

Lecture 25:

Introduction to Machine Learning:

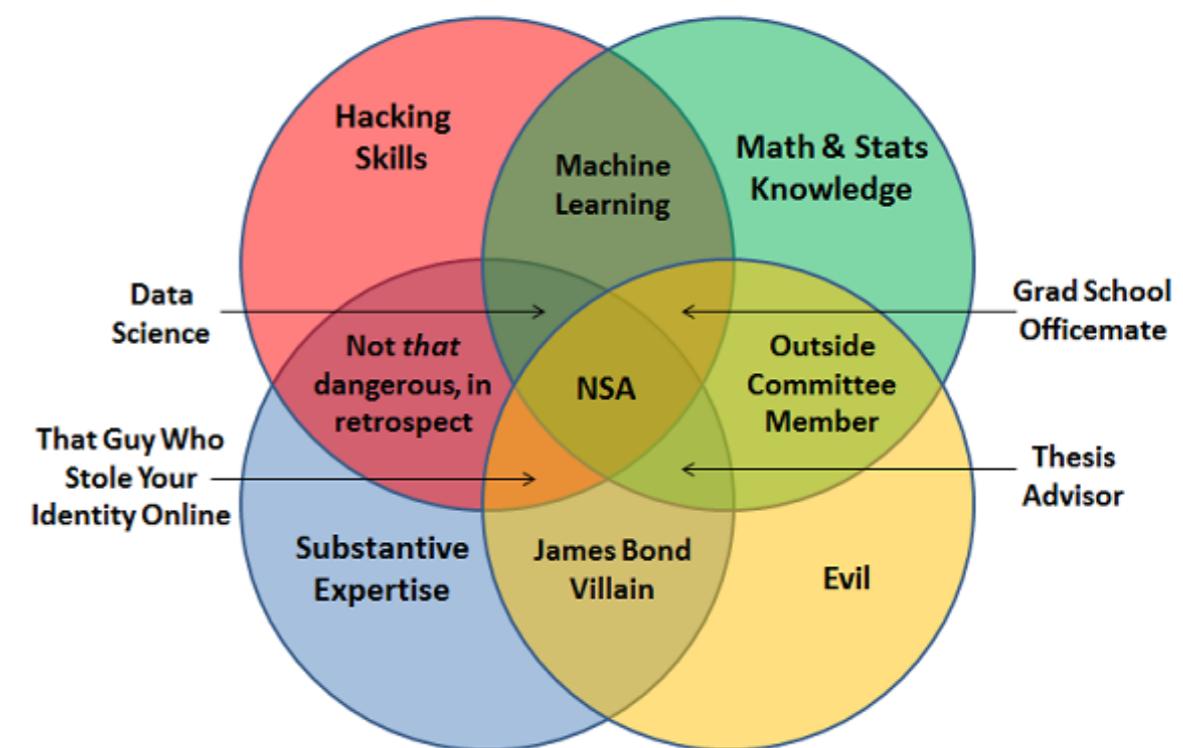
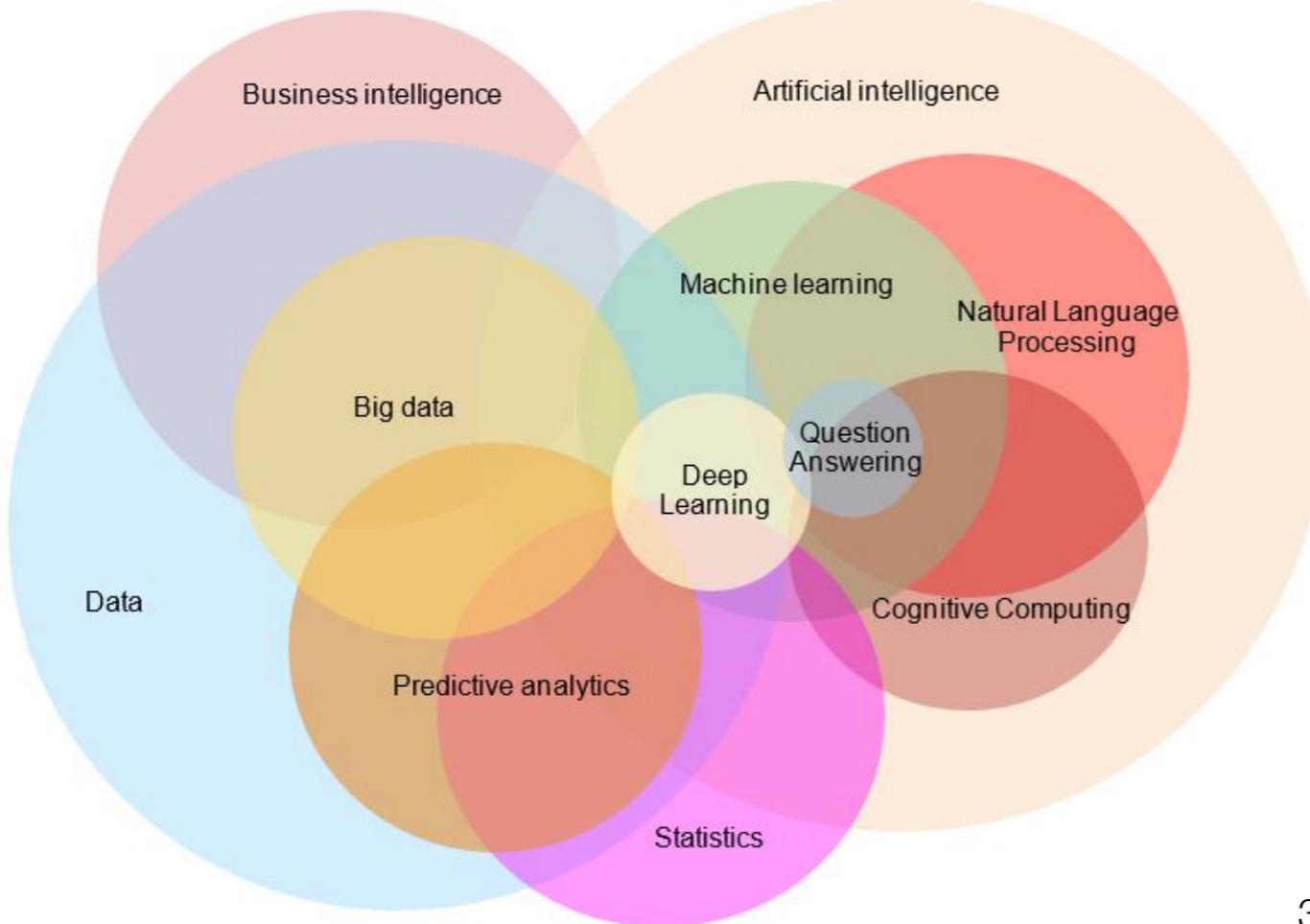
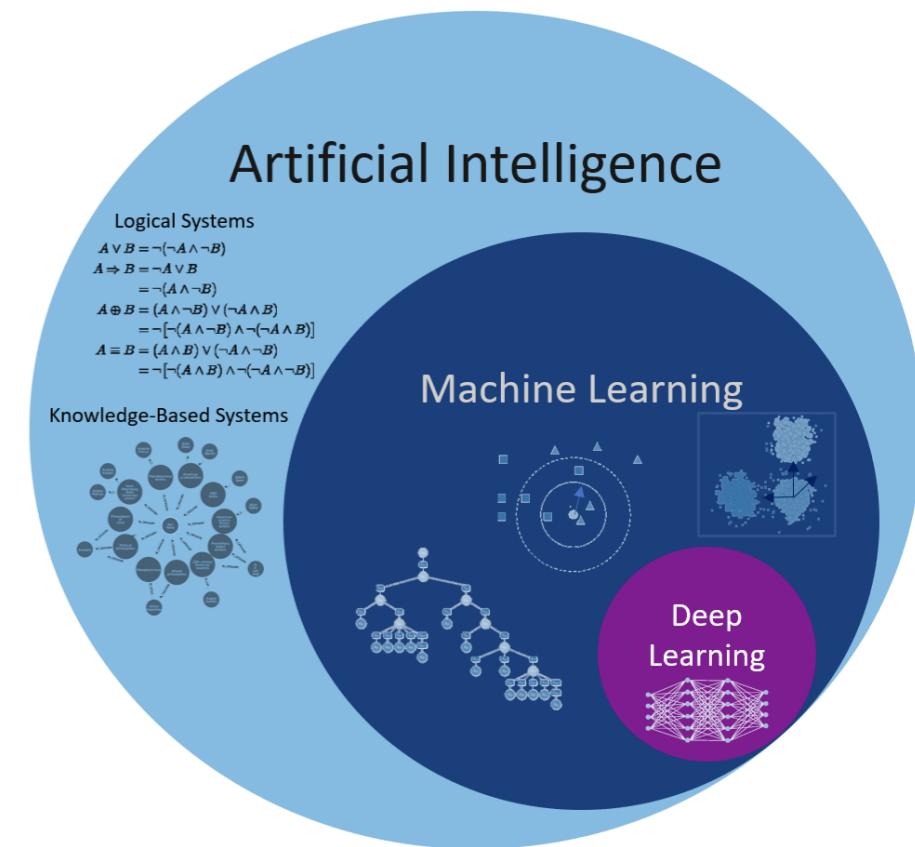
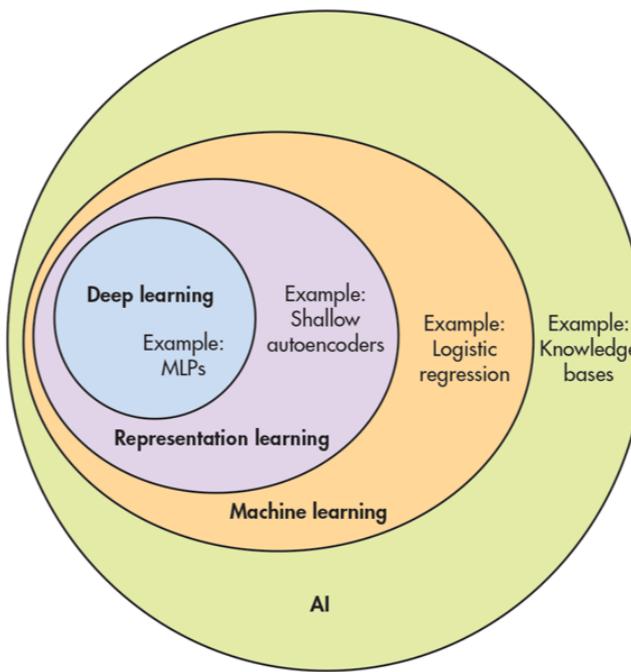
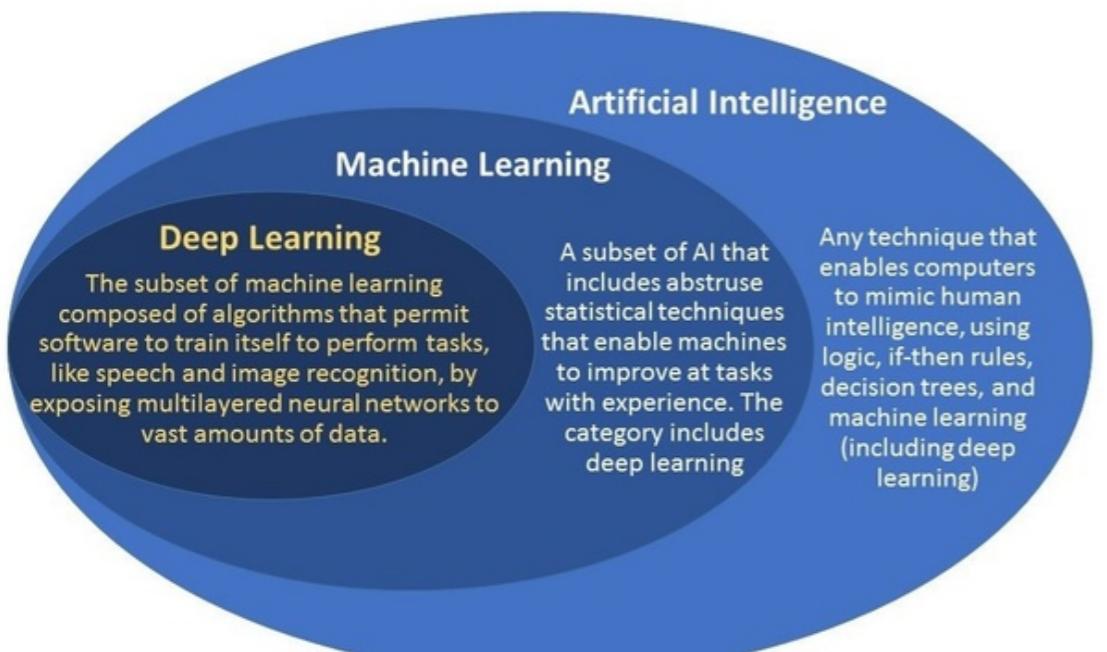
From Linear Regression to Deep Learning

Amir Farbin

Overview

- What is Machine Learning?
- Machine Learning
 - A Formulation
 - Linear Techniques → Kernel Techniques → Decision Trees → Neutral Networks → Deep Learning
 - Bayesian vs Frequentist

AI vs ML vs DL



MOVE OVER, CODERS— PHYSICISTS WILL SOON RULE SILICON VALLEY

Most of the graduate students and postdocs in this room are probably headed in this direction...

it's happening across Silicon Valley. Because structurally and technologically, ***the things that just about every internet company needs to do are more and more suited to the skill set of a physicist.***

But this is a particularly ripe moment for physicists in computer tech, thanks to the rise of machine learning, where machines learn tasks by analyzing vast amounts of data. This ***new wave of data science and AI is something that suits physicists right down to their socks.***

these neural networks are really just math on an enormous scale, mostly linear algebra and probability theory.

Chris Bishop, who heads Microsoft's Cambridge research lab, ... ***"There is something very natural about a physicist going into machine learning," he says, "more natural than a computer scientist."***

Physicists know how to handle data—at MIT, Cloudant's founders handled massive datasets from the Large Hadron Collider—and building these enormously complex systems requires its own breed of abstract thought.

They come because they're suited to the work. And they come because of the money. As Boykin says: "The salaries in tech are arguably absurd." But they also come because there are so many hard problems to solve.

Machine learning will change not only how the world analyzes data but how it *builds software*.

In other words, ***all the physicists pushing into the realm of the Silicon Valley engineer is a sign of a much bigger change to come. Soon, all the Silicon Valley engineers will push into the realm of the physicist.***

AI vs ML vs DL

- ***Artificial Intelligence***: Any technique that mimics human behavior
 - Code, Logic, Symbolic systems, Knowledge Bases
- ***Machine Learning***: Any technique that learns from experience (aka Data)
 - Logistic regression (aka fits), Decision Trees, Clustering, Kernel Methods
 - ***Representation Learning***: Techniques that learn representations of data amenable to specific or general tasks
 - Shallow Auto-encoders
 - ***Neural Networks***: Biologically inspired ML
 - ***Deep Learning***: Multi-layered Neural Networks
 - MLP, DNN, CNN, RNN, ...

Supervised ML

- ML Algorithm: Input → ML → Target
- When you have a well defined target and training set with targets.
- **Tasks: Classification**, Classification with missing inputs, Regression, Transcription, Machine Translation, Structured Output
- “Traditional” Techniques:
 - Linear/Logistic Regression
 - Support Vector Machines
 - Decision Trees

Un-supervised ML

- ML Algorithm: Input → ML → ????
- When you do not have a target or a training set with the target.
- **Tasks:** Clustering, Anomaly Detection, Imputation of Missing Values, Synthesis & Sampling, Denoising, Density Estimation
- “Traditional” Techniques:
 - Principle Component Analysis
 - k-means Clustering

Ingredients of ML

- **Problem Formulation.** Specify:
 - **Data Set:** Inputs/Outputs
 - **ML technique:** $F(\text{Input} \mid \text{Parameters}) = \text{Output}$
 - **Target: Cost (aka loss) function**
 - *Supervised:* Compare F vs Ground Truth Output
 - *Unsupervised:* e.g. Cluster like inputs
 - *Semi-supervised:* $F(\text{Input}) = \text{Input}$
 - **Training: Optimization**
 - Choose how to find best parameters

A bit of Formalism

Data Formulation

- Lets formalize what we mean by a **dataset** with a Probabilistic Model:
 - Assumption: Observed **Data is a mixture of M different populations**
 - Data set of N **data points**, $\{\{x_d\}_i\}$
 - each $\{x_d\}_i$ consisting of
 - d **observations** $\{x_d\}$
 - probability f_j of uniquely coming from one of M **classes**
 - each class has label c_j is indexed by j
 - dependent on parameters $\{a_k\}_j$ (**some parameters of interest, some nuisance parameters**)
 - Dependent on other parameters $\{\beta_l\}$
 - $\implies P(\{x\}|\theta) = P(\{x\}|\{f_j, c_j, \{\{a_k\}_j\}, \{\beta_l\}\}) = \sum_j f_j P(\{x\}|c_j, \{a_k\}_j, \{\beta_l\})$
 - Examples:
 - **Students, Customers, ...**
 - **Pictures (e.g. dogs and cats) ...**

What is it good for?

- If we know $P(X|\theta)$, what is it good for?
 - ***Prediction***: Assume $\theta \implies$ distribution of $\{x_d\}$.
 - ***Classification***: Observation $\{x_d\} \implies$ most likely class c
 - ***Regression***: Dataset $\{\{x_d\}_i\} \implies$ parameters of interest $\{a_j^k\}$ or $\{\beta_l\}$
 - ***Hypothesis test***: Dataset $\{\{x_d\}_i\} \implies$ is H_1 true (or H_0 null hypothesis)

Data Analysis

- Objectives:
 - **Searches** (hypothesis testing): Likelihood Ratio Test (Neyman-Pearson lemma)
 - **Limits** (confidence intervals): Also based on Likelihood
 - **Measurements**: Maximum Likelihood Estimate

$$\frac{P(x|H_1)}{P(x|H_0)} > k_\alpha$$

- **Likelihood**

$$p(\{x\}|\theta) = \text{Pois}(n|\nu(\theta)) \prod_{e=1}^n p(x_e|\theta)$$

- n Independent Events (e) with Identically Distributed Observables ($\{x\}$)
- Significant part of Data Analysis is **approximating the likelihood** as best as we can.

Obtaining the Likelihood

- How do we obtain $P(X|\theta)$?
 - In HEP, we have precise ***algorithmic simulations*** that generate $\{x_d\}$ given θ .
 - We estimate P by comparing observed x_d with simulated $\{x_d\}$.
 - We can build ***analytical first principle models***. *Matrix Element Method* is such a technique.
 - But it's technically difficult, computationally expensive, and only tractable with physics and detector simplifications.
 - We can use ML to learn P from simulation or data.

Likelihood Approximations

- Need $P(\{x_d\} | \theta)$ of an observed event (i). The better we do, the more sensitive our measurements.
- So we simulate a lot of events and generally use histograms (a crude *Probability Density Estimator (PDE) technique*)
 - $\{x_d\} = \{100M \text{ Detector Channels}\}$ or even { particle 4-vectors } are too high dimensional.
 - Curse of dimensionality... more on this later
 - Instead we derive $\{x_d\} = \{ \text{small set of physics motivated observables} \} \rightarrow \textbf{Lose information.}$
 - **Isolate signal** dominating regions of $\{x_d\} \rightarrow \textbf{Lose efficiency.}$
 - Sometimes use **ML-based classifiers** to further reduce dimensionality and improve significance
 - **Profile the likelihood** in 1 or 2 (ideally uncorrelated) observables.

Machine Learning

Basic Formulation

- **AI Algorithm:** $\text{Input} \longrightarrow \text{AI Algorithm} \longrightarrow \text{Output}$
 - $X \longrightarrow F(\theta) \longrightarrow y: y = F(X|\theta)$
 - X : observable inputs
 - θ : Model parameters (most likely learned)
 - y : observable outputs
- **Typical ML task:** Isolate signal in real data $\{x_d\}_i$
 - **Formulate problem:** $X = x_d, y = 0$ or 1 if $c = \text{background or signal}$
 - **Obtain training data:** obtain labeled Dataset $\{x_d, c\}_i$ via simulation, hand labeling, ...
 - **Separate dataset:** into *Train*, *Test*, and possibly *Validation* subsets
 - **Choose:** Pick an ML algorithm F
 - **Train:** Use labeled Dataset $\{x_d, c\}_i$ to obtain θ
 - **Test:** Estimate $P(F(X|\theta)|c_j)$ using Test sample
 - **Optimize:** Select $F(X|\theta) > \text{cut}$ to optimize sensitivity for a hypothesis test
 - $P(F(X|\theta)| \text{signal}) / P(F(X|\theta)| \text{background})$
 - **Validate:** Obtain signal / background efficiency using Validation sample
 - **Apply:** to data $\{x_d\}_i$

ML Models (Linear)

- Linear Models: $F(\vec{x}) = \mathbf{W}\vec{x} + \vec{b}$

- Solve equation $\vec{y} = \mathbf{W}\vec{x} + \vec{b}$

- for “weights” w in matrix W and biases b

- by minimizing $\frac{1}{N} \sum_i^N (F(\vec{x}|\mathbf{W}, \vec{b}) - \vec{y})^2$

- Note

- W is d by m matrix

- x is d component vector

- y is m component vector

- Simple example: b = 0, W is 1 by d

- \Rightarrow Analytic solution

$$W = (X^T X)^{-1} X^T Y$$

$$X = \begin{pmatrix} \vec{x}_1 \\ \vec{x}_2 \\ \vdots \\ \vec{x}_N \end{pmatrix} \quad Y = \begin{pmatrix} \vec{y}_1 \\ \vec{y}_2 \\ \vdots \\ \vec{y}_N \end{pmatrix}$$

ML Models (LDA)

- Linear Discriminant Analysis (Fisher Discriminant)
 - Assume 2 classes: 0 and 1
 - Means μ_0, μ_1 and covariance matrices Σ_0, Σ_1
 - Model: $\vec{y} = \mathbf{W}\vec{x}$
 - Again W is 1 by d, b=0
 - Maximize:
$$S = \frac{(\mathbf{W} \cdot \vec{\mu}_0 - \mathbf{W} \cdot \vec{\mu}_1)^2}{\mathbf{W}^T \Sigma_0 \mathbf{W} - \mathbf{W}^T \Sigma_1 \mathbf{W}}$$
 - Maximal separation between means
 - Smallest possible variance
 - Analytic Solution: $\mathbf{W} = (\Sigma_0 + \Sigma_1)^{-1} (\vec{\mu}_0 - \vec{\mu}_1)$

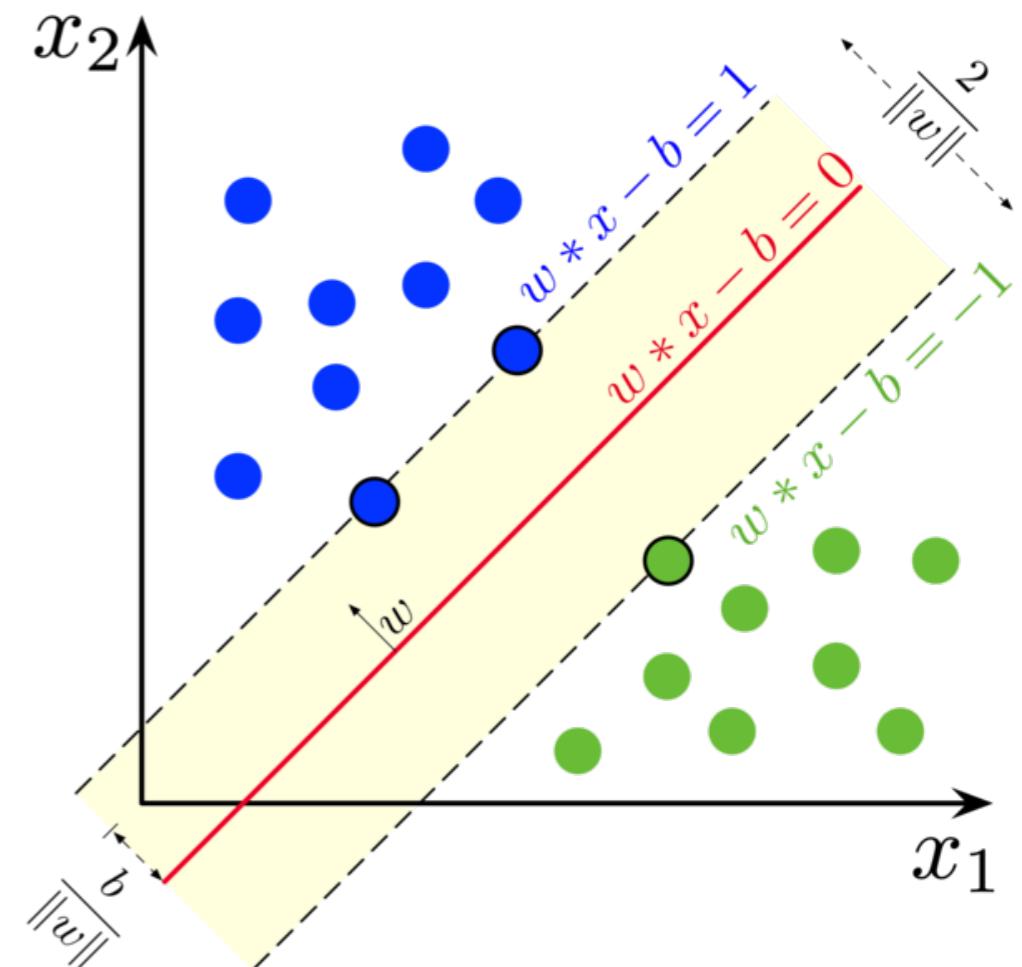
ML Models (Linear SVM)

- Support Vector Machines:

- Same setup... $\vec{y} = \vec{W} \cdot \vec{x} + \vec{b}$
- Note that this is an equation for a hyperplane —> formulate problem as finding the hyperplane that optimally separates two classes.
- Maximize boundary between two classes \Rightarrow plane depends on points closest to the plane / most difficult to separate... the *Support Vectors*.
- Solving this problems is equivalent to solving a dual problem of solving for a_i

$$f(\vec{x}) = \left(\sum_i^N a_i y_i \vec{x}_i \right) \cdot \vec{x} + \vec{b}$$

- Where $y_i = \{+1, -1\}$ depending on class.
- $a_i \geq 0$.
- Dot product measures similarity.
 - \rightarrow Sign of output dependent on sign of the most similar example in training set.
 - Support vectors still most important for defining boundary

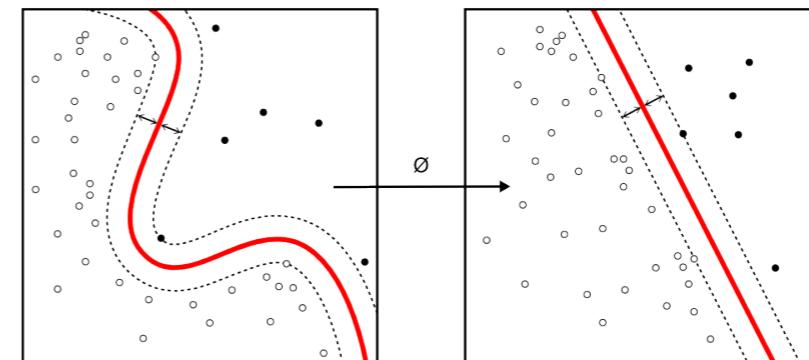


ML Models (Kernel SVM)

- If boundary not linear \Rightarrow Kernel Trick:

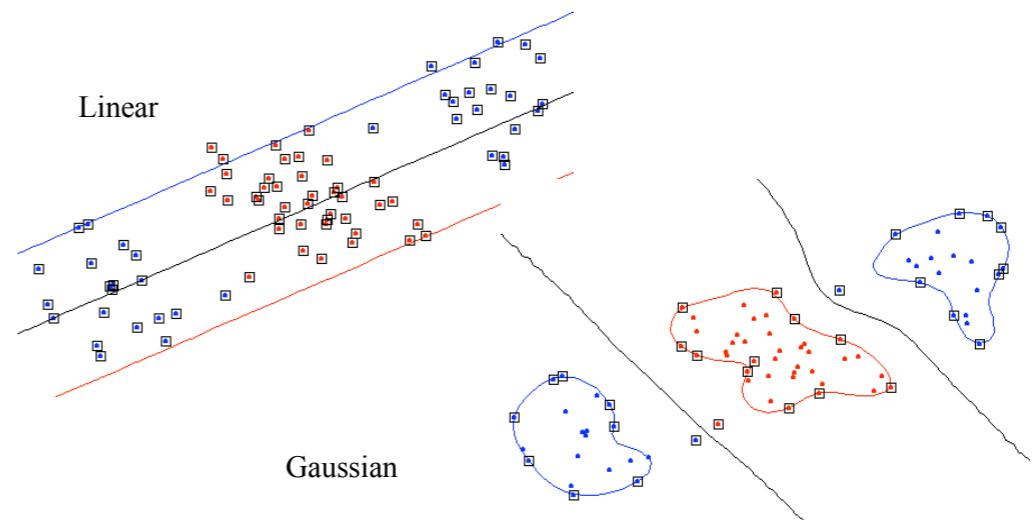
- Replace dot product: $f(\vec{x}) = \sum_i^N a_i y_i k(\vec{x}_i, \vec{x}) + \vec{b}$ $k(\vec{x}_i, \vec{x}) = \phi(\vec{x}_i) \cdot \phi(\vec{x})$

- Example: if x is in radial coordinates, then kernel gets back proper dot product.

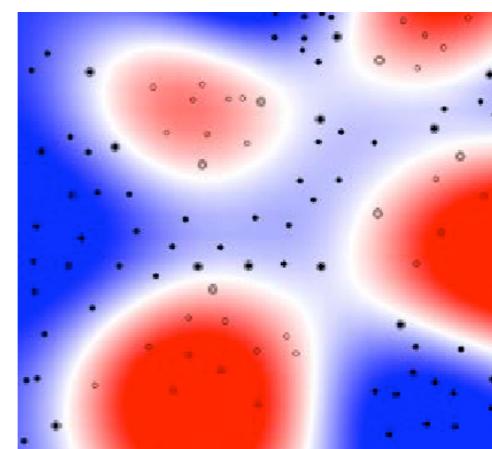


- Example: Gaussian Kernels

- Euclidian Distance in the exponential
- If close to an example in training data \Rightarrow large value
- Equivalent to template matching



Radial Basis Functions

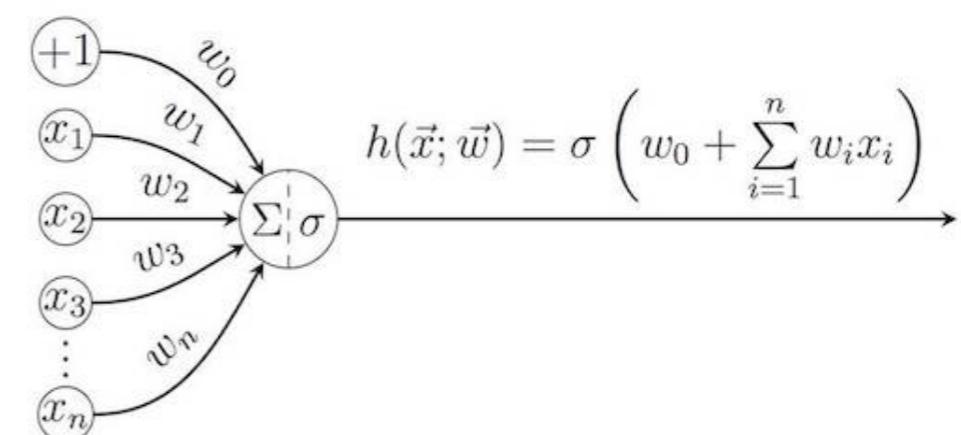
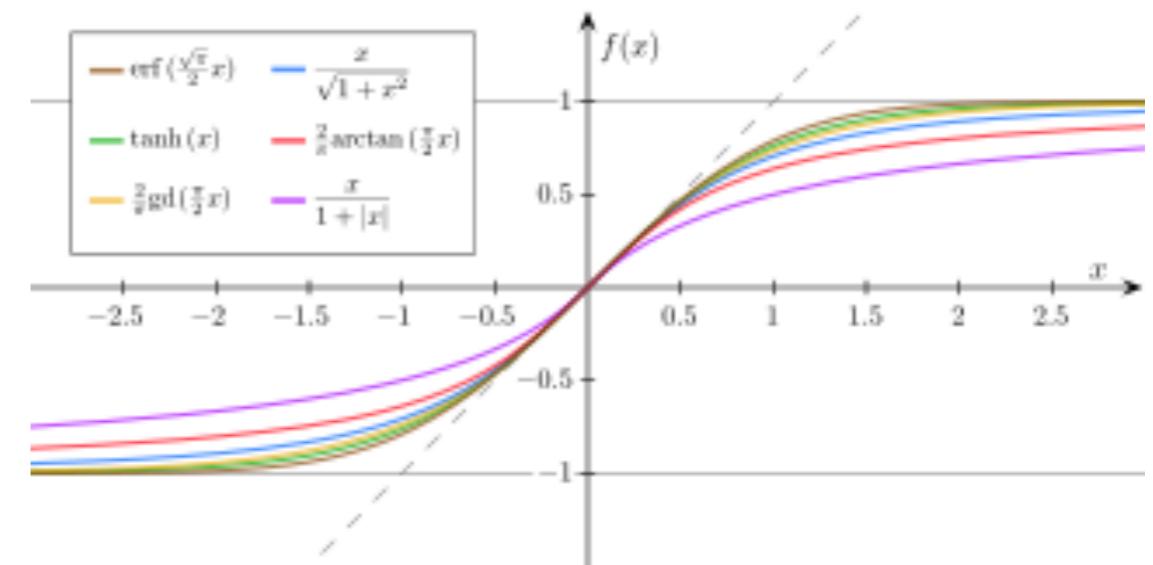


Features

- $X \rightarrow \phi(X) \rightarrow$ Linear Model
- ϕ are features or representation derived from X
- ϕ is:
 - Generic, e.g. polynomials \rightarrow e.g. SVM
 - Manually constructed \rightarrow Feature Engineering
 - Learned \rightarrow Feature Learning

Artificial Neural Network

- A simple one layer NN
 - $F(\mathbf{X} | \mathbf{a} = \mathbf{W}, \mathbf{b}) = f(\mathbf{W} \mathbf{X} + \mathbf{b})$
 - \mathbf{W}, \mathbf{b} = “weights”, “biases”
 - $f(x)$ = “activation function”
 - Must be non-linear.
 - Universal Computation Theorem.



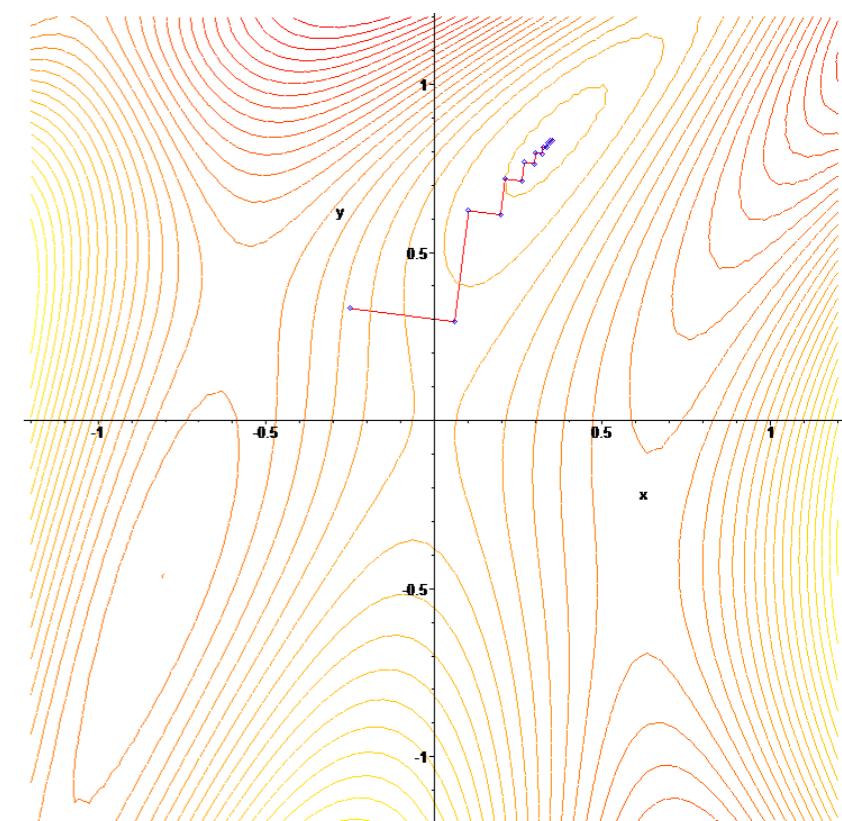
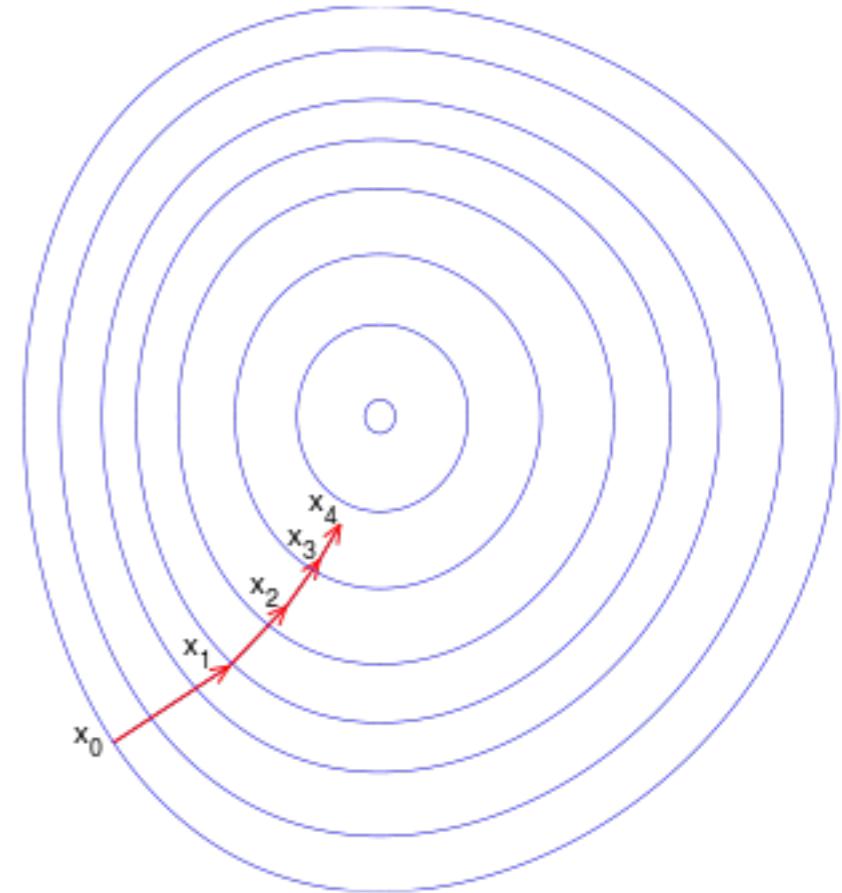
Training == Optimization

- Training = Minimizing cost function w.r.t. parameters $\vec{\alpha}$

$$C[F(\vec{X}_{train}|\vec{\alpha}), \vec{Y}_{train}] \equiv C(\vec{\alpha})$$

- Gradient Decent (Newton's Method):
 - Gradient points to direction of maximal change.
 - Iterate (ϵ sets the step size == *Learning Rate*)

$$\vec{\alpha}_{i+1} = \vec{\alpha}_i - \epsilon \nabla C(\vec{\alpha})$$

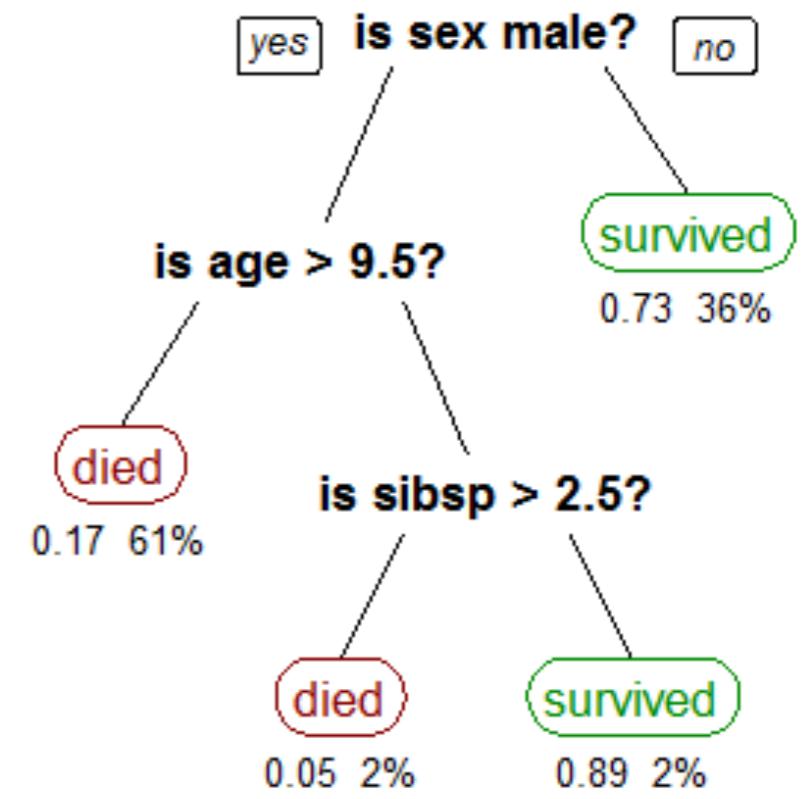


Bayesian vs Frequentist

- ***Supervised Learning:***
 - Data: (X, Y)
 - True: $f^*(X) = Y$
 - Learn: $f(X|\theta) \sim f^*$
- ***Frequentist:***
 - (X, Y) random.
 - Ideal θ exists.
 - Estimate θ .
- ***Bayesian:***
 - (X, Y) fixed.
 - θ is random.
 - Learn $p(\theta)$
- ***Bayesian Learning:***
 - For example: θ is Gaussian distributed \Rightarrow learn μ, σ
 - Allows you to propagate uncertainties \Rightarrow estimate uncertainty on output Y

Decision Trees

- Doesn't fit the $F(\mathbf{X} | \mathbf{W}, \mathbf{b})$ formulation.
- Powerful technique
 - Random Forest
 - Boosting
- Up to recently, in HEP, Boost Decision Trees (BDTs) on well constructed/chosen features
 - have been the best performing techniques, and
 - become the standard to beat,

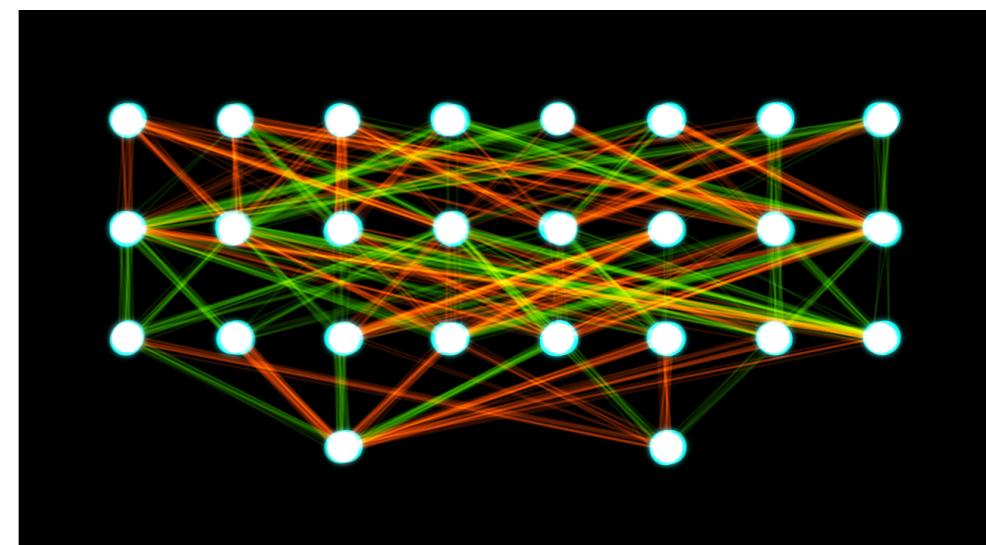
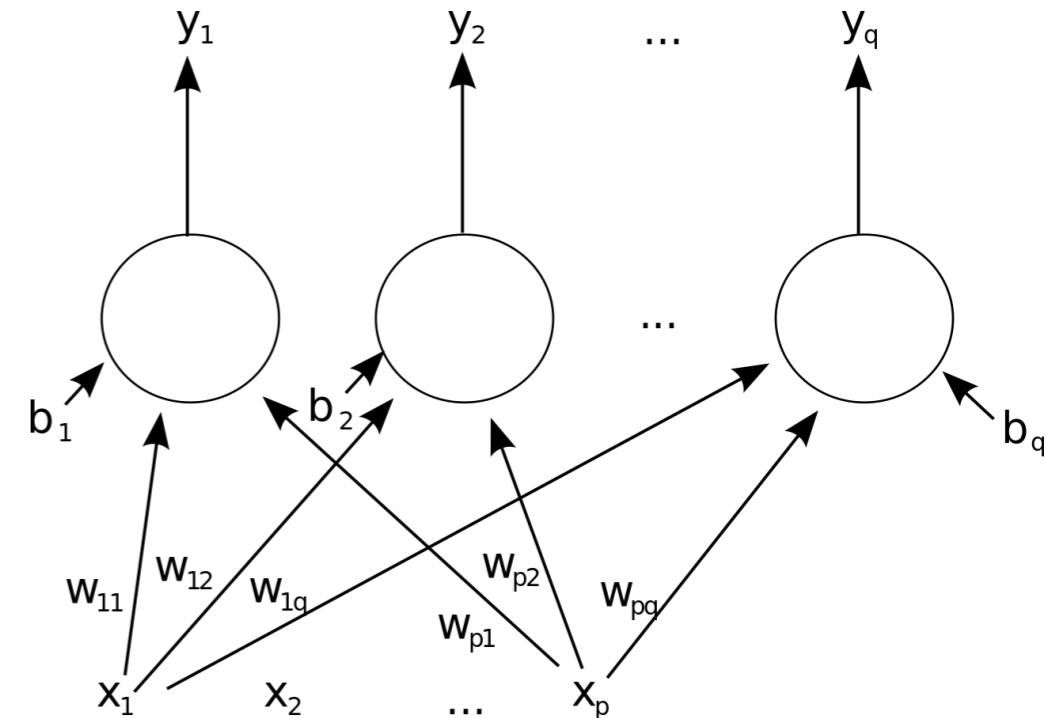


Titanic Survivors

Deep Learning

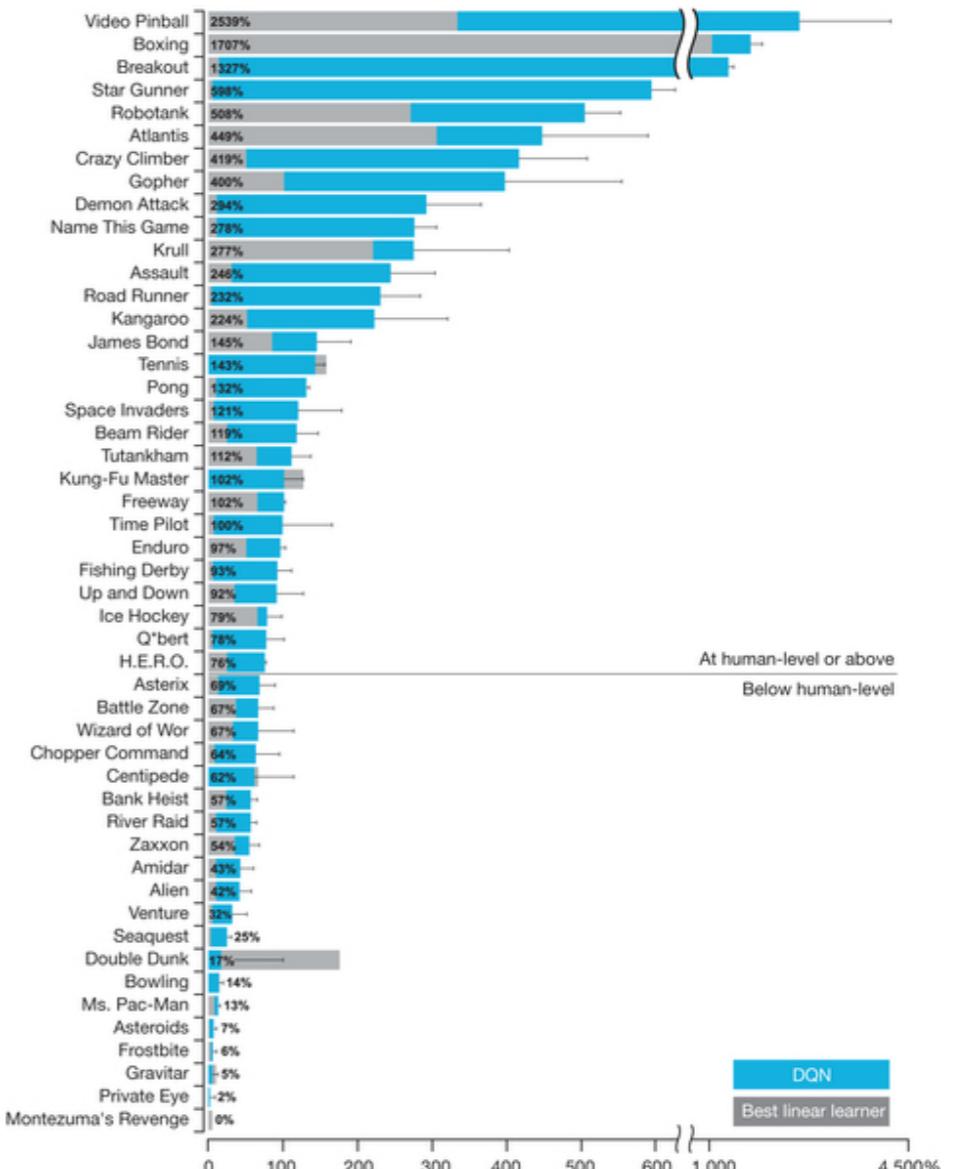
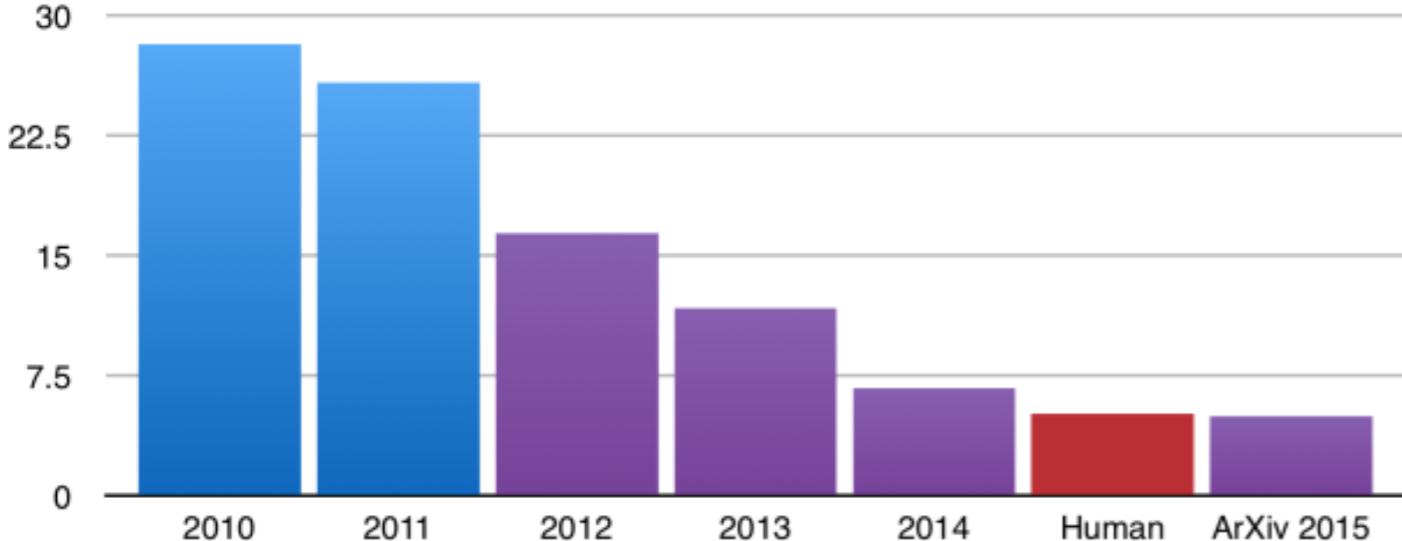
Artificial Neural Networks

- **Biologically inspired computation**, (first attempts in 1943)
 - **Probabilistic Inference**: e.g. signal vs background
 - **Universal Computation Theorem** (1989)
- Multi-layer (**Deep**) Neutral Networks:
 - Not a new idea (1965), just impractical to train. **Vanishing Gradient problem** (1991)
 - Solutions:
 - New techniques: e.g. better activation or layer-wise training
 - **More training**: big training datasets and lots of computation ... **big data and GPUs**
 - **Deep Learning Renaissance**. First DNN in HEP (2014).
 - **Amazing Feats**: Audio/Image/Video recognition, captioning, and generation. Text (sentiment) analysis. Language Translation. Video game playing agents.
 - **Rich field**: Variety of architectures, techniques, and applications.

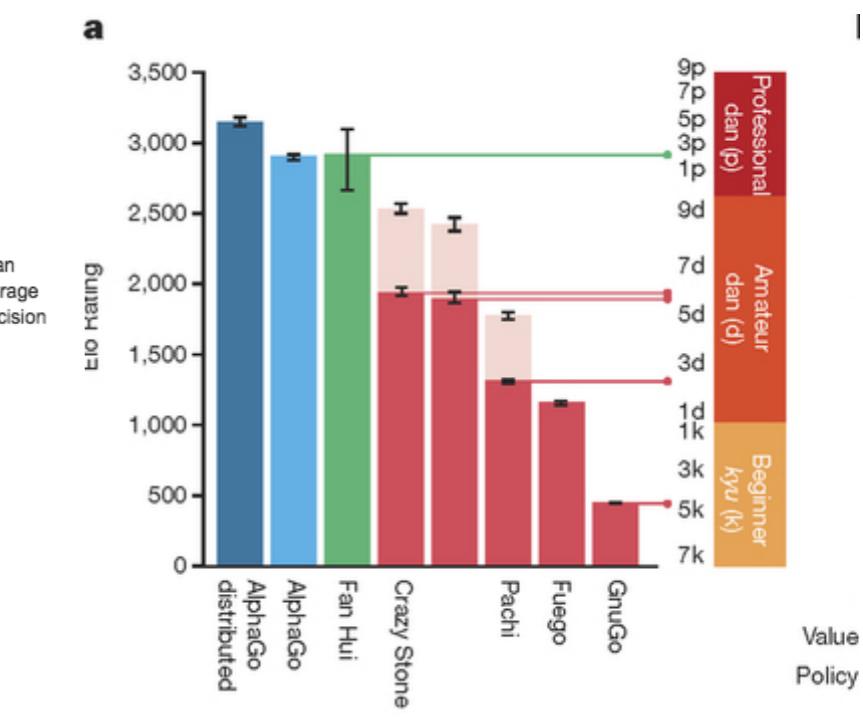
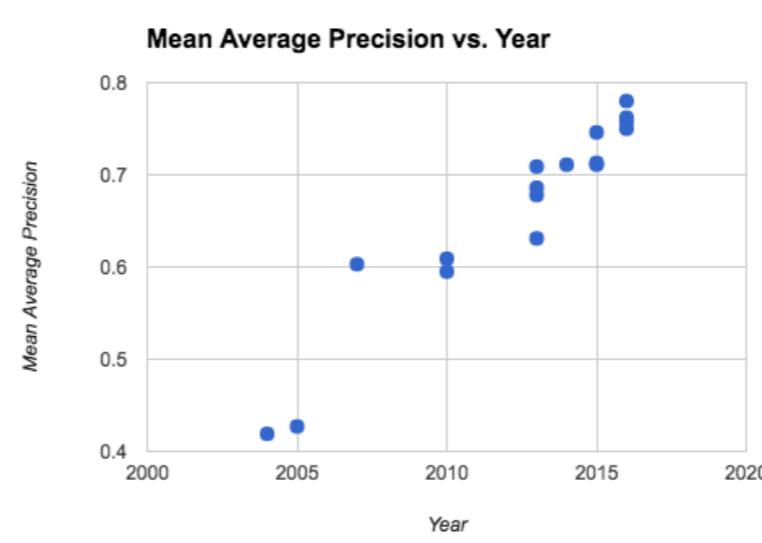
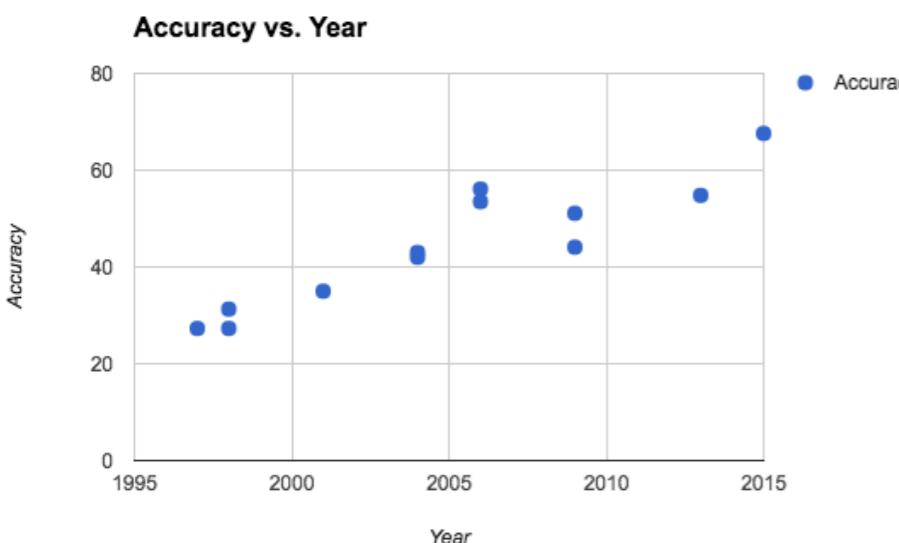
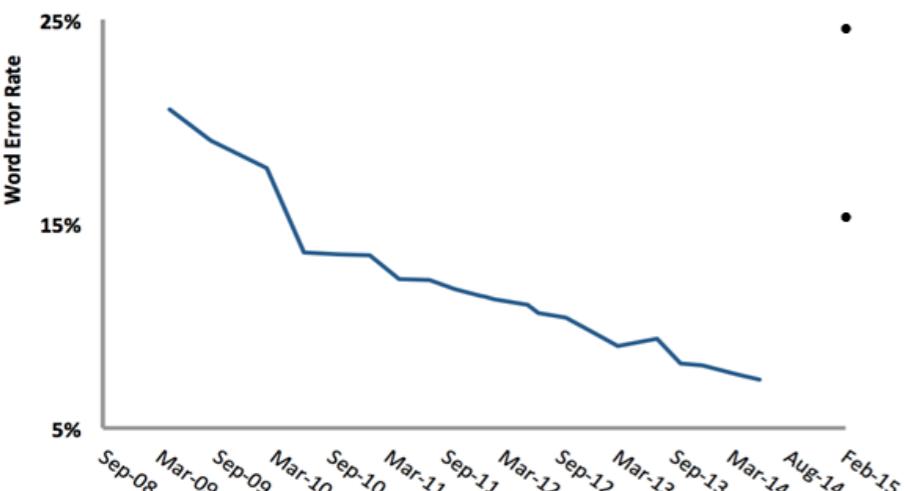


Images from Wikipedia

ILSVRC top-5 error on ImageNet

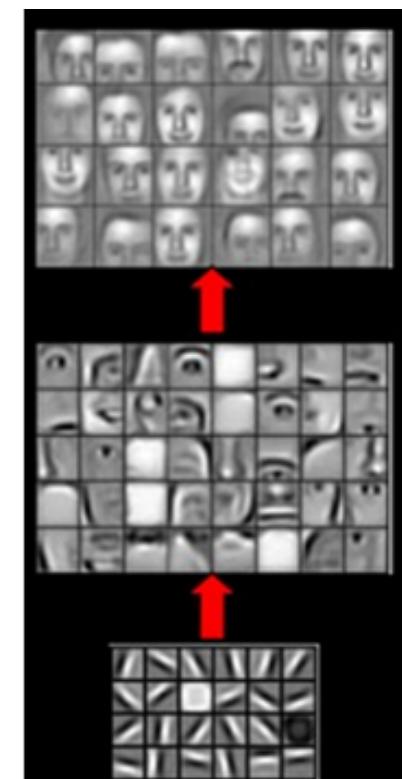
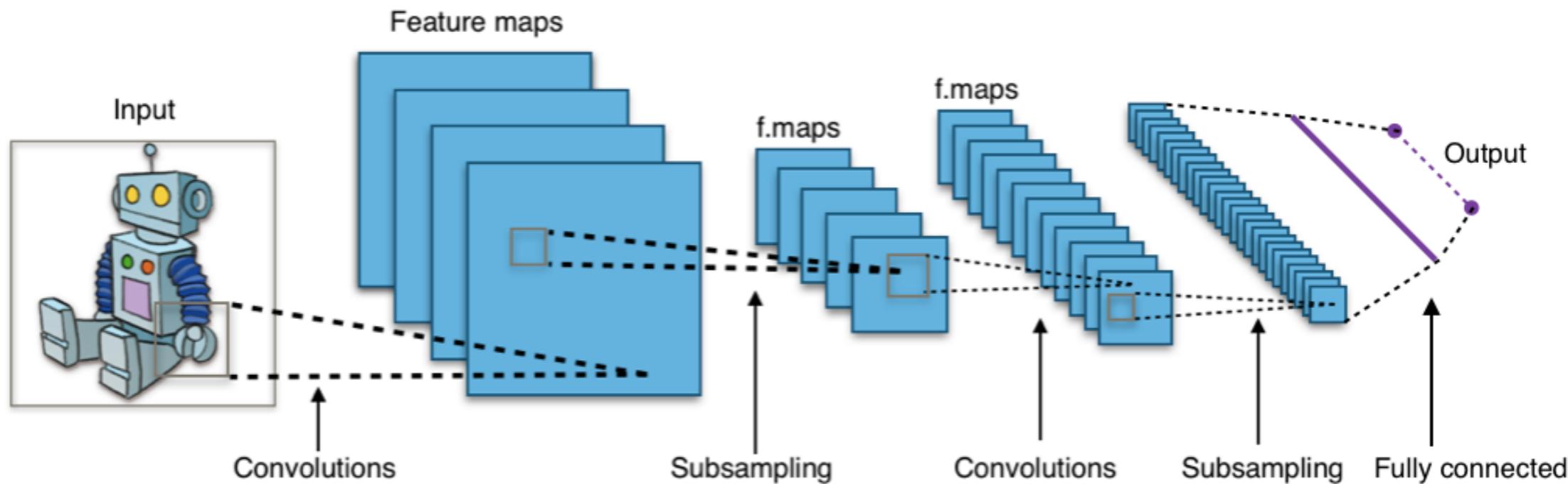


- Continuous server ASR word error rate (WER) reduction ~18% / year: combination of algorithms, data, and computing
- Deep learning (DNNs) is driving recent performance improvements in ASR and meaning extraction

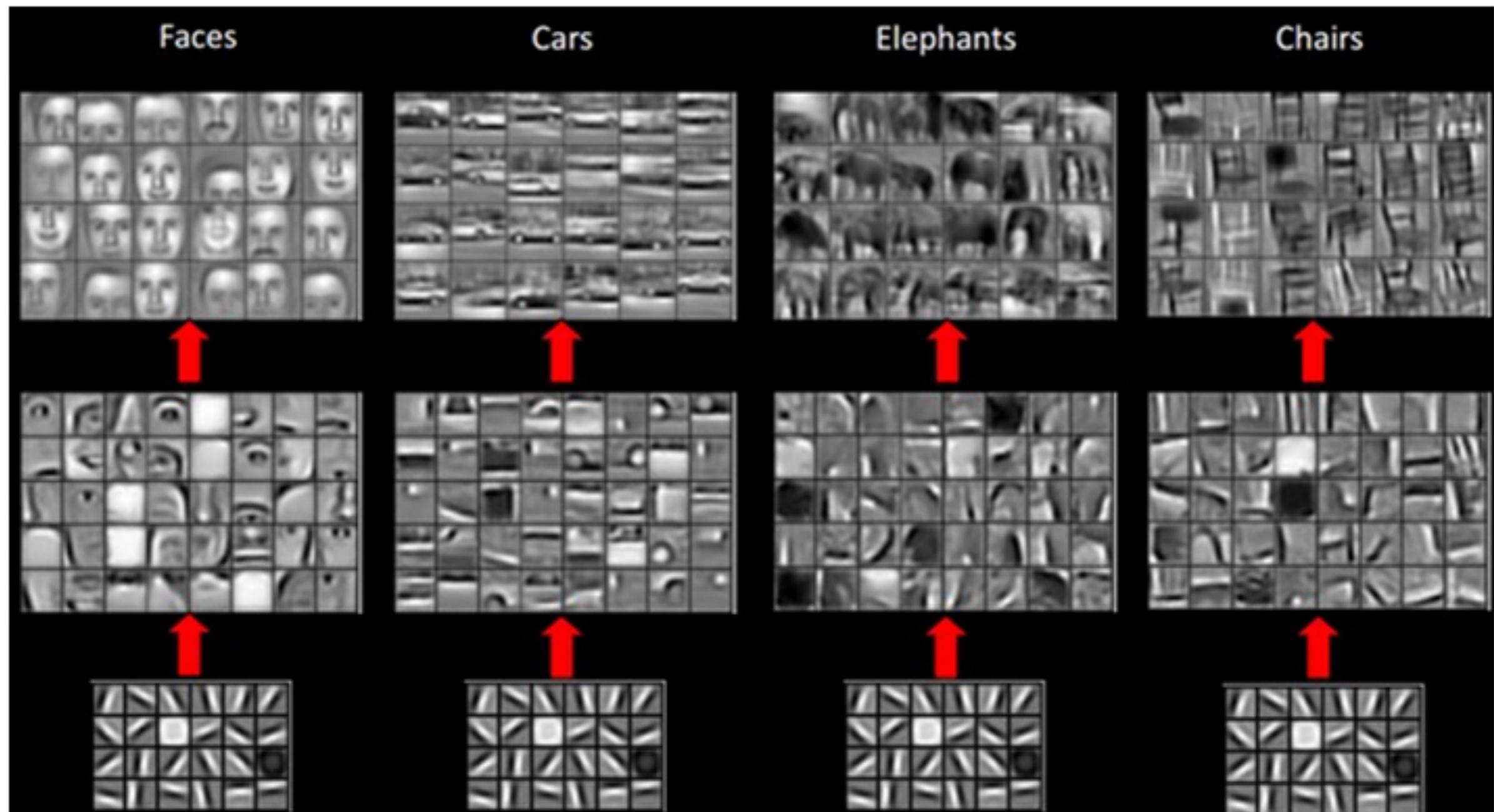


Feature Learning

- **Feature Engineering:** e.g. Event Reconstruction ~ Feature Extraction, Pattern Recognition, Fitting, ...
- Deep Neural Networks can **Learn Features** from **raw data**.
- Example: **Convolutional Neural Networks** - Inspired by visual cortex
 - **Input:** Raw data... for example 1D = Audio, 2D = Images, 3D = Video
 - **Convolutions** ~ learned feature detectors
 - **Feature Maps**
 - **Pooling** - dimension reduction / invariance
 - **Stack:** Deeper layers recognize higher level concepts.

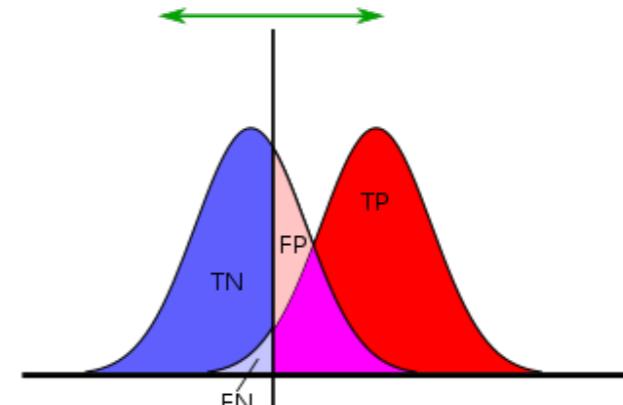
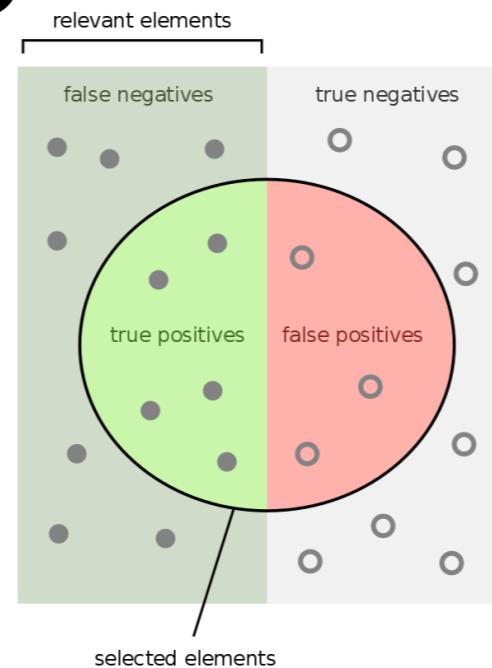


Deep Neutral Networks

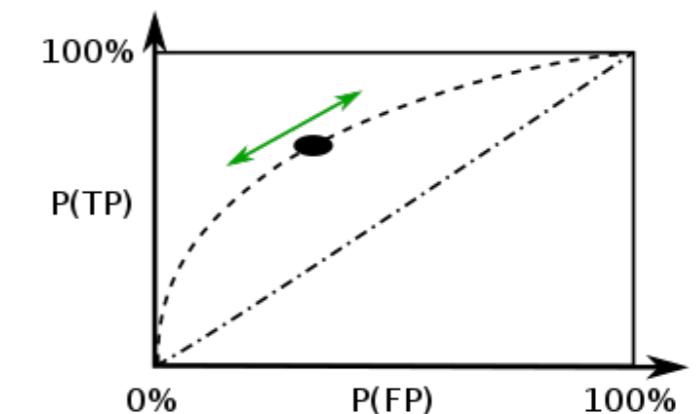


Assessing Performance

- **True Positive Rate (TPR):** Efficiency
- **False Positive Rate (FPR):** Background efficiency, 1/Background Rejection
- **Receiver Operator Curve (ROC):** TPR vs FPR
- **Area Under Curve (AUC):** Area under ROC



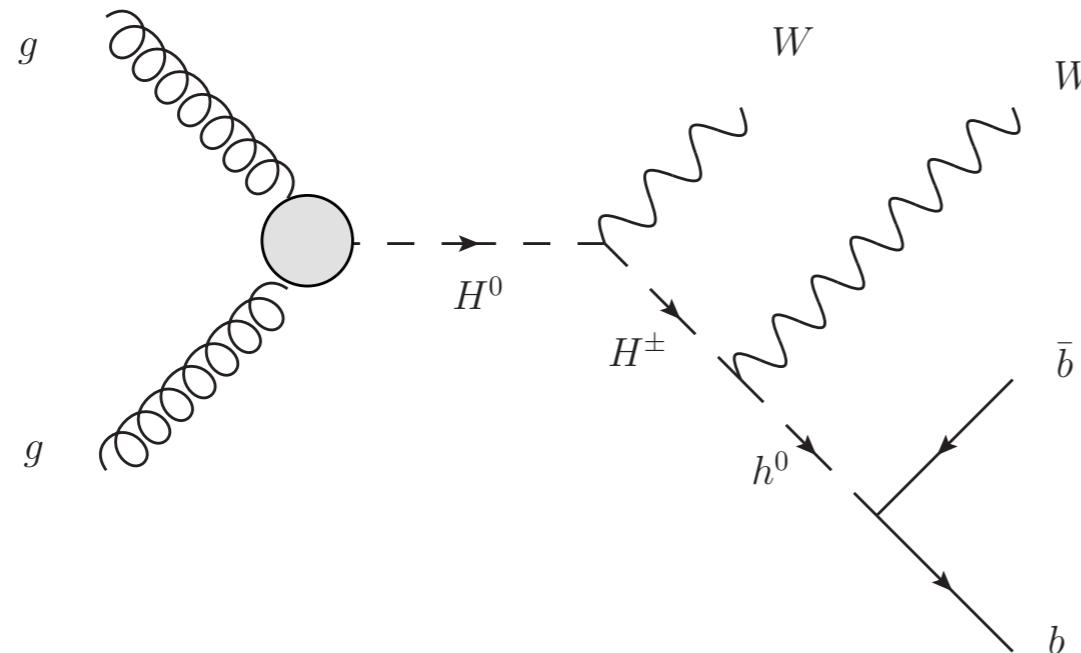
| | |
|----|----|
| TP | FP |
| FN | TN |



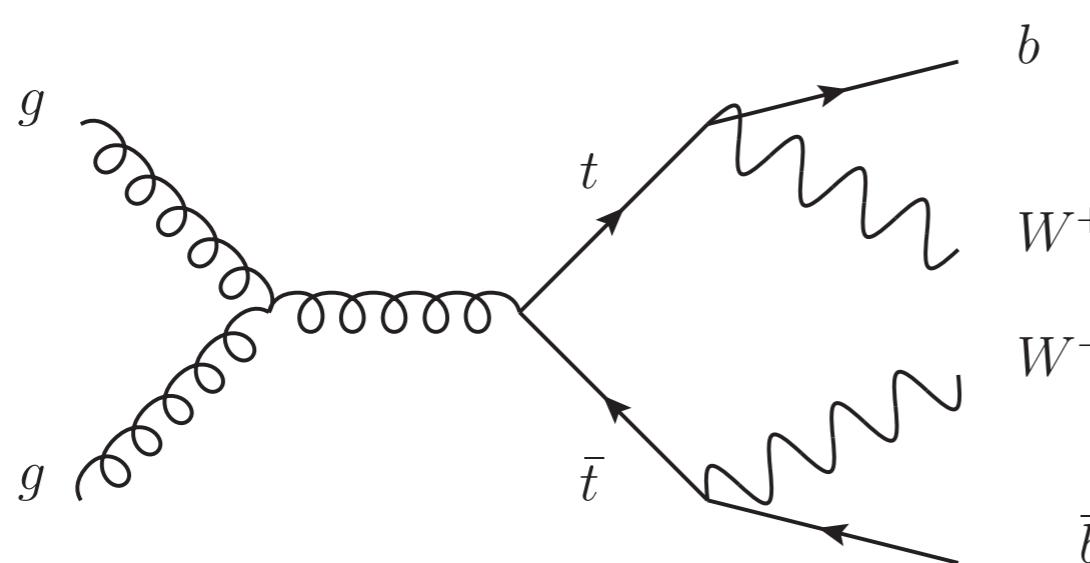
| | | True condition | | Prevalence = $\frac{\sum \text{Condition positive}}{\sum \text{Total population}}$ | Accuracy (ACC) = $\frac{\sum \text{True positive} + \sum \text{True negative}}{\sum \text{Total population}}$ |
|---------------------|------------------------------|---|---|---|--|
| | | Condition positive | Condition negative | | |
| Predicted condition | Predicted condition positive | True positive | False positive, Type I error | Positive predictive value (PPV), Precision = $\frac{\sum \text{True positive}}{\sum \text{Predicted condition positive}}$ | False discovery rate (FDR) = $\frac{\sum \text{False positive}}{\sum \text{Predicted condition positive}}$ |
| | Predicted condition negative | False negative, Type II error | True negative | False omission rate (FOR) = $\frac{\sum \text{False negative}}{\sum \text{Predicted condition negative}}$ | Negative predictive value (NPV) = $\frac{\sum \text{True negative}}{\sum \text{Predicted condition negative}}$ |
| | | True positive rate (TPR), Recall, Sensitivity, probability of detection, Power = $\frac{\sum \text{True positive}}{\sum \text{Condition positive}}$ | False positive rate (FPR), Fall-out, probability of false alarm = $\frac{\sum \text{False positive}}{\sum \text{Condition negative}}$ | Positive likelihood ratio (LR+) = $\frac{\text{TPR}}{\text{FPR}}$ | Diagnostic odds ratio (DOR) = $\frac{\text{LR+}}{\text{LR-}}$ |
| | | False negative rate (FNR), Miss rate = $\frac{\sum \text{False negative}}{\sum \text{Condition positive}}$ | Specificity (SPC), Selectivity, True negative rate (TNR) = $\frac{\sum \text{True negative}}{\sum \text{Condition negative}}$ | Negative likelihood ratio (LR-) = $\frac{\text{FNR}}{\text{TNR}}$ | |
| | | | | | F_1 score = $2 \cdot \frac{\text{Precision} \cdot \text{Recall}}{\text{Precision} + \text{Recall}}$ |

DEEP LEARNING IN HEP

Baldi, Sadowski, Whiteson
arxiv:1402.4735

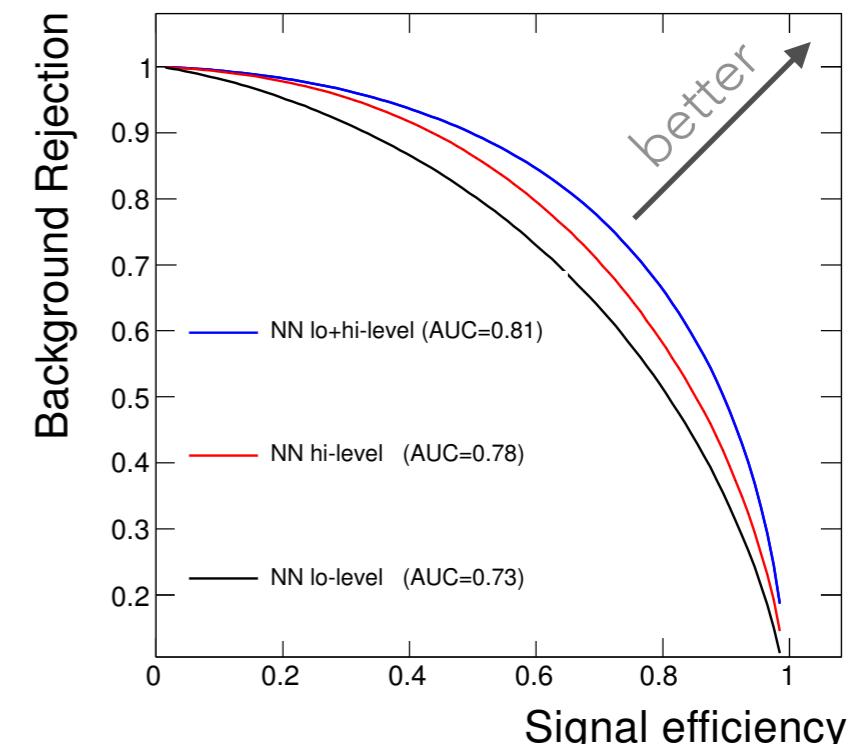


(a)

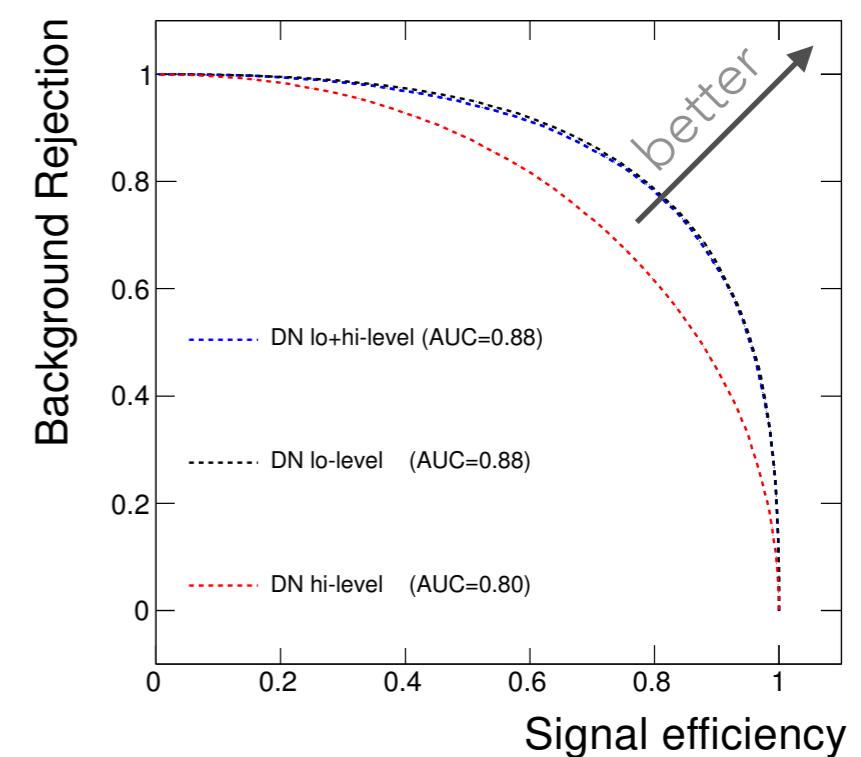


(b)

Slide from Kyle Cranmer



(a)



Data Representation

- Data are stored in “tensors”.
 - Basically an N- Dimensional Array with a “shape”
 - shape = (): Scalar
 - shape = (N,): Vector
 - shape = (N,M): Matrix
 - shape = (N₁, N₂, N₃, ..., N_R): Rank R Tensor
 - Inputs: **X**
 - Can be arbitrary shape. Typically first dimension is the example index (usually an “event” or collision in HEP)
 - Example: Let’s say your examples are students, and your data is their age, sex, years at University, undergrad/grad, and department
 - X = [[20, 0, 2, 0, 4] , # 20 year old, 0=male, 2=junior, 0=undergrad, 4=computer science
[25, 1, 2, 1, 3] , # 25 year old, 1=female, 2=3rd year, 0=grad, 4=physics
[23, 0, 0, 1, 3]] # 25 year old, 1=make, 2=1st year, 0=grad, 4=physics
 - X[0] = [20, 0, 2, 0, 4]: the first student’s data.
 - X[0][3] = 1. This is a graduate student
 - Outputs : **Y**
 - Can be arbitrary shape. Typically first dimension is the example index (usually an “event” or collision in HEP)
 - Example: Y = 0/1, student does not / does know python

Machine Learning Problem Formulation

- Split *Datasets*:
 - $(\mathbf{X}_{\text{train}}, \mathbf{Y}_{\text{train}})$ = training dataset
 - $(\mathbf{X}_{\text{test}}, \mathbf{Y}_{\text{test}})$ = test dataset
 - (\mathbf{X}) = unlabeled data
- Set *Goal*:
 - *Inference* algorithm/function $F(\mathbf{X} | \mathbf{a}) = \mathbf{Y}_{\text{predict}}$.
 - F can be a heuristic. e.g. if (computer science student) then (student knows python).
 - F can be anything
 - \mathbf{a} are parameters of the function, for Neural Networks, these are weights.
 - Note that in a simple classification problem, $\mathbf{Y}_{\text{train}}$ can be 0 or 1 for any example. But $\mathbf{Y}_{\text{predict}}$ will usually be between 0 and 1.
- *Training*: (for Neural Networks)
 - Optimize (usually a minimization) a *cost function* $F(\mathbf{X} | \mathbf{a}) = C(F(\mathbf{X}_{\text{train}} | \mathbf{a}), \mathbf{Y}_{\text{train}})$ w.r.t. \mathbf{a}
 - For example, $C = [F(\mathbf{X} | \mathbf{a}) - \mathbf{Y}_{\text{train}}]^2$
 - $\mathbf{a}_{\text{trained}}$ = result of training
- Validation:
 - Compute cost function on test data $C(F(\mathbf{X}_{\text{test}} | \mathbf{a}_{\text{trained}}), \mathbf{Y}_{\text{test}})$
 - Other metrics. For example:
 - Select $\mathbf{Y}_{\text{test}}=1$ and see how often $F(\mathbf{X}_{\text{test}} | \mathbf{a}_{\text{trained}}) > 0.5$
- Inference:
 - $\mathbf{Y}_{\text{predict}} = F(\mathbf{X} | \mathbf{a}_{\text{trained}})$