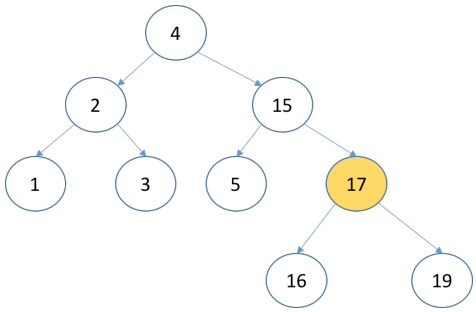
CSCI 3104-Spring 2016: Assignment #5.

Assigned date: Monday, 2/29/2016,
Due date: Tuesday, 3/8/2016, before class
Maximum Points: 50 points (includes 5 points for legibility).

Note: This assignment *must be turned in on paper, before class.* Please do not email: it is very hard for us to keep track of email submissions. Further instructions are on the moodle page.

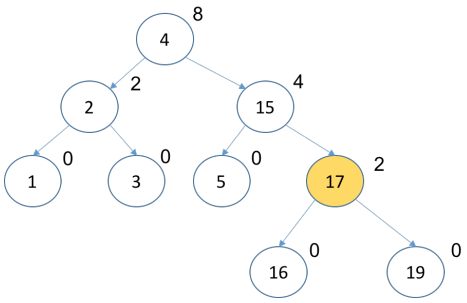
P1 (20 points) You are given binary search tree (BST) T with n nodes with depth $d \leq 2\log_2(n)$. For each node x, its key is denoted $x.key$.
(A) Write an algorithm given a BST T and a node x, to counts how many nodes have keys strictly less than that of node $x.key$. Assume all nodes in the tree have unique keys.

Example: For the tree below, consider the node with key 17.



There are 7 keys that are less than 17.

(B) The tree T is now *augmented* by adding for each node x a field $x.s$ that denotes the number of nodes in the subtree below it. The value of $x.s$ for each node s is shown alongside the node in the picture below.



Provide an algorithm for solving the problem in **(A)** assuming that the tree is augmented. Your algorithm should now run in time proportional to the depth of the node x.
P2 (15 points) We are given a list of intervals $[I_1, \ldots, I_n]$ that denote reserved times for a room. The times start at $t = 0$ and end at $t = M$ for a fixed number M.

Each interval I_i is of the form (ℓ_i, u_i) where $0 \leq \ell_i < u_i \leq M$.

Given the list of intervals provide all the time intervals for which the room is free. Your algorithm must run in time $\Theta(n \log(n))$ or less. Write down the high level steps of your algorithm **and** provide pseudocode.
Example: Inputs $M = 100, n = 6$ and intervals are

$$[I_1 : (0, 10), I_2 : (8, 9), I_3 : (9, 15), I_4 : (17, 25), I_5 : (25, 35), I_6 : (75, 100)$$

Your output free times should be

$$J_1 : (15, 17), J_2 : (35, 75)$$

Hint: Sort the intervals by the finish times.
P3 (15 points) Office hours will take place between time $T = 0$ to $T = 60\mathsf{mins}$ and all students can potentially attend. Each student S_i writes to the professor in advance by specifying a time interval (ℓ_i, u_i) when he/she will attend office hours. Note that $0 \leq \ell_i < u_i \leq 60$.

Given the time intervals for the n students $I_1, \ldots, I_n$, find the maximum number of students who will be at the office hours at any point in time. Your algorithm should run in time $\Theta(n \log_2(n))$.
Example: Inputs $n = 5$ students and intevals are

$$I_1 : (0, 15), I_2 : (15, 20), I_3 : (0, 22), I_4 : (19, 20), I_5 : (21, 40)$$

The maximum number of students at any time instant is 3. If a student i arrives at time t, and another student j leaves at the same time t, then we assume that the leaving happens before the arrival.