

Exercise



Code under analysis



```
char FixFilter(int L, char *E){
    int sum = 0;
    int length = 1 << L; // computes  $2^L$ 
    for (i = 0; i < length; i++){
        sum = sum + E[i]; // computes the sum of samples
    }
    sum = sum >> L; // computes the average = divides the sum by  $2^L$ 
    return sum;
}
```

Assumption: $L \leq 8$

Code under analysis

```
char FixFilter(int L, char *E){
    int sum = 0;
    int length = 1 << L;
    for (i = 0; i < length; i++){
        sum = sum + E[i];
    }
    sum = sum >> L;
    return sum;
}
```

fixFilter:

```
    stmfd r13!,{r3-r6,r14} (a)    // writes 5 registers
    mov    r3,#1             (b)
    mov    r3,r3, lsl r0     (c)
    mov    r6,r0             (d)
    mov    r0,#0             (e)
    mov    r4,#0             (f)
```

for_filter:

```
    cmp    r4,r3             (g)
    bcs    end_filter        (h)
    ldrb   r5,[r1,r4]        (i)    // reads one register
    add    r0,r0,r5          (j)
    add    r4,r4,#1          (k)
    b      for_filter        (l)
```

end_filter:

```
    mov    r0,r0,lsr r6      (m)
    ldmfdd r13!,{r3-r6,r14} (n)    // reads 5 registers
```

Build the CFG



fixFilter:

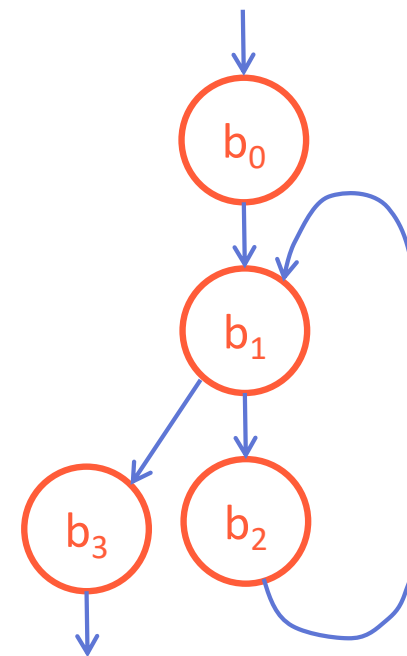
stmfd r13!,{r3-r6,r14}	(a)	
mov r3,#1	(b)	
mov r3,r3, lsl r0	(c)	b_0
mov r6,r0	(d)	
mov r0,#0	(e)	
mov r4,#0	(f)	

for_filter:

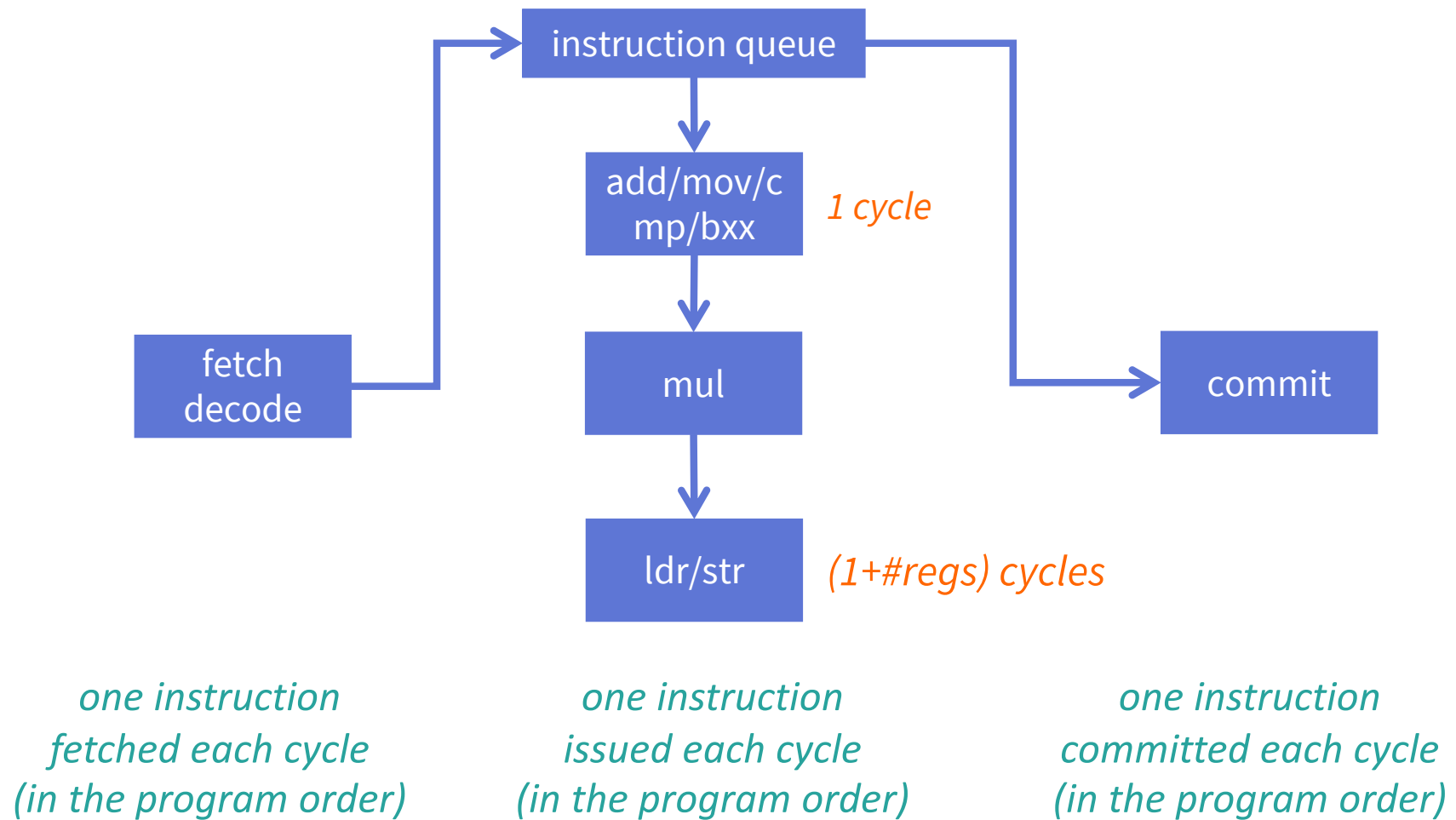
cmp r4,r3	(g)	b_1
bcs end_filter	(h)	
ldrb r5,[r1,r4]	(i)	
add r0,r0,r5	(j)	
add r4,r4,#1	(k)	b_2
b for_filter	(l)	

end_filter:

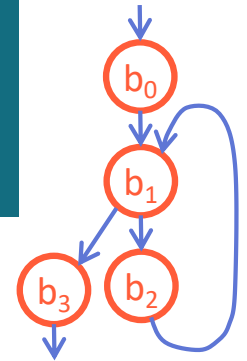
mov r0,r0,lsr r6	(m)	
ldmfd r13!,{r3-r6,r14}	(n)	b_3



Pipeline model



Execution cost of block b_1



```

fixFilter:
    stmfd    r13!,{r3-r6,r14}    (a)
    mov      r3,#1                (b)
    mov      r3,r3, lsl r0        (c)  $b_0$ 
    mov      r6,r0                (d)
    mov      r0,#0                (e)
    mov      r4,#0                (f)


---


for_filter:
    cmp      r4,r3                (g)  $b_1$ 
    bcs      end_filter           (h)


---


    ldrb     r5,[r1,r4]           (i)
    add      r0,r0,r5             (j)  $b_2$ 
    add      r4,r4,#1             (k)
    b        for_filter           (l)


---


end_filter:
    mov      r0,r0,lsr r6         (m)  $b_3$ 
    ldmdfd   r13!,{r3-r6,r14}     (n)
  
```

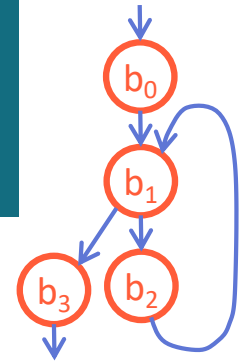
b_1 after b_0

F															
A															
Mu															
Me															
C															

b_1 after b_2

F															
A															
Mu															
Me															
C															

Execution cost of block b_1



```

fixFilter:
    stmfd    r13!,{r3-r6,r14}    (a)
    mov      r3,#1                (b)
    mov      r3,r3, lsl r0        (c)  $b_0$ 
    mov      r6,r0                (d)
    mov      r0,#0                (e)
    mov      r4,#0                (f)


---


for_filter:
    cmp      r4,r3                (g)  $b_1$ 
    bcs      end_filter           (h)


---


    ldrb     r5,[r1,r4]           (i)
    add      r0,r0,r5             (j)  $b_2$ 
    add      r4,r4,#1             (k)
    b        for_filter           (l)


---


end_filter:
    mov      r0,r0,lsr r6         (m)  $b_3$ 
    ldmbd    r13!,{r3-r6,r14}    (n)
  
```

b_1 after b_0

$t_0 = 13$ cycles

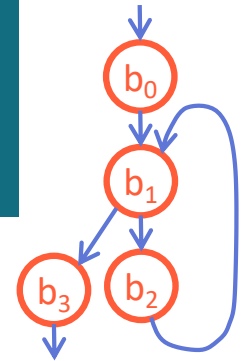
$t_{0-1} = 2$ cycles

F	a	b	c	d	e	f	g	h										
A			b	c	d	e	f	g	h									
Mu																		
Me		a	a	a	a	a	a											
C								a	b	c	d	e	f	g	h			

b_1 after b_2

F																		
A																		
Mu																		
Me																		
C																		

Execution cost of block b_1



```

fixFilter:
    stmfd    r13!,{r3-r6,r14}    (a)
    mov      r3,#1                (b)
    mov      r3,r3, lsl r0        (c)  $b_0$ 
    mov      r6,r0                (d)
    mov      r0,#0                (e)
    mov      r4,#0                (f)


---


for_filter:
    cmp      r4,r3                (g)  $b_1$ 
    bcs      end_filter           (h)


---


    ldrb     r5,[r1,r4]           (i)
    add      r0,r0,r5             (j)  $b_2$ 
    add      r4,r4,#1             (k)
    b        for_filter           (l)


---


end_filter:
    mov      r0,r0,lsr r6         (m)  $b_3$ 
    ldmdf    r13!,{r3-r6,r14}     (n)
  
```

b_1 after b_0

$t_0 = 13$ cycles

$t_{0-1} = 2$ cycles

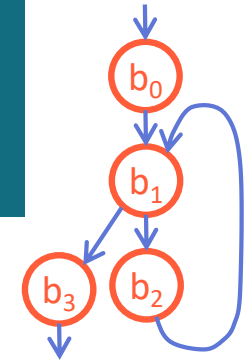
F	a	b	c	d	e	f	g	h										
A			b	c	d	e	f	g	h									
Mu																		
Me		a	a	a	a	a	a											
C								a	b	c	d	e	f	g	h			

b_1 after b_2

$t_{2-1} = 3$ cycles

F	i	j	k	l	m	n	g	h										
A				j	k	l		g	h									
Mu																		
Me		i	i															
C				i	j	k	l		g	h								

Execution cost of block b_2



```

fixFilter:
    stmfd    r13!,{r3-r6,r14}    (a)
    mov      r3,#1                (b)
    mov      r3,r3, lsl r0        (c)  $b_0$ 
    mov      r6,r0                (d)
    mov      r0,#0                (e)
    mov      r4,#0                (f)


---


for_filter:
    cmp      r4,r3                (g)  $b_1$ 
    bcs      end_filter           (h)


---


    ldrb     r5,[r1,r4]           (i)
    add      r0,r0,r5              (j)  $b_2$ 
    add      r4,r4,#1              (k)
    b        for_filter           (l)

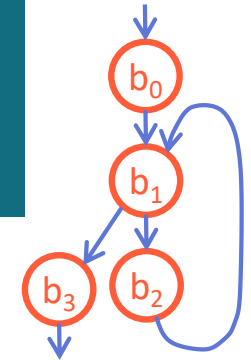

---


end_filter:
    mov      r0,r0,lsr r6          (m)  $b_3$ 
    ldmbd    r13!,{r3-r6,r14}     (n)
  
```

b_2 after b_1

F																		
A																		
Mu																		
Me																		
C																		

Execution cost of block b_2



```

fixFilter:
    stmfd    r13!,{r3-r6,r14}    (a)
    mov      r3,#1                (b)
    mov      r3,r3, lsl r0        (c)  $b_0$ 
    mov      r6,r0                (d)
    mov      r0,#0                (e)
    mov      r4,#0                (f)


---


for_filter:
    cmp      r4,r3                (g)  $b_1$ 
    bcs      end_filter           (h)


---


    ldrb     r5,[r1,r4]           (i)
    add      r0,r0,r5             (j)  $b_2$ 
    add      r4,r4,#1             (k)
    b        for_filter           (l)


---

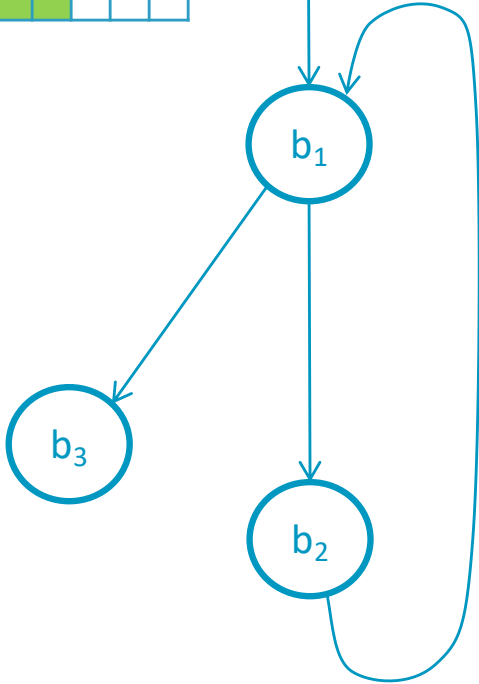
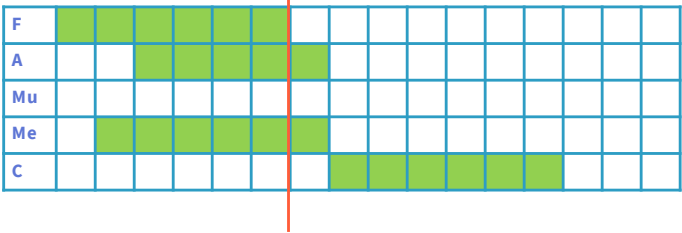

end_filter:
    mov      r0,r0,lsr r6         (m)  $b_3$ 
    ldmdfd   r13!,{r3-r6,r14}     (n)
  
```

b_2 after b_1

$t_{1-2} = 5$ cycles

F	g	h	i	j	k	l												
A		g	h															
Mu						j	k	l										
Me				i	i													
C			g	h		i	j	k	l									

Using abstract interpretation



```
fixFilter:
    stmfd    r13!,{r3-r6,r14}    (a)
    mov      r3,#1                 (b)
    mov      r3,r3, lsl r0        (c)
    mov      r6,r0                (d)
    mov      r0,#0                 (e)
    mov      r4,#0                 (f)
```

```
fixFilter:
    stmfd    r13!,{r3-r6,r14}    (a)
    mov      r3,#1                (b)
    mov      r3,r3, lsl r0        (c)
    mov      r6,r0                (d)
    mov      r0,#0                (e)
    mov      r4,#0                (f)
```

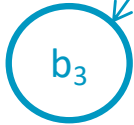
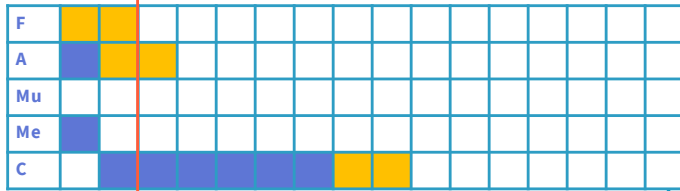
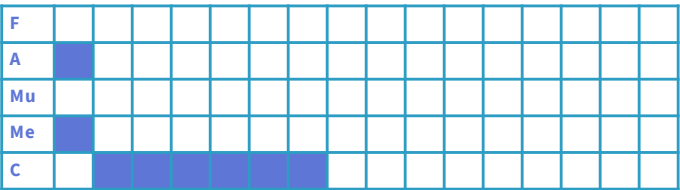
for_filter:		
cmp	r4,r3	(g)
bcs	end_filter	(h)
<hr/>		
ldrb	r5,[r1,r4]	(i)
add	r0,r0,r5	(j)
add	r4,r4,#1	(k)
b	for_filter	(l)

for_filter:			
cmp	r4,r3	(g)	b_1
bcs	end_filter	(h)	
ldrb	r5,[r1,r4]	(i)	b_2
add	r0,r0,r5	(j)	
add	r4,r4,#1	(k)	
b	for_filter	(l)	

```
end_filter:
    mov     r0,r0,lsr r6           (m)
    ldmfd   r13!,{r3-r6,r14}      (n)
```

```
end_filter:
    mov     r0,r0,lsr r6           (m)
    ldmdfd  r13!,{r3-r6,r14}      (n)
```

Using abstract interpretation



stmfd	r13!,{r3-r6,r14}	(a)	
mov	r3,#1	(b)	
mov	r3,r3, lsl r0	(c)	b_0
mov	r6,r0	(d)	
mov	r0,#0	(e)	
mov	r4,#0	(f)	

cmp	r4,r3	(g)	b ₁
bcs	end_filter	(h)	
ldrb	r5,[r1,r4]	(i)	b ₂
add	r0,r0,r5	(j)	
add	r4,r4,#1	(k)	
b	for_filter	(l)	

```
mov    r0,r0,lsr r6      (m)  b3
ldmfd  r13!,{r3-r6,r14}  (n)
```

Using abstract interpretation

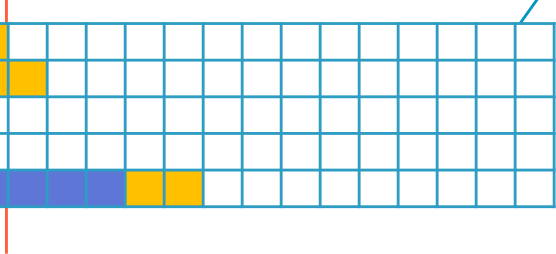
[illegible][illegible]

stmfd	r13!,{r3-r6,r14}	(a)	
mov	r3,#1	(b)	
mov	r3,r3, lsl r0	(c)	b ₀
mov	r6,r0	(d)	
mov	r0,#0	(e)	
mov	r4,#0	(f)	

cmp	r4,r3	(g)	b ₁
bcs	end_filter	(h)	
ldrb	r5,[r1,r4]	(i)	b ₂
add	r0,r0,r5	(j)	
add	r4,r4,#1	(k)	
b	for_filter	(l)	

```
mov    r0,r0,lsr r6      (m)  b3
ldmfd  r13!,{r3-r6,r14}  (n)
```

Using abstract interpretation

[illegible][illegible][illegible]

stmfd	r13!,{r3-r6,r14}	(a)
mov	r3,#1	(b)
mov	r3,r3, lsl r0	(c)
mov	r6,r0	(d)
mov	r0,#0	(e)
mov	r4,#0	(f)

cmp	r4,r3	(g)
bcs	end_filter	(h)
ldrb	r5,[r1,r4]	(i)
add	r0,r0,r5	(j)
add	r4,r4,#1	(k)
b	for_filter	(l)

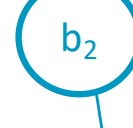
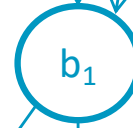
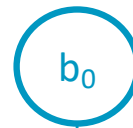
```
mov    r0,r0,lsr r6      (m)
ldmfd  r13!,{r3-r6,r14}  (n)
```

Using abstract interpretation

after b_0

[illegible]

after b_1

[illegible][illegible]

after b_2

[illegible]

fixFilter:

stmfd	r13!,{r3-r6,r14}	(a)	
mov	r3,#1	(b)	
mov	r3,r3, lsl r0	(c)	b_0
mov	r6,r0	(d)	
mov	r0,#0	(e)	
mov	r4,#0	(f)	

```
for_filter:
```

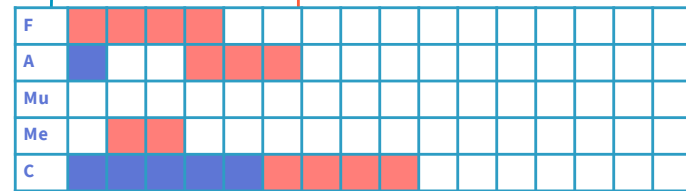
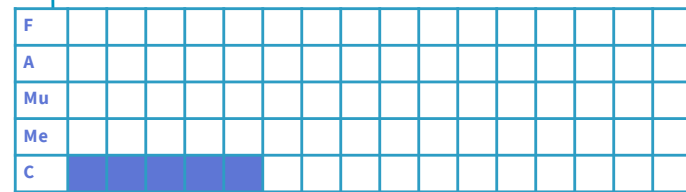
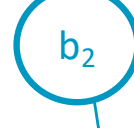
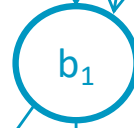
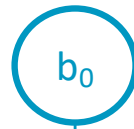
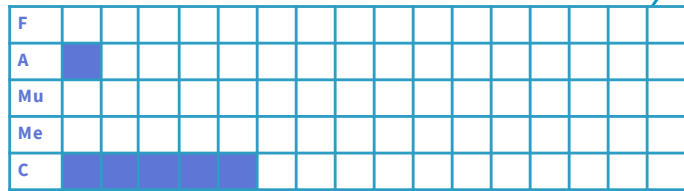
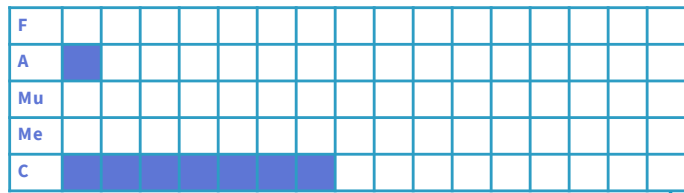
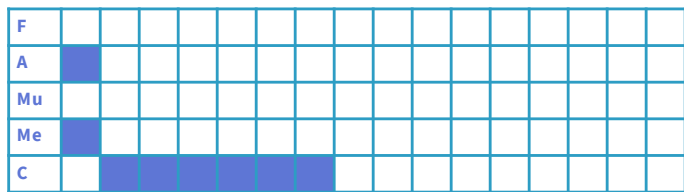
```
cmp    r4,r3          (g)  b1
bcs    end_filter     (h)
```

ldrb	r5,[r1,r4]	(i)	b ₂
add	r0,r0,r5	(j)	
add	r4,r4,#1	(k)	
b	for_filter	(l)	

end_filter:

```
mov    r0,r0,lsr r6      (m)  b3
ldmfd  r13!,{r3-r6,r14}  (n)
```

Using abstract interpretation



stmfd	r13!,{r3-r6,r14}	(a)	
mov	r3,#1	(b)	
mov	r3,r3, lsl r0	(c)	b ₀
mov	r6,r0	(d)	
mov	r0,#0	(e)	
mov	r4,#0	(f)	

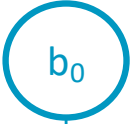
cmp	r4,r3	(g)	b ₁
bcs	end_filter	(h)	
ldrb	r5,[r1,r4]	(i)	b ₂
add	r0,r0,r5	(j)	
add	r4,r4,#1	(k)	
b	for_filter	(l)	

```

mov     r0,r0,lsr r6      (m)  b3
ldmfd   r13!,{r3-r6,r14} (n)

```


Using abstract interpretation

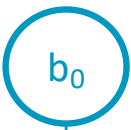
[illegible][illegible][illegible][illegible][illegible][illegible]

stmfd	r13!,{r3-r6,r14}	(a)	
mov	r3,#1	(b)	
mov	r3,r3, lsl r0	(c)	b_0
mov	r6,r0	(d)	
mov	r0,#0	(e)	
mov	r4,#0	(f)	

cmp	r4,r3	(g)	b ₁
bcs	end_filter	(h)	
ldrb	r5,[r1,r4]	(i)	b ₂
add	r0,r0,r5	(j)	
add	r4,r4,#1	(k)	
b	for_filter	(l)	

```
mov     r0,r0,lsr r6      (m)  b3
ldmfd   r13!,{r3-r6,r14} (n)
```

Using abstract interpretation

[illegible][illegible][illegible][illegible][illegible][illegible][illegible]

stmfd	r13!, {r3-r6, r14}	(a)
mov	r3, #1	(b)
mov	r3, r3, lsl r0	(c)
mov	r6, r0	(d)
mov	r0, #0	(e)
mov	r4, #0	(f)

cmp	r4,r3	(g)
bcs	end_filter	(h)
ldrb	r5,[r1,r4]	(i)
add	r0,r0,r5	(j)
add	r4,r4,#1	(k)
b	for_filter	(l)

```
mov    r0,r0,lsr r6      (m)
ldmfd  r13!,{r3-r6,r14}  (n)
```

18

Using abstract interpretation

[illegible][illegible][illegible][illegible][illegible][illegible][illegible][illegible]

stmfd	r13!,{r3-r6,r14}	(a)	
mov	r3,#1	(b)	
mov	r3,r3, lsl r0	(c)	b_0
mov	r6,r0	(d)	
mov	r0,#0	(e)	
mov	r4,#0	(f)	

cmp	r4,r3	(g)	b ₁
bcs	end_filter	(h)	
ldrb	r5,[r1,r4]	(i)	b ₂
add	r0,r0,r5	(j)	
add	r4,r4,#1	(k)	
b	for_filter	(l)	

```
mov     r0,r0,lsr r6      (m)  b3
ldmfd   r13!,{r3-r6,r14} (n)
```

Using abstract interpretation

[illegible][illegible][illegible][illegible][illegible][illegible][illegible][illegible][illegible]

stmfd	r13!, {r3-r6, r14}	(a)	
mov	r3, #1	(b)	
mov	r3, r3, lsl r0	(c)	b_0
mov	r6, r0	(d)	
mov	r0, #0	(e)	
mov	r4, #0	(f)	

cmp	r4,r3	(g)	b ₁
bcs	end_filter	(h)	
ldrb	r5,[r1,r4]	(i)	b ₂
add	r0,r0,r5	(j)	
add	r4,r4,#1	(k)	
b	for_filter	(l)	

```
mov     r0,r0,lsr r6      (m)  b3
ldmfd   r13!,{r3-r6,r14} (n)
```

Using abstract interpretation

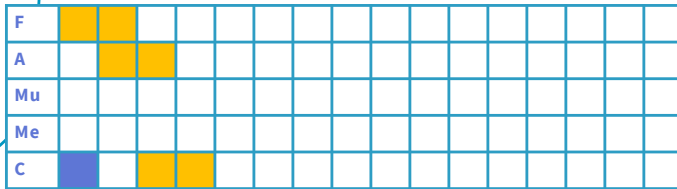
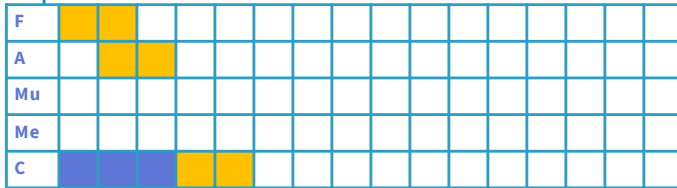
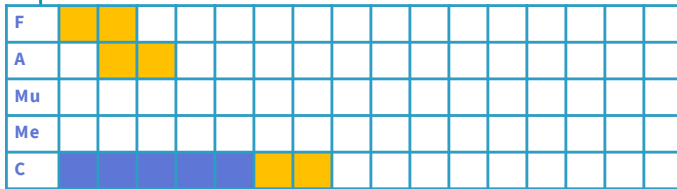
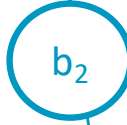
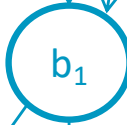
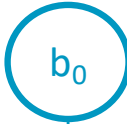
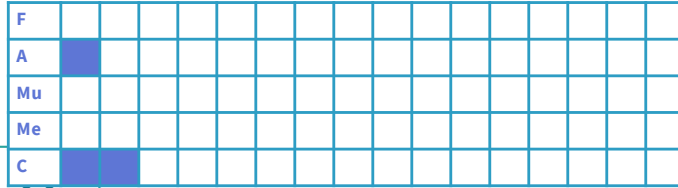
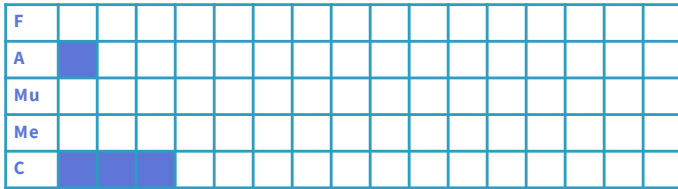
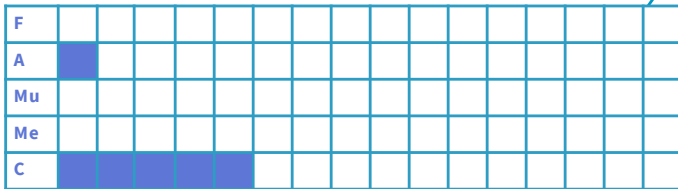
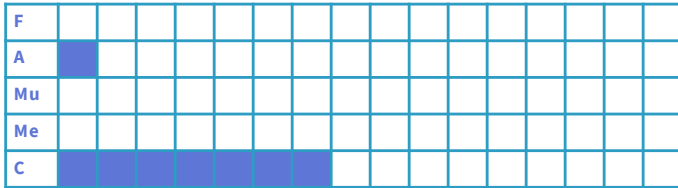
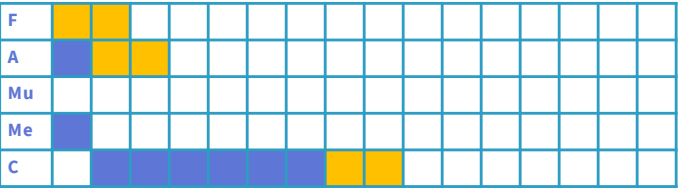
[illegible][illegible][illegible][illegible][illegible][illegible][illegible][illegible]

stmfd	r13!,{r3-r6,r14}	(a)	
mov	r3,#1	(b)	
mov	r3,r3, lsl r0	(c)	b_0
mov	r6,r0	(d)	
mov	r0,#0	(e)	
mov	r4,#0	(f)	

cmp	r4,r3	(g)	b ₁
bcs	end_filter	(h)	
ldrb	r5,[r1,r4]	(i)	b ₂
add	r0,r0,r5	(j)	
add	r4,r4,#1	(k)	
b	for_filter	(l)	

```
mov    r0,r0,lsr r6      (m)  b3
ldmfd  r13!,{r3-r6,r14}  (n)
```

Using abstract interpretation



fixFilter:

stmfd	r13!, {r3-r6, r14}	(a)	
mov	r3, #1	(b)	
mov	r3, r3, lsl r0	(c)	b_0
mov	r6, r0	(d)	
mov	r0, #0	(e)	
mov	r4, #0	(f)	

```
for_filter:
```

```
cmp    r4,r3          (g)  b1
bcs    end_filter     (h)
```

ldrb	r5,[r1,r4]	(i)	b ₂
add	r0,r0,r5	(j)	
add	r4,r4,#1	(k)	
b	for_filter	(l)	

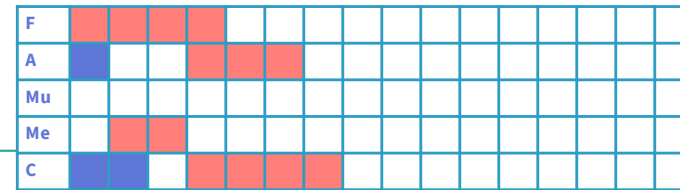
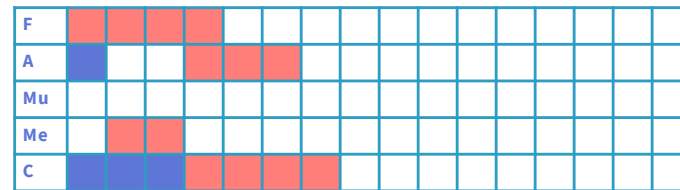
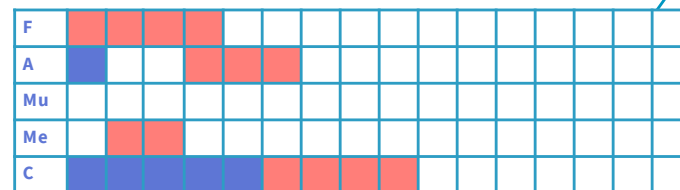
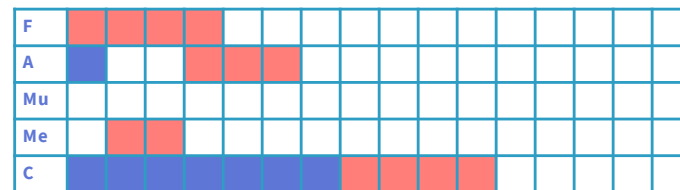
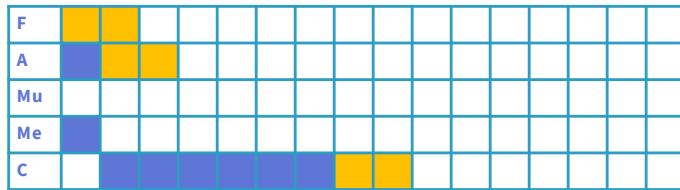
end_filter:

```
mov     r0,r0,lsr r6      (m)  b3
ldmfd   r13!,{r3-r6,r14} (n)
```

$t_{0-1} = 2$ cycles

$$t_{2-1} \leq 3 \text{ cycles}$$

Using abstract interpretation

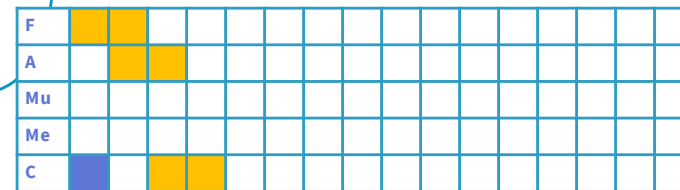
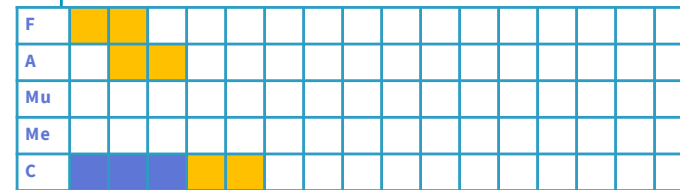
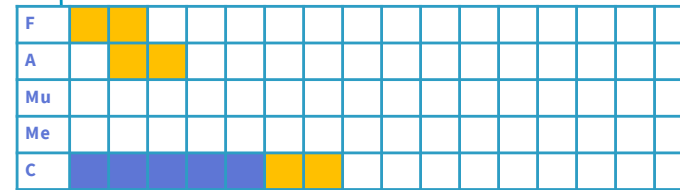


b_0

b_1

b_2

$t_{1-2} \leq 5$ cycles



fixFilter:

stmfd	r13!,{r3-r6,r14}	(a)	
mov	r3,#1	(b)	
mov	r3,r3, lsl r0	(c)	b_0
mov	r6,r0	(d)	
mov	r0,#0	(e)	
mov	r4,#0	(f)	

for_filter:

cmp	r4,r3	(g)	b_1
bcs	end_filter	(h)	
ldrb	r5,[r1,r4]	(i)	
add	r0,r0,r5	(j)	b_2
add	r4,r4,#1	(k)	
b	for_filter	(l)	

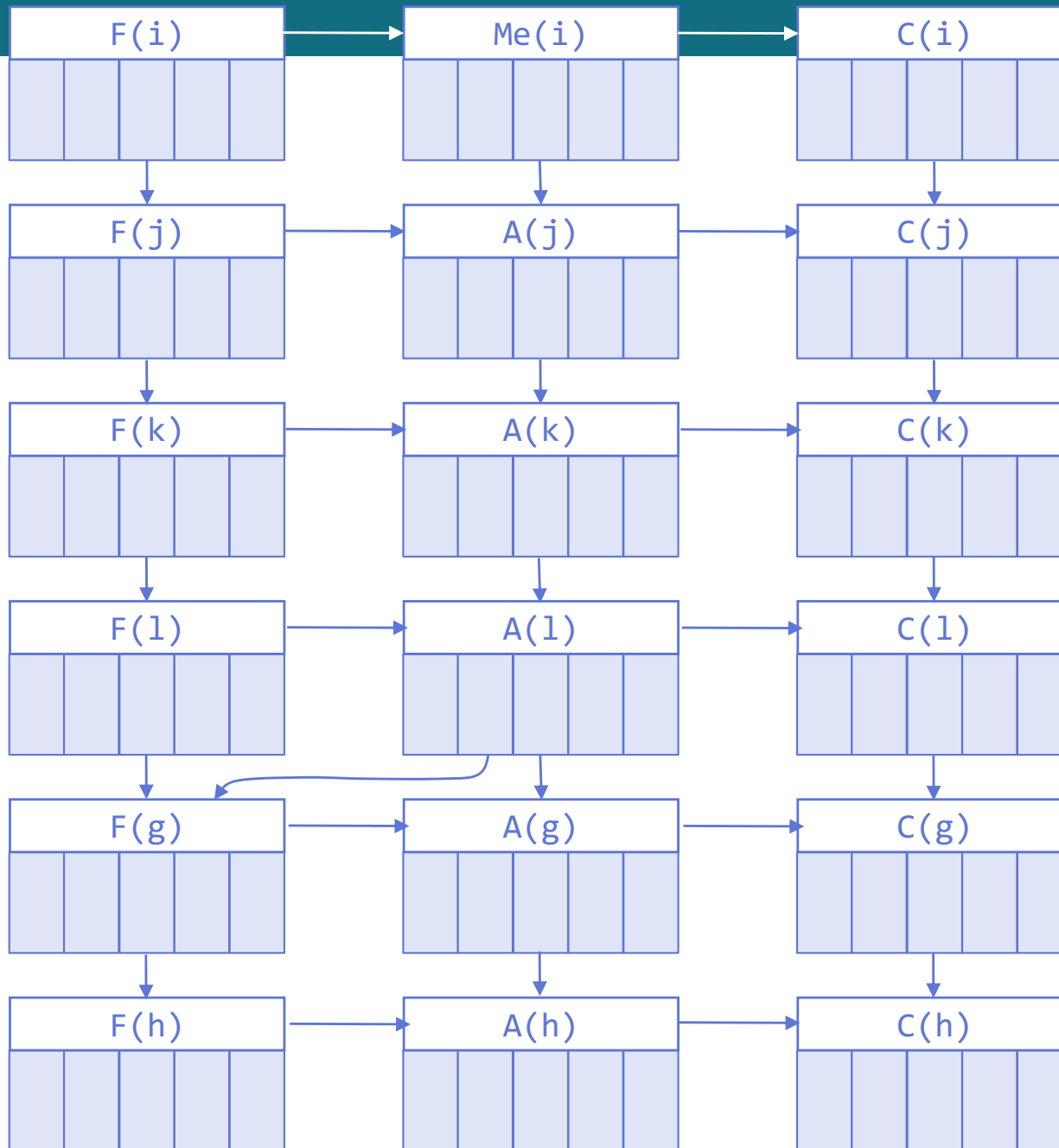
end_filter:

mov	r0,r0,lsr r6	(m)	b_3
ldmfd	r13!,{r3-r6,r14}	(n)	

