402432

Roll No ____________________ [Total No. of Pages : 2

NS3S-1563-A-23/24

B.Sc. III Semester (NEP) Degree Examination

MATHEMATICS

Ordinary Differential Equations and Real Analysis -I

Paper : MATDSCT 3.1

Time : 2 Hours Maximum Marks : 60

Instructions to Candidates:

Answer All the Sections.

SECTION - A

I. Answer any FIVE of the following questions. (5×2=10)

1. a) Solve $xp^2-(x-y)p-y=o$

b) Solve $ydx-xdy+3x^2y^2e^{x^3}=0$

c) Solve $(D^3-3D^2+4)\,Y=0$

d) Solve $\frac{dx}{z}=\frac{dy}{-z}=\frac{dz}{z^2+(x+y)^2}$

e) Using the definition show that $\lim_{n\to\infty}\frac{2n+3}{n+5}=2$

f) Test the convergence of $\sum_{n=1}^{\infty}\mathrm{Tan}\left(\frac{1}{n}\right)$

g) Apply the Leibnit'z test to discuss the convergence of alternate series

$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+----$

SECTION - B

Answer any FOUR of the following questions. (4×5=20)

2. Test for the exactness and solve. $(e^y+1)\cos x\,dx+e^y\sin y\,dy=0$

3. Solve $y^2\log y=xpy+p^2$

4. Solve $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=x^2e^{3x}$.

5. Show that monotonically increasing sequence which is bounded above converges to its least upper bound

6. Discuss the convergence of the series $\sum_{n=1}^{\infty}\left[\left(\frac{n+1}{n}\right)-\left(\frac{n+1}{n}\right)^{n+1}\right]^{-n}$

7. Test the series $1-\frac{1}{4}+\frac{1}{7}-\frac{1}{10}+---$ for

   i) Convergence

   ii) Absolute convergence

   iii) Conditional convergence.

SECTION - C

III. Answer any THREE of the following questions. (3×10=30)

8. a) Reduce the differential equation $(px-y)(x-py)=2p$ to Clairaut's form by the substitution $x^2=u$ and $y^2=v$; Hence find its general solution.

   b) Find the orthogonal trajectories of the family of parabolas $y^2=4ax$

9. Solve $(2+3x)^2\frac{d^2y}{dx^2}+3(2+3x)\frac{dy}{dx}-36y=3x^2+4x+1$

10. a) Verify the condition of integrability and solve $3x^2dx+3y^2dy-(x^3+y^3+e^{2z})dz=0$

    b) Solve $\frac{dx}{x^2-yz}=\frac{dy}{y^2-zx}=\frac{dz}{z^2-xy}$

11. Discuss that the sequence $\{x_n\}$ defined by $x_1=1,\ x_{n+1}=\frac{2x_n+3}{x_n+2}\ \forall.n\geq 1$ converges to $\sqrt{3}$

12. Test the convergence of the series $\frac{1}{2}.\frac{x^3}{3}+\frac{1.3}{2.4}.\frac{x^5}{5}+\frac{1.3.5}{2.4.6}.\frac{x^7}{7}+...(x>0)$