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Peihua QIU

Surface estimation is important in many applications. When conventional smoothing procedures (e.g., running averages, local polynomial kernel smoothing procedures, smoothing spline procedures) are used for estimating jump surfaces from noisy data, jumps are blurred at the same time when noise is removed. In recent years, new smoothing methodologies have been proposed in the statistical literature for detecting jumps in surfaces and for estimating jump surfaces with jumps preserved. We provide a review of these methodologies. Because a monochrome image can be considered a jump surface of the image intensity function, with jumps at the outlines of objects, edge detection and image restoration problems in image processing are closely related to the jump surface estimation problem in statistics. We also review major methodologies on edge detection and image restoration, and discuss connections and differences among these methods and related methods in the statistical literature.

KEY WORDS: Adaptive smoothing; Bilateral filtering; Deblurring; Denoising; Edge detection; Image reconstruction; Image restoration; Jump detection; Jump location curve; Jump-preserving surface estimation; Nonparametric regression; Smoothing.

1. INTRODUCTION

Regression analysis provides a major statistical tool for building functional relationships between a response variable and an explanatory variable (or an explanatory variable vector in multivariate cases). When this tool was first suggested by Galton, Pearson, and some other pioneers more than 100 years ago (Stanton 2001), the true regression function is assumed to be *linear*. Linear regression models were later generalized to various *parametric* regression models, including the Box–Cox regression model (Box and Cox 1964). Since the late 1950s, parametric regression models have been further generalized to *nonparametric* regression models (e.g., Nadaraya 1964; Parzen 1962; Rosenblatt 1956, 1969; Watson 1964). A typical two-dimensional (2-D) nonparametric regression model can be written as

$$Z_{ij} = f(x_i, y_j) + \varepsilon_{ij}, \qquad i = 1, 2, \ldots, n_1; j = 1, 2, \ldots, n_2, \quad (1)$$

where x and y are two explanatory variables, $\{Z_{ij}, i = 1, 2, \ldots, n_1; j = 1, 2, \ldots, n_2\}$ are observations of the response variable Z observed at design points $\{(x_i, y_j), i = 1, 2, \ldots, n_1; j = 1, 2, \ldots, n_2\}$ in design space $[0, 1] \times [0, 1]$, f is a bivariate regression function denoting a true regression surface, and $\{\varepsilon_{ij}, i = 1, 2, \ldots, n_1; j = 1, 2, \ldots, n_2\}$ are independent and identically distributed (iid) random errors with mean 0 and unknown variance σ^2.

In conventional nonparametric regression analysis, f often is assumed to be a *continuous* function. In model (1), the continuity assumption implies that the true regression surface f can change slightly when the values of x and y change slightly, which is reasonable in many applications. Several conventional smoothing procedures have been proposed in the literature for estimating a continuous regression function from noisy data, including the local polynomial kernel smoothing procedures, smoothing splines, regression splines, wavelet transformation methods, and so forth. (For nice overviews of these methods, see Fan and Gijbels 1996; Härdle 1990; Müller 1988; Qiu 2005; Wahba 1990.)

In some applications, however, the regression function f may have jumps in the design space, which often represent structural changes of the related process; for instance, the image intensity function of an image is discontinuous at the outlines of objects, and the equi-temperature surfaces in high sky or deep ocean are often discontinuous. It is easy to check that estimated regression functions by most conventional nonparametric regression procedures will not converge to the true regression function f at the jump positions, which implies that conventional nonparametric regression analysis cannot handle the regression problem properly when f has jumps. Handling such a problem properly requires some new statistical tools.

In recent years, some smoothing procedures have been suggested in the statistical literature for estimating jump regression functions from noisy data. (For 1-D methodologies, see Gijbels and Goderniaux 2004; Gijbels, Lambert, and Qiu 2007; Hall and Titterington 1992; McDonald and Owen 1986; Müller 1992; Qiu 1991, 1994, 2003; Qiu, Asano, and Li 1991; Qiu and Yandell 1998; Wang 1995; Wu and Chu 1993; and the references cited therein.) In this article we focus on 2-D problems, because they are directly related to image analysis, as discussed later. We review some recent methodologies on jump surface estimation.

An important application of jump surface estimation is image processing. A monochrome digital image, generated by a uniform sampling scheme in digitization of the spatial location, consists of pixels spaced regularly in rows and columns. This image can be well described by the 2-D regression model (1). More specifically, in model (1), x_i denotes the ith row of pixels, y_j denotes the jth column, f is the image intensity function, $f(x_i, y_j)$ and Z_{ij} are the true and observed image intensity levels at the (i, j)th pixel, and ε_{ij} denotes the random noise at that pixel. The image intensity function f has jumps at various places, called *edges* in the image processing literature. Therefore, edge detection and edge-preserving image restoration in image processing are essentially the same problems as jump detection and jump-preserving surface estimation in regression analysis, although two different sets of terminologies are used in the two research areas. In this article we also review major edge detection and image restoration procedures.

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Our discussion here is kept concise. We address mainly the main ideas of some fundamental methodologies in the related areas, their major strengths and limitations, connections and differences between jump surface estimation methods in the statistical literature and the related methods in the image processing literature, and some important open problems for future research. A more detailed background introduction and detailed discussion about traditional methodologies on jump curve/surface estimation and image analysis have been given in earlier work (Qiu 2005).

The article is organized as follows. Jump detection in 2-D regression surfaces is discussed in Section 2; jump-preserving surface estimation, in Section 3; edge detection, in Section 4; image denoising, in Section 5; and image deblurring problems, in Section 6. Some remarks conclude the article in Section 7.

2. ESTIMATION OF JUMP LOCATION CURVES OF TWO–DIMENSIONAL SURFACES

Jumps in a regression surface often represent important surface structures; therefore, it is important to detect their positions accurately from observed data. In 2-D cases, jump positions usually form curves in the design space, called the *jump location curves* (JLCs) by Qiu (1997). (For a formal definition of JLCs, see Qiu 1998.) In this article, for simplicity, we assume that design points are regularly spaced in the design space $[0, 1] \times [0, 1]$, as specified in model (1). But most existing procedures introduced in this section and the next section also can work well in cases when design points are irregularly spaced with some homogeneity properties and when the design space is a general connected region in R^2.

This section is organized in two parts. Section 2.1 introduces some early jump detection procedures requiring the assumption that the number of JLCs is known. Section 2.2 discusses some procedures for detecting arbitrary JLCs.

2.1 Jump Detection When the Number of Jump Location Curves Is Known

Early methodologies proposed in the statistical literature for estimating JLCs treat JLCs as curves in the design space and try to estimate them by some other curves in the same space. They usually assume that the number of JLCs is known and that the JLCs satisfy certain smoothness conditions. For instance, when it is assumed that there is a single JLC with the expression $y = \phi(x)$, for $x \in [0, 1]$, Korostelev and Tsybakov (1993) suggested a minimax estimator of the JLC, by approximating the JLC with a piecewise polynomial function and by estimating the polynomial coefficients with maximum likelihood estimation. In such cases, Rudemo and Stryhn (1994) suggested estimating the function ϕ by a step function with its coefficients estimated by maximum likelihood. Both works made some generalizations to cases with multiple JLCs having mathematical expressions. In the case with a single JLC that is a "smooth, closed, and simple" curve, O'Sullivan and Qian (1994) suggested estimating the JLC by the minimizer of a contrast statistic, defined by the difference between observation averages inside and outside a candidate JLC, searched in a sufficiently rich class of candidate JLCs, including the true JLC as a member.