$t_{0-1} = 2$ cycles

$t_{2-1} \leq 3$ cycles

Using execution graphs

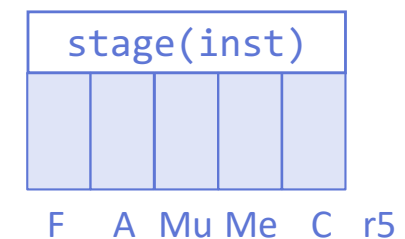


```

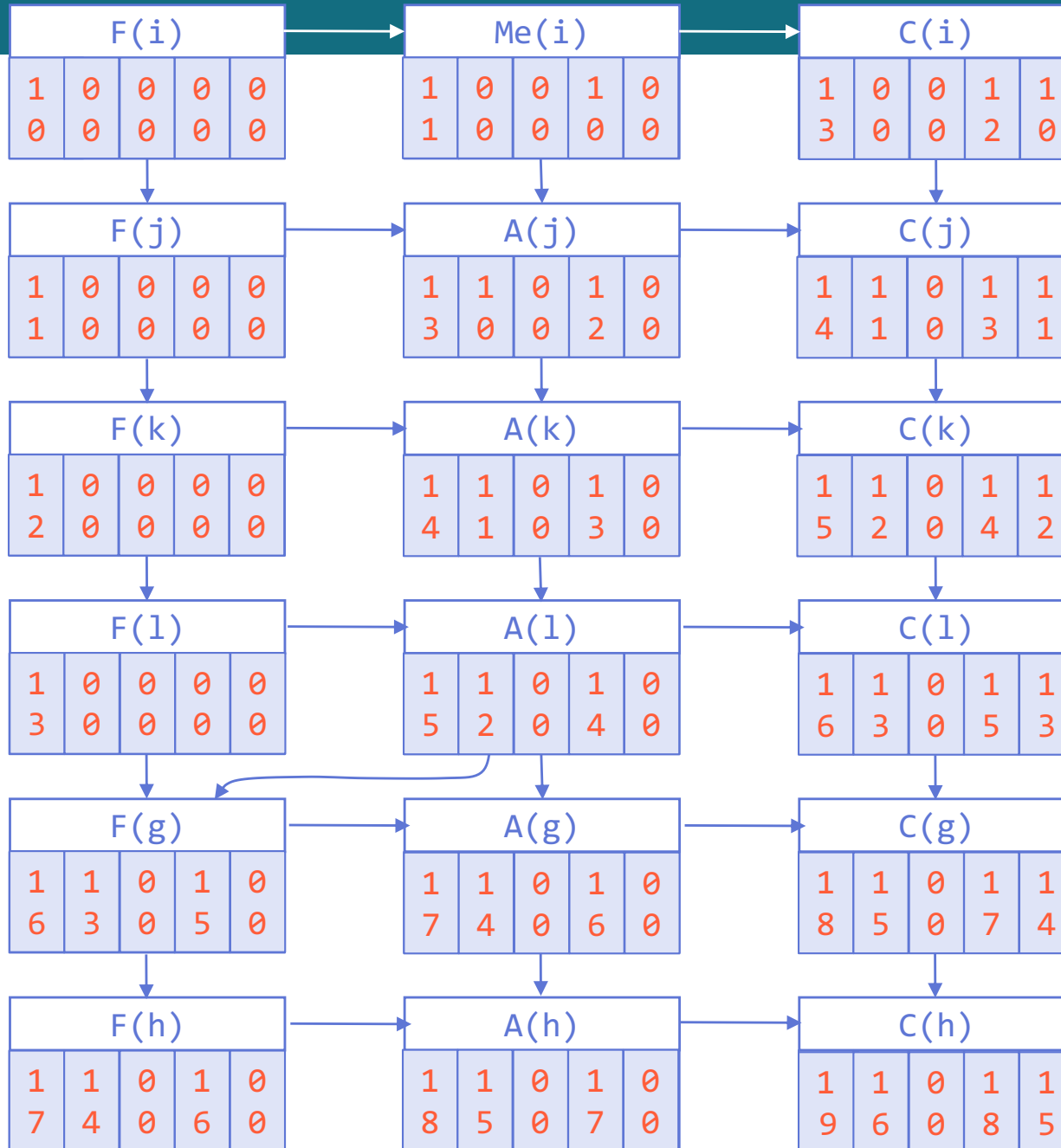
fixFilter:
    stmfd    r13!,{r3-r6,r14}    (a)
    mov      r3,#1                (b)
    mov      r3,r3, lsl r0        (c) b0
    mov      r6,r0                (d)
    mov      r0,#0                (e)
    mov      r4,#0                (f)
for_filter:
    cmp      r4,r3                (g) b1
    bcs      end_filter           (h)
    ldrb      r5,[r1,r4]          (i)
    add      r0,r0,r5              (j) b2
    add      r4,r4,#1              (k)
    b        for_filter           (l)
end_filter:
    mov      r0,r0,lsr r6         (m) b3
    ldmfid   r13!,{r3-r6,r14}    (n)
    
```

Resources:

pipeline stages: F, A, Mu, Me, C
 registers: r5



Using execution graphs

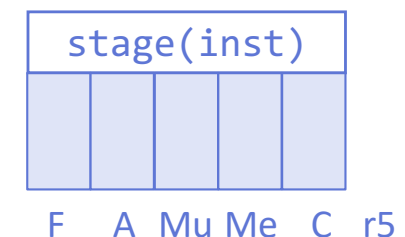


```

fixFilter:
    stmfd    r13!,{r3-r6,r14}    (a)
    mov      r3,#1                (b)
    mov      r3,r3, lsl r0        (c)
    mov      r6,r0                (d)
    mov      r0,#0                (e)
    mov      r4,#0                (f)
for_filter:
    cmp      r4,r3                (g)
    bcs      end_filter           (h)
    ldrb     r5,[r1,r4]           (i)
    add      r0,r0,r5             (j)
    add      r4,r4,#1             (k)
    b        for_filter           (l)
end_filter:
    mov      r0,r0,lsr r6         (m)
    ldmdf    r13!,{r3-r6,r14}    (n)
    
```

Resources:

pipeline stages: F, A, Mu, Me, C
registers: r5



Using execution graphs



C(l)				
1	1	0	1	1
6	3	0	5	3

$$t_{C(l)} = \max(t_F+6 , t_A+3 , t_{Me}+5 , t_C+3)$$

C(h)				
1	1	0	1	1
9	6	0	8	5

$$t_{C(h)} = \max(t_F+9 , t_A+6 , t_{Me}+8 , t_C+5)$$

F	A	Mu	Me	C

$$t_{C(h)} - t_{C(l)} = \max(9-6 , 6-3 , 8-5 , 5-3)$$

$$t_{2-1} \leq 3 \text{ cycles}$$

ILP formulation



$$\max T = 13 x_0 + 2 x_{0-1} + 5 x_{1-2} + 3 x_{2-1} + 8 x_{1-3}$$

$$x_0 = 1$$

$$x_0 = x_{0-1}$$

$$x_1 = x_{0-1} + x_{2-1}$$

$$x_1 = x_{1-2} + x_{1-3}$$

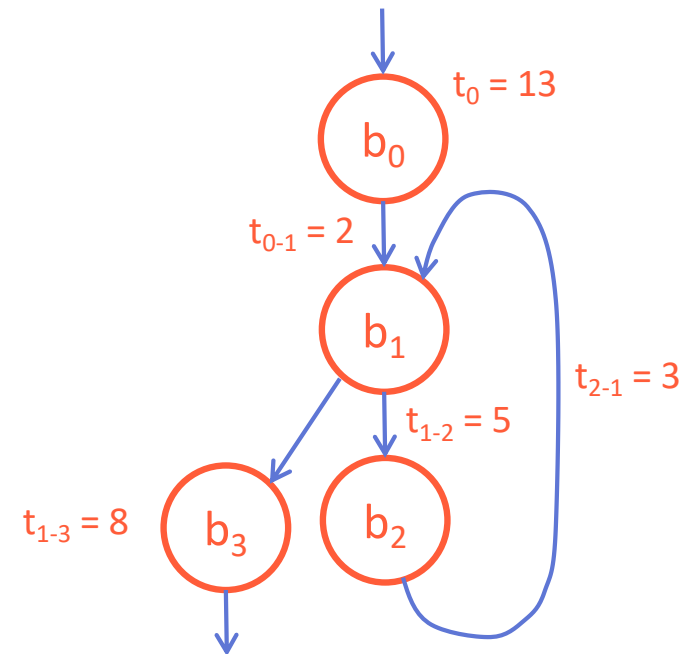
$$x_2 = x_{1-2}$$

$$x_2 = x_{2-1}$$

$$x_3 = x_{1-3}$$

$$x_3 = 1$$

$$x_{2-1} \leq 256$$



Instruction cache analysis



Assumptions:

- 32-byte 2-way set associative cache
- 8-byte cache lines (memory blocks)
- 4-byte instruction codes
- function stored at address 0

fixFilter:

$L_0 \rightarrow S_0$	stmfd	r13!, {r3-r6, r14}	(a)	} b_0
	mov	r3, #1	(b)	
$L_1 \rightarrow S_1$	mov	r3, r3, lsl r0	(c)	
	mov	r6, r0	(d)	
$L_2 \rightarrow S_0$	mov	r0, #0	(e)	
	mov	r4, #0	(f)	

for_filter:

$L_3 \rightarrow S_1$	cmp	r4, r3	(g)	} b_1
	bcs	end_filter	(h)	
$L_4 \rightarrow S_0$	ldrb	r5, [r1, r4]	(i)	} b_2
	add	r0, r0, r5	(j)	
$L_5 \rightarrow S_1$	add	r4, r4, #1	(k)	
	b	for_filter	(l)	

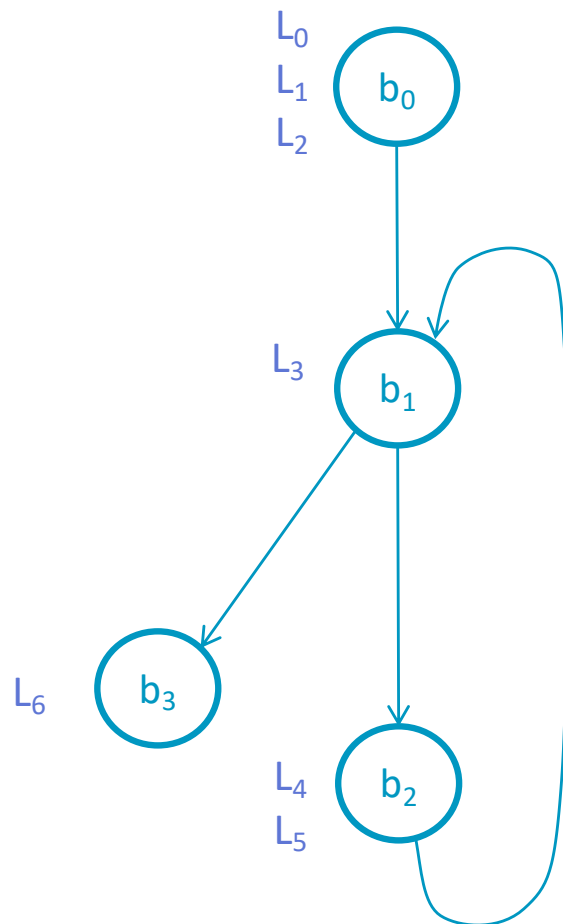
end_filter:

$L_6 \rightarrow S_0$	mov	r0, r0, lsr r6	(m)	} b_3
	ldmfd	r13!, {r3-r6, r14}	(n)	

