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MAT-I-001
B.Sc. I Semester (NEP) Degree Examination
MATHEMATICS
Algebra - I and Calculus - I
Paper: MATDSCT - 1.1

Time : 2 Hours **Maximum Marks : 60**

Instructions to Candidates:

Answer **All** the Sections.

SECTION - A

Answer any FIVE of the following questions. **(5×2=10)**

1. a) Reduce the Matrix $\begin{pmatrix} 1 & 5 & 4 \\ 2 & 8 & 6 \\ 3 & 22 & 7 \end{pmatrix}$ to the echelon form.

b) Find the eigen values of $\begin{pmatrix} 3 & 4 \\ 3 & 2 \end{pmatrix}$.

c) Find the angle between radius vector and tangent for the curve $r\sec^2\left(\theta/2\right) = 2$.

d) Show that $f(x) = \begin{cases} x.\sin\left(\frac{1}{x}\right) at\ x \neq 0 \\ 0 \qquad\quad x = 0 \end{cases}$ is continuous at $x = 0$.

e) Evaluate $\lim_{x\to 0}(\cot x - \operatorname{cosec} x)$

f) Find the n^{th} derivative of $\cos 3x.\cos 2x.\cos x$.

g) Find the double points on the curve

$x^3 + x^2 + y^2 - x - 4y + 3 = 0$.

SECTION - B

Answer any FOUR of the following questions. (4×5=20)

2. Verify system

$$x+y+z=-4$$
$$x-2y+3z=5$$
$$4x+3y+4z=7$$ for consistant and hence solve

3. Show that the following curves are Orthogonal

$r^2.\cos 2\theta = a^2$ *and* $r^2.\sin 2\theta = b^2$

4. Verify Rolle's Theorem for the function

$f(x)=\log\left(\frac{x^2+mn}{mx+nx}\right)$ over *[m, n]* where $0 < m < n$

5. Prove that every differentiable function is continuous but not conversely.

6. If $Y=\text{Sin}^{-1}\left(\frac{2x}{1+x^2}\right)$ then show that $(1+x^2)Y_{n+2}+2(n+1)xY_{n+1}+n(n+1)Y_n=0.$

7. Find the derivative of arc for the curve

$r=a(1-\cos\theta)$

SECTION - C

Answer any THREE of the following questions. (3×10=30)

8. Show that $A=\begin{pmatrix}1 & 2 & 1\\ -1 & 0 & 3\\ 3 & -1 & 1\end{pmatrix}$ satisfies the Cayley Hamilton theorem and hence find inverse.

9. Find the Centre of curvature and evolute of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$

10. a) Show that $\log(1-x) = -(x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + ...)$ using Maclaurin's expansion.

b) Evaluate $\lim_{\theta \to \pi/2} (\text{Sin}\,\theta)^{\text{Tan}\,\theta}$

11. a) Prove that $D^n\left[e^{ax}.\sin(bx+c)\right] = (a^2+b^2)^{n/2}.e^{ax}\sin\left[n\,\text{Tan}^{-1}(b/a) + (bx+c)\right]$

b) Find the n^{th} derivative of $\frac{3x}{2x^2 - x - 1}$

12. Trace the curve $Y^2 = x^2\left(\frac{3a-x}{a+x}\right)$