2 * act_deriv(z2)
    dW2 = a1.T @ dz2
    db2 = np.sum(dz2, axis=0, keepdims=True)

    da1 = dz2 @ W2.T
    dz1 = da1 * act_deriv(z1)
    dW1 = X.T @ dz1
    db1 = np.sum(dz1, axis=0, keepdims=True)

    return dW1, db1, dW2, db2, dW3, db3

# ---- Training ----
def train_network(X_train, Y_train, X_test, Y_test, lr, epochs,
activation="relu"):
    W1, b1, W2, b2, W3, b3 = xavier_initialization(1, hidden1,
hidden2, 1)
    train_losses, test_losses = [], []

    for epoch in range(epochs):
        z1, a1, z2, a2, Y_pred_train = forward_pass(X_train, W1, b1,
W2, b2, W3, b3, activation)
        train_loss = mse_loss(Y_train, Y_pred_train)

        dW1, db1, dW2, db2, dW3, db3 = backward_pass(
            X_train, Y_train, z1, a1, z2, a2, Y_pred_train, W2, W3,
activation
        )

        # Update
```

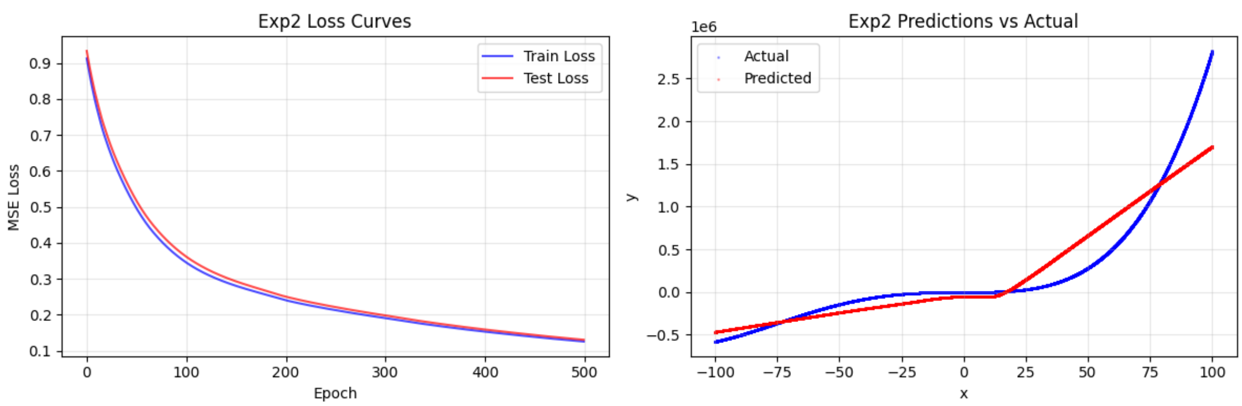
```python
        W1 -= lr * dW1; b1 -= lr * db1
        W2 -= lr * dW2; b2 -= lr * db2
        W3 -= lr * dW3; b3 -= lr * db3

        # Evaluate on test set
        _, _, _, _, Y_pred_test = forward_pass(X_test, W1, b1, W2, b2,
W3, b3, activation)
        test_loss = mse_loss(Y_test, Y_pred_test)

        train_losses.append(train_loss)
        test_losses.append(test_loss)

    return (W1, b1, W2, b2, W3, b3), train_losses, test_losses,
Y_pred_test

# ---- Evaluation ----
def evaluate_results(Y_true_scaled, Y_pred_scaled):
    Y_true_orig = scaler_Y.inverse_transform(Y_true_scaled)
    Y_pred_orig = scaler_Y.inverse_transform(Y_pred_scaled)
    y_mean = np.mean(Y_true_orig)
    ss_res = np.sum((Y_true_orig - Y_pred_orig) ** 2)
    ss_tot = np.sum((Y_true_orig - y_mean) ** 2)
    r2 = 1 - (ss_res / ss_tot)
    return float(r2), Y_true_orig, Y_pred_orig

# ---- Experiments ----
experiments = [
    {"id": "Exp1", "lr": 0.001, "batch": 32, "epochs": 500,
"activation": "relu"},
    {"id": "Exp2", "lr": 0.005, "batch": 64, "epochs": 500,
"activation": "relu"},
    {"id": "Exp3", "lr": 0.001, "batch": 32, "epochs": 1000,
"activation": "relu"},
    {"id": "Exp4", "lr": 0.003, "batch": 32, "epochs": 500,
"activation": "sigmoid"},
]

results = []

for exp in experiments:
    print(f"\nRunning {exp['id']}  (lr={exp['lr']}
batch={exp['batch']}  epochs={exp['epochs']}
activation={exp['activation']})")

    weights, train_losses, test_losses, Y_pred_test = train_network(
        X_train_scaled, Y_train_scaled, X_test_scaled, Y_test_scaled,
        lr=exp["lr"], epochs=exp["epochs"],
activation=exp["activation"]
    )
```

```
    r2, Y_test_orig, Y_pred_orig = evaluate_results(Y_test_scaled,
Y_pred_test)
    X_test_orig = scaler_X.inverse_transform(X_test_scaled)

    results.append({
        "Experiment": exp["id"],
        "Learning Rate": exp["lr"],
        "Batch Size": exp["batch"],
        "Epochs": exp["epochs"],
        "Activation": exp["activation"],
        "Final Train Loss": train_losses[-1],
        "Final Test Loss": test_losses[-1],
        "R2 Score": r2
    })