Cache analysis by Abstract Interpretation



MUST analysis

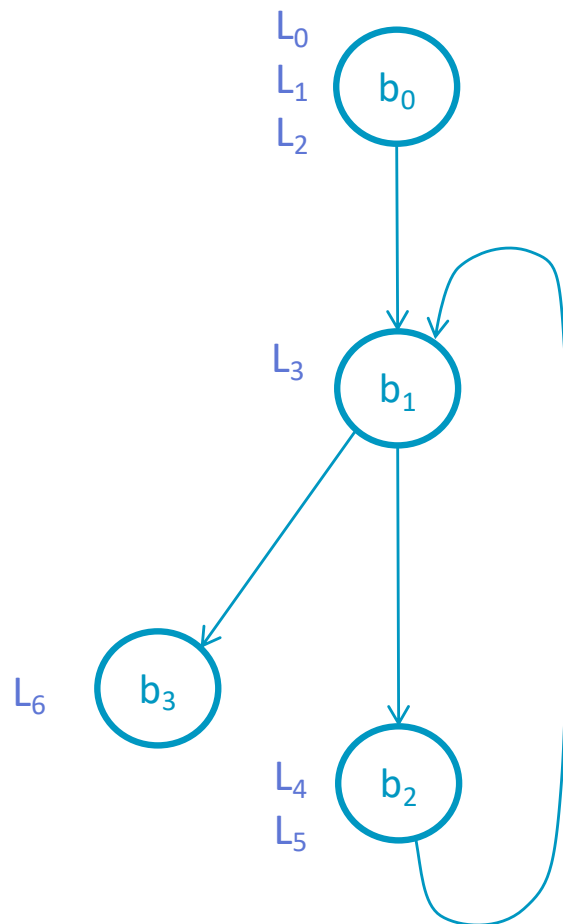


	<i>before</i>		<i>after</i>		
	set 0	set 1	set 0	set 1	
b_0	<div></div>	<div></div>	<div></div>	<div></div>	age 0 age 1
b_1	<div></div>	<div></div>	<div></div>	<div></div>	age 0 age 1
b_2	<div></div>	<div></div>	<div></div>	<div></div>	age 0 age 1
b_3	<div></div>	<div></div>	<div></div>	<div></div>	age 0 age 1

Cache analysis by Abstract Interpretation



MUST analysis

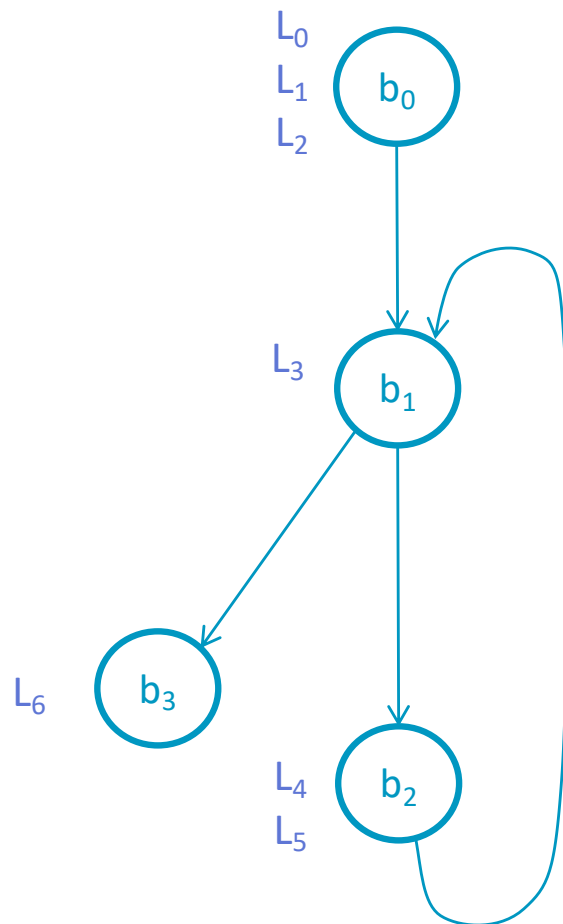


	<i>before</i>		<i>after</i>		
	set 0	set 1	set 0	set 1	
b_0	<div></div>	<div></div>	<div>L_2</div>	<div>L_1</div>	age 0
	<div></div>	<div></div>	<div>L_0</div>	<div></div>	age 1
b_1	<div></div>	<div></div>	<div></div>	<div></div>	age 0
	<div></div>	<div></div>	<div></div>	<div></div>	age 1
b_2	<div></div>	<div></div>	<div></div>	<div></div>	age 0
	<div></div>	<div></div>	<div></div>	<div></div>	age 1
b_3	<div></div>	<div></div>	<div></div>	<div></div>	age 0
	<div></div>	<div></div>	<div></div>	<div></div>	age 1

Cache analysis by Abstract Interpretation



MUST analysis

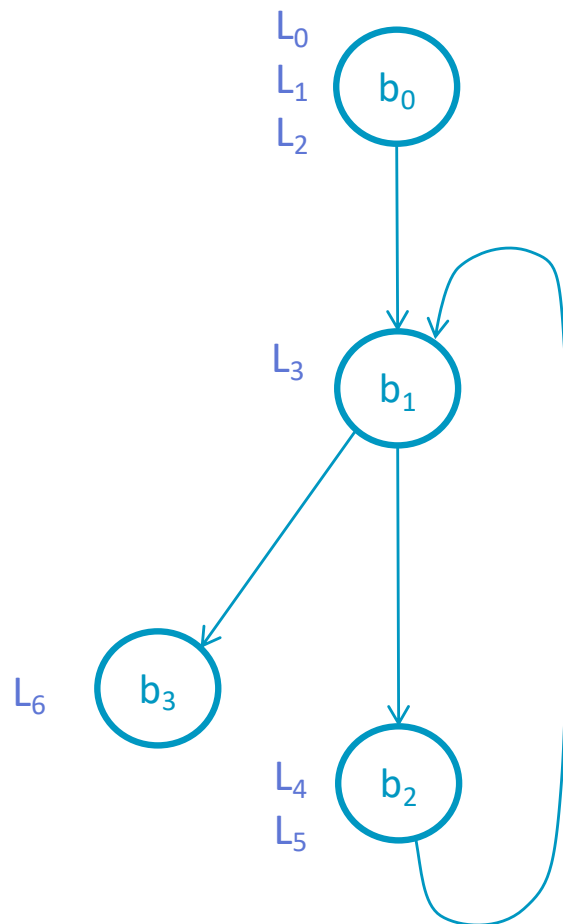


	<i>before</i>		<i>after</i>		
	set 0	set 1	set 0	set 1	
b_0	<div></div>	<div></div>	<div>L_2</div>	<div>L_1</div>	age 0
	<div></div>	<div></div>	<div>L_0</div>	<div></div>	age 1
b_1	<div>L_2</div>	<div>L_1</div>	<div>L_2</div>	<div>L_3</div>	age 0
	<div>L_0</div>	<div></div>	<div>L_0</div>	<div>L_1</div>	age 1
b_2	<div></div>	<div></div>	<div></div>	<div></div>	age 0
	<div></div>	<div></div>	<div></div>	<div></div>	age 1
b_3	<div></div>	<div></div>	<div></div>	<div></div>	age 0
	<div></div>	<div></div>	<div></div>	<div></div>	age 1

Cache analysis by Abstract Interpretation



MUST analysis

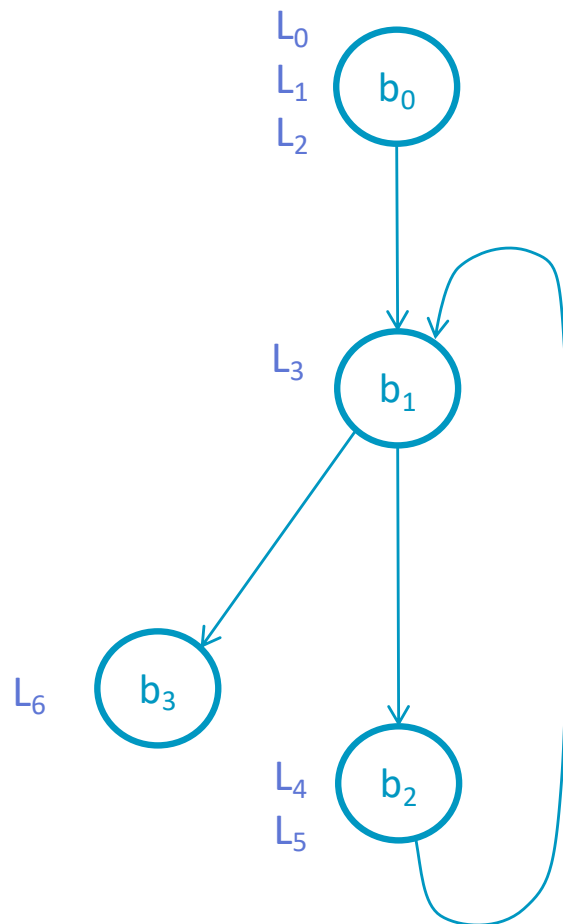


	<i>before</i>		<i>after</i>		
	set 0	set 1	set 0	set 1	
b_0	<div></div>	<div></div>	<div>L_2</div>	<div>L_1</div>	age 0
	<div></div>	<div></div>	<div>L_0</div>	<div></div>	age 1
b_1	<div>L_2</div>	<div>L_1</div>	<div>L_2</div>	<div>L_3</div>	age 0
	<div>L_0</div>	<div></div>	<div>L_0</div>	<div>L_1</div>	age 1
b_2	<div>L_2</div>	<div>L_3</div>	<div>L_4</div>	<div>L_5</div>	age 0
	<div>L_0</div>	<div>L_1</div>	<div>L_2</div>	<div>L_3</div>	age 1
b_3	<div></div>	<div></div>	<div></div>	<div></div>	age 0
	<div></div>	<div></div>	<div></div>	<div></div>	age 1

Cache analysis by Abstract Interpretation



MUST analysis

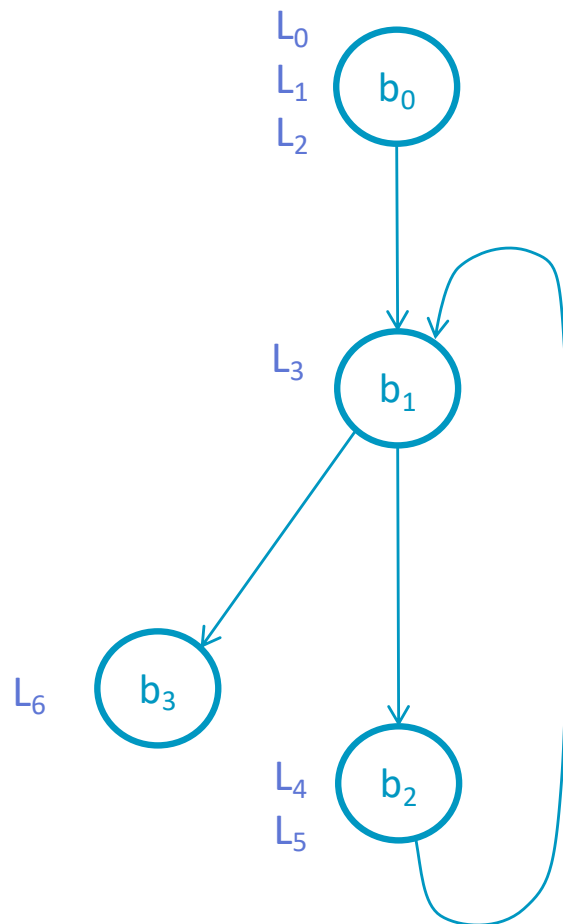


	<i>before</i>		<i>after</i>		
	set 0	set 1	set 0	set 1	
b_0	<div></div>	<div></div>	<div>L_2</div>	<div>L_1</div>	age 0
	<div></div>	<div></div>	<div>L_0</div>	<div></div>	age 1
b_1	<div>L_2</div>	<div>L_1</div>	<div>L_2</div>	<div>L_3</div>	age 0
	<div>L_0</div>	<div></div>	<div>L_0</div>	<div>L_1</div>	age 1
b_2	<div>L_2</div>	<div>L_3</div>	<div>L_4</div>	<div>L_5</div>	age 0
	<div>L_0</div>	<div>L_1</div>	<div>L_2</div>	<div>L_3</div>	age 1
b_3	<div>L_2</div>	<div>L_3</div>	<div>L_6</div>	<div>L_3</div>	age 0
	<div>L_0</div>	<div>L_1</div>	<div>L_2</div>	<div>L_1</div>	age 1

Cache analysis by Abstract Interpretation



MUST analysis

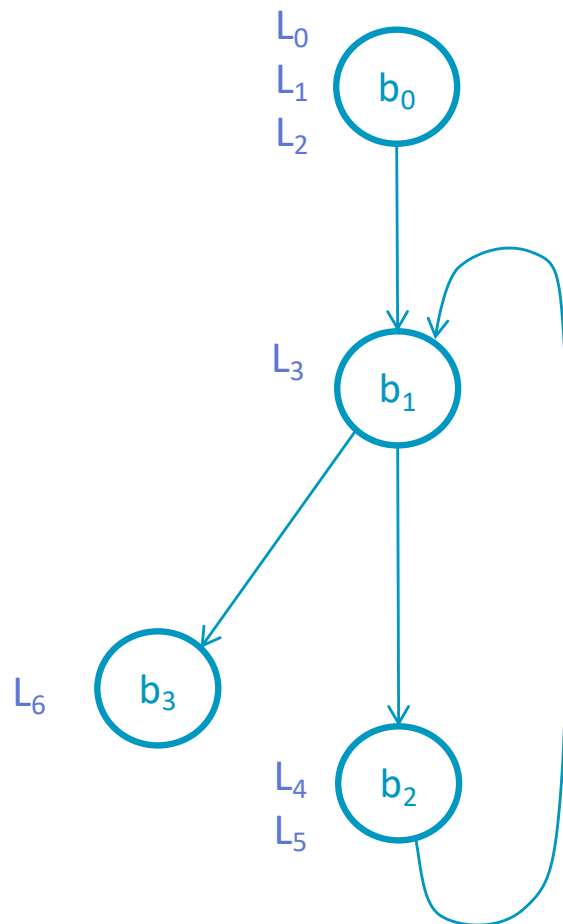


	<i>before</i>		<i>after</i>		
	set 0	set 1	set 0	set 1	
b_0	<div></div>	<div></div>	<div>L_2</div>	<div>L_1</div>	age 0
	<div></div>	<div></div>	<div>L_0</div>	<div></div>	age 1
b_1	<div></div>	<div></div>	<div></div>	<div>L_3</div>	age 0
	<div>L_2</div>	<div></div>	<div>L_2</div>	<div></div>	age 1
b_2	<div>L_2</div>	<div>L_3</div>	<div>L_4</div>	<div>L_5</div>	age 0
	<div>L_0</div>	<div>L_1</div>	<div>L_2</div>	<div>L_3</div>	age 1
b_3	<div>L_2</div>	<div>L_3</div>	<div>L_6</div>	<div>L_3</div>	age 0
	<div>L_0</div>	<div>L_1</div>	<div>L_2</div>	<div>L_1</div>	age 1