To get a rough idea how such methods work, next we introduce two methods, of Müller and Song (1994a) and Qiu (1997), in more detail. Both methods assume a single JLC. Müller and Song's jump detection criterion $\Delta(\theta, x, y)$ is defined by a difference of two averages of the observations located in two one-sided, square-shaped neighborhoods of a given point (x, y) along a direction θ. The two neighborhoods are required to be a certain distance apart. The absolute jump magnitudes along the true JLC Γ are assumed to be bounded above 0. Then their estimator $\widehat{\Gamma}$ of Γ is defined by the maximizer of

$$\sup_{\Gamma' \in \Xi} \left[\inf_{(x,y)\in\Gamma'} \left(\sup_{\theta\in[-\pi/2,\pi/2]} |\Delta(\theta, x, y)| \right) \right], \tag{2}$$

where Ξ denotes a sufficiently rich class of candidate JLCs, including Γ as a member. When the JLC has the expression $y = \phi(x)$, Qiu suggested a so-called *rotational difference kernel estimator* (RDKE) of the JLC, using two *rotational kernel functions*, $K_j(\theta, x, y; h_n, p_n)$, for $j = 1, 2$, obtained by rotating the supports $[-h_n/2, h_n/2] \times [0, p_n]$ and $[-h_n/2, h_n/2] \times [-p_n, 0)$ of two one-sided kernel functions $K_j^*(x/h_n, y/p_n)$, for $j = 1, 2$, an angle θ counterclockwise about the origin, where h_n and p_n are two bandwidth parameters and $n = n_1 n_2$. Then the jump detection criterion is defined by

$$\begin{aligned} M_{RDKE}&(\theta, x, y) \\ &= \frac{1}{n_1 n_2 h_n p_n} \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} Z_{ij} \big[K_2(\theta, x_i - x, y_j - y; h_n, p_n) \\ &\quad - K_1(\theta, x_i - x, y_j - y; h_n, p_n) \big], \end{aligned} \tag{3}$$

where $(x, y) \in [b_n, 1 - b_n] \times [b_n, 1 - b_n]$ and $b_n = \sqrt{h_n^2/4 + p_n^2}$. The RDKE of $\phi(x)$ is defined by

$$\theta^*(x, y) = \arg \max_{\theta\in[-\pi/2,\pi/2]} |M_{RDKE}(\theta, x, y)|;$$

$$\widehat{\phi}(x) = \arg \max_{b_n \le y \le 1-b_n} \big| M_{RDKE}(\theta^*(x, y), x, y) \big|.$$

Note that procedures (2) and (3) can be generalized in several different ways. For instance, when multiple JLCs are present, to use Müller and Song's procedure (2), the design space can be partitioned into a sufficiently large number of disjoint regions so that Müller and Song's procedure can be used in each region. To decrease the computational burden, instead of searching all possible directions at a given point for a gradient direction [cf. expression (2)], we could search only the x- and y-axis directions (see Müller and Song 1994b for related discussions). For procedure (3), Garlipp and Müller (2004) corrected a mistake in the work of Qiu (1997) and proposed a robust version using M-kernel estimation. Such robust jump detection procedures are useful when there are outliers or "salt-and-pepper" noise in observed data. Garlipp and Müller (2006) further extended that robust jump detection procedure for detecting linearly or circularly shaped objects. Other methods for estimating JLCs when the number of JLCs is assumed known include several tracking algorithms proposed by Hall and coauthors (e.g., Hall, Peng, and Rau 2001; Hall, Qiu, and Rau 2007; Hall and Rau 2000, 2002), methods based on wavelet transformations (e.g., Wang 1998), methods by smoothing splines (e.g., Shiau 1985), and others.

2.2 Detection of Arbitrary Jump Location Curves

A major reason why some methods mentioned in the previous section require that the number of JLCs be known and some other quite restrictive assumptions is due to the belief that JLCs are curves and should be estimated by curves. We know that curves have the global nature that any two points on a curve are connected by some other points on the same curve. Because of this, conditions imposed on the JLCs by these methods are also global, in the sense that they should be satisfied at all points on the JLCs. An alternative but more flexible description of the JLCs is that points on the JLCs constitute a *pointset* in the design space that can be estimated by another pointset in the same design space. Because a pointset need not form curves, it can be described and estimated more flexibly. Using this concept, several procedures have been proposed in the literature for estimating arbitrary JLCs, some of which are described here.

Qiu and Yandell (1997) suggested estimating arbitrary JLCs based on local linear estimation. At any design point (x_i, y_j), for $\ell + 1 \le i \le n_1 - \ell$ and $\ell + 1 \le j \le n_2 - \ell$, we consider a square-shaped neighborhood $N(x_i, y_j)$ with width $k = 2\ell + 1$, where $0 < \ell \ll \min(n_1, n_2)$ is an integer. A least squares (LS) plane is then fitted in this neighborhood, and the fitted LS plane is denoted by

$$\widehat{Z}_{ij}(x, y) = \widehat{\beta}_0^{(i,j)} + \widehat{\beta}_1^{(i,j)}(x - x_i) + \widehat{\beta}_2^{(i,j)}(y - y_j) \quad \text{for } (x, y) \in N(x_i, y_j).$$

Then the gradient direction of the fitted plane is $\vec{v}_{ij} = (\widehat{\beta}_1^{(i,j)}, \widehat{\beta}_2^{(i,j)})$. On two sides of (x_i, y_j) along this direction, let (x_{N1}, y_{N1}) and (x_{N2}, y_{N2}) be two design points whose neighborhoods $N(x_{N1}, y_{N1})$ and $N(x_{N2}, y_{N2})$ are just next to the neighborhood $N(x_i, y_j)$. Then a jump detection criterion is defined by

$$\delta_{ij} = \min\{\|\vec{v}_{ij} - \vec{v}_{N1}\|, \|\vec{v}_{ij} - \vec{v}_{N2}\|\}, \tag{4}$$

where $\vec{v}_{N1}$ and $\vec{v}_{N2}$ are gradient vectors of the fitted LS planes in $N(x_{N1}, y_{N1})$ and $N(x_{N2}, y_{N2})$, and $\|\cdot\|$ is the Euclidean norm. The set of design points $\{(x_i, y_j) : \delta_{ij} > b, i = (3k+1)/2, \ldots, n_1 - (3k-1)/2, j = (3k+1)/2, \ldots, n_2 - (3k-1)/2\}$ is then used as an estimator of the JLCs, where b is a threshold parameter properly chosen by some hypothesis testing arguments. Due to the nature of local smoothing and thresholding on which the foregoing procedure is based, two kinds of deceptive jumps may be detected, either close to the true JLCs or scattered in the whole design space. Qiu and Yandell also suggested two modification procedures to delete these deceptive jumps.

Qiu (2002) proposed an alternative estimator of the JLCs that attempts to simplify the computation involved in the RDKE procedure (3). Note that the RDKE procedure detects a possible jump at a given point by searching all possible directions, which is computationally expensive. To overcome this limitation, Qiu (2002) suggested searching only the x- and y-directions at any point (x, y) by the criterion

$$M_n(x, y) = \max\left\{|M_{RDKE}(0, x, y)|, |M_{RDKE}(\pi/2, x, y)|\right\},$$

where M_{RDKE} is as defined in (3). Then the pointset

$$\widehat{D}_n = \{(x_i, y_j) : M_n(x_i, y_j) > u_n\} \tag{5}$$

is used as an estimator of the pointset of the true JLCs, denoted as D, where u_n is a threshold parameter. Qiu proved that as long as the two bandwidths h_n and p_n [cf. eq. (3)] are chosen such that $\lim_{\min(n_1,n_2)\to\infty} h_n/p_n = 0$ and other regularity conditions are satisfied, $\widehat{D}_n$ is a strongly consistent estimator of D in Hausdorff distance, defined by

$$d_H(\widehat{D}_n, D) = \max\left\{\sup_{\mathbf{s}_1 \in \widehat{D}_n} \inf_{\mathbf{s}_2 \in D} \|\mathbf{s}_1 - \mathbf{s}_2\|, \sup_{\mathbf{s}_1 \in D} \inf_{\mathbf{s}_2 \in \widehat{D}_n} \|\mathbf{s}_1 - \mathbf{s}_2\|\right\}.$$

Regarding the performance measurement of a pointset estimator, Qiu (2002) pointed out two limitations of the Hausdorff distance for that purpose. One limitation is that it is difficult to compute, and the second is that it is sensitive to individual points in the related pointsets due to the supremums/infimums involved in its definition. To overcome these limitations, Qiu suggested an alternative performance measure, $d^*(\widehat{D}_n, D)$, which is a weighted average of the proportion of the continuity points detected among all continuity design points and the proportion of the true jump points missed by the jump detection procedure. But, this alternative measure has its own limitations. For instance, it depends on the two proportions mentioned earlier; however, it does not depend on relative distances between points in the two pointsets. In the extreme case when $\widehat{D}_n$ and D were disjoint, $d^*(\widehat{D}_n, D)$ would be a constant, no matter how far away $\widehat{D}_n$ is from D, which obviously is unreasonable. Due to the importance of a good performance measure for comparing different jump detectors, much more research is needed on this topic.

The idea of estimating JLCs by a pointset allows us to handle arbitrary JLCs. As long as the points in the pointset are close enough to the true JLCs and the points are dense enough as well, the visual effect of the estimator should be reasonably good. Otherwise, however, the pointset estimator will not be acceptable. Furthermore, in certain applications it is better to estimate JLCs by curves instead of pointsets. For instance, in bioinformatics, before generating gene expression data from spotted microarray images, spot boundaries separating foregrounds from backgrounds should be estimated (e.g., Qiu and Sun 2007; Yang, Buckley, Dudoit, and Speed 2002). If spot boundaries were estimated by pointsets, then foreground and background pixels would not be properly defined even after jump detection; consequently, gene expression data could not be generated. For these reasons, it might be an interesting research problem to replace the pointset estimator by some curves in certain cases, after the pointset estimator is obtained. In the context of image segmentation for microarray images, Qiu and Sun (2006) suggested a postsmoothing algorithm for this purpose.

Note that almost all existing jump detection procedures in the statistical literature detect jumps based on estimation of the first-order derivatives of the true surface f in one way or another. As discussed in Section 4, second-order derivatives of f also include useful information for jump detection. It might be an interesting problem to make proper use of this information in jump detection, together with the useful information included in the first-order derivatives. Recently, Sun and Qiu (2007) proposed such a jump detection procedure using both the first-order and second-order derivatives; nonetheless, more research is needed in this direction. A related problem, detecting jumps in derivatives of f, has not yet been properly addressed in the statistical literature. For instance, jumps in the first-order derivatives correspond to sharp angles in the surface, which are often called roof edges in image analysis (see the related discussion in Sec. 4). Detection of such jumps, or detection of such jumps together with jumps in f, should be important in certain applications.

3. JUMP–PRESERVING SURFACE ESTIMATION

In some applications, our ultimate goal is to estimate the underlying regression surface from noisy data. When the surface has jumps, conventional 2-D smoothing procedures are not appropriate, because they will blur jumps when removing noise, as discussed in Section 1. Therefore, new smoothing procedures for jump-preserving surface estimation are needed.

There are two potential approaches to estimation of surfaces with jumps preserved. In the first approach, jumps are first detected by a jump detector, as described in Section 2, and then the surface is estimated properly after accommodating the detected jumps. We call this type of jump surface estimation method the *direct* method. In the second approach, jumps are not detected explicitly; rather, smoothing procedures adapt to the underlying jump structure automatically. Such methods are called *indirect* methods. In a specific application, if specification of jump locations is important, besides surface estimation, then direct methods are appropriate. The microarray image analysis problem mentioned in Section 2 fits to this scenario. If our ultimate goal is surface estimation, then it is desirable to preserve jumps in the estimation process, because they are important structures of the true surface; but if our major research interest is not knowing the jump locations, then indirect methods may be more convenient. Many image denoising applications can be classified into this category; see Section 5 for a related discussion.

In the simple case with a single JLC with expression $y = \phi(x)$, the jump surface can be easily estimated by one of the following two direct methods:

1. The JLC is first estimated with a curve, as discussed in Section 2.1, and then the surface is estimated as usual in design subregions separated by the estimated JLC.
2. The JLC and the corresponding jump magnitude function $C(x)$ are first estimated by univariate functions $\widehat{\phi}(x)$ and $\widehat{C}(x)$; then the modified data $\{Z_{ij} - \widehat{C}(x_i)I(y_j > \widehat{\phi}(x)), i = 1, 2, \ldots, n_1; j = 1, 2, \ldots, n_2\}$ are computed; and finally the continuity part of the surface is estimated as usual from the modified data, where $I(\cdot)$ is the indicator function and the data $\{(x_i, y_j, Z_{ij}), i = 1, 2, \ldots, n_1; j = 1, 2, \ldots, n_2\}$ are assumed to be from model (1).

Obviously, such direct methods also can be applied to some more general cases, such as when multiple JLCs exist, each JLC can be expressed by a univariate function of x, and all JLCs do not cross one another.

In a general case with arbitrary JLCs, the JLCs are often estimated by a pointset that may not form curves in the design space, as discussed in Section 2.2. In such cases, neither one of the foregoing direct methods would work. To overcome this difficulty, Qiu (1998) suggested a three-stage direct method. In the first stage, jump candidate points are detected using a jump detector, such as (4). Then, in the second stage, if enough jump points are detected in a neighborhood of a given design point, then a local principal component (PC) line is fitted through them, which provides a first-order approximation to a possible JLC in the neighborhood. In the third stage, observations on the same side of the PC line as the given point are combined using a weighted average procedure to estimate the surface at the given point. By this method, if there are no jump candidate points in the neighborhood, then all observations in the neighborhood are actually used in surface estimation. On the other hand, if a JLC exists in the neighborhood, then only those observations on one specific side of the PC line are used. Thus blurring around the jump locations is avoided.

