

CSCI 3104-Spring 2016: Assignment #2.

Assigned date: Monday 1/25/2016,

Due date: Thursday, 2/4/2016, before class

Maximum Points: 50 points (includes 5 points for legibility).

Note: This assignment *must be turned in on paper, before class*. Please do not email: it is very hard for us to keep track of email submissions. Further instructions are on the moodle page.

P1 (Divide and Conquer Maxmin, 25 points) We wish to implement an algorithm to solve the following problem:

Input: An array a of size n .

Output: \max , \min the maximum and minimum numbers, respectively in array a .

(A) Write the pseudocode for an algorithm that runs in $\Theta(n)$ time (by scanning the array from left to right).

(B) Consider a divide and conquer scheme:

1. Split the array into two equal parts.
2. Recursively compute the maximum and minimum of each part.
3. Combine the results to find maximum and minimum of the original array.

Write the pseudocode of the divide and conquer scheme using recursion. In doing so, also specify the base case.

(C) Write the recurrence relation $T(n)$ that characterizes the running time of your divide and conquer scheme.

(D) Solve $T(n)$ either by expanding or using the master method to derive a Θ bound.

P2 (Divide and Conquer Majority, 25 points) An element e is a majority element in an array a of size n if and only if it occurs $\lceil \frac{n}{2} \rceil + 1$ or more times.

We wish to implement an algorithm to solve the following problem:

Input: An array a of size n .

Output: A number e if it is the majority element of array a and NONE if a has no majority element.

We use the following divide and conquer scheme:

1. Split a into two equal parts.
2. Recursively find whether a majority element exists for each part or NONE.
3. Combine the results of the recursive calls.

(A) Suppose array \mathbf{a} of size n is divided into two parts $\mathbf{a1}$ and $\mathbf{a2}$. Let $\mathbf{e1}$ and $\mathbf{e2}$ be the majority elements of $\mathbf{a1}$ and $\mathbf{a2}$, respectively. What is the majority element of the overall array \mathbf{a} ?

(B) Write the pseudocode for the divide and conquer algorithm above. Pay special attention to the base case.

(C) Derive and solve the recurrence for the running time $T(n)$.