Day 1 (of the rest of my life)

Annie Lu

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Problem 4 Suppose that $A, B \in M_{n,n}(\mathbb{C})$. Show that the characteristic polynomials of AB and BA are equal. *Hint:* One approach is to first show that it holds when B is invertible.

Solution:

Problem 12 Assume $[0,1] = \bigcup_{n=1}^{\infty} I_n$ where $I_n = [a_n, b_n] \neq \emptyset$ and

$$I_n \cap I_m = \emptyset$$

whenever $n \neq m$.

(a) Let $E = \{a_n : n \geq 1\} \cup \{b_n : n \geq 1\}$ be the set of endpoints of the intervals above. Prove E is closed.

(b) Prove no such family of intervals $\{I_n\}$ can exist.

Solution: