

## 1. Dataset Format

You already started this right — for each topic (like Trigonometry), create markdown datasets with:

- Topic Overview
- Prerequisites & links to them
- Real-world applications
- Concepts explained in steps
- Formulas with derivations
- Guided problems (easy ➡ challenging)
- Common mistakes
- Mini quizzes or checks for understand

# Algebra Instructional Dataset (For AI Math Tutor)

## Prerequisites

- Basic arithmetic (addition, subtraction, multiplication, division)
- Understanding of numbers: integers, fractions, decimals
- Understanding of mathematical operations and order of operations (PEMDAS)

## Why Learn Algebra?

Algebra helps you solve problems where numbers are unknown. It's used in computer science, engineering, economics, and everyday problem-solving. For example, calculating the total cost of groceries, analyzing phone bills, or determining how long a trip will take.

## Core Concepts

### 1. Variables and Constants

- Variable: A symbol (usually a letter) that represents an unknown value.
- Constant: A fixed value (e.g., 3, -7, 2.5).

## 2. Expressions

- Combination of variables, constants, and operators (e.g.,  $3x + 2$ ,  $a^2 - 5a + 6$ ).
- You cannot "solve" an expression, but you can simplify or evaluate it.

## 3. Equations

- Equation: A statement that two expressions are equal (e.g.,  $2x + 3 = 9$ ).
- Solving an equation means finding the value of the variable(s) that make the equation true.

## 4. Linear Equations

- Form:  $ax + b = 0$
- Example:  $2x + 4 = 10 \rightarrow x = 3$
- Graph: A straight line

## 5. Quadratic Equations

- Form:  $ax^2 + bx + c = 0$
- Solving methods:

Factoring

Quadratic Formula

Completing the square

## 6. Factorization

- Rewriting an expression as a product of its factors
- Example:  $x^2 + 5x + 6 = (x + 2)(x + 3)$



# Teaching Flow (AI System Prompt Guide)

## Step 1: Connect to the Student

"Let's explore Algebra together! It's like solving puzzles using math. First, let me check — do you understand what a variable is?"

## Step 2: Explain with Examples

"Think of  $x$  as a box that hides a number. If I say  $x + 3 = 5$ , we're asking: 'What number plus 3 gives 5?' The answer is 2. So,  $x = 2$ ."

### Step 3: Explore Expressions

"Can you simplify the expression  $3x + 2x$ ? Yes, it becomes  $5x$  because you're adding like terms."

### Step 4: Solve Equations Step-by-Step

"Let's solve  $2x + 4 = 10$ :

Subtract 4 from both sides  $\rightarrow 2x = 6$

Divide both sides by 2  $\rightarrow x = 3$ "

### Step 5: Practice Factorization

"Try factoring  $x^2 + 7x + 10$ . Which two numbers multiply to 10 and add to 7?"

### Step 6: Apply Quadratic Formula

"For any equation in form  $ax^2 + bx + c = 0$ , use this formula:

$$[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} ]$$

### Step 7: Real-Life Application

"You earn \$10 per hour. If you want to earn \$150, how many hours must you work? Set up an equation:  $10x = 150$ . What's  $x$ ?"

## ! Common Mistakes

- Forgetting to perform the same operation on both sides of the equation
- Mixing up signs (especially during subtraction and multiplication)
- Incorrectly simplifying expressions (e.g.,  $3x + 2 \neq 5x$ )
- Factoring errors (e.g., wrong pair of factors)

## Tips and Strategies

- Always isolate the variable
- Check your solution by plugging it back into the equation
- Use the discriminant  $b^2 - 4ac$  to determine the nature of quadratic roots
- Practice mental math and simple operations

## Practice Problems

**Linear Equations:**

1. Solve:  $3x + 5 = 14$
2. Solve:  $7x - 4 = 3x + 8$

### **Expressions:**

3. Simplify:  $5x - 2x + 7$
4. Evaluate:  $2a + 3b$  for  $a = 2$ ,  $b = 3$

### **Quadratic Equations:**

5. Factor:  $x^2 + 6x + 8$
6. Solve using the quadratic formula:  $x^2 - 3x - 10 = 0$



### **Reflection Questions**

- What is the difference between an expression and an equation?
- Why is factoring useful in solving quadratic equations?
- How can we use algebra in real life?



## **Geometry Instructional Dataset**



### **Prerequisite Knowledge**

Before learning Geometry, students should be familiar with:

- Basic arithmetic (addition, subtraction, multiplication, division)
- Understanding of numbers and operations
- Simple algebraic concepts like variables and expressions

## Topic Overview

Geometry is the branch of mathematics that deals with the study of shapes, sizes, relative positions of figures, and the properties of space. It is used in everyday life through architectural designs, engineering structures, and even in nature.

## Learning Objectives

By the end of this topic, students should be able to:

- Understand different types of shapes and their properties
- Use formulas to calculate perimeter, area, and volume
- Identify angles and apply angle theorems
- Understand coordinate geometry and graph geometric figures

## Concept 1: Points, Lines, and Angles

### Subtopics:

- Point, Line, Line Segment, Ray
- Types of Angles (Acute, Right, Obtuse, Straight)
- Complementary and Supplementary Angles

### Key Formulas & Facts:

- Complementary angles sum to  $90^\circ$
- Supplementary angles sum to  $180^\circ$

### How a Teacher Teaches This:

"Let's start with the basics of geometry—points and lines. A point represents a location. A line extends in both directions infinitely. Then we move on to angles. What happens when two lines meet? They form angles. Can anyone tell me what we call two angles that add up to 90 degrees?"

### Sample Problems:

1. Find the complement of a  $40^\circ$  angle.
2. Two angles are supplementary. One is  $70^\circ$ . Find the other.

## Concept 2: Triangles

### Subtopics:

- Types (Equilateral, Isosceles, Scalene)
- Pythagorean Theorem

- Triangle Inequality Theorem
- Area of a Triangle

#### **Key Formulas & Facts:**

- Area =  $\frac{1}{2} \times \text{base} \times \text{height}$
- $a^2 + b^2 = c^2$  (Right triangle)
- The sum of the angles in a triangle =  $180^\circ$

#### **How a Teacher Teaches This:**

"Imagine cutting a sandwich diagonally. What shape do you get? A triangle! Triangles are everywhere. Let's talk about the special rules of triangles, like the Pythagorean theorem. We'll try this with numbers to see how it works."

#### **Sample Problems:**

1. Find the area of a triangle with a base of 10 cm and a height of 5 cm.
2. A right triangle has legs of 3 cm and 4 cm. Find the hypotenuse.

### **Concept 3: Quadrilaterals**

#### **Subtopics:**

- Parallelogram, Square, Rectangle, Rhombus, Trapezium
- Properties of each
- Area and Perimeter

#### **Key Formulas & Facts:**

- Area of a rectangle = length  $\times$  width
- Area of square =  $\text{side}^2$
- Perimeter = the sum of all sides

#### **How a Teacher Teaches This:**

"Let's draw some 4-sided figures! Each has its own special properties. A square has equal sides and all right angles. A trapezium has one pair of parallel sides. Can you identify them in the room around you?"

#### **Sample Problems:**

1. Find the area of a square with a side of 6 cm.
2. A rectangle has a length of 8 cm and a width of 3 cm. Find the perimeter.

### **Concept 4: Circles**

### Subtopics:

- Radius, Diameter, Circumference
- Area of a Circle
- Arcs and Sectors

### Key Formulas & Facts:

- Circumference =  $2\pi r$
- Area =  $\pi r^2$
- Diameter =  $2r$

### How a Teacher Teaches This:

"Have you ever drawn a perfect circle with a compass? That's geometry in action! Circles are all about the radius and diameter. Let's use the formulas to determine how far you would walk around a circular garden."

