# Algebra Instructional Dataset (For AI Math Tutor)

## 🧠 Prerequisites

- Basic arithmetic (addition, subtraction, multiplication, division)
- Understanding of numbers: integers, fractions, decimals
- Understanding of mathematical operations and order of operations (PEMDAS)

## 🎯 Why Learn Algebra?

Algebra helps you solve problems where numbers are unknown. It's used in computer science, engineering, economics, and everyday problem-solving. For example, calculating the total cost of groceries, analyzing phone bills, or determining how long a trip will take.

## 📚 Core Concepts

### 1. Variables and Constants

- Variable: A symbol (usually a letter) that represents an unknown value.
- Constant: A fixed value (e.g., 3, -7, 2.5).

### 2. Expressions

- Combination of variables, constants, and operators (e.g., $3x + 2$, $a^2 - 5a + 6$).
- You cannot "solve" an expression, but you can simplify or evaluate it.

### 3. Equations

- Equation: A statement that two expressions are equal (e.g., $2x + 3 = 9$).
- Solving an equation means finding the value of the variable(s) that make the equation true.

### 4. Linear Equations

- Form: $ax + b = 0$
- Example: $2x + 4 = 10 \rightarrow x = 3$
- Graph: A straight line

### 5. Quadratic Equations

- Form: $ax^2 + bx + c = 0$
- Solving methods:

  Factoring

Quadratic Formula

Completing the square

## 6. Factorization

- Rewriting an expression as a product of its factors
- Example: $x^2 + 5x + 6 = (x + 2)(x + 3)$

# 🧑‍🏫 Teaching Flow (AI System Prompt Guide)

### Step 1: Connect to theStudent

"Let's explore Algebra together! It's like solving puzzles using math. First, let me check — do you understand what a variable is?"

### Step 2: Explain with Examples

"Think of x as a box that hides a number. If I say x + 3 = 5, we're asking: 'What number plus 3 gives 5?' The answer is 2. So, x = 2."

### Step 3: Explore Expressions

"Can you simplify the expression 3x + 2x? Yes, it becomes 5x because you're adding like terms."

### Step 4: Solve Equations Step-by-Step

"Let's solve 2x + 4 = 10:

Subtract 4 from both sides → 2x = 6

Divide both sides by 2 → x = 3"

### Step 5: Practice Factorization

"Try factoring $x^2 + 7x + 10$. Which two numbers multiply to 10 and add to 7?"

### Step 6: Apply Quadratic Formula

"For any equation in form $ax^2 + bx + c = 0$, use this formula:

[ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ]

### Step 7: Real-Life Application

"You earn $10 per hour. If you want to earn $150, how many hours must you work? Set up an equation: 10x = 150. What's x?"

## ❗ Common Mistakes

- Forgetting to perform the same operation on both sides of the equation
- Mixing up signs (especially during subtraction and multiplication)
- Incorrectly simplifying expressions (e.g., $3x + 2 \neq 5x$)
- Factoring errors (e.g., wrong pair of factors)

## 🔑 Tips and Strategies

- Always isolate the variable
- Check your solution by plugging it back into the equation
- Use the discriminant $b^2 - 4ac$ to determine the nature of quadratic roots
- Practice mental math and simple operations

## 🧪 Practice Problems

**Linear Equations:**

1. Solve: $3x + 5 = 14$
2. Solve: $7x - 4 = 3x + 8$

**Expressions:**

3. Simplify: $5x - 2x + 7$
4. Evaluate: $2a + 3b$ for $a = 2$, $b = 3$

**Quadratic Equations:**

5. Factor: $x^2 + 6x + 8$
6. Solve using the quadratic formula: $x^2 - 3x - 10 = 0$

## 🧠 Reflection Questions

- What is the difference between an expression and an equation?
- Why is factoring useful in solving quadratic equations?
- How can we use algebra in real life?