Data Structures

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Realdolmen

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Who am I?

- Master of Engineering: Computer Science (KUL)
  - Computational informatics
- RealdoImen: acADDemICT in 09/2015
- Current project: Planning infrastructure @ Infrabel

Outline

Outline

What are Data Structures?

Data Structure

A way in which data are stored for efficient search and retrieval. Different data structures are suited for different problems.[1]

- Data type $\neq$ data structure
- `java.util.HashSet` vs. hash table
- `array` vs. array

[1] Encyclopædia Britannica

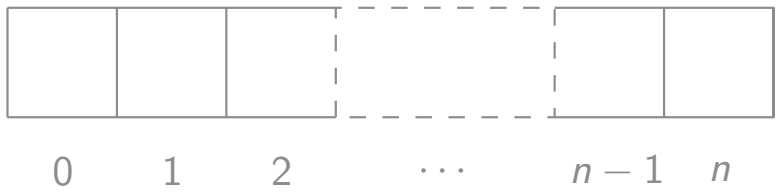
Outline

Array

Definition

- An indexed set of related elements.[2]
- An assemblage of items that are randomly accessible by integers, the index.[3]

- Example: linear array

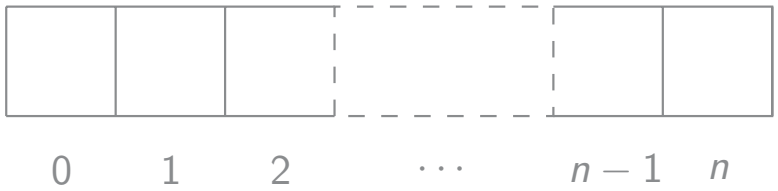


[2] Oxford Dictionary
[3] National Institute of Standards & Technology

Array

Operations

- `get`
- `set`
- `indexOf`



Array

Operations

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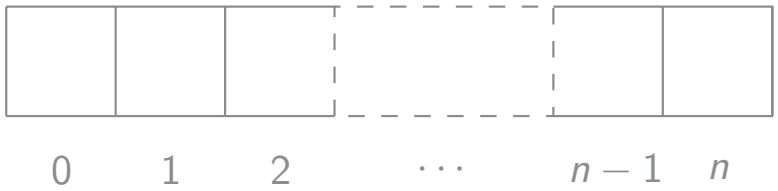
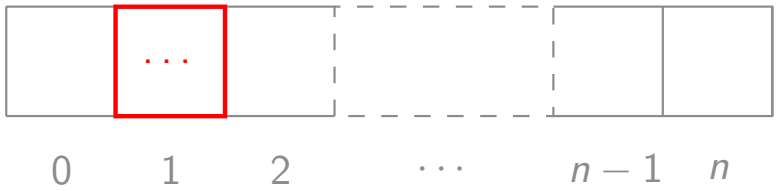


`get(1)`

Array

Operations

- `get`
- `set`
- `indexOf`

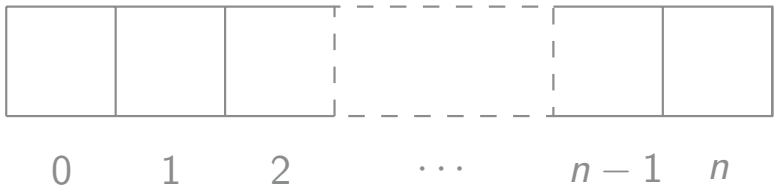


`get(1)`

Array

Operations

- `get` $O(1)$
- `set`
- `indexOf`



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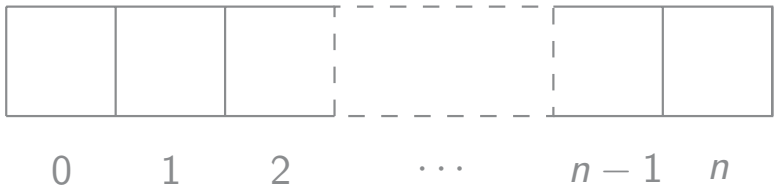

`set(2)`

Array

Operations

- `get` $O(1)$
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- `indexOf`

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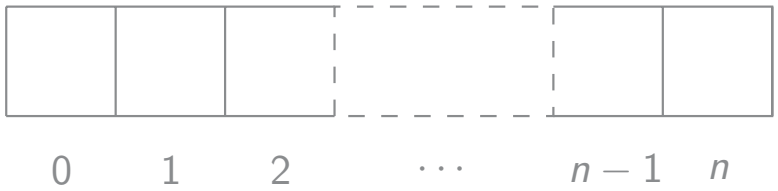

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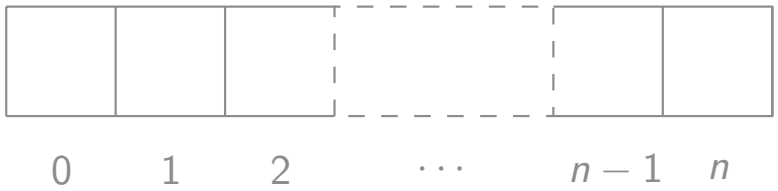
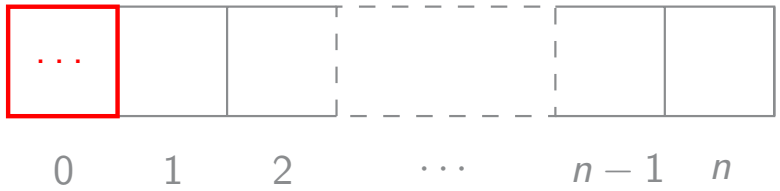


`indexOf(object)`

Array

Operations

- `get` $O(1)$
- `set` $O(1)$
- `indexOf`



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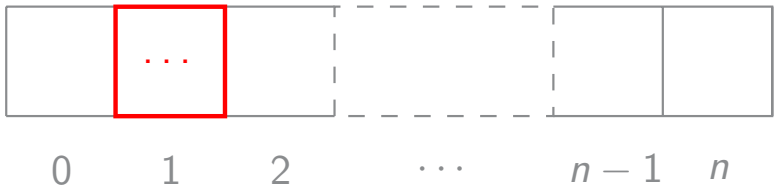

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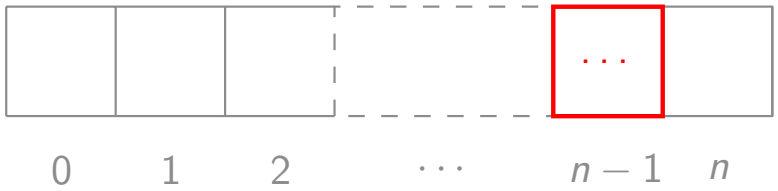

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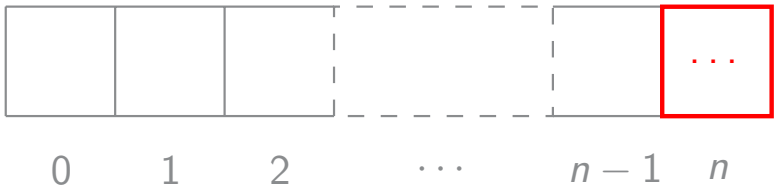

`indexOf(object)`

Array

Operations

- `get` $O(1)$
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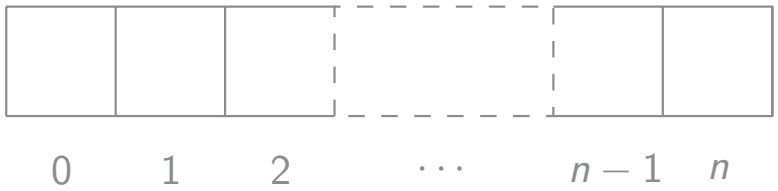
- `indexOf` $O(n)$



Outline

Linked List

Definition

A linked list is a data structure in which the objects are arranged in a linear order. Unlike arrays in which the linear order is determined by indices, the order is determined by a pointer in each object.[4]

- Different types: singly, doubly, multiply, circular, ...
- Example: doubly linked list

[4] Introduction to Algorithms By Cormen, Leierson, Rivest & Stein

Linked List

Operations

- `add/remove first/last`
- `get/insertAt`
- `indexOf`

Linked List

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- `add/remove first/last`
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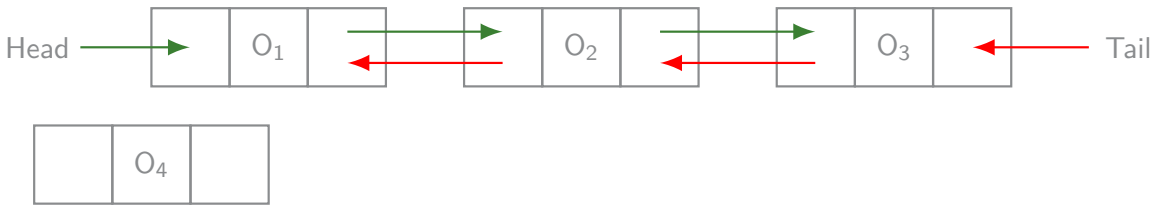
`addFirst(`O_4`)`

Linked List

Operations

- `add/remove first/last`
- `get/insertAt`
- `indexOf`



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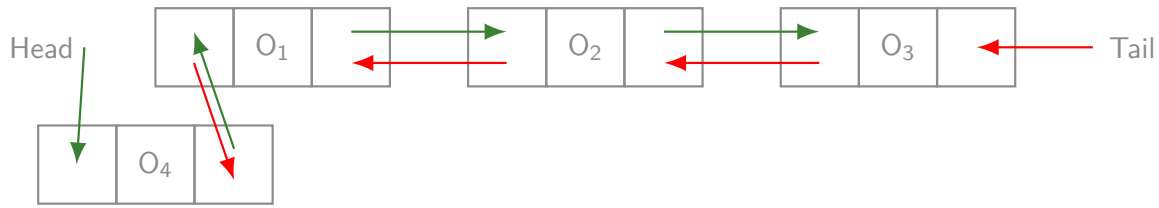

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Operations

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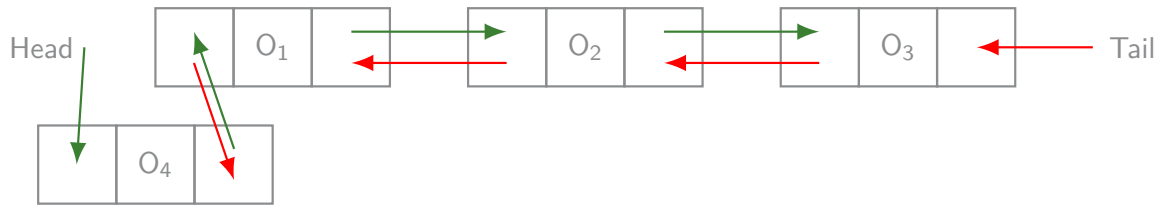


`addFirst(`O_4`)`

Linked List

Operations

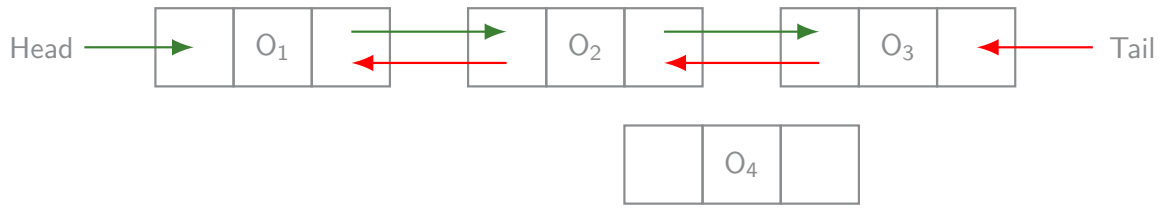
- `add/remove first/last` $O(1)$
- `get/insertAt`
- `indexOf`

`insertAt(2)`

Linked List

Operations

- `add/remove first/last` $O(1)$
- `get/insertAt`
- `indexOf`



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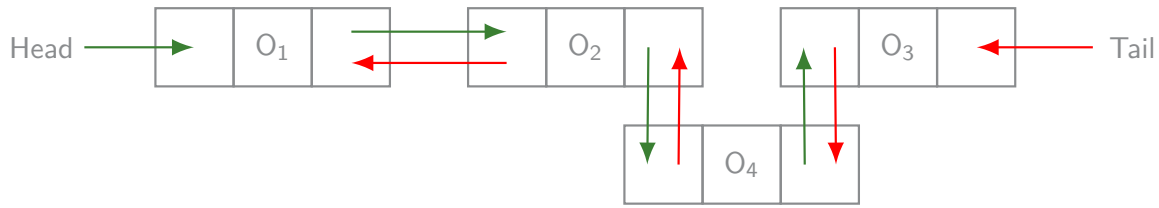
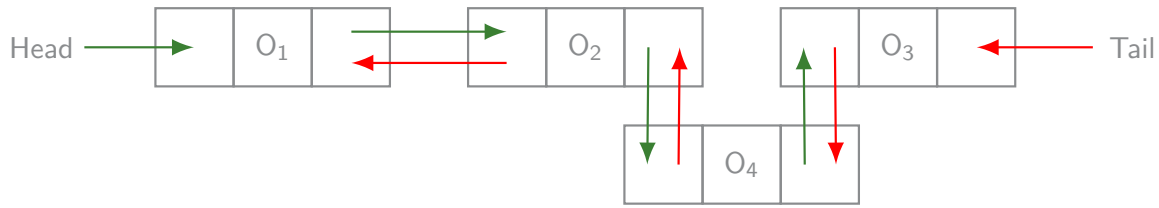


`insertAt(2)`

Linked List

Operations

- `add/remove first/last` $O(1)$
- `get/insertAt` $O(n)$
- `indexOf`



`insertAt(2)`

Linked List

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- `add/remove first/last` $O(1)$
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`indexOf(`O_2`)`

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Operations

- `add/remove first/last` $O(1)$
- `get/insertAt` $O(n)$
- `indexOf` $O(n)$

`indexOf(`O_2`)`

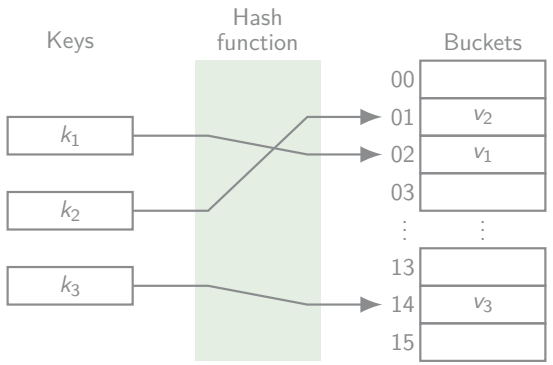
Outline

Hash Table

Definition

A dictionary in which keys are mapped to array positions by hash functions.[5]

- Hash functions: determinism, uniformity, defined range, data normalisation, non-invertible, perfect, ...
- Collisions resolution: chaining, open addressing, ...
- Example:

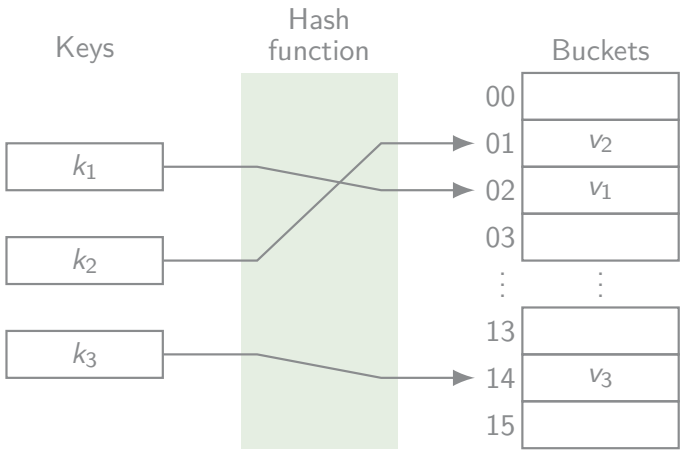


[5] National Institute of Standards & Technology

Hash Table

Operations

- `put`
- `remove`
- `get`



Hash Table

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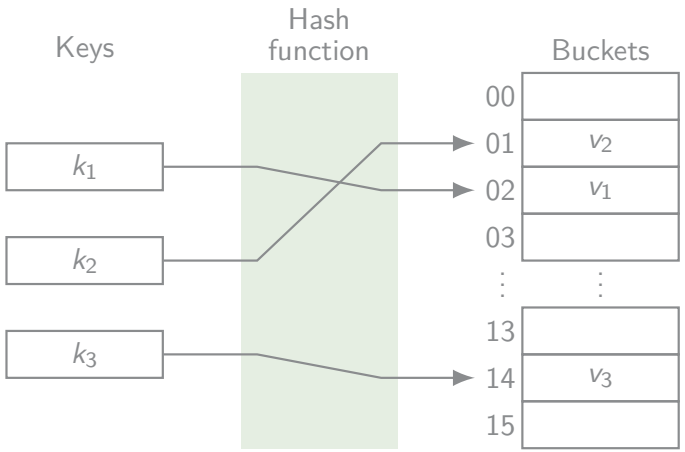
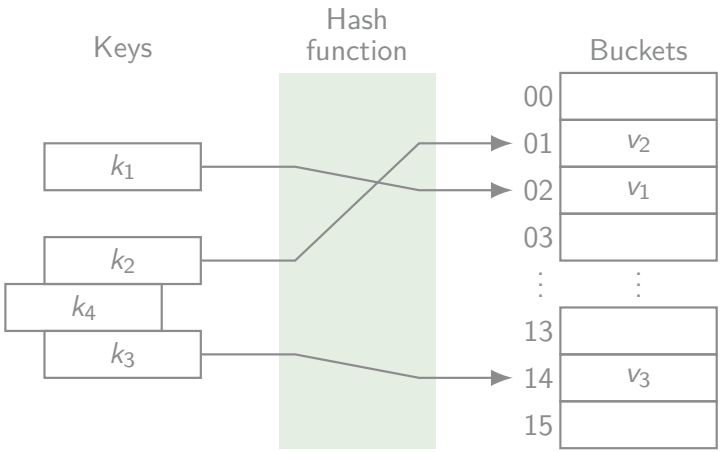


`put(`O_4`)`

Hash Table

Operations

- `put`
- `remove`
- `get`

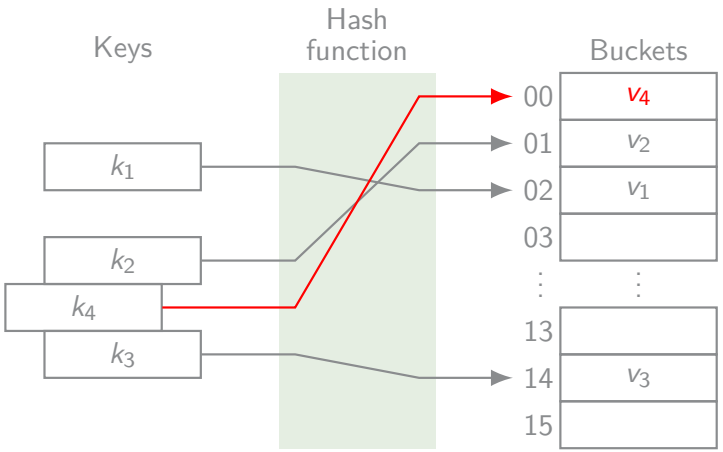


`put(O4)`

Hash Table

Operations

- `put`
- `remove`
- `get`

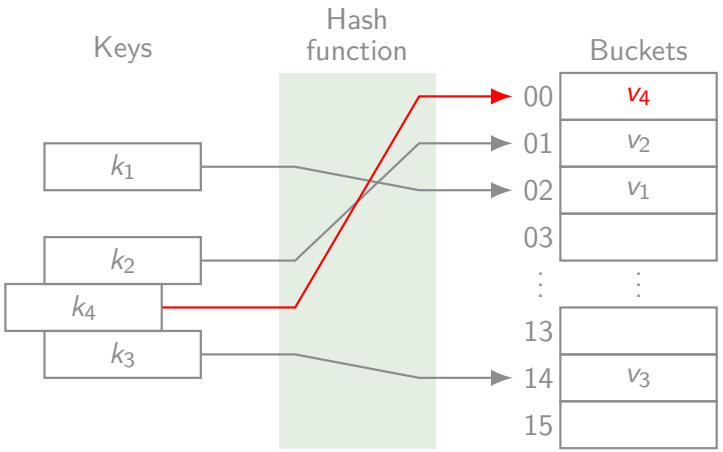


`put(`O_4`)`

Hash Table

Operations

- `put` $O(1)$ / $O(n)$
- `remove`
- `get`



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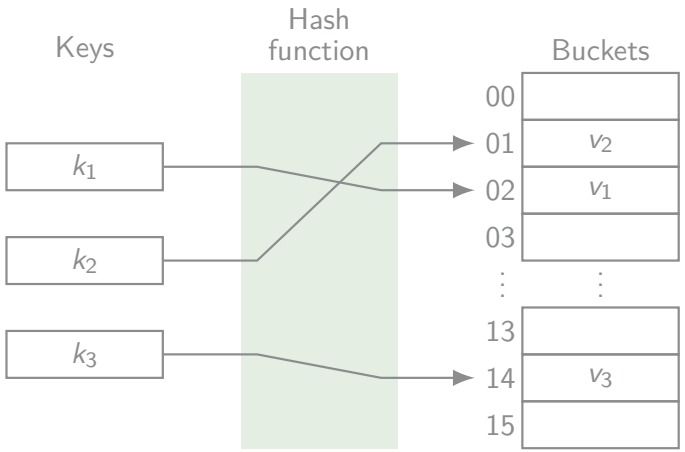

`remove(`O_2`)`

Hash Table

Operations

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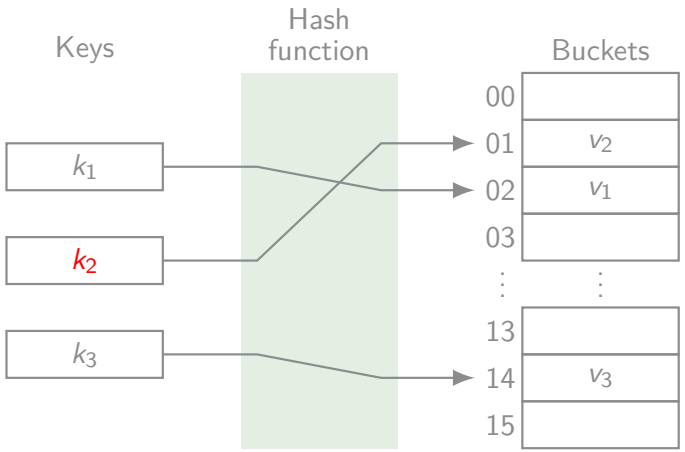

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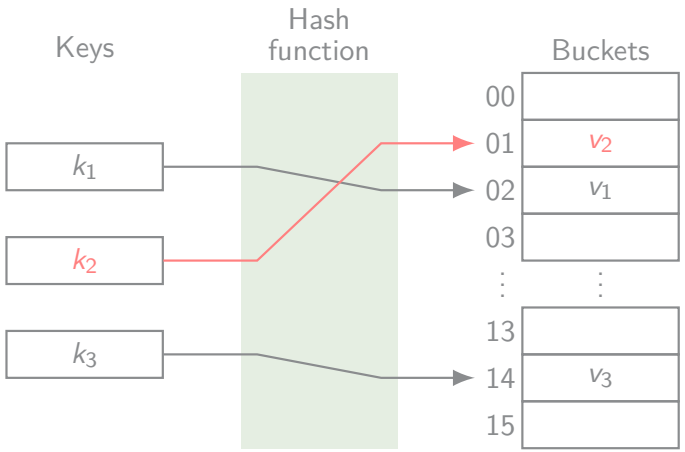

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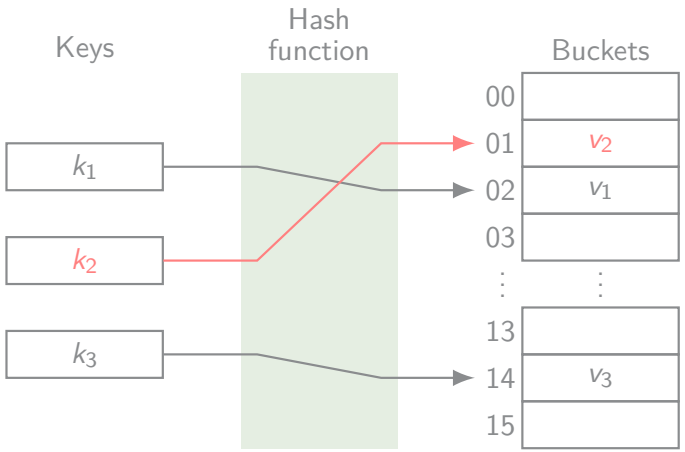

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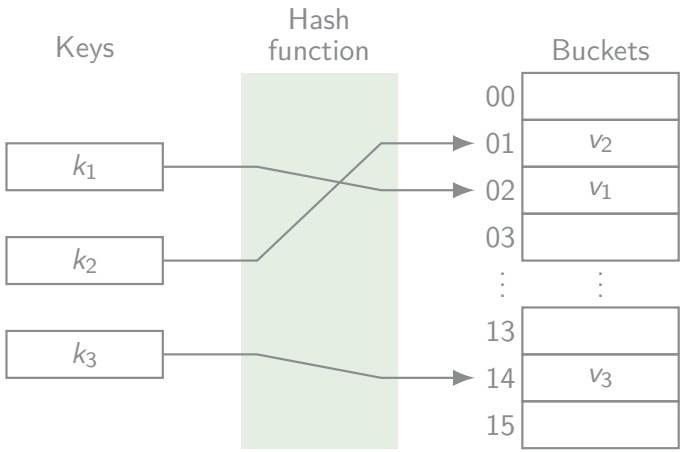

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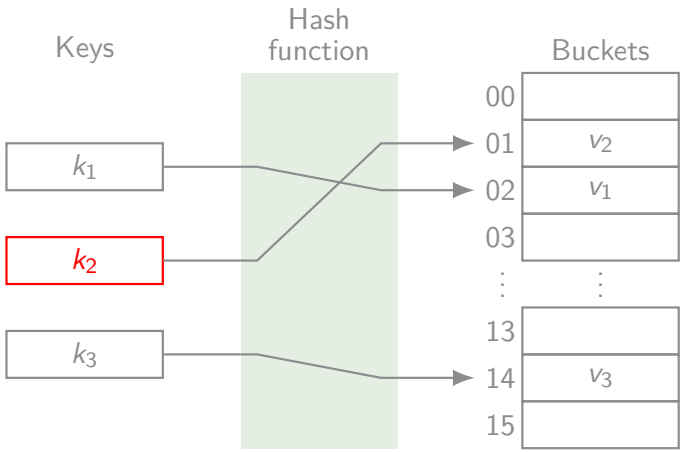

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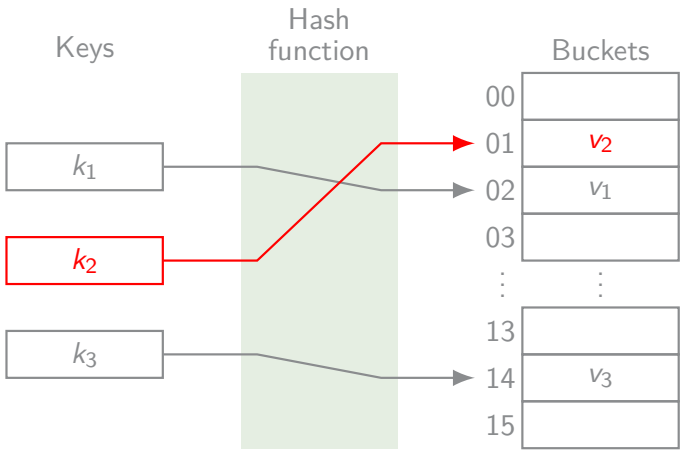

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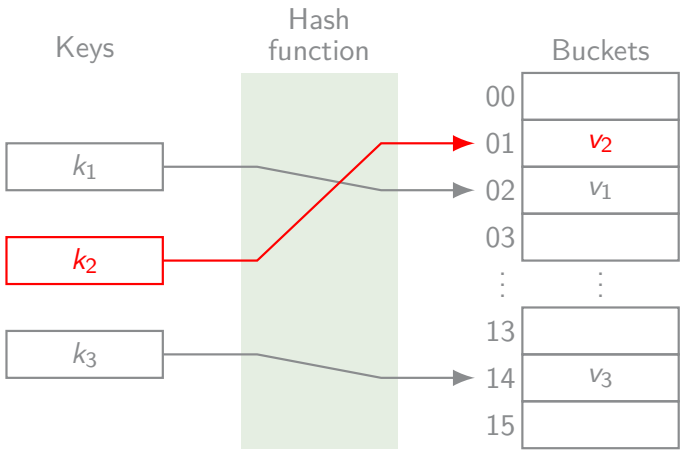


`get(`O_2`)`

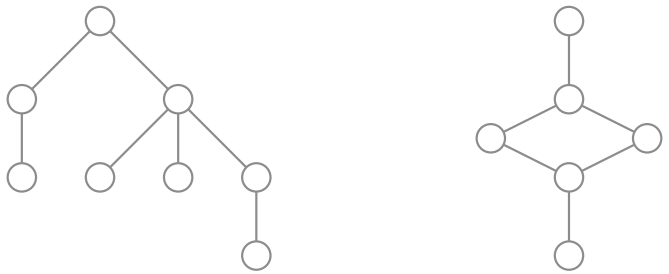
Outline

Tree

Definition

A data structure made up of nodes or vertices and edges without having any cycle. A tree that is not empty consists of a root node and potentially many levels of additional nodes that form a hierarchy.

- Depth, binary, (nearly) complete, ...
- Example:



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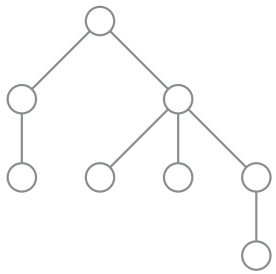
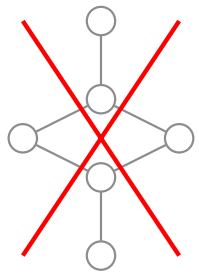




Binary Heap

Definition (Heap)

A complete tree where every node has a key more extreme (greater or less) than or equal to the key of its parent.[6]

Definition (Binary Heap)

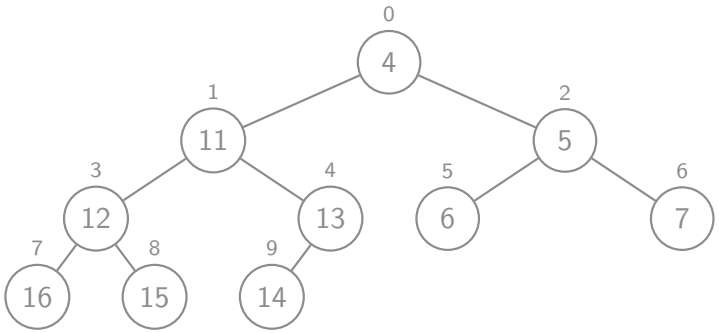
A binary heap data structure is an array object that we can view as a nearly complete binary tree that satisfies the min-heap or max-heap property.[7]

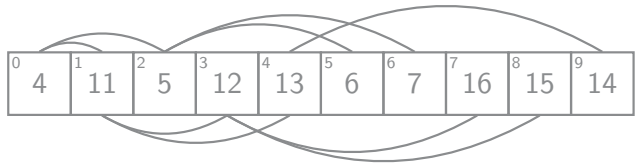
[6] National Institute of Standards & Technology

[7] Introduction to Algorithms By Cormen, Leierson, Rivest & Stein

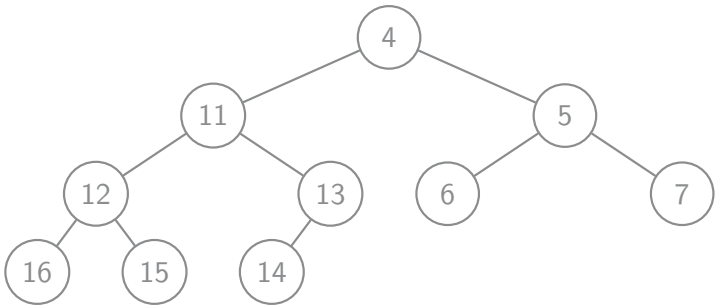
Binary Min-Heap

- `Parent(n)` $\lfloor \frac{n-1}{2} \rfloor$
- `Left(n)` $2n+1$
- `Right(n)` $2(n+1)$

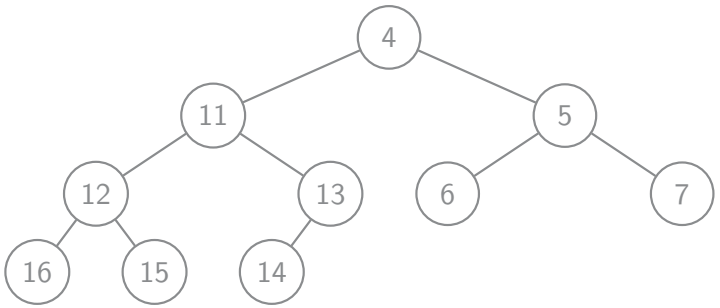




Binary Min-Heap

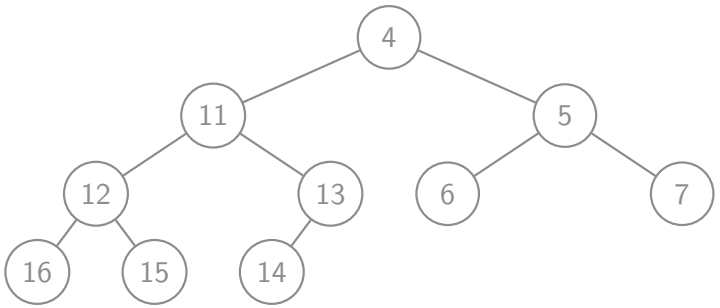


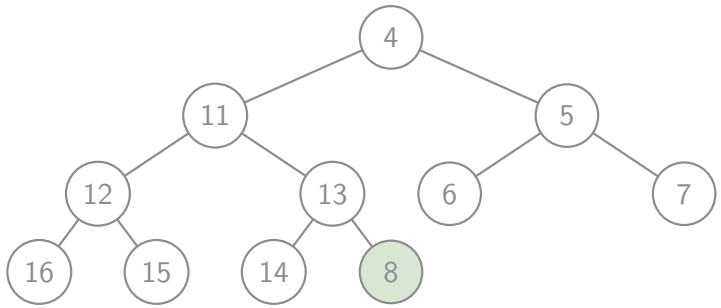
Binary Min-Heap



add 8

Binary Min-Heap





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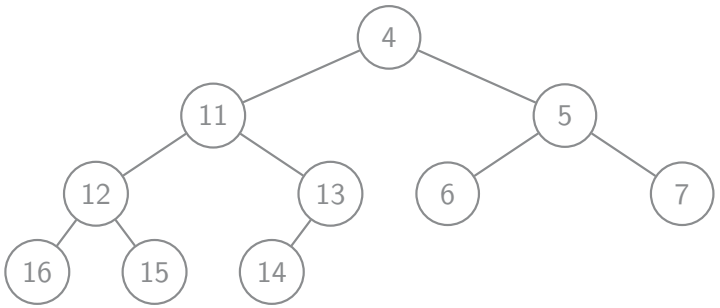
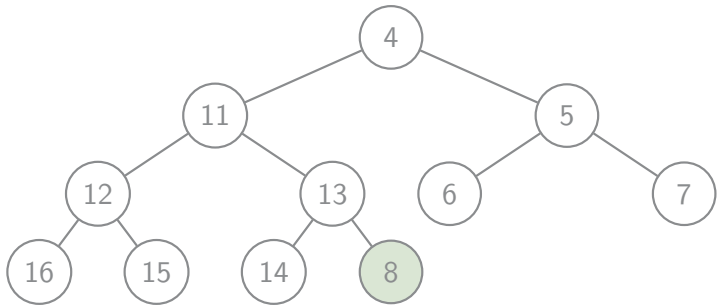
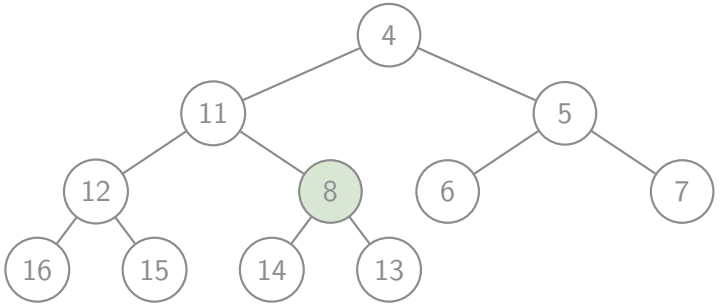
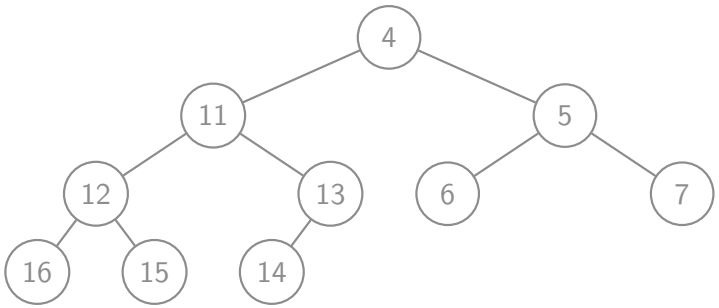
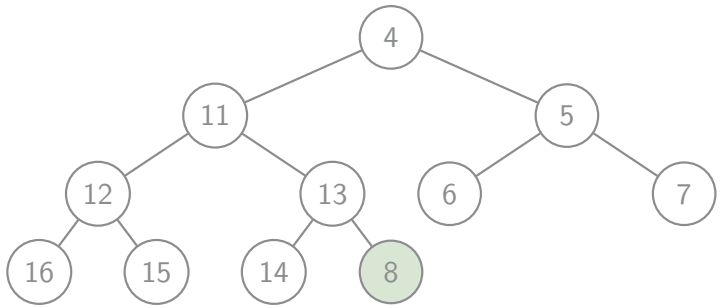


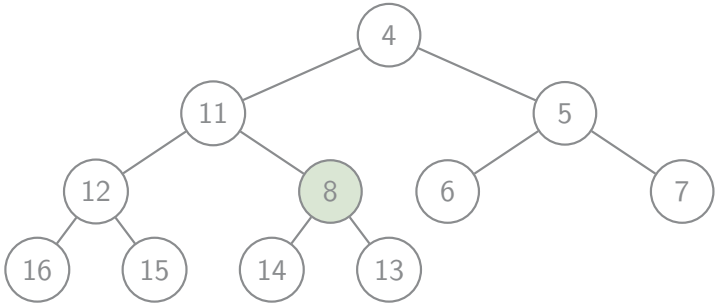


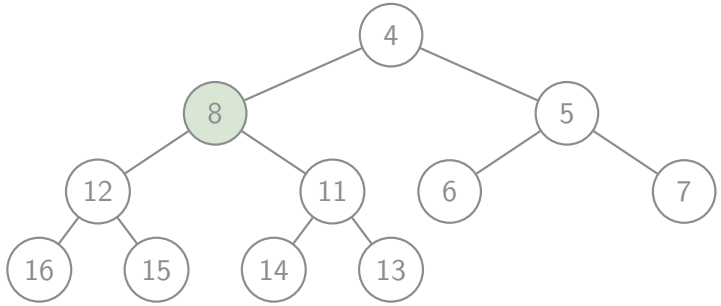


Binary Min-Heap









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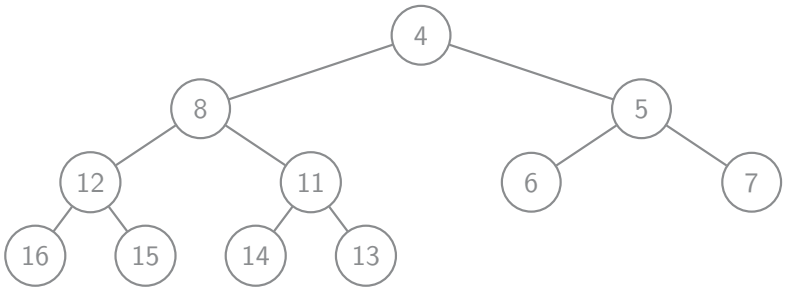
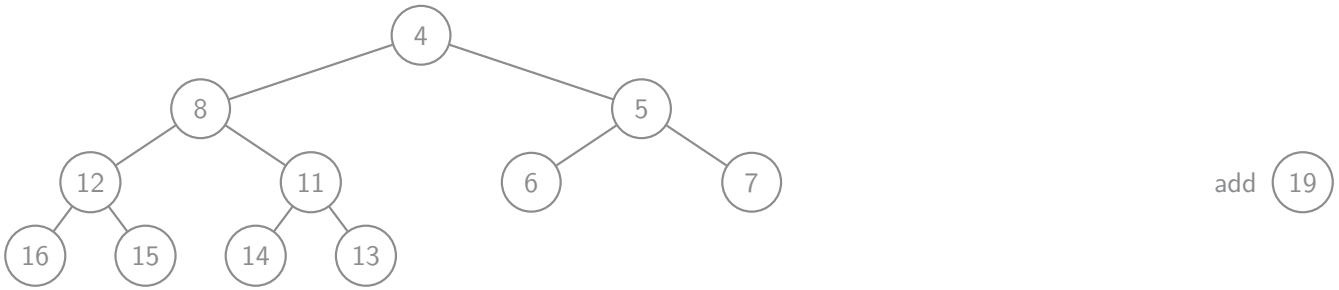


Binary Min-Heap



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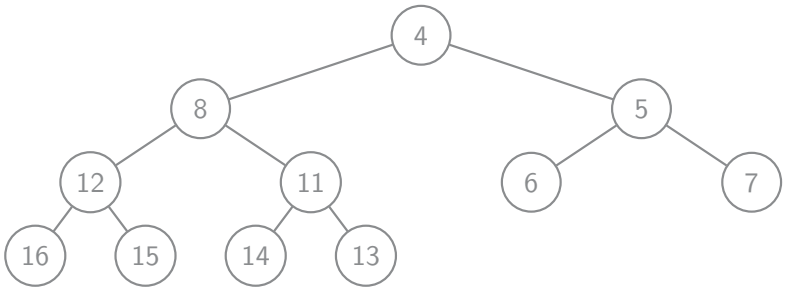

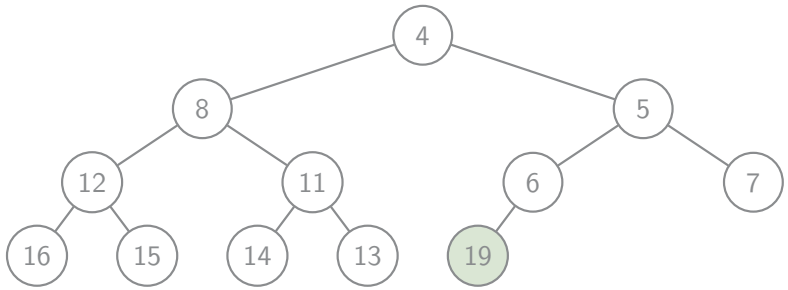

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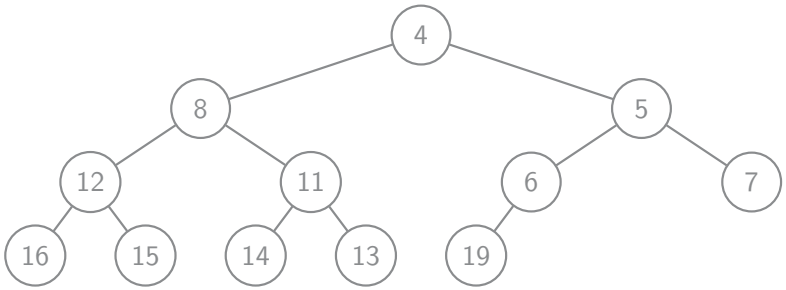

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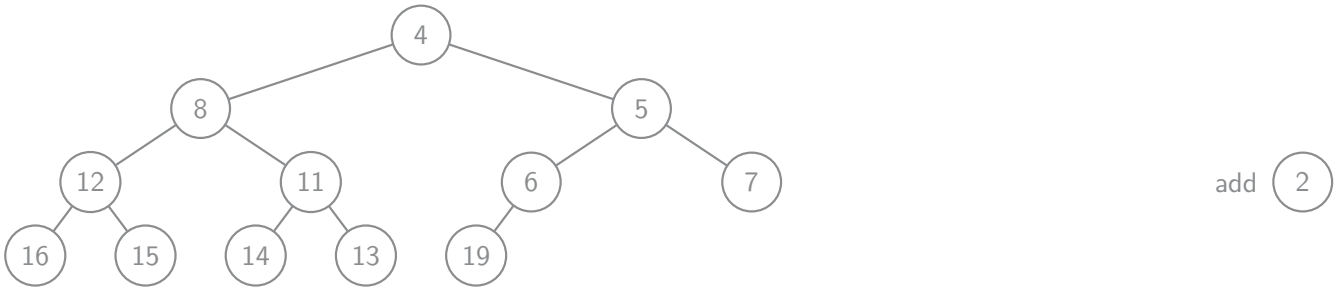

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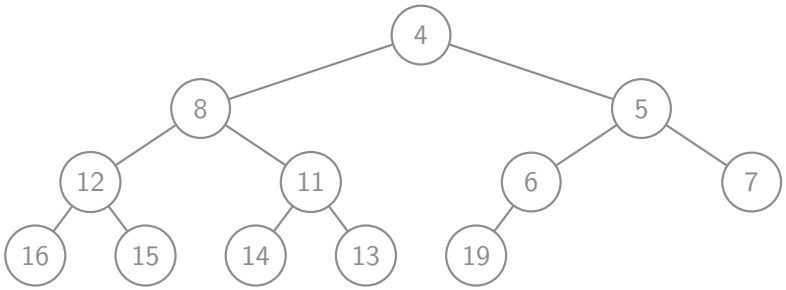

