Homeless Shelter Optimization to Allocate New Shelters

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Team 1:
MGSC 662: Decision Analytics

Agenda

- Introduction and Project Description
- Data Collection
- Data Preparation
- Modelling
  - Maximize Homeless to Current Shelters
  - Minimize Cost of New Shelters
- Important Insights

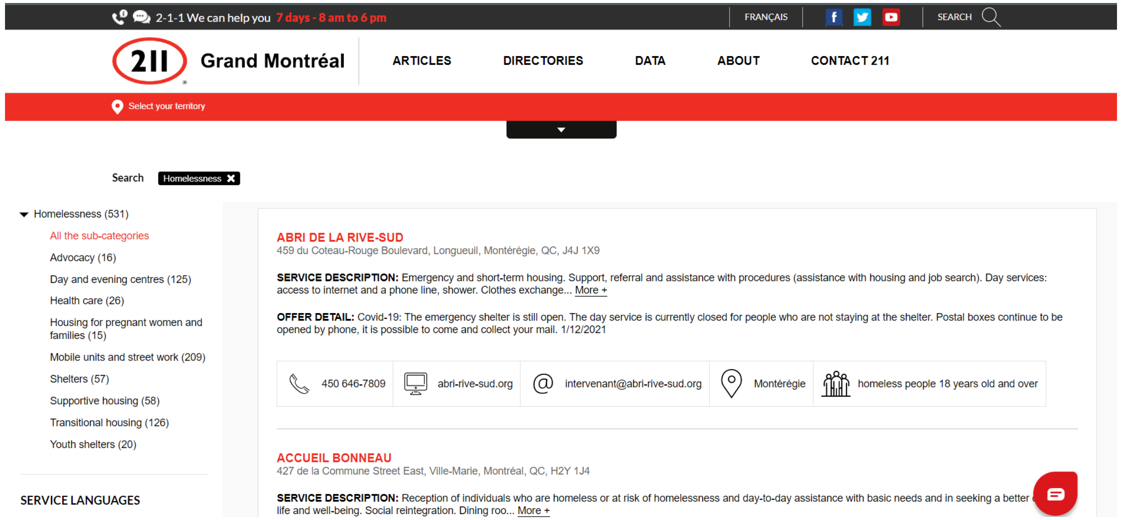
What is the Problem?

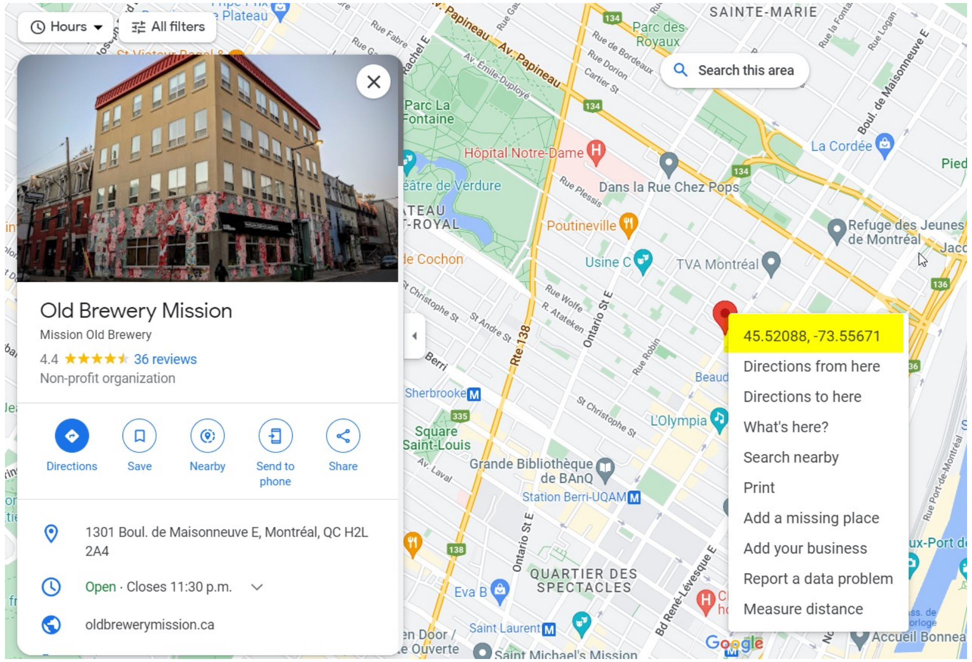
- In 2018, total homeless population in Montreal was 3149 people
- Homelessness has increased by 20% in 2020 due to COVID-19
- Capacity of current homeless shelters cannot support demand
- Goal of our project is to determine best location for new homeless shelters to account for demand
- Reduce homelessness in Montreal

Old Brewery Mission – Homeless Shelter in Montreal

Data Collection

- Current Capacity of Homeless Shelters on Montreal Island
- Homeless Population (Hot Spots)
  - Homeless Encampments
  - Metro Stations
- New Shelter Locations
- Latitude and Longitude from Google Maps
- Total Expenses from Financial Statements





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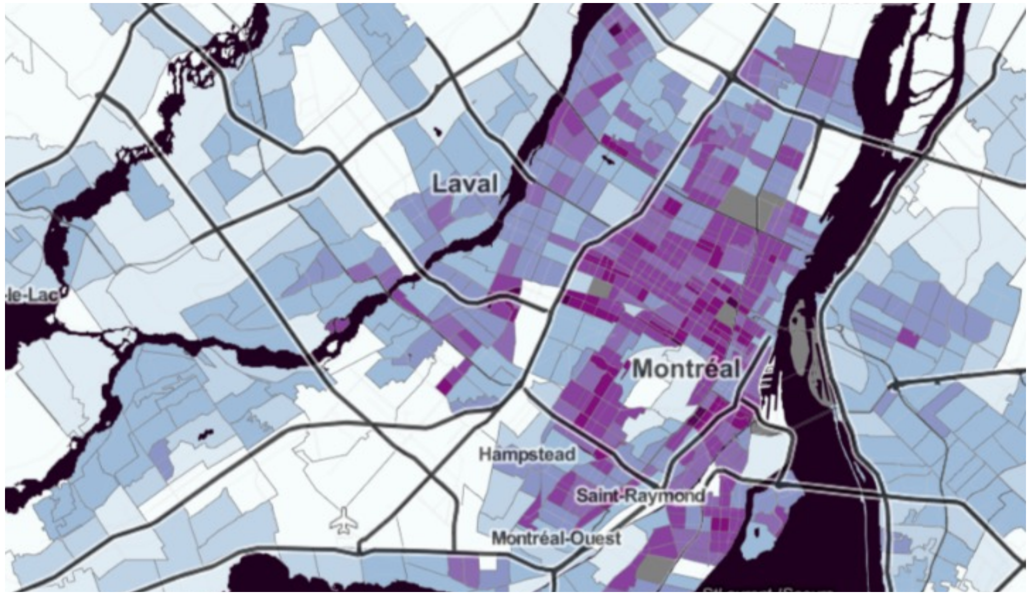
Data Preparation

- Distributed homeless population 50/50 metro stations and encampments
- Encampments equally allocated except West Island
- Metro Stations used closest census tract population per square km
  - Scale: Location density / min(density)
- Total * scale / sum(all scale)
  - For each metro station



Data Preparation

- Total Expenses Adjustments
  - Multiple Locations
  - Missing Values
  - New Shelter
    - Cost per Homeless
    - Fixed Cost
- Haversine Distance
  - Calculates angular distance along a sphere based on latitude and longitude
  - Between hot spots to new and existing shelters

Beds	Annual Cost	Total Beds	Adjusted Costs
12	$8,931,650.00	97	$1,104,946.39
49	$8,931,650.00	97	$4,511,864.43
15	$8,931,650.00	97	$1,381,182.99
21	$8,931,650.00	97	$1,933,656.19

- φ_1, φ_2 are the latitude of point 1 and latitude of point 2,
- λ_1, λ_2 are the longitude of point 1 and longitude of point 2.

$$
\begin{aligned}
d &= 2r \arcsin\left(\sqrt{\operatorname{hav}(\varphi_2-\varphi_1) + (1-\operatorname{hav}(\varphi_1-\varphi_2)-\operatorname{hav}(\varphi_1+\varphi_2))\cdot\operatorname{hav}(\lambda_2-\lambda_1)}\right)\\
&= 2r \arcsin\left(\sqrt{\sin^2\left(\frac{\varphi_2-\varphi_1}{2}\right) + \left(1-\sin^2\left(\frac{\varphi_2-\varphi_1}{2}\right)-\sin^2\left(\frac{\varphi_2+\varphi_1}{2}\right)\right)\cdot\sin^2\left(\frac{\lambda_2-\lambda_1}{2}\right)}\right)\\
&= 2r \arcsin\left(\sqrt{\sin^2\left(\frac{\varphi_2-\varphi_1}{2}\right) + \cos\varphi_1\cdot\cos\varphi_2\cdot\sin^2\left(\frac{\lambda_2-\lambda_1}{2}\right)}\right).
\end{aligned}
$$

Model Development

The project was split into two phases

- Maximise the homeless people from their existing locations into the homeless shelters

- Minimize the cost of housing the unallocated homeless population from each hotspot (encampment/metro station)

Model 1 – Maximising Occupancy

Decision Variables :

$x_{i,j,k}$: *homeless people allocated from hotspot i to shelter j for gender k {0 = female, 1 = male} (Integer)*

Parameters :

B_j : *Bed capacity of shelter j*

F_i : *Number of Females in hotspot i*

M_i: *Number of Males in hotspot i*

s : *Total number of existing shelters*

h : *Total number of hotspot locations*

F_i: *Number of Females in hotspot i*

M_i: *Number of Males in hotspot i*

$d_{i,j}$: *The Haversine distance between hotspot i and shelter j*

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Model 1 – Objective Function

We try to maximise the number of people moving from their respective hotspots to the shelters

$$Max\left(\sum_{i=0}^{h} \sum_{j=0}^{s} \sum_{k=0}^{1} x_{i,j,k} \right)$$

Model 1 - Constraints

- Female only shelters can house only females
- Male only shelters can house only males
- Open shelters can house anyone
- A shelter can house no more people than its capacity

$$\sum_{i=0}^{h} x_{i,j,0} \leq B_j \; for \; j = \{38, 39, \; \ldots 66\}$$

$$\sum_{i=0}^{h} x_{i,j,1} = 0 \; for \; j = \{38, 39, \; \ldots 66\}$$

$$\sum_{i=0}^{h} x_{i,j,1} \leq B_j \; for \; j = \{67, 68, \; \ldots 80\}$$

$$\sum_{i=0}^{h} x_{i,j,0} = 0 \; for \; j = \{67, 68, \; \ldots 80\}$$

$$\sum_{i=0}^{h} \sum_{i=0}^{k} x_{i,j,k} \leq B_j \; for \; j = \{0, 1, \ldots \ldots 37\}$$

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Model 1 - Constraints

- A hotspot can't have more people than the assumed distribution

$$\sum_{j=0}^{s} x_{i,j,1} \leq M_i \quad for\ i\ in\ \{0, 1, \ldots\ h\}$$

$$\sum_{j=0}^{s} x_{i,j,0} \leq F_i \quad for\ i\ in\ \{0, 1, \ldots\ h\}$$

Model 1 - Constraints

- An individual can't travel more than 4 kilometers

$$x_{i,j,k} * (4 - d_{i,j}) \geq 0 \;\; for \;\; i = \{0, 1, \;\ldots h\} \;\; for \;\; j = \{0, 1, \;\ldots s\} \;\; for \;\; k = \{0, 1\}$$

- Non-negativity constraints

$$x_{i,j,k} \geq 0$$

Model 1 - Results

Out of the total homeless population of 3566, 3142 people were allocated to different housing shelters with the following locations having unallocated individuals,

Station	Latitude	Longitude	Homeless	Remaining
Plamondon	45.50484	-73.6379	27	19
Cote-Sainte-Catherine	45.50436	-73.6349	67	67
Snowdon	45.49569	-73.6266	21	21
Villa-Maria	45.49184	-73.617	29	11
Sherbrooke	45.52889	-73.569	35	1
Jarry	45.5539	-73.6266	44	41
Cremazie	45.55631	-73.6404	29	20
Cote-des-Neiges	45.50821	-73.6239	29	20
Universite-de-Montreal	45.51927	-73.6218	27	19
Edouard-Montpetit	45.52312	-73.6143	32	22
Outremont	45.52986	-73.6191	23	16
Acadie	45.5313	-73.6246	17	17
Parc	45.54236	-73.6287	33	23
De Castelnau	45.54717	-73.6218	33	33
Notre Dame De Grace	45.461922	-73.620111	195	95