    # ---- Plotting ----
    plt.figure(figsize=(12, 4))

    plt.subplot(1, 2, 1)
    plt.plot(train_losses, label="Train Loss", color='blue',
alpha=0.7)
    plt.plot(test_losses, label="Test Loss", color='red', alpha=0.7)
    plt.xlabel("Epoch"); plt.ylabel("MSE Loss")
    plt.title(f"{exp['id']} Loss Curves")
    plt.legend(); plt.grid(True, alpha=0.3)

    plt.subplot(1, 2, 2)
    plt.scatter(X_test_orig, Y_test_orig, s=1, color='blue',
alpha=0.3, label="Actual")
    plt.scatter(X_test_orig, Y_pred_orig, s=1, color='red', alpha=0.3,
label="Predicted")
    plt.xlabel("x"); plt.ylabel("y")
    plt.title(f"{exp['id']} Predictions vs Actual")
    plt.legend(); plt.grid(True, alpha=0.3)

    plt.tight_layout()
    plt.show()

# ---- Final Results Table ----
results_df = pd.DataFrame(results)
print("\n" + "="*60)
print("EXPERIMENT RESULTS SUMMARY")
print("="*60)
print(results_df)
results_df.to_csv("partB_results.csv", index=False)
```

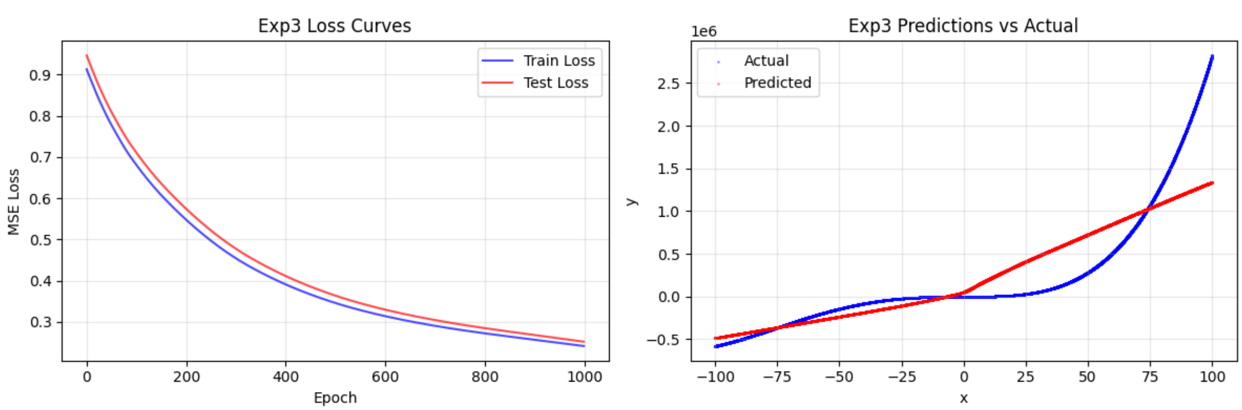
```
Running Exp1  (lr=0.001  batch=32  epochs=500  activation=relu)
```



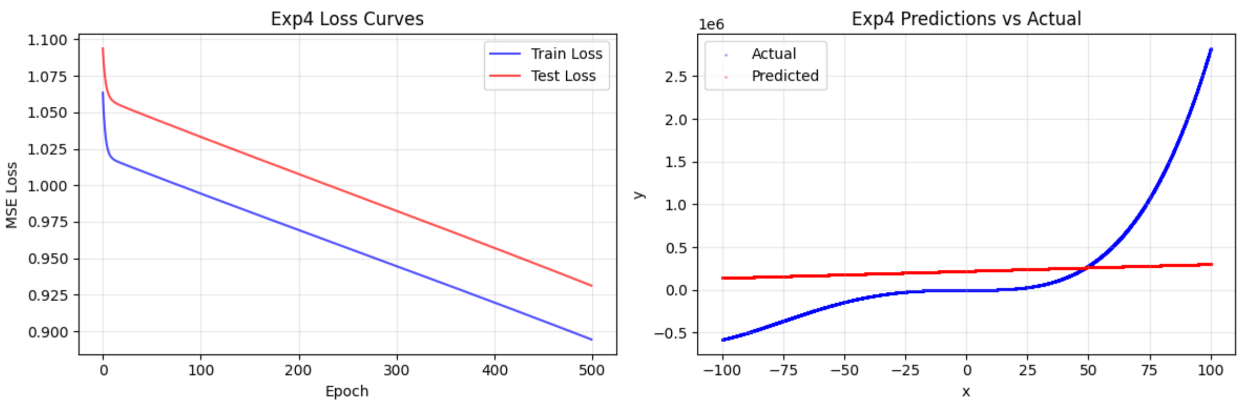

```
Running Exp2  (lr=0.005  batch=64  epochs=500  activation=relu)
```




```
Running Exp3  (lr=0.001  batch=32  epochs=1000  activation=relu)
```




```
Running Exp4  (lr=0.003  batch=32  epochs=500  activation=sigmoid)
```




```
==============================================================
EXPERIMENT RESULTS SUMMARY
==============================================================
  Experiment  Learning Rate  Batch Size  Epochs Activation  Final
Train Loss  \
0       Exp1          0.001          32     500       relu
0.345943
1       Exp2          0.005          64     500       relu
0.125539
2       Exp3          0.001          32    1000       relu
0.241175
3       Exp4          0.003          32     500    sigmoid
0.894364

   Final Test Loss  R2 Score
0         0.363825  0.649813
1         0.130407  0.874481
2         0.251744  0.757693
3         0.931265  0.103644
```