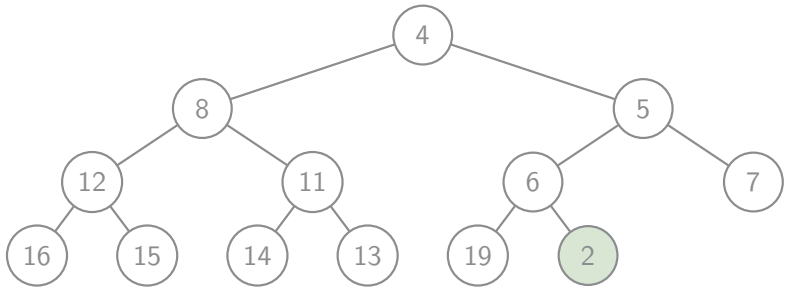

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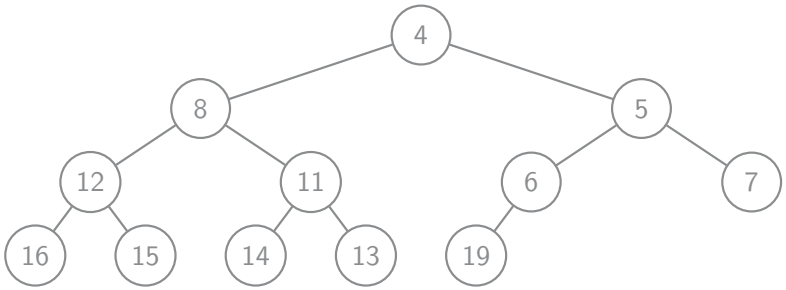

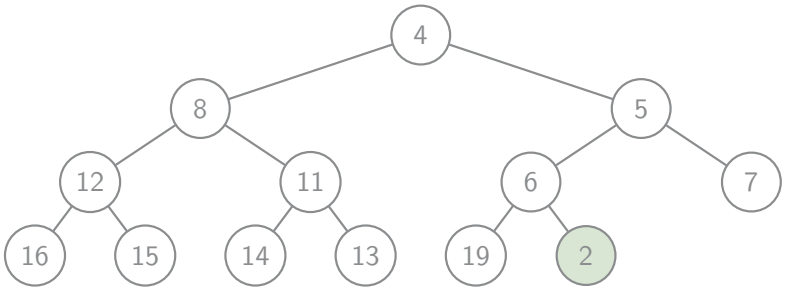

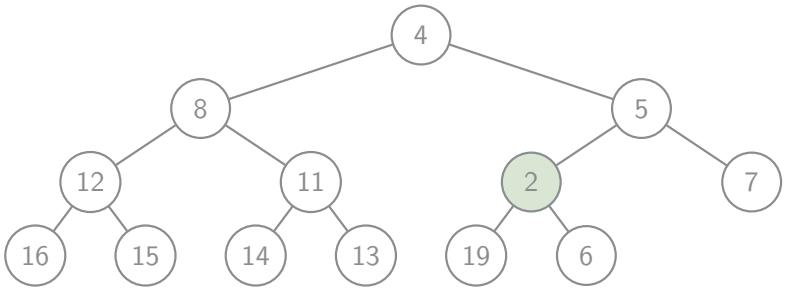

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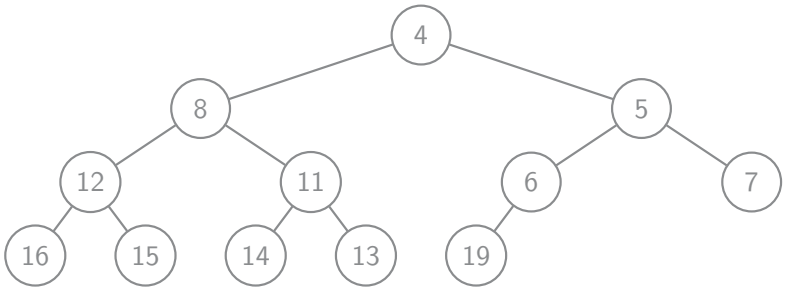

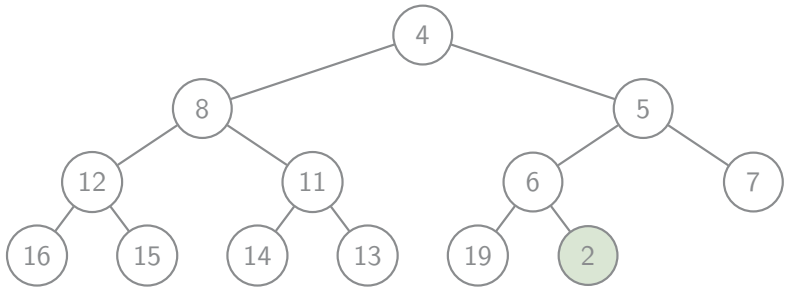

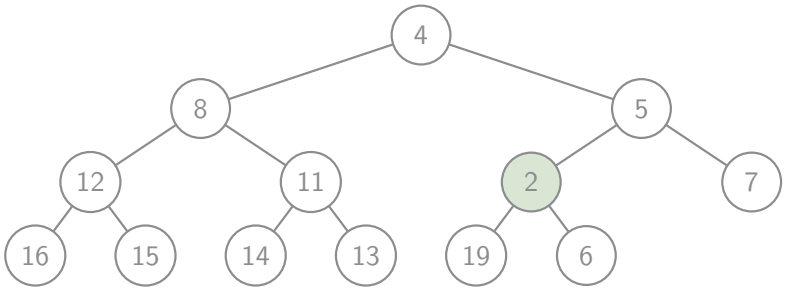

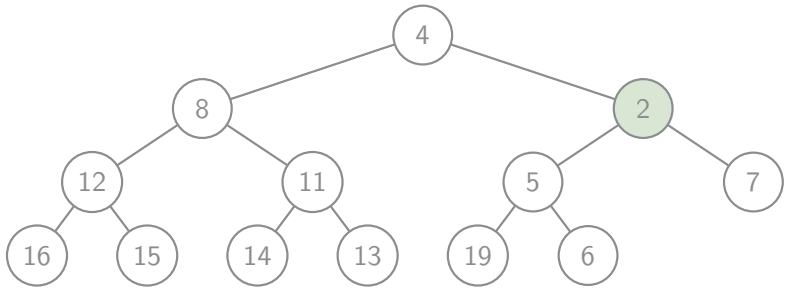

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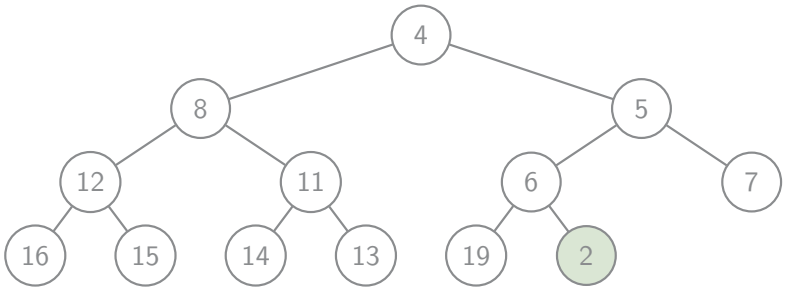
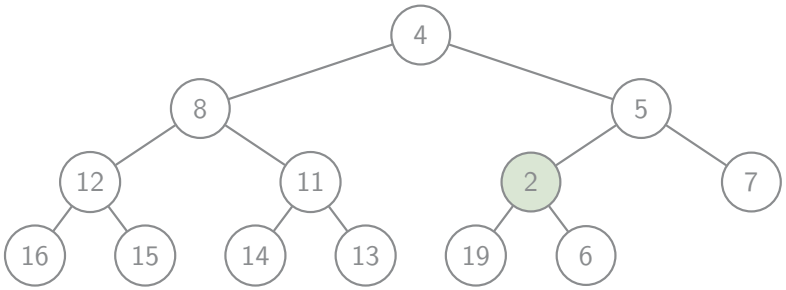
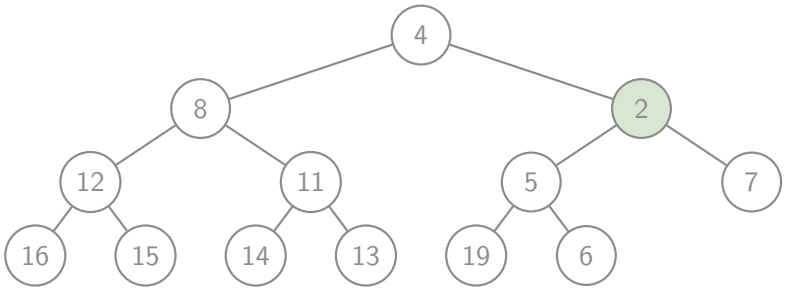
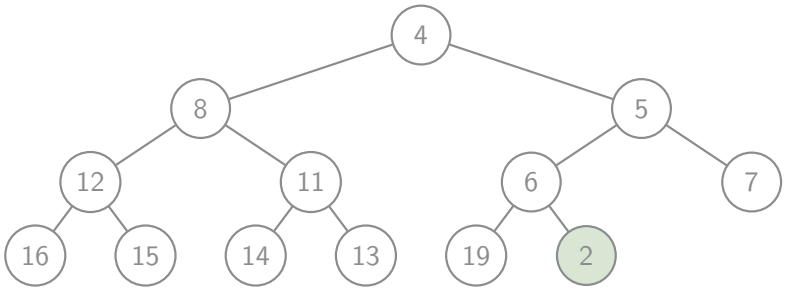
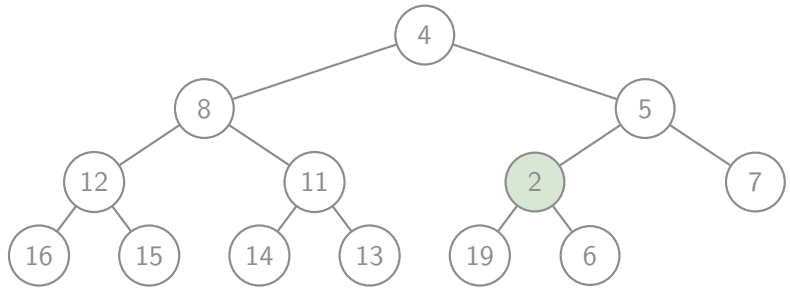


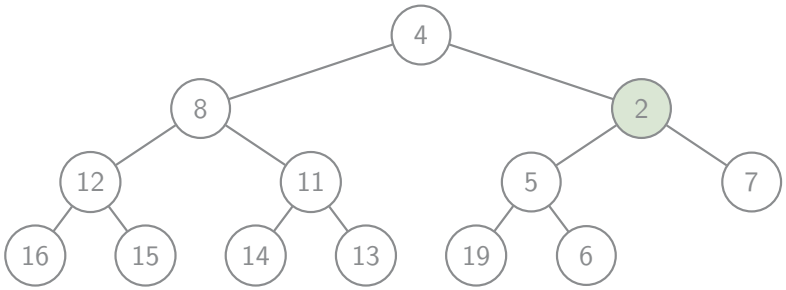


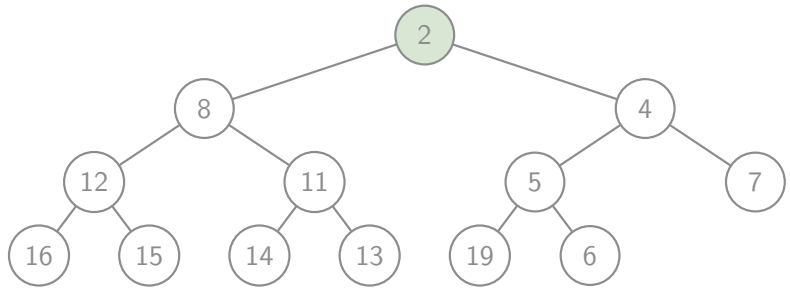


Binary Min-Heap









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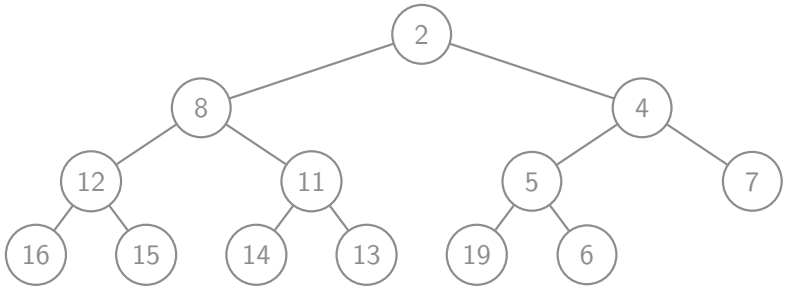

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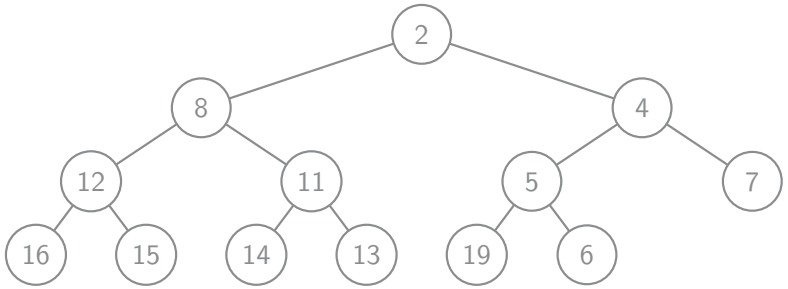
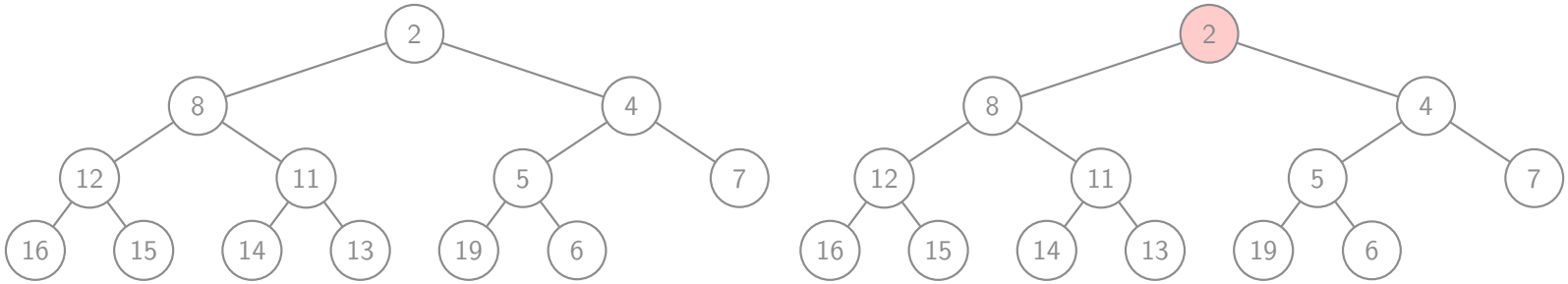


`poll`

Binary Min-Heap



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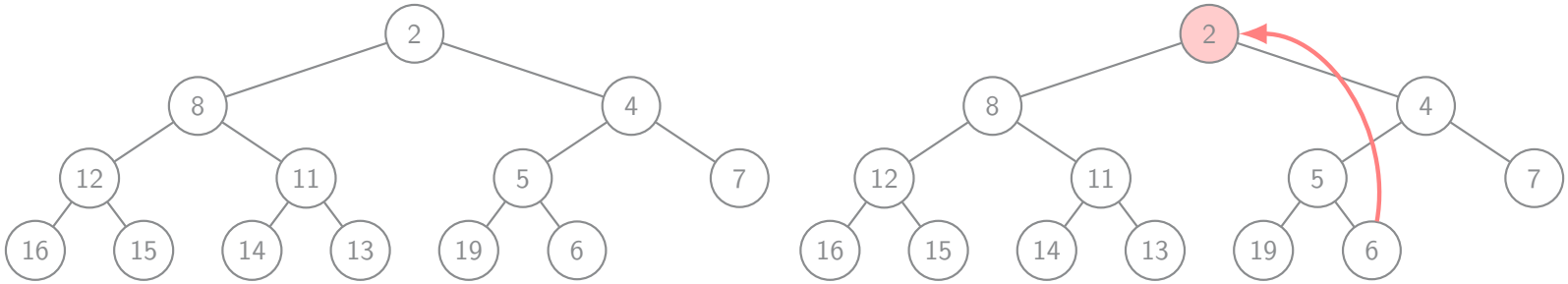
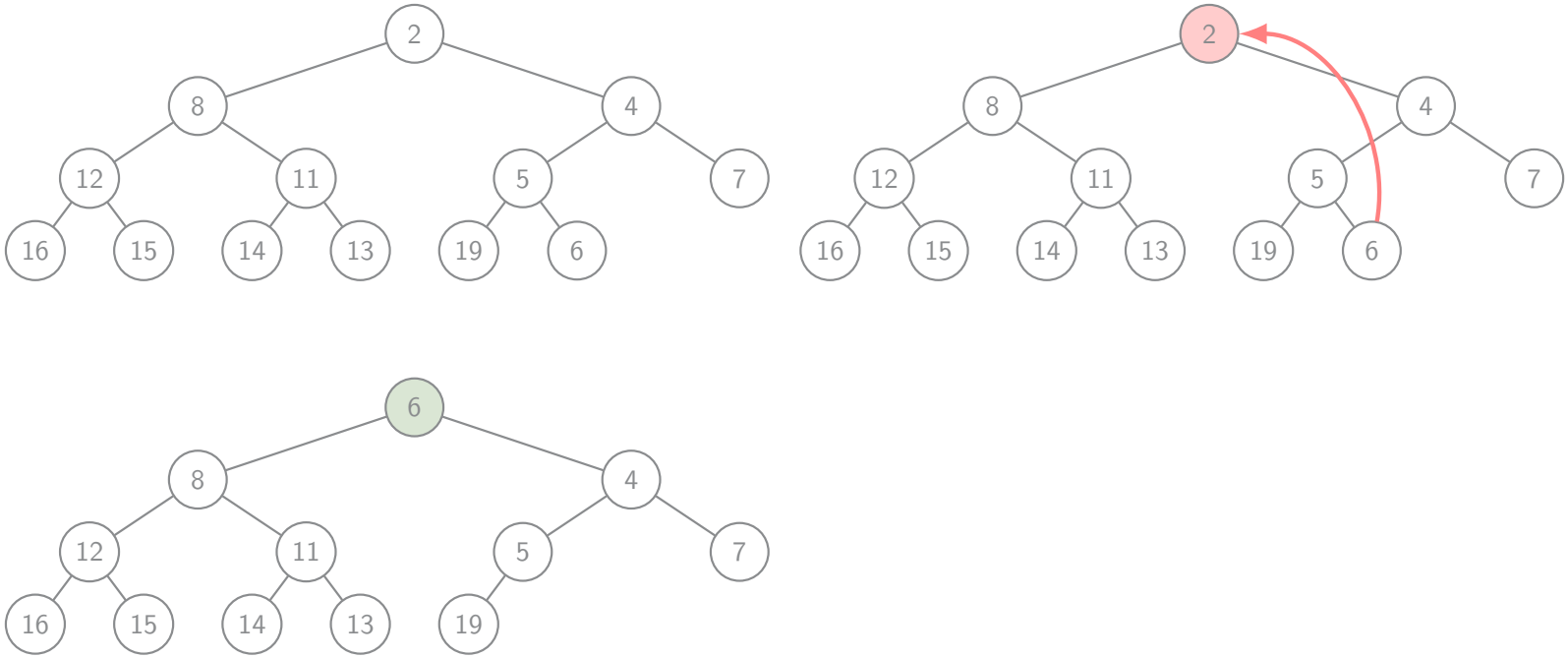
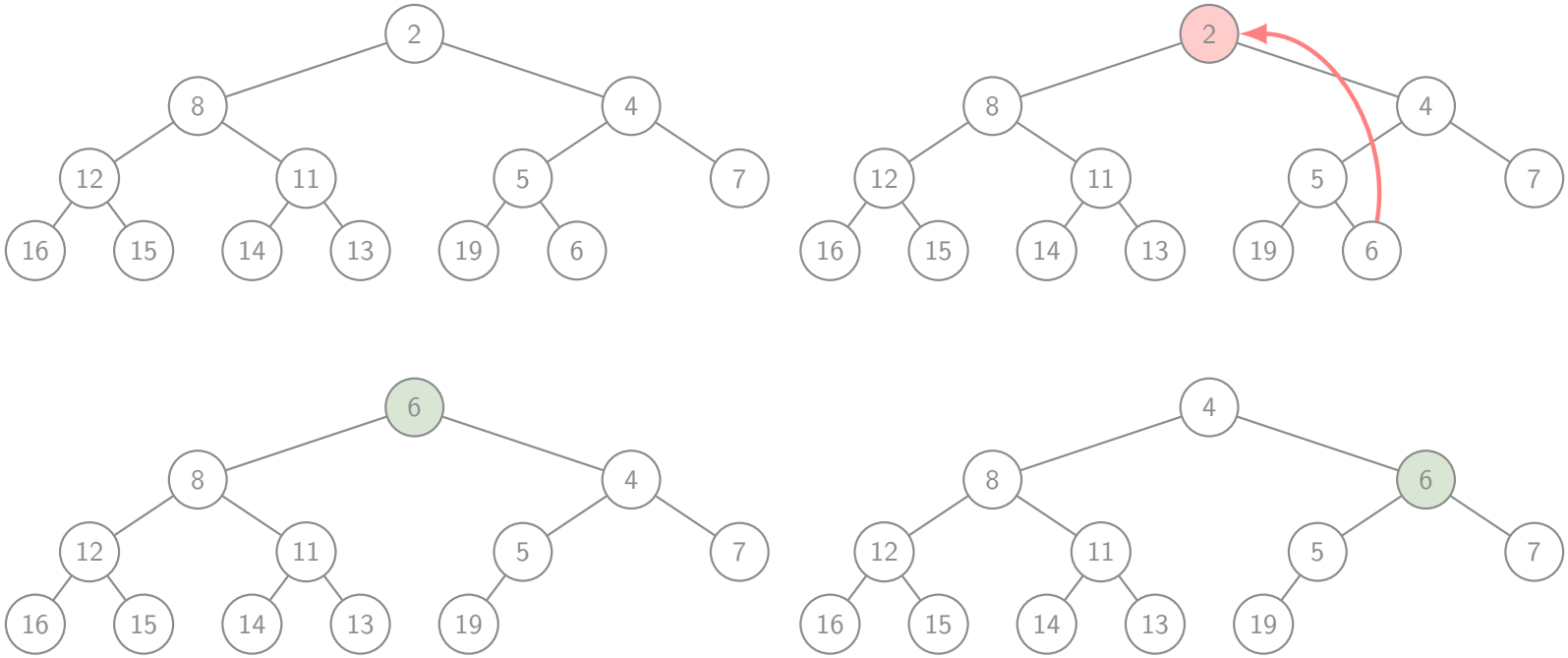


Binary Min-Heap



Binary Min-Heap



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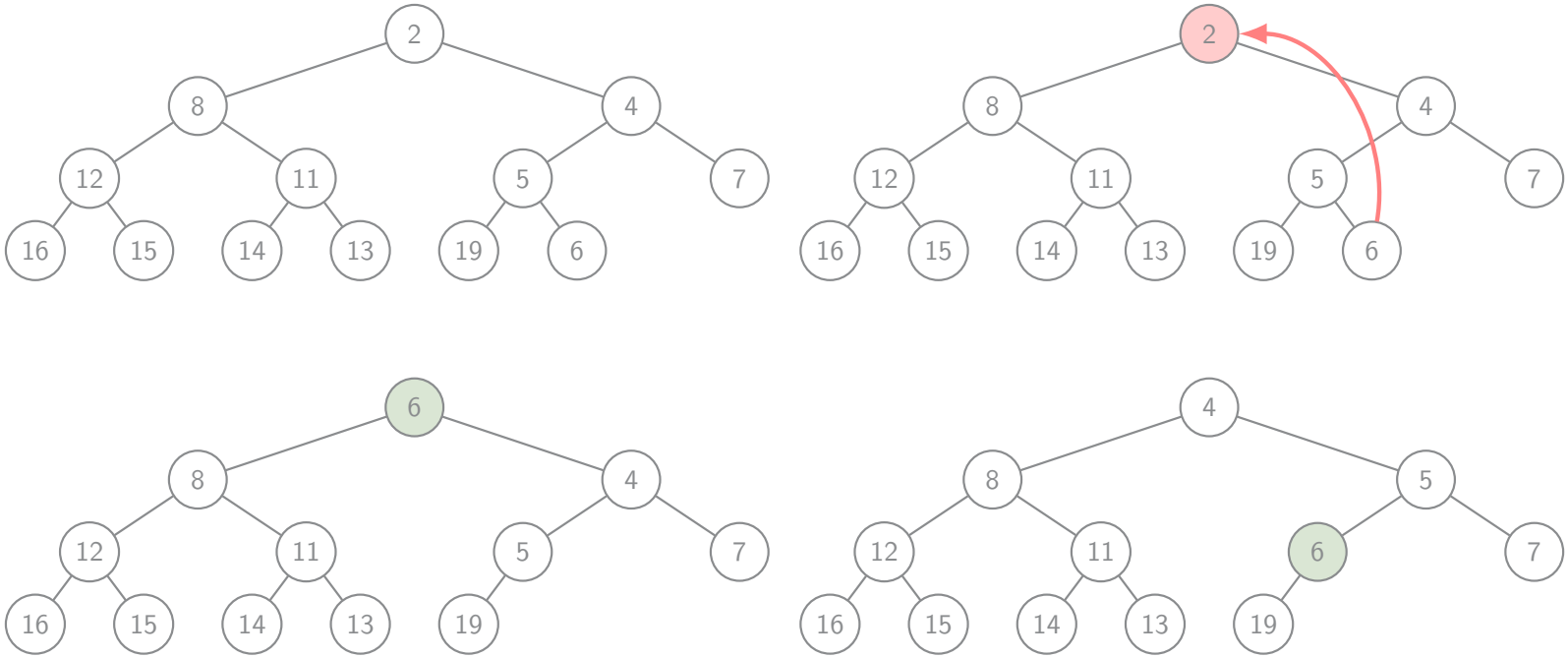


Binary Min-Heap

Operations

- `insert`
- `removeAt`
- `peek`
- `poll`

Binary Min-Heap

Operations

- `insert` $O(\log n)$
- `removeAt` $O(\log n)$
- `peek` $O(1)$
- `poll` $O(\log n)$

Binary Min-Heap

Operations

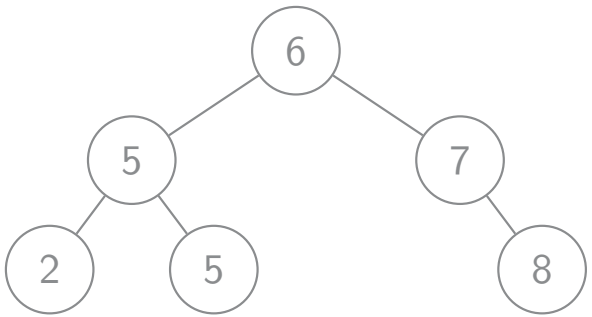
- `insert` $O(\log n)$
- `removeAt` $O(\log n)$
- `peek` $O(1)$
- `poll` $O(\log n)$

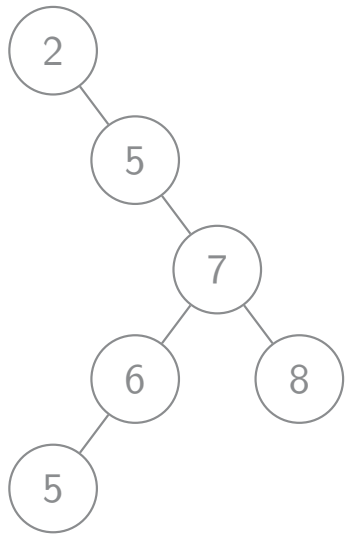
- Heapsort
- Frequently used in Priority Queues

Binary Search Tree

Definition

A binary tree in which the left child $\leq$ the parent and the right child $\geq$ the parent.





Binary Search Tree

Operations

- `insert` $O(\log n)$ / $O(n)$
- `delete` $O(\log n)$ / $O(n)$
- `search` $O(\log n)$ / $O(n)$

Red-Black Tree

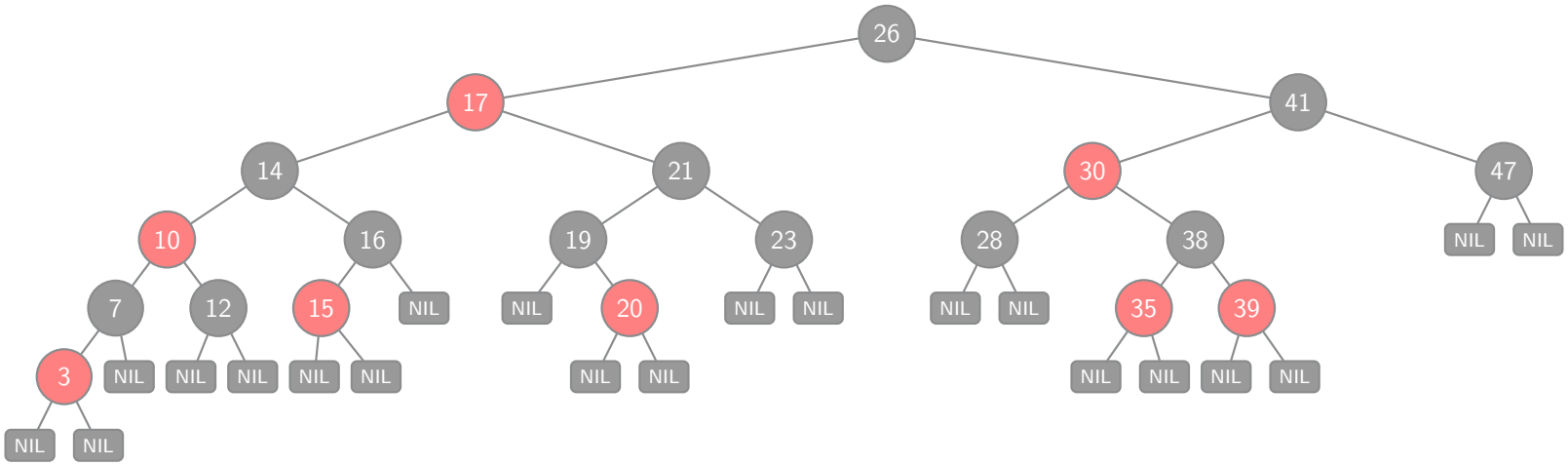
- Binary search tree
- Approximately balanced
- NIL leaves
- Red-black properties

Red-Black Tree

- Binary search tree
- Approximately balanced
- NIL leaves
- Red-black properties
  - Every node is either red or black
  - Root is black
  - Every leaf is black
  - If a node is red, its children are black
  - For each node, all paths to its descendant leaves contain the same number of black nodes

Red-Black Tree

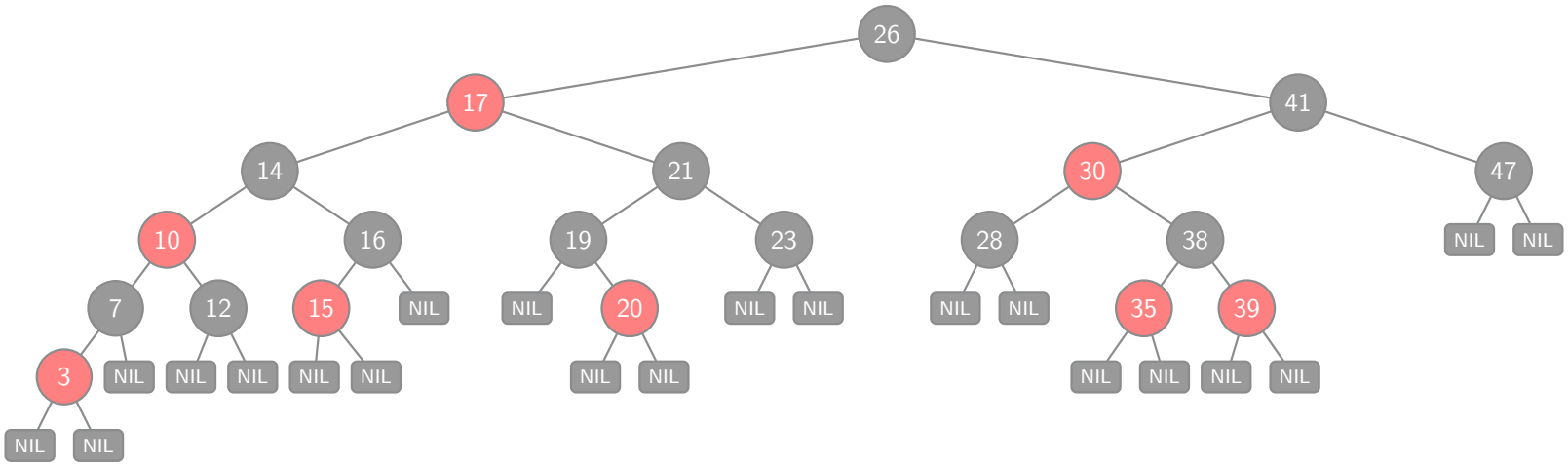
- Node is either red or black
- Every leaf is black
- Root is black
- If red, children are black
- $\forall$ node: all paths to its leaves have the same number of black nodes