Recently, several indirect methods have been proposed in the statistical literature. The so-called sigma filter and M-smoother, discussed by Chu, Glad, Godtliebsen, and Marron (1998), are both based on robust estimation. The surface estimator of the sigma filter is defined by

$$\widehat{f}_S(x_i, y_j) = \left[\sum_{s=1}^{n_1}\sum_{t=1}^{n_2} Z_{st} K\left(\frac{x_s - x_i}{h_n}, \frac{y_t - y_j}{h_n}\right) L\left(\frac{Z_{st} - Z_{ij}}{g_n}\right)\right] \Big/ \left[\sum_{s=1}^{n_1}\sum_{t=1}^{n_2} K\left(\frac{x_s - x_i}{h_n}, \frac{y_t - y_j}{h_n}\right) L\left(\frac{Z_{st} - Z_{ij}}{g_n}\right)\right], \tag{6}$$

where K and L are two density kernel functions and h_n and g_n are two bandwidths. Clearly, $\widehat{f}_S(x_i, y_j)$ is a weighted average of all observations; the weights are determined not only by the distance from individual design points (x_s, y_t) to the given design point (x_i, y_j), but also by the difference between individual observations Z_{st} and the observation Z_{ij} at (x_i, y_j). If (x_s, y_t) and (x_i, y_j) are on two different sides of a JLC, then the absolute difference between Z_{st} and Z_{ij} would be large. Consequently, Z_{st} would receive small weights in computing $\widehat{f}_S(x_i, y_j)$; thus jumps are preserved to some degree by (6). The surface estimator of the M-smoother is defined by the solution of

$$\min_{a \in R} \sum_{s=1}^{n_1}\sum_{t=1}^{n_2} \rho(Z_{st} - a) K\left(\frac{x_s - x_i}{h_n}, \frac{y_t - y_j}{h_n}\right),$$

where $\rho(z)$ is a robust loss function chosen to grow slowly when z tends to $\pm\infty$.

The basic idea of both the sigma filter and M-smoother is to assign small weights to observations located on the other side of a JLC around a given design point. But those observations still receive some weight in the weighted average; therefore, certain blurring is inevitable by these methods (although the degree of blurring has been greatly reduced). This issue is especially important when the noise level in the observed data is high, or when an attempt is made to preserve small jumps. Polzehl and Spokoiny (2000) suggested an iterative version of the sigma filter by which the aforementioned problem might be alleviated to a certain degree after some iterations. But we do not know much about the theoretical properties of that iterative procedure, and much more research is needed.

Recently, Qiu (2004) suggested an indirect method based on local piecewise linear kernel smoothing. At a given point, a square-shaped neighborhood is considered. Then, four surface estimators are obtained by local linear kernel smoothing in the four quadrants of the neighborhood. If the range of the four estimators is larger than a threshold value, then the final surface estimator is defined by the one with the smallest weighted residual sum of squares of the related fitted local plane. Otherwise, the final surface estimator is defined by the simple average of the four estimators. This procedure preserves jumps well, but its surface estimator is a bit noisy in continuity regions of the surface. Several improvements have been proposed recently; for instance, Gijbels et al. (2006) suggested fitting a plane in a circular neighborhood $N(x, y)$ of a given point (x, y) by local linear kernel smoothing, obtaining a conventional surface estimator $\widehat{a}_c(x, y)$ and a gradient estimator $G(x, y)$. Then the neighborhood $N(x, y)$ is divided into two halves by a line passing through (x, y) and perpendicular to the gradient direction $G(x, y)$. Two one-sided surface estimators $\widehat{a}_1(x, y)$ and $\widehat{a}_2(x, y)$ are obtained in the two halves of $N(x, y)$ by local linear kernel smoothing. The final surface estimator is defined by

$$\widehat{f}(x, y) = \begin{cases} \widehat{a}_c(x, y) & \text{if } \mathrm{dif}(x, y) \le u \\ \widehat{a}_1(x, y) & \text{if } \mathrm{dif}(x, y) > u \text{ and} \\ & \quad \mathrm{WRMS}_1(x, y) < \mathrm{WRMS}_2(x, y) \\ \widehat{a}_2(x, y) & \text{if } \mathrm{dif}(x, y) > u \text{ and} \\ & \quad \mathrm{WRMS}_1(x, y) > \mathrm{WRMS}_2(x, y) \\ \dfrac{\widehat{a}_1(x, y) + \widehat{a}_2(x, y)}{2} & \\ & \text{if } \mathrm{dif}(x, y) > u \text{ and} \\ & \quad \mathrm{WRMS}_1(x, y) = \mathrm{WRMS}_2(x, y), \end{cases} \tag{7}$$

where $\mathrm{dif}(x, y) = \max\{\mathrm{WRMS}_c(x, y) - \mathrm{WRMS}_1(x, y), \mathrm{WRMS}_c(x, y) - \mathrm{WRMS}_2(x, y)\}$, $\mathrm{WRMS}_c(x, y)$, $\mathrm{WRMS}_1(x, y)$, and $\mathrm{WRMS}_2(x, y)$ denote the weighted residual mean squares of the corresponding fitted local planes and u is a threshold parameter. An alternative two-stage surface estimator has been suggested by Qiu (2006). In the first stage, the fitted surface is defined by the one of $\widehat{a}_c(x, y)$, $\widehat{a}_1(x, y)$, and $\widehat{a}_2(x, y)$ with the smallest WRMS value. In the second stage, estimated surface values at the design points obtained in the first stage are used as new data, and the foregoing procedure is applied to this data in the same way, except that one of the three estimators is selected based on their estimated variances in the second stage. The final surface estimator preserves jumps well and removes noise efficiently.

The surface estimator (7) and the alternative estimator of Qiu (2006) still have much room for improvement. For instance, estimator (7) uses a constant threshold value u in the entire design space, which may not be reasonable in cases where the noise level changes with location. Both procedures divide the circular neighborhood of a given point into two halves for preserving jumps in the entire design space. To preserve corners of JLCs, estimator (7) was generalized by Gijbels et al. (2006) by using two opposite sectors of constant size in the circular neighborhood when constructing the two one-sided estimators. An ideal surface estimation procedure should have the ability to adapt to not only the presence of jumps, but also the shape of the underlying JLCs, which has not been achieved by most existing procedures.

Another issue that has not been well addressed in the literature concerns preservation of other important features of the regression surface, including peaks and valleys, in surface estimation. So far, most procedures have discussed preservation of jumps in f only. In some applications, especially those with reasonably large sample sizes, such as image analysis, preservation of peaks, valleys, and other important features should be important and possible to achieve.

4. EDGE DETECTION IN IMAGE PROCESSING

In the computer science literature, many different image processing techniques have been proposed for various purposes (e.g., Gonzalez and Woods 1992; Rosenfeld and Kak 1982). In this and the next two sections, we focus on three types of image processing techniques: those for edge detection, those for image denoising, and those for image deblurring. These techniques are widely used in applications and closely related to the jump surface estimation methods discussed in the previous two sections.

In the literature, several different types of edges are defined (cf. Qiu 2005, chap. 6). *Step edges* refer to places in an image at which the image intensity function has jumps; *roof edges* refer to places at which the first-order derivatives of the image intensity function have jumps. Other types of edges include ramp edges, spike edges, line edges, texture edges, and others. Most edge detection techniques in the literature are used for detecting step edges, because they often represent outlines of objects or some sudden structural changes of the related process. For this reason, edges mentioned in this and the next two sections are referred to step edges if there is no further specification.

Most existing edge detectors are based on estimation of the first-order and second-order derivatives of the image intensity function, because they include useful information about edges. Figure 1(a) displays a 1-D profile of a step edge that is slightly blurred during image acquisition. Its first- and second-order derivatives are displayed in Figures 1(b) and 1(c). It can be seen that the first-order derivative peaks at the step edge, the second-order derivative equals zero at the step edge, and the second-order derivative changes its sign on two sides of the edge. In the literature, the second and third properties are called *zero-crossing* properties of the second-order derivative. In the 2-D setup, the directional first- and second-order derivatives in the direction perpendicular to a given edge segment have similar properties.

Edge detectors based on first-order derivatives usually work as follows. Let $f(x, y)$ be the image intensity function. The magnitude of its gradient vector,

$$M_f(x, y) = \sqrt{[f'_x(x, y)]^2 + [f'_y(x, y)]^2}, \tag{8}$$

should be large if the pixel (x, y) is on an edge segment. Thus, $M_f(x, y)$ can be used for edge detection once $f'_x(x, y)$ and $f'_y(x, y)$ are estimated properly. Many masks have been proposed in the literature for estimating f'_x and f'_y using directional differences, including Sobel masks, Roberts operators, Prewitt masks, Frei–Chen masks, truncated pyramid operators, derivatives of Gaussian (DoG) operators, and others (see, e.g., Gonzalez and Woods 1992, chap. 4). After estimating $M_f(x, y)$, we still need to choose a threshold value for identifying detected edge pixels. In applications, we usually consider a sequence of threshold values and choose the one giving results with the best visual impression.

Obviously, the aforementioned edge detectors are similar in nature to the jump detectors discussed in Section 2, because

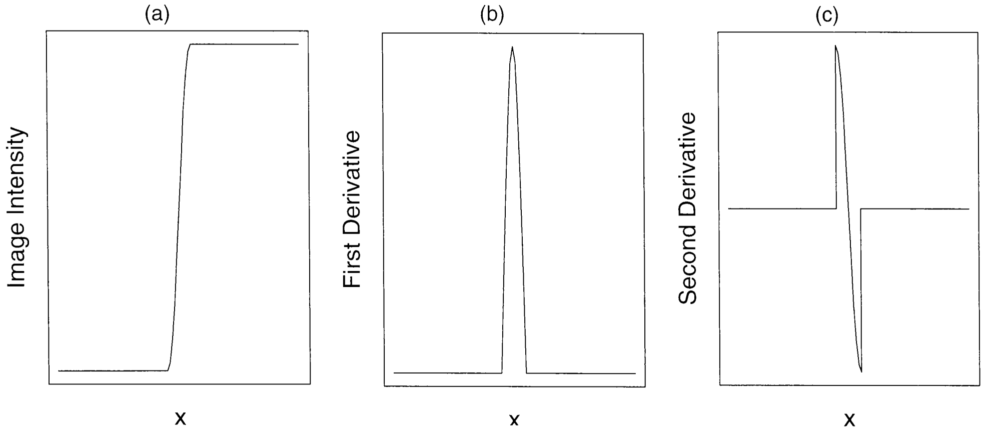

Figure 1. Properties of the First-Order and Second-Order Derivatives Around a Step Edge. (a) 1-D profile of a step edge which is slightly blurred. (b) First-order derivative of the 1-D profile. (c) Second-order derivative of the 1-D profile.

all are based on estimation of first-order derivatives of f. But there are several important differences between the two groups of methods. First, edge detectors usually use masks of fixed sizes; commonly used sizes are 3×3, 5×5, and so forth. With such small sizes, edge detectors are easy to compute and use, but their ability to remove noise is limited. In comparison, window widths of most jump detectors in the statistical literature are chosen by data-driven procedures, such as the cross-validation and bootstrap procedures. Second, most masks used in the image processing literature are square-shaped. However, as pointed out by Qiu (2002), to successfully detect edges with different curvatures, the x-direction and y-direction masks should be narrow along the x- and y-axis directions, respectively; see the related discussion about the jump detector (5) in Section 2.2. Third, most edge detectors make use of the spatial regularity of pixel locations and thus cannot be applied to applications with irregular design points. In comparison, most jump detectors in statistics do not have this limitation. Other differences between the two groups of methods include weight assignments, threshold selection, and so forth.

We now discuss edge detection based on second-order derivatives. A commonly used operator for this purpose is the following Laplacian operator:

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}.$$

This has the zero-crossing properties around a given edge pixel (x_0, y_0) in the following sense: $\nabla^2 f(x, y)$ would be positive on one side of the edge segment, negative on the other side, and zero at (x_0, y_0) or at some place(s) between (x_0, y_0) and its neighboring pixels. Compared with edge detectors based on the first-order derivatives of f, the Laplacian operator has the advantage of localizing the detected edges well; that is, its detected edges are usually thinner than those detected by edge detectors based on the first-order derivatives. It is quite sensitive to noise, however. To balance localization of detected edges and denoising, Marr and Hildreth (1980) proposed the *Laplacian of Gaussian* (LoG) edge detector, by which the image is presmoothed by a weighted averaging procedure with the weights determined by a 2-D Gaussian density $G(x, y; s)$, where s is a scale parameter, and then edges are detected by applying the Laplacian operator to the presmoothed image. (For related discussions, see Haralick 1984; Nalwa and Binford 1986; Qiu and Bhandarkar 1996; and references cited therein.) It should be pointed out that although LoG is less sensitive to noise compared with the Laplacian operator, it has its own limitations. One major limitation is caused by the presmoothing step. At the same time that noise is removed in that step, edges (especially small ones) are blurred; consequently, localization of the detected edges is sacrificed to some degree. To overcome this limitation, the Gaussian presmoother should be replaced by an edge-preserving smoother. Some jump-preserving surface estimation methods discussed in Section 3 might be helpful for this.

Canny (1986) suggested a benchmark edge detector based on the following three criteria for measuring edge detection performance: (a) There should be a low probability of missing real edge pixels, and a low probability of detecting false edge pixels; (b) detected edge pixels should have good localization; and (c) there should not be multiple responses to a single true edge pixel. After describing these criteria with mathematical expressions, Canny derived an optimal edge detector for detecting step edges, using the so-called *variational* approach. This detector can be well approximated by the DoG and LoG operators mentioned earlier. The literature contains several modifications and generalizations of Canny's edge detection criteria, for instance, Petrou and Kittler (1991) adapted Canny's criteria for detecting roof edges and ramp edges, and Demigny and Kamlé (1997) proposed a discrete version for Canny's criteria. More recent discussions about performance assessment of edge detectors have been given by Heath, Sarkar, Sanocki, and Bowyer (1997, 1998), Shin, Goldgof, and Bowyer (2001), Yitzhaky and Peli (2003), and the references cited therein.

Tan, Gelfand, and Delp (1992) formulated the edge detection problem as a problem of cost minimization. Their edge estimator is defined as the edge configuration that minimizes a cost function, defined as a weighted average of the following five cost factors: edge curvature, dissimilarity of the regions separated by the edges, number of edge pixels, fragmentation of the edges, and edge thickness. After these five cost factors are properly defined, Tan el al. suggested searching for the optimal edge estimator among all possible candidates by a simulated annealing algorithm. An alternative, genetic algorithm-based optimization procedure for finding the optimal edge estimator was suggested by Bhandarkar, Zhang, and Potter (1994).

The literature contains many more edge detectors than those just discussed. For instance, as a byproduct, Markov random field (MRF) modeling approaches and diffusion approaches (which are mainly for image restoration and are described in more detail in the next section) also can generate detected edge images (see Hansen and Elliot 1982; Perona and Malik 1990). Shiau (1985, 1987) and Shiau, Wahba, and Johnson (1986) suggested using the *partial smoothing spline* method for estimating certain discontinuous curves and surfaces. This general method has been applied to several image analysis problems, including estimation of object boundaries and detection of boundary corners (cf. Chen and Chin 1993), and we believe that it has great potential for use in edge detection.