Cache analysis by Abstract Interpretation



MUST analysis

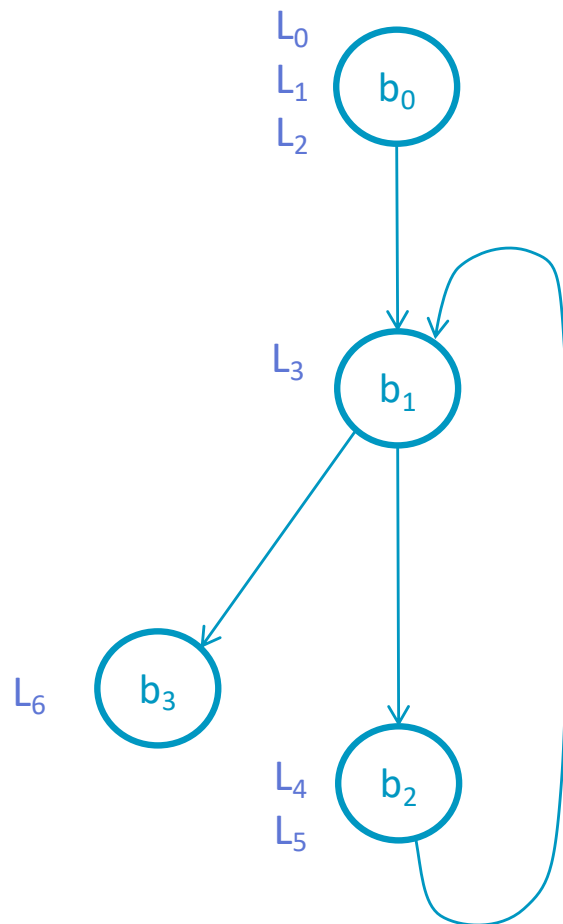


	<i>before</i>		<i>after</i>		
	set 0	set 1	set 0	set 1	
b_0	<div></div>	<div></div>	<div>L_2</div>	<div>L_1</div>	age 0
	<div></div>	<div></div>	<div>L_0</div>	<div></div>	age 1
b_1	<div></div>	<div></div>	<div></div>	<div>L_3</div>	age 0
	<div>L_2</div>	<div></div>	<div>L_2</div>	<div></div>	age 1
b_2	<div></div>	<div>L_3</div>	<div>L_4</div>	<div>L_5</div>	age 0
	<div>L_2</div>	<div></div>	<div></div>	<div>L_3</div>	age 1
b_3	<div>L_2</div>	<div>L_3</div>	<div>L_6</div>	<div>L_3</div>	age 0
	<div>L_0</div>	<div>L_1</div>	<div>L_2</div>	<div>L_1</div>	age 1

Cache analysis by Abstract Interpretation



MUST analysis

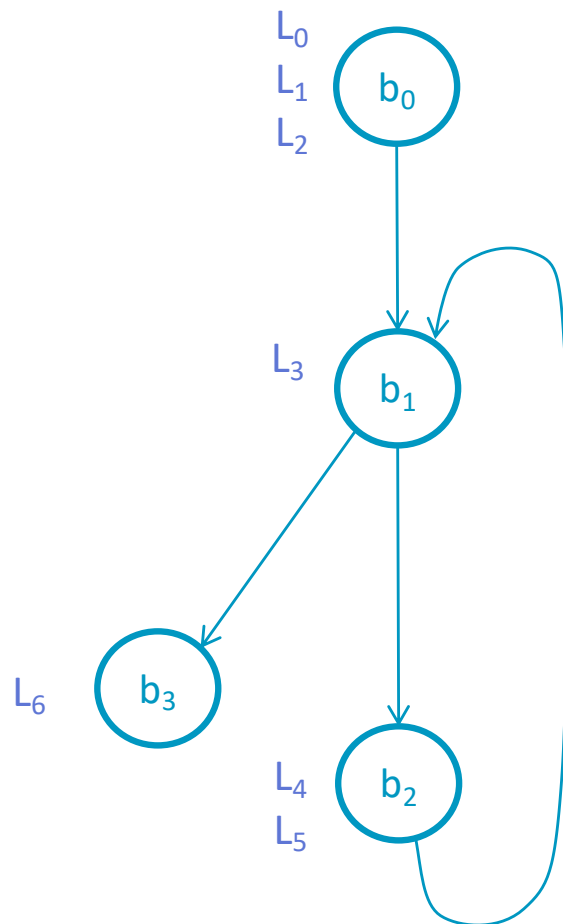


	before		after		
	set 0	set 1	set 0	set 1	
b_0	<div></div>	<div></div>	<div>L_2</div>	<div>L_1</div>	age 0 age 1
b_1	<div><div></div><div>L_2</div></div>	<div></div>	<div><div></div><div>L_2</div></div>	<div>L_3</div>	age 0 age 1
b_2	<div><div></div><div>L_2</div></div>	<div>L_3</div>	<div>L_4</div>	<div>L_5</div>	age 0 age 1
b_3	<div><div></div><div>L_2</div></div>	<div>L_3</div>	<div>L_6</div>	<div>L_3</div>	age 0 age 1

Cache analysis by Abstract Interpretation



MUST analysis

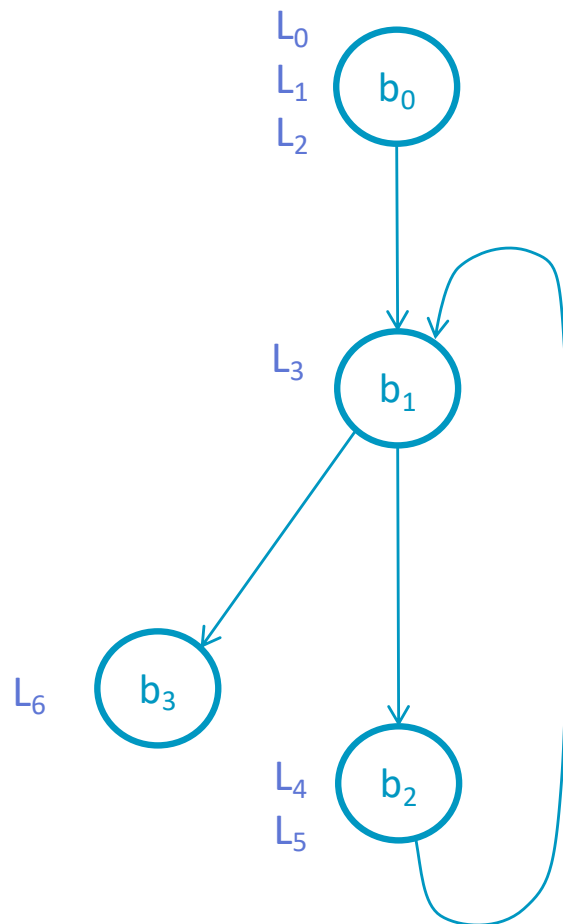


	before		after		
	set 0	set 1	set 0	set 1	
b_0	<div></div>	<div></div>	<div>L_2</div>	<div>L_1</div>	age 0 age 1
b_1	<div></div>	<div></div>	<div></div>	<div>L_3</div>	age 0 age 1
b_2	<div></div>	<div>L_3</div>	<div>L_4</div>	<div>L_5</div>	age 0 age 1
b_3	<div></div>	<div>L_3</div>	<div>L_6</div>	<div>L_3</div>	age 0 age 1

Cache analysis by Abstract Interpretation



MUST analysis



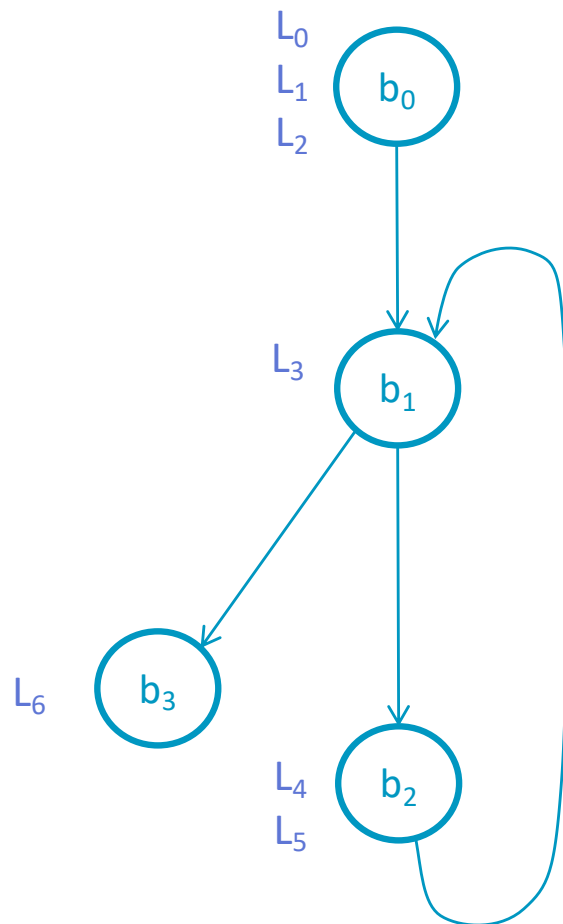
	<i>before</i>		<i>after</i>		
	set 0	set 1	set 0	set 1	
b_0	<div></div>	<div></div>	<div>L_2</div>	<div>L_1</div>	age 0
	<div></div>	<div></div>	<div>L_0</div>	<div></div>	age 1
b_1	<div></div>	<div></div>	<div></div>	<div>L_3</div>	age 0
	<div></div>	<div></div>	<div></div>	<div></div>	age 1
b_2	<div></div>	<div>L_3</div>	<div>L_4</div>	<div>L_5</div>	age 0
	<div></div>	<div></div>	<div></div>	<div>L_3</div>	age 1
b_3	<div></div>	<div>L_3</div>	<div>L_6</div>	<div>L_3</div>	age 0
	<div>L_2</div>	<div></div>	<div></div>	<div></div>	age 1