### Sample Problems:

1. Find the circumference of a circle with a radius of 7 cm.
2. What is the area of a circle with a diameter of 10 cm?

## Concept 5: Coordinate Geometry

### Subtopics:

- Cartesian Plane
- Distance Formula
- Midpoint Formula
- Plotting Shapes

### Key Formulas & Facts:

- Distance =  $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Midpoint =  $((x_1 + x_2)/2, (y_1 + y_2)/2)$

### How a Teacher Teaches This:

"Let's plot points on a grid! The x-axis goes left and right; the y-axis goes up and down. If you have two points, how far apart are they? Let's use the distance formula to calculate it."

### Sample Problems:

1. Find the distance between (1,2) and (4,6).
2. Find the midpoint of points (3,4) and (7,8).

## Summary Questions for Practice

1. Name two types of quadrilaterals and list one property of each.
2. What's the difference between radius and diameter?
3. Plot a triangle on a coordinate grid and calculate its area.

## Next Topic Recommendation

If the student understands Geometry, suggest moving on to:

- Trigonometry
- Mensuration
- Geometry in 3D

# Trigonometry: Full Instructional Guide

## 1. Introduction to Trigonometry

**Objective:** Understand what trigonometry is and why it's important.

Trigonometry is the study of relationships between the angles and sides of triangles. It plays a crucial role in fields like architecture, engineering, astronomy, and physics.

## 2. Prerequisites

Before diving into trigonometry, students should be comfortable with:

- Algebra (equation solving, manipulating expressions)
- Geometry (properties of triangles, Pythagorean theorem)
- Basic arithmetic and number operations

## 3. Real-Life Applications

- Measuring the height of a building without climbing it
- Calculating distances in astronomy
- Analyzing waves and oscillations



## 4. Key Terms and Definitions

- Angle: Formed by two rays with a common endpoint
- Right Triangle: A triangle with one  $90^\circ$  angle
- Hypotenuse: The side opposite the right angle in a right triangle
- Opposite side: The side opposite a given angle
- Adjacent side: The side next to a given angle (not the hypotenuse)

## 5. Trigonometric Ratios

These are defined for a right-angled triangle:

- Sine (sin) = Opposite / Hypotenuse
- Cosine (cos) = Adjacent / Hypotenuse
- Tangent (tan) = Opposite / Adjacent

## 6. Teaching Sequence

### Step 1: Labelling Triangle Sides

Use a right triangle diagram and help students label the hypotenuse, opposite, and adjacent sides based on a given angle.

### Step 2: Introducing Trigonometric Ratios

Explain and write down the three basic ratios using the labelled triangle. Use the acronym SOH-CAH-TOA for a memory aid.

### Step 3: Practice Problems

Example:

- If the opposite side is 3 cm and the hypotenuse is 5 cm, what is  $\sin(\theta)$ ?
- $\sin(\theta) = 3/5 = 0.6$

### Step 4: Using a Calculator

Show students how to calculate sine, cosine, and tangent values using a calculator. Emphasize checking whether the calculator is in degree or radian mode.

### Step 5: Solving Triangles

Guide students to find unknown angles and sides using trigonometric ratios.

## 7. The Unit Circle

Introduce the unit circle as a way to extend trigonometry beyond right triangles. Cover:

- Definition (circle with radius 1 centred at the origin)
- Coordinates representing  $\cos(\theta)$  and  $\sin(\theta)$
- Special angles ( $0^\circ$ ,  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$ ,  $90^\circ$ )

## 8. Common Mistakes to Address

- Confusing opposite and adjacent sides
- Forgetting to check the calculator mode
- Using degrees instead of radians or vice versa

## 9. Practice Set (Basic Level)

1. In a right triangle, if the opposite side is 4 and the adjacent is 3, find  $\tan(\theta)$ .
2. Given  $\sin(\theta) = 0.8$ , find the angle  $\theta$  (use calculator).
3. Find the hypotenuse if the opposite side is 5 and  $\sin(\theta) = 0.6$ .

## 10. Advanced Topics (for Extension)

- Trigonometric Identities
- Graphs of sine, cosine, and tangent functions
- Inverse trigonometric functions
- Solving real-world problems using trigonometry

## 11. Review & Recap

- Create a summary diagram showing SOH-CAH-TOA
- Provide a mini-quiz
- Encourage group problem-solving activities

## 12. Transition Topics

After mastering Trigonometry, students should move on to:

- Trigonometric identities
- Coordinate geometry
- Vectors and forces

Note: This structured dataset can be broken into smaller .md or .jsonl chunks with metadata for RAG use.

# Functions and Graphs - Teaching Dataset

## Prerequisite Knowledge

- Understanding of variables and expressions
- Basic algebraic manipulation (solving linear equations)
- Familiarity with coordinate system (conceptual only)

## Learning Objectives

- Understand what a function is
- Learn how to determine the domain and range
- Recognize types of functions (linear, quadratic, etc.)
- Learn function notation and evaluation
- Understand increasing/decreasing behaviour

## Topic Breakdown

### 1. What is a Function?

A function is a rule that assigns each input exactly one output.

Example: If (  $f(x) = x + 3$  ), then:

- (  $f(2) = 5$  )
- (  $f(-1) = 2$  )

Explain: Think of a function as a machine: You put in a number (input), it performs a rule, and gives back an output.

### 2. Function Notation

- (  $f(x)$  ) is read as "f of x"
- It means the function f with input x

Example: If (  $f(x) = 2x - 4$  ), then:

- (  $f(3) = 2(3) - 4 = 2$  )
- (  $f(-1) = 2(-1) - 4 = -6$  )

### 3. 🌐 Domain and Range

- Domain: All valid inputs (x-values)
- Range: All possible outputs (f(x)-values)

Example: For (  $f(x) = x^2$  ):

- Domain: All real numbers
- Range: (  $f(x) \geq 0$  )

### 4. 🧱 Types of Functions

#### ♦ Linear Functions

Form: (  $f(x) = mx + b$  )

- Straight line
- Constant rate of change (slope = m)

Example: (  $f(x) = 2x + 1$  )

#### ♦ Quadratic Functions

Form: (  $f(x) = ax^2 + bx + c$  )

- Parabolic shape (U-shaped curve)
- Has a vertex and an axis of symmetry

Example: (  $f(x) = x^2 - 4$  )

#### ♦ Constant Functions

Form: (  $f(x) = c$  )

- Always returns the same output

### 5. 📈 Increasing vs Decreasing

- A function is increasing when the output rises as x increases.
- A function is decreasing when the output drops as x increases.

Example:

- (  $f(x) = x^2$  ) is decreasing on (  $(-\infty, 0]$  ) and increasing on (  $[0, \infty)$  )

### 6. 🛠️ Practice Problems (With Steps)

**Q1:** Evaluate (  $f(x) = 3x - 2$  ) at (  $x = 4$  )

**Answer:** (  $f(4) = 3(4) - 2 = 10$  )

**Q2:** What is the domain of (  $f(x) = \frac{1}{x-2}$  )?

**Answer:** All real numbers except (  $x = 2$  )

**Q3:** Identify type: (  $f(x) = -x^2 + 5x + 2$  )

**Answer:** Quadratic Function

**Q4:** Is (  $f(x) = 2x + 3$  ) increasing or decreasing?

**Answer:** Increasing (positive slope)



## Teaching Strategy

- Begin by explaining functions as real-world input-output machines
- Use lots of everyday examples: vending machines, formulas, apps
- Break into segments: input/output, notation, domain/range
- Relate to previous algebra skills
- Reinforce with small concept-check questions



## Real-Life Applications

- Cost functions in economics (e.g., total cost = fixed + variable)
- Converting temperature ( $^{\circ}\text{F} = 1.8 \times ^{\circ}\text{C} + 32$ )
- Predictive models in statistics