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MAT-II-001
B.Sc. II Semester (NEP) Degree Examination
MATHEMATICS
Algebra - II And Calculus - II
Paper : MAT-DSCT - 2.1

Time : 2 Hours **Maximum Marks : 60**

Instructions to Candidates:

Answer All the sections.

SECTION - A

Answer any FIVE of the following. **(5×2=10)**

1. a. If S and T be two subsets of R such that $S \subseteq T$ and $S \neq \phi$ then prove that If T is bounded above then $SupS \leq SupT$.

b. Find the order of each element of multiplicative group G where G = {1,-1,i,-i}.

c. Find all the right cosets of the subgroup H = {0,3} in the group $(Z_6, +_6)$.

d. If $x = r\cos\theta$, $y = r\sin\theta$ then prove that $\left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2 = 1$.

e. If $u = \frac{x+y}{2}$, $v = \frac{x-y}{2}$ then show that $\frac{\partial(u,v)}{\partial(x,y)} = \frac{1}{2}$.

f. Evaluate $\int_c (x+y)dx + (y-x)dy$ along the parabola $y^2 = x$ from (1,1) to (4,2).

g. Evaluate $_0\int^1 {}_0\int^1 \frac{dxdy}{\sqrt{(1-x^2)(1-y^2)}}$.

SECTION - B

Answer any FOUR of the following. **(4×5=20)**

2. Show that A set A is a neighbourhood of a point a If and only if there exists a positive integer n such that the open interval $\left(a - \frac{1}{n}, a + \frac{1}{n}\right) \subset A$.

3. Show that every subgroup of cyclic group is a cyclic.

4. Show that a subset H of a group G is subgroup of G if and only if $HH^{-1}=H$.

5. If $u = Tan^{-1}(x^4 + y^4 + x^3y + xy^3)$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$.

6. If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$ Then show that $\frac{r(x,y,z)}{r(r,\theta,\phi)} = r^2\sin\theta$.

7. Evaluate $\iint_R x^2y^2\,dxdy$ where R is the circle $x^2 + y^2 = 1$.

SECTION - C

Answer any THREE of the following. **(3×10=30)**

8. a. Show that every subset of a countable set is countable.
   b. Show that every open set is a union of open intervals.
9. a. State and prove Lagranges theorem.
   b. Find the number of generators of cyclic group of order 60.
10. If $u = x + y$, $v = xy$ then verify $J.J' = 1$.
11. a. Evaluate $\int_0^a\int_0^{\sqrt{a^2-x^2}} x^2\,dydx$ by changing into polar Co-ordinate.

    b. Evaluate $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} xyz\,dx\,dy\,dz$.

12. a. Evaluate $\iint \frac{x^2y^2}{x^2+y^2}\,dx\,dy$ outside the circle $x^2 + y^2 = a^2$ and inside the circle $x^2 + y^2 = b^2 (b > a)$.

    b. Find the volume of the tetrahedron bounded by the Co-ordinate planes and the plane $x+y+z=1$.