424 individuals unallocated

Model 2 – Minimising Cost

Decision Variables :

$x_{i,j}$: *homeless people allocated from hotspot i to potential shelter j (Integer)*

D_i : *dummy variable corresponding to the potential shelters (Binary)*

Parameters :

B_j: *Bed capacity of potential shelter j*

α: *Desired allocation percentage*

R_i : *Number of homeless at hotspot i*

FC_j: *Annual fixed cost of potential shelter j*

HC_j: *Cost per bed or to maintain a bed, constant throughout all potential shelters j*

hl: Number of hotspot locations with unallocated people (14 locations)

ps: Number of potential shelters identified (7 locations)

di,j: The Haversine distance between hotspot i and potential shelter j

M: Big M variable = 1000000

Model 2 – Objective Function

We try to minimize the cost of housing the unallocated people at the remaining hotspots

$$Min\left(\sum_{j=0}^{ps}\sum_{i=0}^{hl} HC_j \times x_{i,j} + \sum_{j=0}^{ps} FC_j \times D_j\right)$$

Model 2 - Constraints

- The potential shelters can't accommodate more people than their capacity

$$\sum_{i=0}^{hl} x_{i,j} \leq \boldsymbol{B}_j \quad for\ j = \{0, 1, \ \ldots \ ps\}$$

- The hotspots can't have more people than the unallocated

$$\sum_{j=0}^{ps} x_{i,j} \leq \boldsymbol{R}_i \quad for\ i = \{0, 1, \ \ldots \ hl\}$$

Model 2 - Constraints

- An individual can't travel more than 4 kilometers

$$x_{i,j} * \left(4 - d_{i,j}\right) \geq 0 \;\; for \;\; i = \{0, 1, \; \dots hl\} \;\; for \;\; j = \{0, 1, \; \dots ps\}$$

- At least α % people need to be allocated

$$\sum_{i=0}^{hl} x_{i,j} \;\geq \alpha \times \sum_{i=0}^{hl} R_i \;\; for \;\; j = \{0, 1, \; \dots \; ps\}$$

- Potential Shelter can only accommodate people if chosen

$$x_{i,j} \leq D_j \times M \;\; for \;\; i = \{0, 1, \dots \; hl\} \;\; for \;\; j = \{0, 1, \dots \; ps\}$$

$$x_{i,j} \;\geq\; 0$$

Model 2 - Results

There was a tradeoff between the minimum percentage of people to be allocated and the least distance an individual can travel.

(3: Stade de Soccer de Montreal, 5: Old Royal Victoria's Hospital, 7 : Hotel Dieu)

Allocation Utilization (%)	Distance (km)	Cost ($)	Number of shelters selected	Shelters selected	Total people assigned	Total people unassigned
100	4	UNFEASIBLE				
95		UNFEASIBLE				
90		$23,165,788	3	3,5,7	383	41
85		$21,780,108	2	3,5	362	62
80		$21,211,188			341	85
75		$12,608,472		3,7	319	106

Model 2 - Extension

Allocation Utilization (%)	Distance (km)	Cost ($)	Number of shelters selected	Shelters selected	Total people assigned	Total people unassigned
100	5	**$16,907,591**	3	3,6,7	424	0
95		$16,364,531			403	21
90		$14,263,512	2	3,7	382	42
85		$13,720,452			361	63
80		$13,151,532			340	85
75		$12,608,472			318	106
100	6	**$16,907,591**	3	3,6,7	424	0
95		$15,651,967		2,3,7	403	21
90		$14,263,512	2	3,7	382	42
85		$13,720,452			361	63
80		$13,151,532			340	84
75		$12,582,612			318	106

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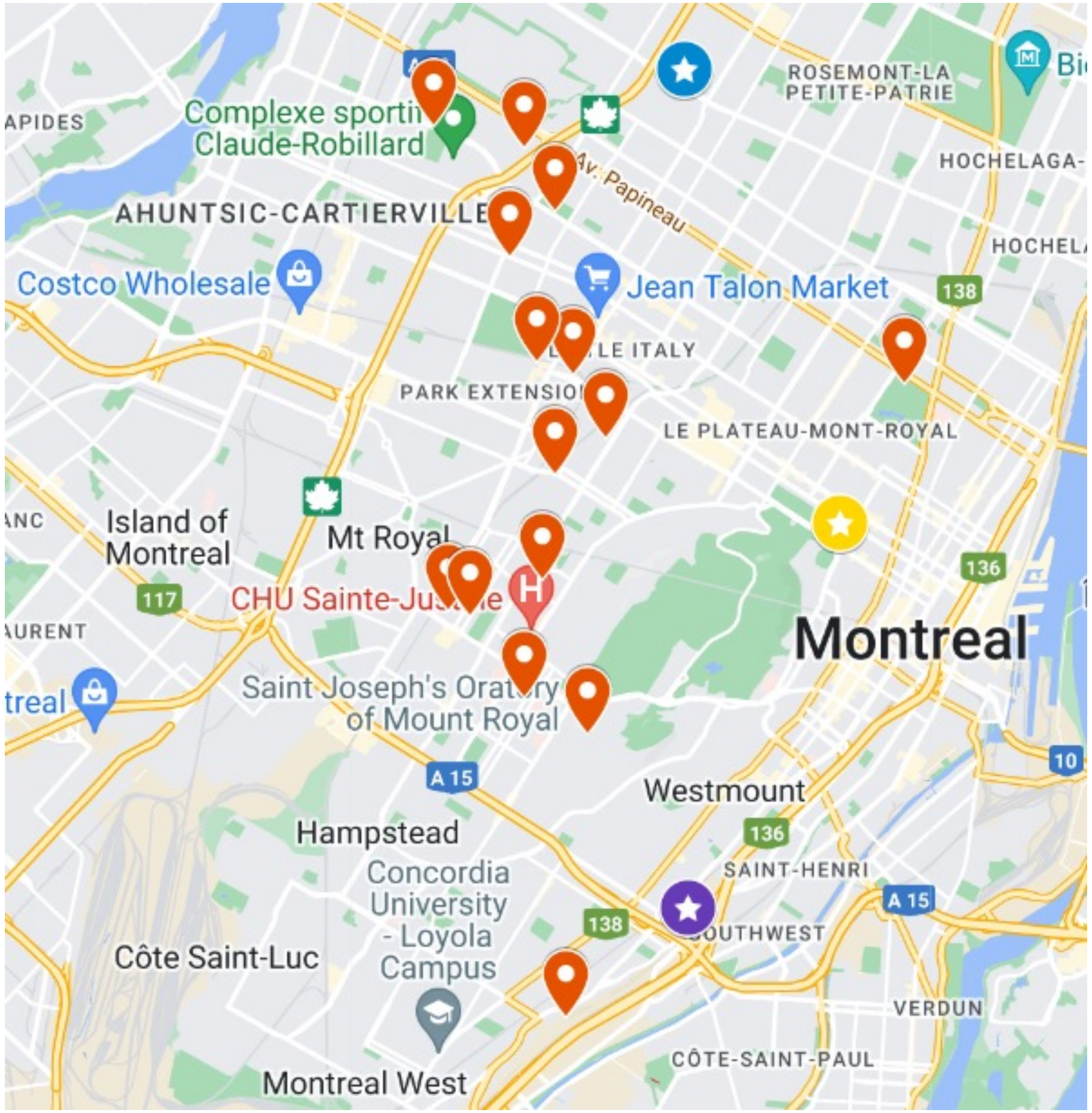
Important Insights

- Based on our models unallocated homeless people are mostly located around the Blue metro line
  - Unallocated: Orange
- New Locations (80/85%):
  - Stade de Soccer de Montreal (Blue)
  - Old Royal Victoria's Hospital (Purple)
- New Locations (75%):
  - Stade de Soccer de Montreal (Blue)
  - Hotel Dieu (Yellow)



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Concluding Remarks

- Our research has introduced an optimization method to simulate homeless distribution and systematically assign unallocated homeless people to new shelter locations
- This has the potential to improve outcomes for the homeless individuals
- These results can be expanded upon and presented to the municipal and provincial governments of Quebec to gain funding for the creation of new shelters

Thank you for listening!

Any Questions?