📘 Functions and Graphs - Teaching Dataset

🧠 Prerequisite Knowledge

- Understanding of variables and expressions
- Basic algebraic manipulation (solving linear equations)
- Familiarity with coordinate system (conceptual only)

🎯 Learning Objectives

- Understand what a function is
- Learn how to determine the domain and range
- Recognize types of functions (linear, quadratic, etc.)
- Learn function notation and evaluation
- Understand increasing/decreasing behaviour

📌 Topic Breakdown

1. 🔍 What is a Function?

A function is a rule that assigns each input exactly one output.

Example: If $f(x) = x + 3$, then:

- $f(2) = 5$
- $f(-1) = 2$

Explain: Think of a function as a machine: You put in a number (input), it performs a rule, and gives back an output.

2. ✍️ Function Notation

- $f(x)$ is read as "f of x"
- It means the function f with input x

Example: If $f(x) = 2x - 4$, then:

- $f(3) = 2(3) - 4 = 2$
- $f(-1) = 2(-1) - 4 = -6$

3. 🌐 Domain and Range

- Domain: All valid inputs (x-values)
- Range: All possible outputs (f(x)-values)

Example: For $f(x) = x^2$:

- Domain: All real numbers
- Range: $f(x) \geq 0$

4. 🧱 Types of Functions

🔹 Linear Functions

Form: $f(x) = mx + b$

- Straight line
- Constant rate of change (slope = m)

Example: $f(x) = 2x + 1$

🔹 Quadratic Functions

Form: $f(x) = ax^2 + bx + c$

- Parabolic shape (U-shaped curve)
- Has a vertex and an axis of symmetry

Example: $f(x) = x^2 - 4$

🔹 Constant Functions

Form: $f(x) = c$

- Always returns the same output

5. 📈 Increasing vs Decreasing

- A function is increasing when the output rises as x increases.
- A function is decreasing when the output drops as x increases.

Example:

- $f(x) = x^2$ is decreasing on $(-\infty, 0]$ and increasing on $[0, \infty)$

6. 🛠️ Practice Problems (With Steps)

Q1: Evaluate $f(x) = 3x - 2$ at $x = 4$

Answer: $f(4) = 3(4) - 2 = 10$

Q2: What is the domain of (f(x) = \frac{1}{x-2})?

Answer: All real numbers except (x = 2)

Q3: Identify type: (f(x) = -x^2 + 5x + 2)

Answer: Quadratic Function

Q4: Is (f(x) = 2x + 3) increasing or decreasing?

Answer: Increasing (positive slope)

🎓 Teaching Strategy

- Begin by explaining functions as real-world input-output machines
- Use lots of everyday examples: vending machines, formulas, apps
- Break into segments: input/output, notation, domain/range
- Relate to previous algebra skills
- Reinforce with small concept-check questions

📚 Real-Life Applications

- Cost functions in economics (e.g., total cost = fixed + variable)
- Converting temperature (°F = 1.8 × °C + 32)
- Predictive models in statistics