Day 1 (of the rest of my life)

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Problem 10. Let A be a 3×3 real matrix with $A^3 = I$. Show that A is similar to a matrix of the form

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

for some (real) θ. What values of θ are possible?

Solution:

Problem 5 Let A be an n by n matrix with real entries, and let $\mathbf{b}$ be an n by 1 column vector with real entries. Prove that there exists an n by 1 column vector solution $\mathbf{x}$ to the equation $A\mathbf{x} = \mathbf{b}$ if and only if $\mathbf{b}$ is in orthocomplement of the kernel of the transpose of A.

Solution: