

Your Name: _____

P1 (10 points) An investor has a total budget of B dollars and is considering n possible investments.

1. Investment I_j has a cost c_j and yields a profit p_j .
2. For each investment I_j , the investor can either choose to buy (a single unit of) I_j or choose not to.
3. The total cost of investments cannot exceed the budget.
4. The investor seeks to maximize her profit.

You are given as an input the budget B and the list of n investments with cost and profits.

Investment ID	Cost	Profit
1	c_1	p_1
2	c_2	p_2
\vdots	\vdots	\vdots
n	c_n	p_n

Let $\text{PROFIT}(j, b)$ denote the maximum profit that can be made by considering the first j investments with a budget of b .

(A) What is the value of $\text{PROFIT}(0, b)$? 0

(B) What is the value of $\text{PROFIT}(j, 0)$ for arbitrary j ? 0

(C) What is the value of $\text{PROFIT}(j, b)$ where $b < 0$? $-\infty$

(D) Consider the recurrence for $\text{PROFIT}(j, b)$ where $j \geq 1$ and $0 < b \leq B$.

$$\text{PROFIT}(j, b) : \begin{cases} \text{?}_2 + \text{PROFIT}(j-1, \text{?}_3) & \# \text{ Investor chooses to investment } j \\ \text{?}_4 + \text{PROFIT}(j-1, b) & \# \text{ Investor does not choose investment } j \end{cases}$$

Write down appropriate values for $\text{?}_1, \dots, \text{?}_4$.

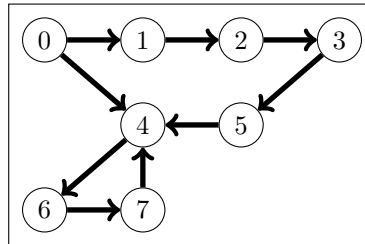
$$\text{PROFIT}(j, b) : \max \begin{cases} p_j + \text{PROFIT}(j-1, b - c_j) & \# \text{ Investor chooses to investment } j \\ 0 + \text{PROFIT}(j-1, b) & \# \text{ Investor does not choose investment } j \end{cases}$$

(E) What values of j, b would you choose for $\text{PROFIT}(j, b)$ to solve the original problem with n investments and budget B ?

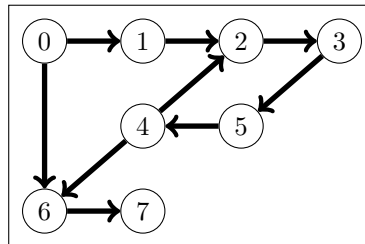
$\text{PROFIT}(n, B)$

P2 (10 points) Consider a DFS for the graph below starting from the node 0. For each node, the adjacent nodes are considered in the increasing order of their IDs.

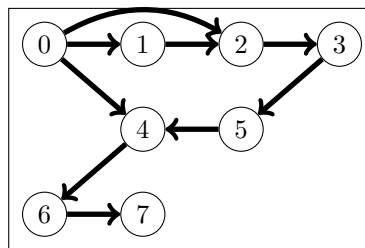
A



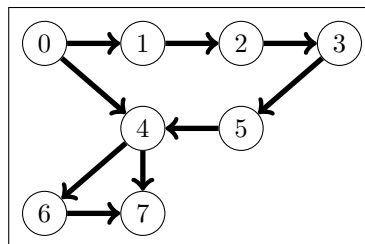
B



C



D



Fill out the table below with the discovery, finish times and parent in the DFS tree for each node. Assume that discovery time for node 0 is 1, as filled out already.

Node	Discovery Time	Finish Time	Parent
0	1	16	NIL
1	2	15	0
2	3	14	1
3	4	13	2
4	6	11	5
5	5	12	3
6	7	10	4
7	8	9	6

Does DFS discover a back edge? If yes, write down the back edge. Otherwise, write down “NO”.

A: Yes, (7, 4) B: Yes, (4, 2) C, D: No!