Cache analysis by Abstract Interpretation



MUST analysis

FIX POINT



before

after

b_0

set 0	set 1

set 0	set 1	
L_2	L_1	age 0
L_0		age 1

b_1

	L_3	age 0
		age 1

b_2

	L_3

L_4	L_5	age 0
	L_3	age 1

b_3

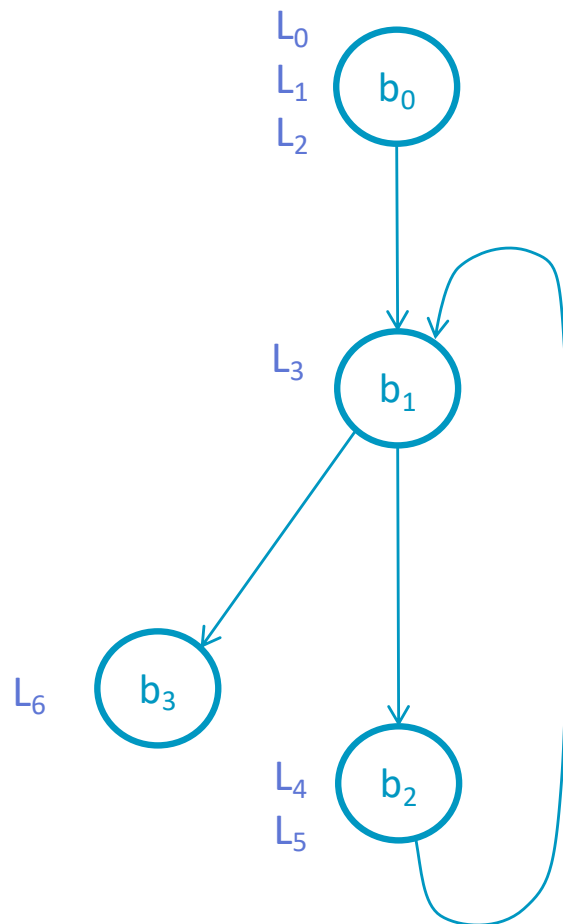
	L_3

L_6	L_3	age 0
		age 1

Cache analysis by Abstract Interpretation



MAY analysis

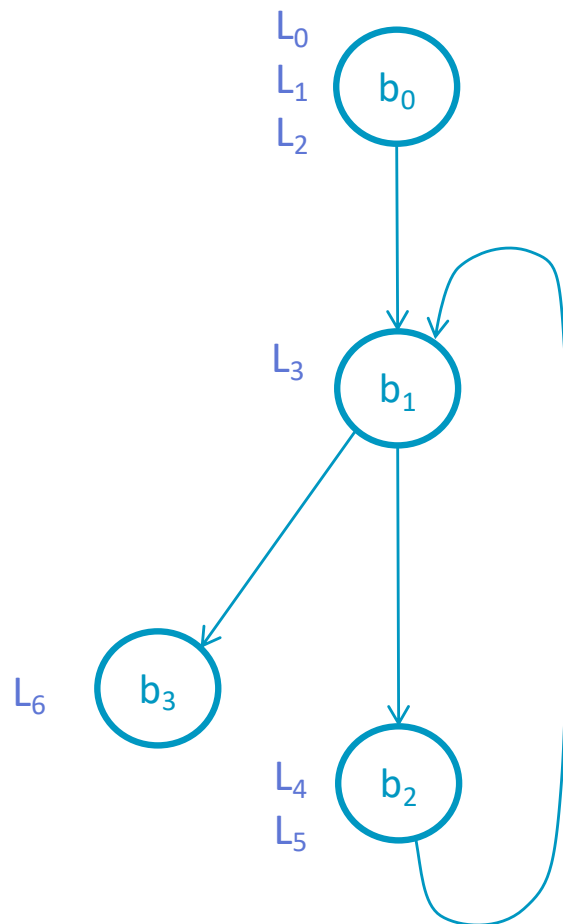


	before		after		
	set 0	set 1	set 0	set 1	
b_0	<div></div>	<div></div>	<div>L_2</div> <div>L_0</div>	<div>L_1</div> <div></div>	age 0 age 1
b_1	<div>L_2</div> <div>L_0</div>	<div>L_1</div> <div></div>	<div>L_2</div> <div>L_0</div>	<div>L_3</div> <div>L_1</div>	age 0 age 1
b_2	<div>L_2</div> <div>L_0</div>	<div>L_3</div> <div>L_1</div>	<div>L_4</div> <div>L_2</div>	<div>L_5</div> <div>L_3</div>	age 0 age 1
b_3	<div>L_2</div> <div>L_0</div>	<div>L_3</div> <div>L_1</div>	<div>L_6</div> <div>L_2</div>	<div>L_3</div> <div>L_1</div>	age 0 age 1

Cache analysis by Abstract Interpretation



MAY analysis

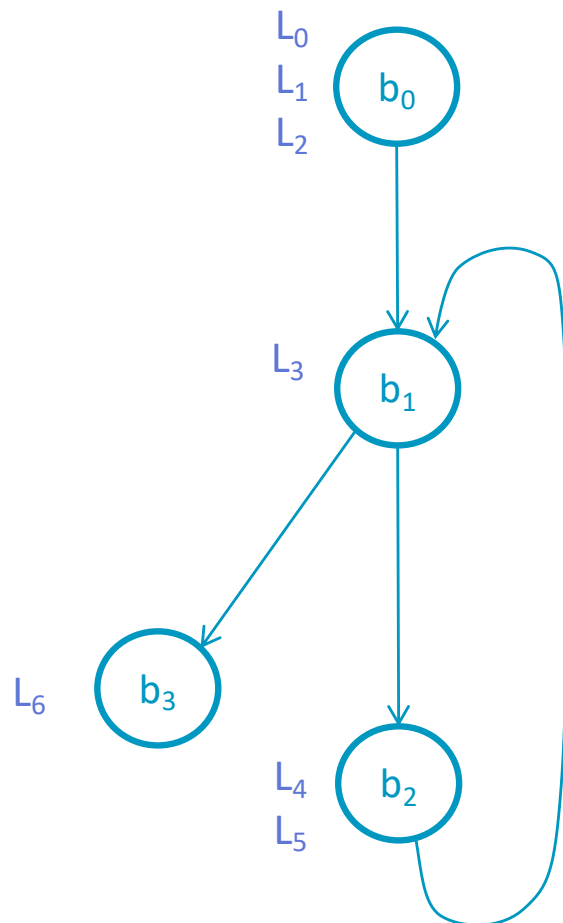


	<i>before</i>		<i>after</i>		
	set 0	set 1	set 0	set 1	
b_0	<div></div>	<div></div>	<div>L_2</div>	<div>L_1</div>	age 0
	<div></div>	<div></div>	<div>L_0</div>	<div></div>	age 1
b_1	<div>$L_{2,4}$</div>	<div>$L_{1,5}$</div>	<div>$L_{2,4}$</div>	<div>L_3</div>	age 0
	<div>L_0</div>	<div>L_3</div>	<div>L_0</div>	<div>$L_{1,5}$</div>	age 1
b_2	<div>L_2</div>	<div>L_3</div>	<div>L_4</div>	<div>L_5</div>	age 0
	<div>L_0</div>	<div>L_1</div>	<div>L_2</div>	<div>L_3</div>	age 1
b_3	<div>L_2</div>	<div>L_3</div>	<div>L_6</div>	<div>L_3</div>	age 0
	<div>L_0</div>	<div>L_1</div>	<div>L_2</div>	<div>L_1</div>	age 1

Cache analysis by Abstract Interpretation



MAY analysis

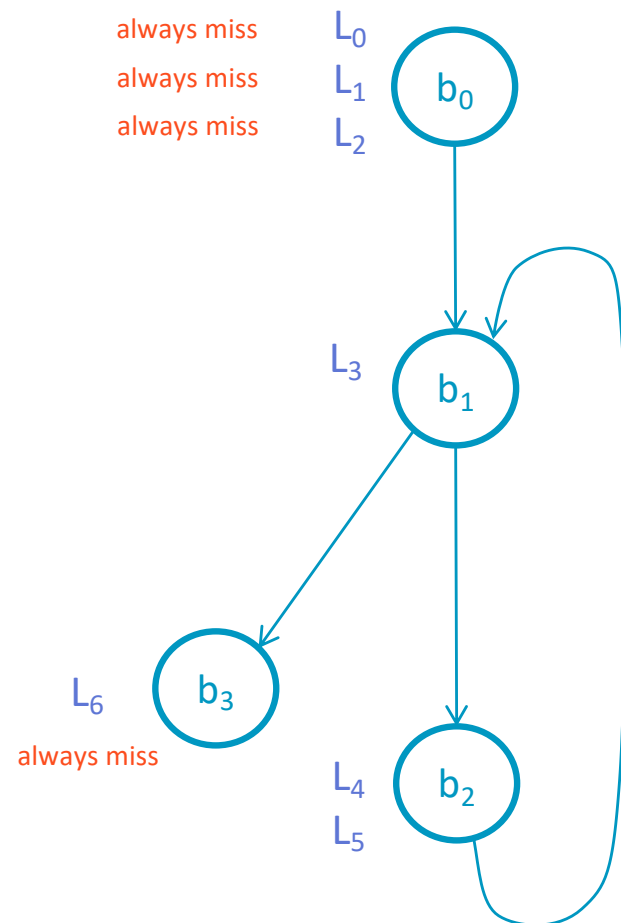


	<i>before</i>		<i>after</i>		
	set 0	set 1	set 0	set 1	
b_0	<div></div>	<div></div>	<div>L_2</div>	<div>L_1</div>	age 0
	<div></div>	<div></div>	<div>L_0</div>	<div></div>	age 1
b_1	<div>$L_{2,4}$</div>	<div>$L_{1,5}$</div>	<div>$L_{2,4}$</div>	<div>L_3</div>	age 0
	<div>L_0</div>	<div>L_3</div>	<div>L_0</div>	<div>$L_{1,5}$</div>	age 1
b_2	<div>$L_{2,4}$</div>	<div>L_3</div>	<div>L_4</div>	<div>L_5</div>	age 0
	<div>L_0</div>	<div>$L_{1,5}$</div>	<div>$L_{2,0}$</div>	<div>L_3</div>	age 1
b_3	<div>L_2</div>	<div>L_3</div>	<div>L_6</div>	<div>L_3</div>	age 0
	<div>L_0</div>	<div>L_1</div>	<div>L_2</div>	<div>L_1</div>	age 1

Cache analysis by Abstract Interpretation



MAY analysis



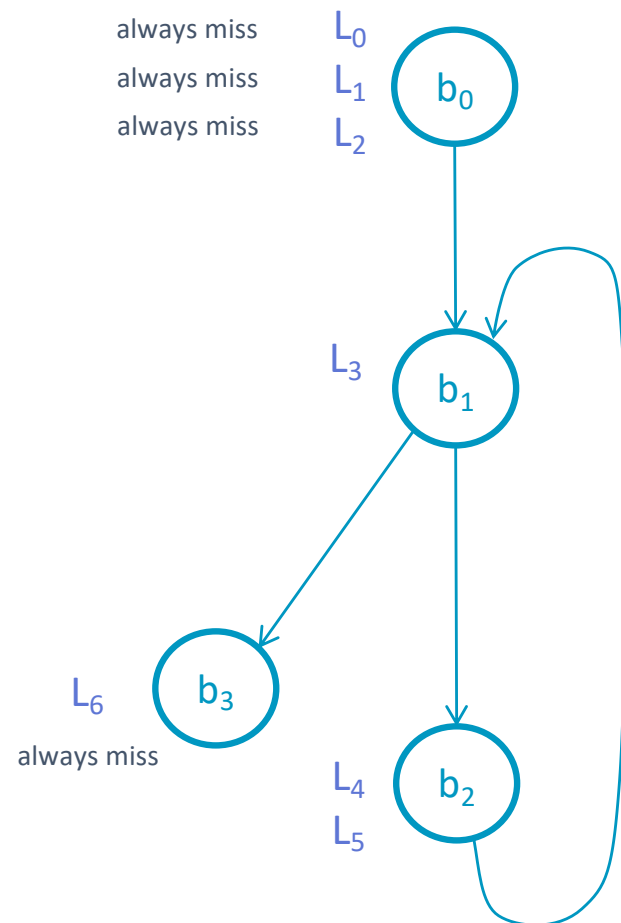
FIX POINT

	before		after		
	set 0	set 1	set 0	set 1	
b_0			L_2 L_0	L_1	age 0 age 1
b_1	$L_{2,4}$ L_0	$L_{1,5}$ L_3	$L_{2,4}$ L_0	L_3 $L_{1,5}$	age 0 age 1
b_2	$L_{2,4}$ L_0	L_3 $L_{1,5}$	L_4 $L_{2,0}$	L_5 L_3	age 0 age 1
b_3	$L_{2,4}$ L_0	L_3 $L_{1,5}$	L_6 $L_{2,4}$	L_3 $L_{1,5}$	age 0 age 1

Cache analysis by Abstract Interpretation



PERSISTENCE analysis

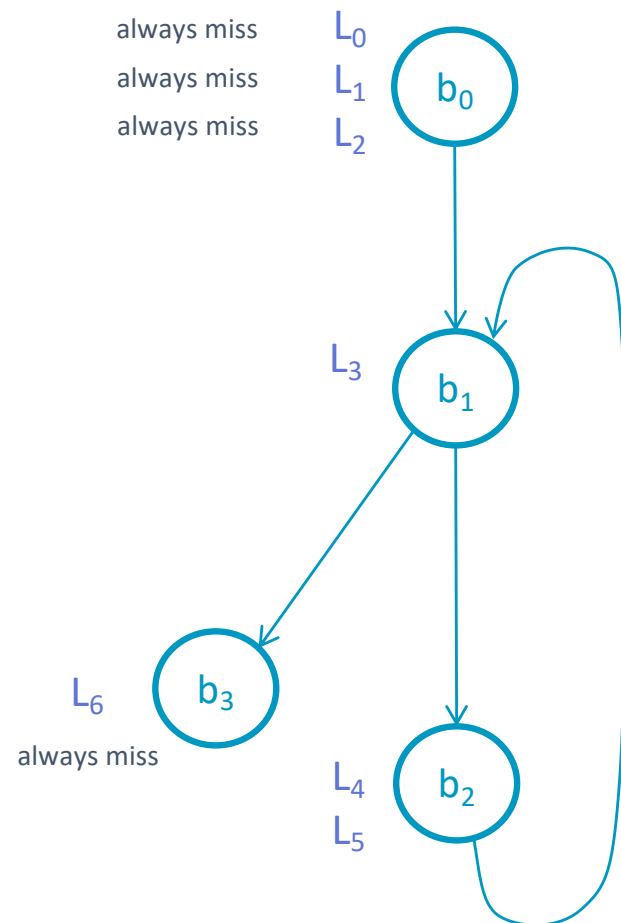


	before		after		
	set 0	set 1	set 0	set 1	
b_0			L_2	L_1	age 0
			L_0		age 1
					evicted
b_1					age 0
					age 1
					evicted
b_2					age 0
					age 1
					evicted
b_3					age 0
					age 1
					evicted