We note that several other image analysis problems, including image segmentation, image enhancement, object boundary detection, pattern recognition, and shape analysis, are closely related to edge detection. Methodologies used for solving these problems might also be helpful for edge detection. For instance, *active contour models* (ACMs), or *snakes*, and geodesic ACMs are used mainly for shape analysis (see, e.g., Caselles, Kimmel, and Sapiro 1997; Chan and Luminita 2001; Kass, Witkin, and Terzopoulos 1988, Malladi, Sethian, and Vemuri 1995). The basic idea of these methods is to evolve a curve, subject to constraints from a given image, to detect object boundaries in that image. As an example, by the snake model, the boundary of an object in the image I is estimated by the solution of

$$\min_C \left\{ \alpha \int_0^1 |C'(s)|^2\,ds + \beta \int_0^1 |C''(s)|\,ds - \lambda \int_0^1 \left| \nabla I(C(s)) \right|^2 ds \right\},$$

where $C(s) : [0, 1] \to R^2$ is a candidate parameterized boundary curve, $\nabla I(C(s))$ is the image gradient assigned at the boundary curve, and α, β, and λ are positive parameters. In the foregoing expression, the first two terms control the smoothness of the candidate boundary curve, and the third term attracts the candidate boundary curve toward the object in the image. In the literature, such a minimization problem is often handled using the *level set method* by solving a partial differential equation (see, e.g., Caselles, Catté, Coll, and Dibos 1993; Osher and Sethian 1988).

5. IMAGE DENOISING

Observed images often are degraded versions of their true images. If pointwise noise and spatial blur are the degradations of our major concern, then the relationship between an observed image Z and its true image f can be described by the following model:

$$Z(x, y) = (h \otimes f)(x, y) + \varepsilon(x, y), \quad \text{for } (x, y) \in \Omega, \tag{9}$$

where $\varepsilon(x, y)$ denotes noise at (x, y), Ω is the design space of the image, and $h \otimes f$ is a spatially degraded version of f, defined by the convolution between a point spread function (psf), h, and the true image, f. Image restoration techniques try to recover the true image f from the observed image Z. Because edges are important structures of the true image, they should be preserved in the image restoration process.

In general, image restoration is challenging, due mainly to the fact that spatial blur and pointwise noise often are mixed up, and removing them simultaneously is difficult. For ease of presentation, we roughly classify all image restoration methods into two categories: those mainly for *denoising* and those mainly for *deblurring*. Denoising methods, discussed in this section, are often used in applications in which there is little blurring in Z. Some denoising methods also can be used in applications when there is substantial blurring in Z, but the blurring mechanism is known. When blurring is our major concern, deblurring methods should be used, as discussed in Section 6.

Image restoration by MRF modeling is an active research area in the image processing literature. Geman and Geman (1984) have provided a general framework for this approach. Suppose that a given image has n_1 rows and n_1 columns of pixels, $\mathbf{X} = \{X_{ij}; i, j = 1, 2, \ldots, n_1\}$ is the matrix of observed image intensities at the pixel sites $\mathcal{S} = \{(i, j); i, j = 1, 2, \ldots, n_1\}$, and $\mathbf{F} = \{F_{ij}, i, j = 1, 2, \ldots, n_1\}$ is the matrix of true image intensities. Between two neighboring pixels in either horizontal or vertical direction, an unobservable binary edge element is assumed, with 1 denoting an existing edge between the two pixels and 0 denoting no edge. The matrix of all such edge elements is called a *line process*, and denoted by $\mathbf{L}$. Geman and Geman assumed that $\mathbf{Y} = (\mathbf{F}, \mathbf{L})$ is an MRF or, equivalently, that its prior distribution is the following Gibbs distribution:

$$P(\mathbf{F} = \mathbf{f}, \mathbf{L} = \mathbf{l}) = \frac{1}{A} \exp\left(-\frac{U(\mathbf{f}, \mathbf{l})}{T} \right),$$

where $U(\mathbf{f}, \mathbf{l})$ is an energy function, A is a normalizing constant, and T is a temperature parameter. Under the assumption that pointwise noise is iid with normal distribution, it is proved that the posterior distribution of $\mathbf{Y}$ given $\mathbf{X} = \mathbf{x}$ is also a Gibbs distribution with energy function $U^P(\mathbf{f}, \mathbf{l})$, where the superscript "P" denotes "posterior." The restored image is defined by one of the maximizers of the posterior distribution or, equivalently, one minimizer of the energy function $U^P(\mathbf{f}, \mathbf{l})$. This estimator is called the maximum a posteriori (MAP) estimator. In this approach, edges are detected and preserved by using the line process $\mathbf{L}$. But in searching for the MAP estimator directly from all image configurations is almost impossible numerically, even for a moderate-sized gray-scale image. To overcome the computational difficulty, Geman and Geman suggested using the Gibbs sampler scheme and an annealing algorithm for obtaining the MAP estimator.

To further simplify the computation involved in the MAP procedure, Besag (1986) suggested the iterated conditional modes (ICM) algorithm for image restoration. The ICM algorithm works iteratively. Its updated estimator of F_{ij} in each iteration is defined by the maximizer of the local conditional probability $P(F_{ij} | \mathbf{X}, \widehat{F}_s, s \in \mathcal{S} \backslash \{(i, j)\})$, instead of the global posterior distribution as in the MAP procedure, where $\widehat{F}_s$ is obtained from the previous iteration. The literature contains other generalizations and modifications of the MAP procedure (see, e.g., Fessler, Erdogan, and Wu 2000; Godtliebsen and Sebastiani 1994; Marroquin, Velasco, Rivera, and Nakamura 2001). Li (1995) has provided a nice overview on this topic.

Usually, MRF methods work well in cases when blurring is involved in the observed image and the blurring mechanism described by the psf h is known. Because of their iterative nature, MRF methods often are relatively difficult to compute, although much effort has been made toward simplifying their computation. For the same reason, studying their theoretical properties is also difficult. Many numerical studies in the literature show that they work well in various applications, although the MRF assumption and some other model assumptions may not be valid in certain applications; see the references previously cited for related discussions.

The image restoration problem also can be formulated as a *regularization* problem, in which the restored image is defined by the minimizer of the following energy function in an appropriate function space:

$$U(g,d)+U(g),$$

where the first term $U(g,d)$ measures the fidelity of a candidate estimator g of the true image intensity function f to the observed image intensity function d (i.e., the data), and $U(g)$ is a smoothness measure of g, which is called a regularizer. To preserve edges, a line process is often used in $U(g)$. Poggio, Torre, and Koch (1985) give a nice review on some early regularization procedures. More recent methods have been discussed by Blake and Zisserman (1987), Green (1990), Lange (1990), Bouman and Sauer (1993), Stevenson, Schmitz, and Delp (1994), Li (1998), and others.