Cache analysis by Abstract Interpretation



PERSISTENCE analysis

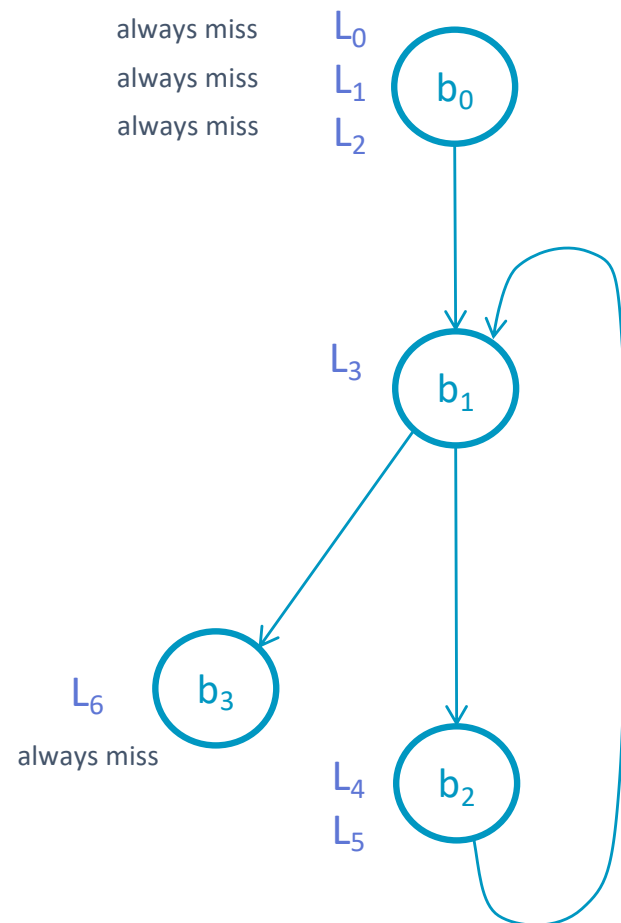


	<i>before</i>		<i>after</i>		
	set 0	set 1	set 0	set 1	
b_0			L_2 L_0	L_1	age 0 age 1 evicted
b_1	L_2 L_0	L_1	L_2 L_0	L_3 L_1	age 0 age 1 evicted
b_2					age 0 age 1 evicted
b_3					age 0 age 1 evicted

Cache analysis by Abstract Interpretation



PERSISTENCE analysis

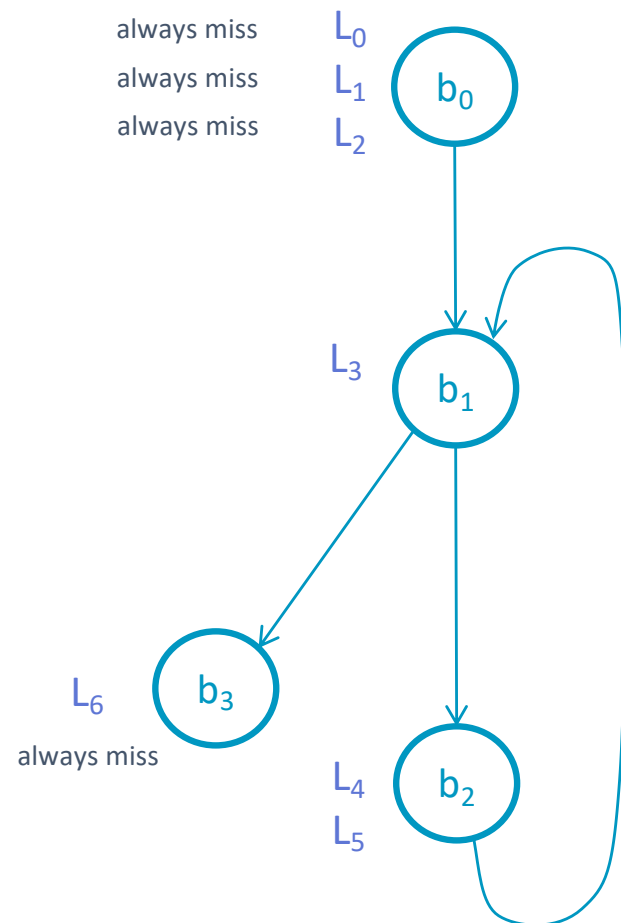


	before		after		
	set 0	set 1	set 0	set 1	
b_0			L_2 L_0	L_1	age 0 age 1 evicted
b_1	L_2 L_0	L_1	L_2 L_0	L_3 L_1	age 0 age 1 evicted
b_2	L_2 L_0	L_3 L_1	L_4 L_2 L_0	L_5 L_3 L_1	age 0 age 1 evicted
b_3					age 0 age 1 evicted

Cache analysis by Abstract Interpretation



PERSISTENCE analysis

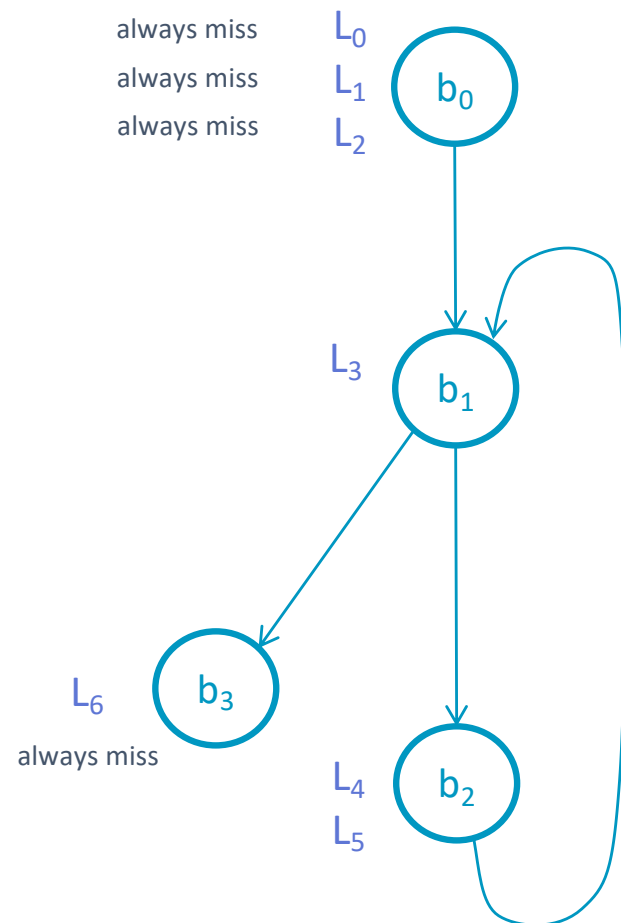


	<i>before</i>		<i>after</i>		
	set 0	set 1	set 0	set 1	
b_0			L_2 L_0	L_1	age 0 age 1 evicted
b_1	L_2 L_0	L_1	L_2 L_0	L_3 L_1	age 0 age 1 evicted
b_2	L_2 L_0	L_3 L_1	L_4 L_2 L_0	L_5 L_3 L_1	age 0 age 1 evicted
b_3	L_2 L_0	L_3 L_1	L_6 L_2 L_0	L_3 L_1	age 0 age 1 evicted

Cache analysis by Abstract Interpretation



PERSISTENCE analysis

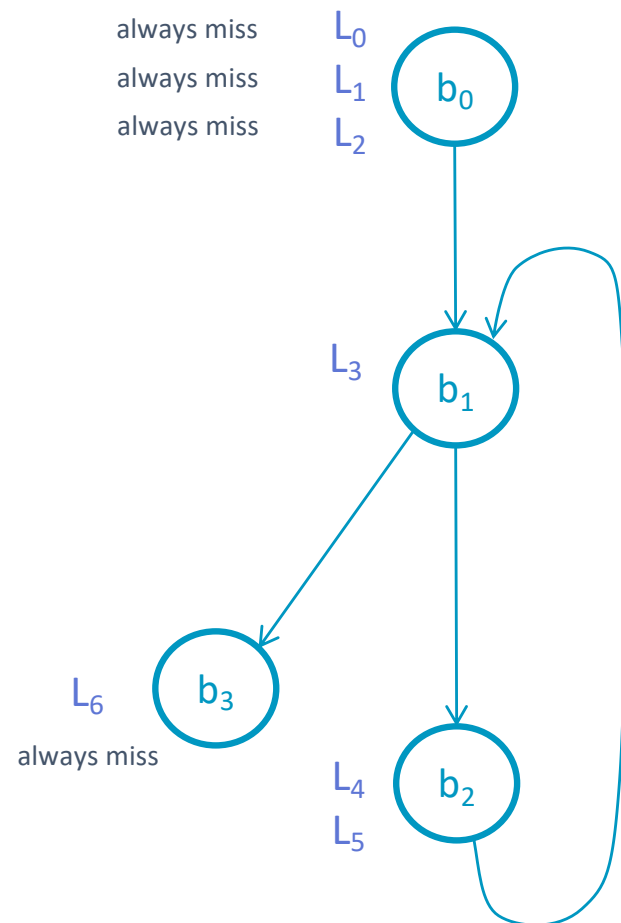


	<i>before</i>		<i>after</i>		
	set 0	set 1	set 0	set 1	
b_0			L_2	L_1	age 0
			L_0		age 1
					evicted
b_1	L_4	L_5	L_4	L_3	age 0
	L_2	L_3	L_2	L_5	age 1
	L_0	L_1	L_0	L_1	evicted
b_2	L_2	L_3	L_4	L_5	age 0
	L_0	L_1	L_2	L_3	age 1
			L_0	L_1	evicted
b_3	L_2	L_3	L_6	L_3	age 0
	L_0	L_1	L_2	L_1	age 1
			L_0		evicted

Cache analysis by Abstract Interpretation



PERSISTENCE analysis



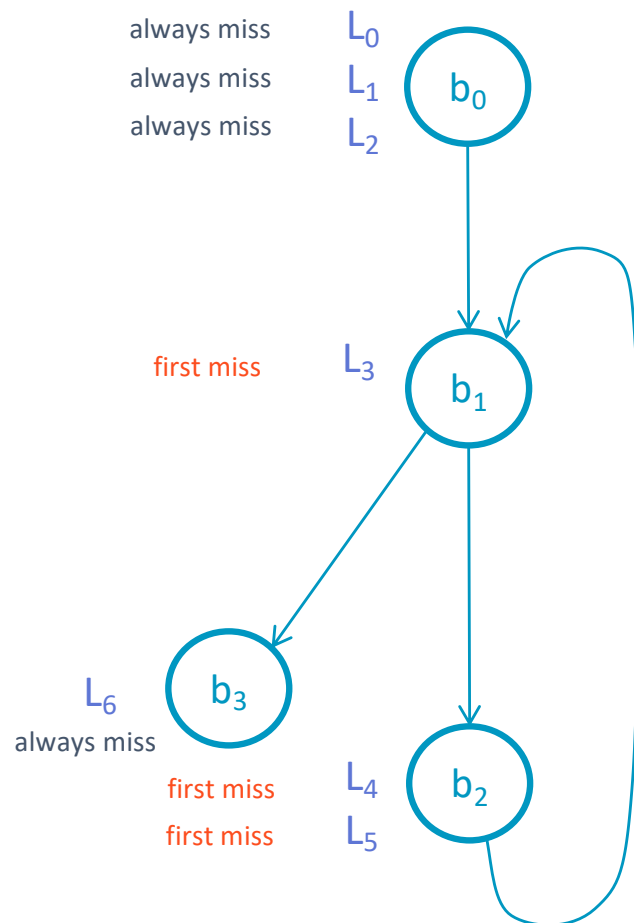
	<i>before</i>		<i>after</i>		
	set 0	set 1	set 0	set 1	
b_0			L_2	L_1	age 0
			L_0		age 1
					evicted
b_1	L_4	L_5	L_4	L_3	age 0
	L_2	L_3	L_2	L_5	age 1
	L_0	L_1	L_0	L_1	evicted
b_2	L_4	L_3	L_4	L_5	age 0
	L_2	L_5	L_2	L_3	age 1
	L_0	L_1	L_0	L_1	evicted
b_3	L_2	L_3	L_6	L_3	age 0
	L_0	L_1	L_2	L_1	age 1
			L_0		evicted

Cache analysis by Abstract Interpretation



PERSISTENCE analysis

FIX POINT



	before		after		
	set 0	set 1	set 0	set 1	
b_0			L_2 L_0	L_1	age 0 age 1 evicted
b_1	L_4 L_2 L_0	L_5 L_3 L_1	L_4 L_2 L_0	L_3 L_5 L_1	age 0 age 1 evicted
b_2	L_4 L_2 L_0	L_3 L_5 L_1	L_4 L_2 L_0	L_5 L_3 L_1	age 0 age 1 evicted
b_3	L_4 L_2 L_0	L_3 L_5 L_1	L_6 L_4 $L_{0,2}$	L_3 L_5 L_1	age 0 age 1 evicted

New ILP formulation



$$\max T = 13 x_0 + 2 x_{0-1} + 5 x_{1-2} + 3 x_{2-1} + 8 x_{1-3} \\ + \text{misspenalty} * (3 x_0 + 1 + x_3 + 2)$$

$$x_0 = 1$$

$$x_0 = x_{0-1}$$

$$x_1 = x_{0-1} + x_{2-1}$$

$$x_1 = x_{1-2} + x_{1-3}$$

$$x_2 = x_{1-2}$$

$$x_2 = x_{2-1}$$

$$x_3 = x_{1-3}$$

$$x_3 = 1$$

$$x_{2-1} \leq 256$$