Local median filtering is a popular presmoothing tool in image processing because of its ability to preserve edges while removing noise (Gallagher and Wise 1981; Huang 1981). By conventional local median filtering, the restored image intensity at a given pixel equals the sample median of the observed image intensities in a neighborhood. Usually, square-shaped neighborhoods are used in applications. Sometimes cross-shaped, X-shaped, or other types of neighborhoods are also used to better preserve line edges, angles, and other image details. There are some generalizations of the conventional local median filtering; among these, weighted median filtering has been found useful in several applications (e.g., Arce 1991; Haavisto, Gabbouj, and Neuvo 1991). From a statistical viewpoint, we know that median filters are sensitive to noise; therefore, it is difficult for them to preserve small edges in the presence of noise. As pointed out earlier, for presmoothing purposes, some jump-preserving surface estimation procedures discussed in Section 3 might be more reasonable.

Saint-Marc, Chen, and Medioni (1991) proposed another local smoothing filter, the *adaptive smoothing filter*, based on the idea that edge structure of the image should be taken into account when the image is restored. That filter works iteratively as follows. Let $Z(x,y)$ be the observed image intensity at pixel (x,y) and let $S^{(0)}(x,y)=Z(x,y)$ be the image intensity before smoothing. Then the smoothed image intensity at (x,y) in the $(t+1)$th iteration, for any $t\ge 0$, is defined by

$$S^{(t+1)}(x,y)=\frac{1}{N^{(t+1)}(x,y)}\sum_{i=-1}^{1}\sum_{j=-1}^{1}S^{(t)}(x+i,y+j)\times w^{(t)}(x+i,y+j), \quad (10)$$

where $w^{(t)}(x+i,y+j)=\exp(-(d^{(t)}(x+i,y+j))^2/(2\sigma_D^2))$ are weights, $d^{(t)}(x+i,y+j)$ is the magnitude of image gradient at $(x+i,y+j)$, $\sigma_D>0$ is a scale parameter, and $N^{(t+1)}(x,y)=\sum_{i=-1}^{1}\sum_{j=-1}^{1}w^{(t)}(x+i,y+j)$. By (10), $S^{(t+1)}(x,y)$ is defined by a weighted average of the restored image intensities in the 3×3 neighborhood of (x,y) obtained in the previous iteration, pixels closer to edges would receive less weight because their image gradient magnitudes are usually large, and, consequently, edges are preserved to a certain degree. However, a small amount of blurring is inevitable in (10) due to the fact that pixels located on the different side of the edge, different from the given pixel (x,y), still receive some weight, although relatively small. Iterations might help eliminate blurring, but so far, we have not found any theoretical justifications for this property. This phenomenon of blurring might become worse when larger neighborhoods are used.

A generalization of the foregoing adaptive smoothing filter is the *bilateral filtering* procedure suggested by Tomasi and Manduchi (1998). In this procedure, the restored image intensity at the position $\mathbf{s}=(x,y)$ is defined by

$$\widehat{f}(\mathbf{s})=\frac{1}{N(\mathbf{s})}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}Z(\mathbf{s}^*)K_D(\mathbf{s}^*,\mathbf{s})K_R(Z(\mathbf{s}^*),Z(\mathbf{s}))\,d\mathbf{s}^*, \quad (11)$$

where $\mathbf{s}^*=(x^*,y^*)$ is a nearby point of $\mathbf{s}$, K_D is a nonnegative bivariate kernel function, K_R is a nonnegative univariate kernel function, and $N(\mathbf{s})$ is a normalizing constant. Usually, the two kernel functions in (11) are chosen to be probability density functions of Gaussian distributions; thus the restored image intensity $\widehat{f}(\mathbf{s})$ is a weighted average of the image intensities at nearby pixels, and the weights are determined by the distance from an individual pixel site $\mathbf{s}^*$ to the given pixel site $\mathbf{s}$ and by the difference between the observed image intensities at $\mathbf{s}^*$ and $\mathbf{s}$. Comparing equation (11) with (6) shows that the bilateral filtering procedure discussed in the image processing literature is identical to the sigma filter discussed in the statistical literature, although different terminology is used in the two disciplines. Thus the related discussion in Section 3 about the sigma filter also applies to the bilateral filtering procedure here.

Another popular denoising technique in the computer science literature involves *diffusion equations*, such as

$$\begin{cases}\dfrac{\partial Z(x,y,t)}{\partial t}=\operatorname{div}(cG_Z(x,y,t))\\ \text{subject to } Z(x,y,0)=Z_0(x,y),\end{cases} \quad (12)$$

where "div" is the divergence operator, $Z_0(x,y)$ is an observed image, c is a constant, and $G_Z(x,y,t)$ denotes the gradient of $Z(x,y,t)$. Koenderink (1984) and Hummel (1987) pointed out that the solution of (12) is equivalent to the smoothed image $Z(x,y,t)=(Z_0\otimes\phi)(x,y;t)$, where $\phi(x,y;t)$ denotes the probability density function of the 2-D Gaussian distribution $\mathrm{N}((0,0)',t^2I)$ and $Z_0\otimes\phi$ denotes the convolution between Z_0 and ϕ. In physics, (12) is often used to describe diffusion processes that equilibrate concentration differences without creating or destroying mass. The constant c is called the *diffusion conductance*, or *diffusivity*. The degree of smoothing is controlled by the diffusivity. In the case of (12), all locations in the image, including edge pixels, are smoothed equally in the diffusion process. In such cases, the diffusion equation is homogeneous. To preserve edges during smoothing, or, to smooth within regions separated by edges instead of between such regions, the diffusivity should be ideally 0 at edge positions and 1 within separated regions. In applications, edge positions can be roughly indicated by the image gradient magnitude $|G_Z(x, y, t)|$. Therefore, the following nonhomogeneous diffusivity can be applied to (12) for edge-preserving denoising:

$$c(x, y, t) = g\big(|G_Z(x, y, t)|\big),$$

where g is a decreasing function with $g(0) = 1$. The literature includes several generalizations and modifications of this procedure (see, e.g., Catté, Lions, Morel, and Coll 1992; Charbonnier, Blanc-Féraud, Aubert, and Barlaud 1994; Weickert 1998; Andreu, Ballester, Caselles, and Mazón 2001 for related discussion). The relationships between nonhomogeneous diffusion filtering and both adaptive smoothing and bilateral filtering were discussed by Barash (2002). The relationships between nonhomogeneous diffusion filtering and wavelet shrinkage methods (cf. Chang, Yu, and Vetterli 2000) were studied by Mrázek, Weickert, and Steidl (2003).

6. IMAGE DEBLURRING

The image deblurring problem described by model (9) is generally ill-posed in the sense that (a) there might be many different sets of h and f corresponding to the same observed image, and (b) the inverse problem to estimate f from Z often involves some numerical singularities [cf. (13) and the related discussion]. Therefore, it seems impossible to estimate both h and f properly from Z alone without using any extra information about f, h, or both.

Early image deblurring methods assume that h is known. This assumption is reasonable in some cases, because h can be specified (at least approximately) based on our knowledge about the image acquisition device (e.g., camera). For instance, the *linear blur model* is appropriate for describing h when blurring is caused mainly by relative motion between the image acquisition device and the object, and the *Gaussian blur model* is often used for describing blurring caused by atmospheric turbulence in remote sensing and aerial imaging (see Bates and McDonnell 1986 for detailed descriptions of these models). After h is specified, f can be estimated based on the relationship

$$\mathcal{F}\{Z\}(u, v) = \mathcal{F}\{h\}(u, v)\mathcal{F}\{f\}(u, v) + \mathcal{F}\{\varepsilon\}(u, v), \quad \text{for } (u, v) \in R^2, \tag{13}$$

where $\mathcal{F}\{f\}$ denotes the Fourier transformation of f. By (13), many methods have been proposed in the literature for estimating f, including some noniterative methods, such as inverse filtering, Wiener filtering, and constrained least squares filtering procedures (see, e.g., Gonzalez and Woods 1992, chap. 5), and several iterative methods, such as the Lucy–Richardson procedure, Landweber procedure, Tikhonov–Miller procedure, MAP procedure, maximum entropy procedure, procedures based on EM algorithm, and others (see, e.g., Skilling 1989; Carraso 1999; Figueiredo and Nowak 2003). The Wiener filter, for instance, defines the restored image as

$$\widehat{f}(x, y) = \frac{1}{(2\pi)^2}\mathcal{R}\left\{\int\int \frac{\overline{\mathcal{F}\{h\}(u, v)}}{|\mathcal{F}\{h\}(u, v)|^2 + \alpha(u^2 + v^2)^{\beta/2}} \times \mathcal{F}\{Z\}(u, v)\exp\{i(ux + vy)\}\,du\,dv\right\}, \tag{14}$$

where $\overline{\mathcal{F}\{h\}(u, v)}$ denotes the complex conjugate of $\mathcal{F}\{h\}(u, v)$, $\mathcal{R}\{C\}$ denotes the real part of the complex number C, and $\alpha, \beta > 0$ are two parameters.

In (13), if the noise term is ignored, then the restored image by the Wiener filter should be defined by (14) without the term $\alpha(u^2 + v^2)^{\beta/2}$ in the denominator of the integrant. But in such cases, noise effect would dominate the image estimator, because $\mathcal{F}\{f\}(u, v)$ and $\mathcal{F}\{h\}(u, v)$ usually converge to zero rapidly as $u^2 + v^2$ tends to infinity, and $\mathcal{F}\{\varepsilon\}(u, v)$ converges to zero much less rapidly. How to diminish the noise effect turns out to be a major challenge for image restoration in cases when h is known. Inclusion of $\alpha(u^2 + v^2)^{\beta/2}$ in (14) is mainly for this purpose. The Wiener filter is known to be optimal in minimizing the mean integrated squared error of the restored image when h is assumed known, when the noise is assumed to be Gaussian, and when some other regularity conditions are satisfied (see Gonzalez and Woods 1992, chap. 5).

In many applications, however, it is difficult to specify the psf h completely, based on our previous knowledge about the image acquisition device. Image restoration when h is unknown is often referred to as the *blind image restoration* problem. A number of procedures for solving this problem have been proposed, which can be grouped roughly into two categories. One type of such procedures assumes that h can be described by a parametric model with one or more unknown parameters and the parameters together with the true image are estimated by some algorithms, most of which are iterative (see, e.g., Cannon 1976; Katsaggelos and Lay 1990; Rajagopalan and Chaudhuri 1999; Carasso 2001; Joshi and Chaudhuri 2005). For instance, Carasso (2001) assumed that h belongs to the symmetric Lévy "stable" density family with its Fourier transformation defined by

$$\mathcal{F}\{h\}(u, v) = \exp\{-\xi(u^2 + v^2)^\eta\} \quad \text{for } (u, v) \in R^2,$$

where $\xi > 0$ and $0 < \eta \le 1$ are two parameters (see Hall and Qiu 2007a for a related discussion). The other type of procedure assumes that the true image comprises an object with known finite support; the background is uniformly black, gray, or white; and the psf h satisfies various conditions (see, e.g., Yang, Galatsanos, and Stark 1994; Kundur and Hatzinakos 1998). For instance, Kundur and Hatzinakos (1998) assumed that h has its inverse, both h and its inverse have finite integrations over the real plane, and both the true image f and the psf h are irreducible in the sense that they cannot be expressed as a convolution of two or more component images with finite support.

Apparently, the validity of parametric models used by the first group of methods for describing psf h should be checked in a specific application. If such models do not describe the underlying blurring mechanism well, then the quality of the restored images is called into question, because numerical studies have shown that restored images are quite sensitive to the correct specification of h (e.g., Carasso 2001; Hall and Qiu 2007a). It remains an open problem how to do such a goodness-of-fit model checking. Considering the second group of methods, some of their model assumptions may not be realistic; for instance, some commonly used h functions, including the Gaussian blurring function, do not have integrable inverses.

The literature contains some other blind image deblurring procedures. One group of such methods is based on the *total* *variation* (TV) minimization method first proposed by Rudin, Osher, and Fatemi (1992). For instance, Chan and Wong (1998) formulated the blind deblurring problem as

$$\min_{\widetilde{f},\widetilde{h}}\left\{\frac{1}{2}\int_{\Omega}[(\widetilde{h}\otimes\widetilde{f})(x,y)-Z(x,y)]^2\,dx\,dy\right.$$
$$\left.+\alpha_1\int_{\Omega}|\bigtriangledown\widetilde{f}(x,y)|\,dx\,dy+\alpha_2\int_{\Omega}|\bigtriangledown\widetilde{h}(x,y)|\,dx\,dy\right\}, \quad (15)$$

where Ω is the image design space and α_1 and α_2 are two positive parameters. Solutions of (15) are used as estimators of f and h. Clearly, in (15), the first term measures the goodness-of-fit of the estimators and the second and third terms regularize their total variations. Chan and Wong solved the minimization problem by an iterative algorithm after α_1 and α_2 were properly selected (see You and Kaveh 1996 for a similar procedure). Recently, Hall and Qiu (2007b) suggested estimating the psf h from an observed test image of an imaging device, and then restore other observed images by the same imaging device using the estimated h. The true test image considered in that article has a square block in the middle with a uniform background. But the idea in this also can be applied to cases when no such test images are available but the observed image to deblur has one or more parts in which the true image has step edges at known locations with uniform image intensities on either side of any of those step edges.

7. CONCLUDING REMARKS

In this article we have provided a review of recent methodologies for jump surface estimation, edge detection, and image restoration. From our review, it can be seen that although different terminologies are used in the two disciplines, jump detection and jump surface estimation in statistics are essentially the same problems as edge detection and image restoration in image processing. Therefore, properly combining the strengths of the two groups of methods can benefit both research areas.

Besides statistics, computer sciences, and electronic engineering, surface estimation is a research topic in several other disciplines, including numerical mathematics (e.g., Cheney and Kincaid 1994), computer-aided design (e.g., Farin 1993), meteorology (e.g., Bai and Feng 2003), geology (e.g., Bonan et al. 2002), and others. Some curve/surface estimation techniques proposed in these areas might be helpful in handling jump surface estimation and image analysis. For instance, Bezier curves, commonly used in computer-aided design, might be more flexible than the PC lines used in the second stage of the three-stage jump surface estimation procedure discussed in Section 3, and Bezier surfaces have great potential for local use in detecting and preserving jumps in surface estimation.

From model (9), it can be seen that the image deblurring problem is similar to the deconvolution problem (e.g., Carroll and Hall 1988; Fan 1991) and the errors-in-variables regression problem (e.g., Hall and Qiu 2005; Delaigle, Hall, and Qiu 2006), both of which are extensively discussed in the statistical literature. Although formulations of the three problems are different in one way or another, all involve convolutions and are so-called inverse problems. Therefore, existing methods for handling the deconvolution problem and the errors-in-variables regression problem might be helpful in solving the image deblurring problem.

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