Red-Black Tree

Operations

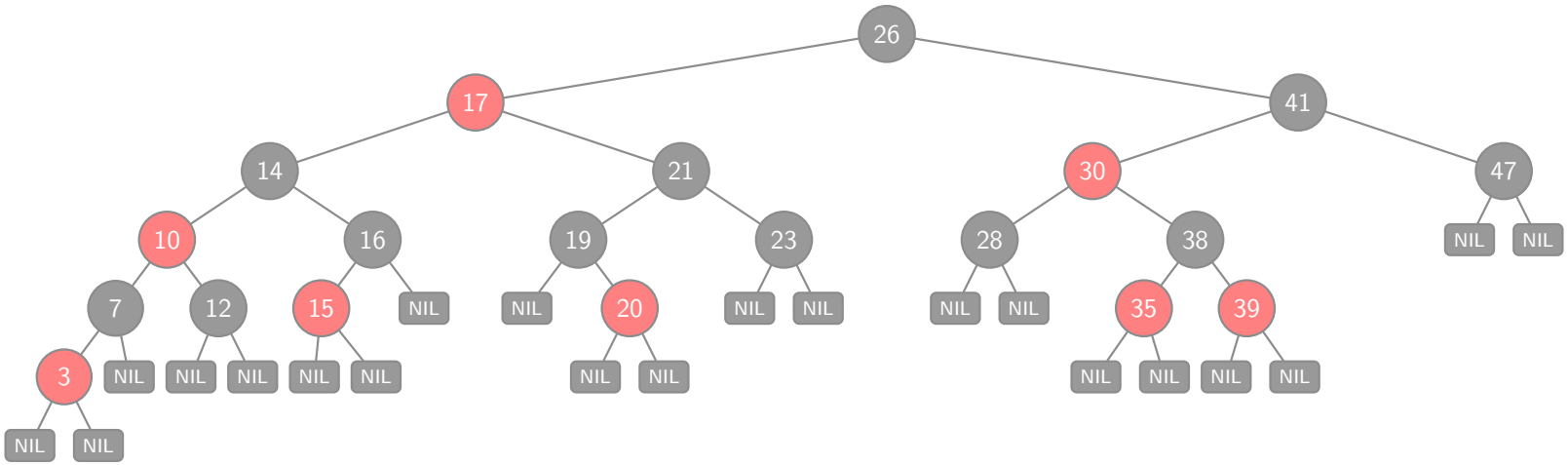
- `insert`
- `delete`
- `search`



Red-Black Tree

Operations

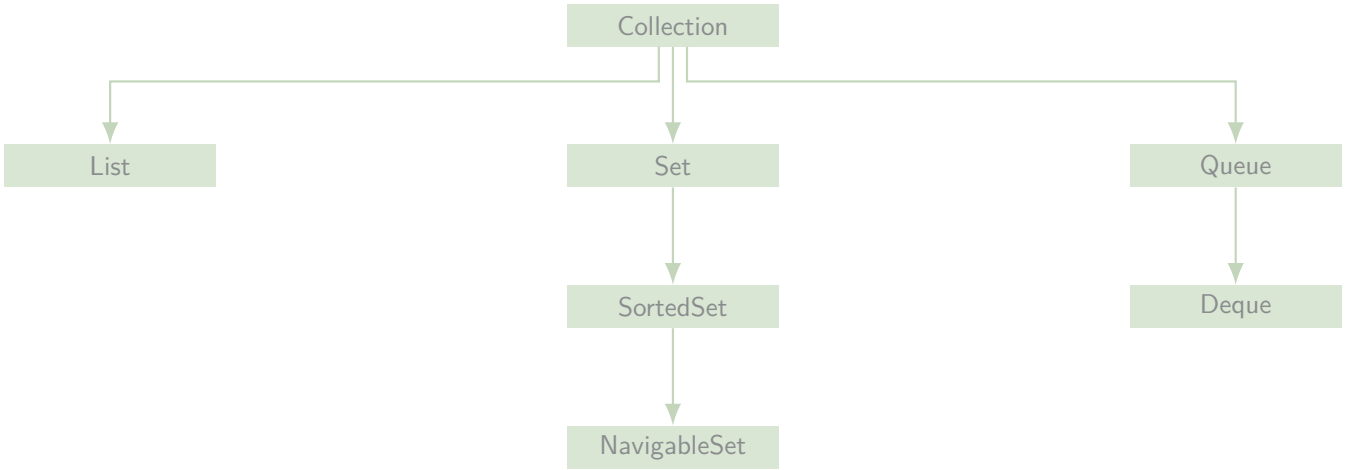
- `insert` $O(\log n)$
- `delete` $O(\log n)$
- `search` $O(\log n)$



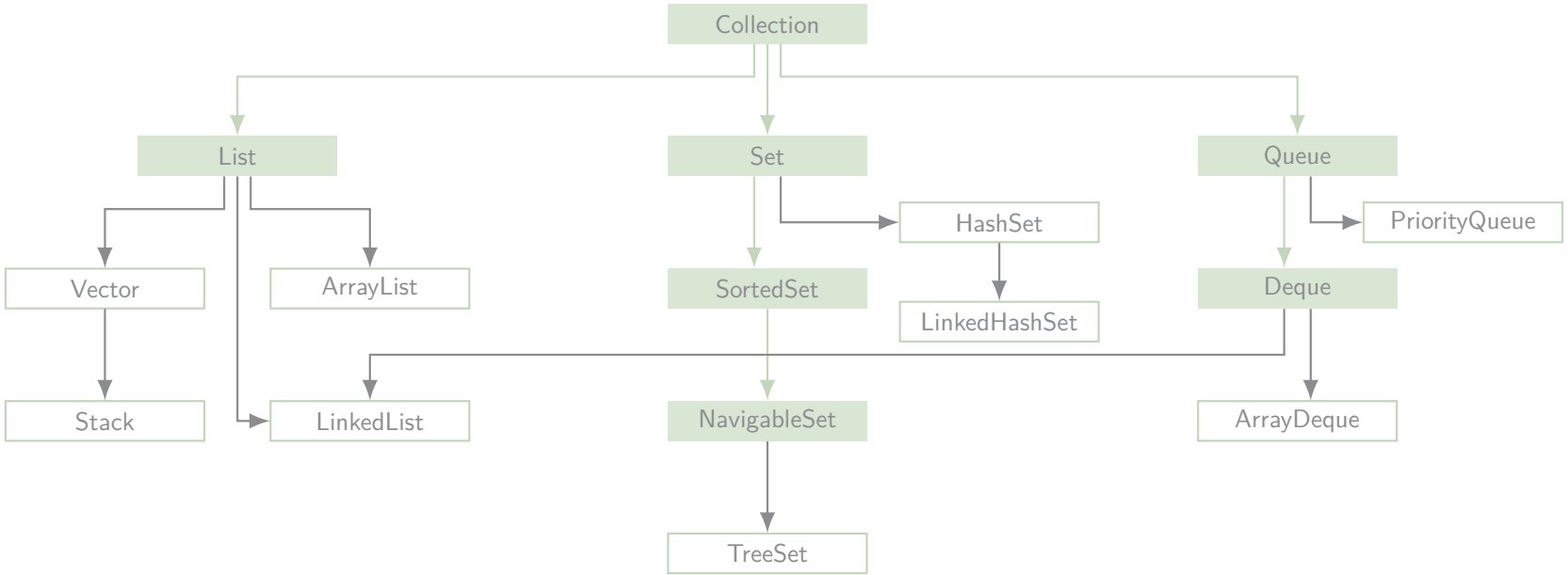
Outline

Outline

Java Collection API



Java Collection API



List Interface

	Impl	`add`	`remove`	`contains`	`get`
LinkedList	linked list				
ArrayList	array				
Vector	array				
Stack	array				

List Interface

	Impl	`add`	`remove`	`contains`	`get`
LinkedList	linked list	$O(1)$			
ArrayList	array				
Vector	array				
Stack	array				

List Interface

	Impl	`add`	`remove`	`contains`	`get`
LinkedList	linked list	$O(1)$	$O(1)$		
ArrayList	array				
Vector	array				
Stack	array				

List Interface

	Impl	`add`	`remove`	`contains`	`get`
LinkedList	linked list	$O(1)$	$O(1)$	$O(n)$	
ArrayList	array				
Vector	array				
Stack	array				

List Interface

	Impl	`add`	`remove`	`contains`	`get`
LinkedList	linked list	$O(1)$	$O(1)$	$O(n)$	$O(n)$
ArrayList	array				
Vector	array				
Stack	array				

List Interface

	Impl	`add`	`remove`	`contains`	`get`
LinkedList	linked list	$O(1)$	$O(1)$	$O(n)$	$O(n)$
ArrayList	array	$O(1)$			
Vector	array				
Stack	array				

List Interface

	Impl	`add`	`remove`	`contains`	`get`
LinkedList	linked list	$O(1)$	$O(1)$	$O(n)$	$O(n)$
ArrayList	array	$O(1)$	$O(n)$		
Vector	array				
Stack	array				

List Interface

	Impl	`add`	`remove`	`contains`	`get`
LinkedList	linked list	$O(1)$	$O(1)$	$O(n)$	$O(n)$
ArrayList	array	$O(1)$	$O(n)$	$O(n)$	
Vector	array				
Stack	array				

List Interface

	Impl	`add`	`remove`	`contains`	`get`
LinkedList	linked list	$O(1)$	$O(1)$	$O(n)$	$O(n)$
ArrayList	array	$O(1)$	$O(n)$	$O(n)$	$O(1)$
Vector	array				
Stack	array				

List Interface

	Impl	`add`	`remove`	`contains`	`get`
LinkedList	linked list	$O(1)$	$O(1)$	$O(n)$	$O(n)$
ArrayList	array	$O(1)$	$O(n)$	$O(n)$	$O(1)$
Vector	array	$O(1)$	$O(n)$	$O(n)$	$O(1)$
Stack	array	$O(1)$	$O(n)$	$O(n)$	$O(1)$

Set Interface

	Impl	`add`	`contains`	`next`
HashSet	hash table			
LinkedHashSet	hash table linked list			
TreeSet	red-black tree			

Set Interface

	Impl	`add`	`contains`	`next`
HashSet	hash table	$O(1)$		
LinkedHashSet	hash table linked list			
TreeSet	red-black tree			

Set Interface

	Impl	`add`	`contains`	`next`
HashSet	hash table	$O(1)$	$O(1)$	
LinkedHashSet	hash table linked list			
TreeSet	red-black tree			

Set Interface

	Impl	`add`	`contains`	`next`
HashSet	hash table	$O(1)$	$O(1)$	$O(h/n)$
LinkedHashSet	hash table linked list			
TreeSet	red-black tree			

Set Interface

	Impl	`add`	`contains`	`next`
HashSet	hash table	$O(1)$	$O(1)$	$O(h/n)$
LinkedHashSet	hash table linked list	$O(1)$		
TreeSet	red-black tree			

Set Interface

	Impl	`add`	`contains`	`next`
HashSet	hash table	$O(1)$	$O(1)$	$O(h/n)$
LinkedHashSet	hash table linked list	$O(1)$	$O(1)$	
TreeSet	red-black tree			

Set Interface

	Impl	`add`	`contains`	`next`
HashSet	hash table	$O(1)$	$O(1)$	$O(h/n)$
LinkedHashSet	hash table linked list	$O(1)$	$O(1)$	$O(1)$
TreeSet	red-black tree			

Set Interface

	Impl	`add`	`contains`	`next`
HashSet	hash table	$O(1)$	$O(1)$	$O(h/n)$
LinkedHashSet	hash table linked list	$O(1)$	$O(1)$	$O(1)$
TreeSet	red-black tree	$O(\log n)$		

Set Interface

	Impl	`add`	`contains`	`next`
HashSet	hash table	$O(1)$	$O(1)$	$O(h/n)$
LinkedHashSet	hash table linked list	$O(1)$	$O(1)$	$O(1)$
TreeSet	red-black tree	$O(\log n)$	$O(\log n)$	

Set Interface

	Impl	`add`	`contains`	`next`
HashSet	hash table	$O(1)$	$O(1)$	$O(h/n)$
LinkedHashSet	hash table linked list	$O(1)$	$O(1)$	$O(1)$
TreeSet	red-black tree	$O(\log n)$	$O(\log n)$	$O(\log n)$

Queue Interface

	Impl	`offer`	`peak`	`poll`
PriorityQueue	binary heap			
ArrayDeque	array			
LinkedList	linked list			

Queue Interface

	Impl	`offer`	`peak`	`poll`
PriorityQueue	binary heap	$O(\log n)$		
ArrayDeque	array			
LinkedList	linked list			

Queue Interface

	Impl	`offer`	`peak`	`poll`
PriorityQueue	binary heap	$O(\log n)$	$O(1)$	
ArrayDeque	array			
LinkedList	linked list			

Queue Interface

	Impl	`offer`	`peak`	`poll`
PriorityQueue	binary heap	$O(\log n)$	$O(1)$	$O(\log n)$
ArrayDeque	array			
LinkedList	linked list			

Queue Interface

	Impl	`offer`	`peak`	`poll`
PriorityQueue	binary heap	$O(\log n)$	$O(1)$	$O(\log n)$
ArrayDeque	array	$O(1)$		
LinkedList	linked list			

Queue Interface

	Impl	`offer`	`peak`	`poll`
PriorityQueue	binary heap	$O(\log n)$	$O(1)$	$O(\log n)$
ArrayDeque	array	$O(1)$	$O(1)$	
LinkedList	linked list			

Queue Interface

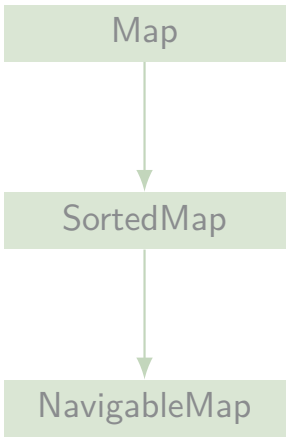
	Impl	`offer`	`peak`	`poll`
PriorityQueue	binary heap	$O(\log n)$	$O(1)$	$O(\log n)$
ArrayDeque	array	$O(1)$	$O(1)$	$O(1)$
LinkedList	linked list			

Queue Interface

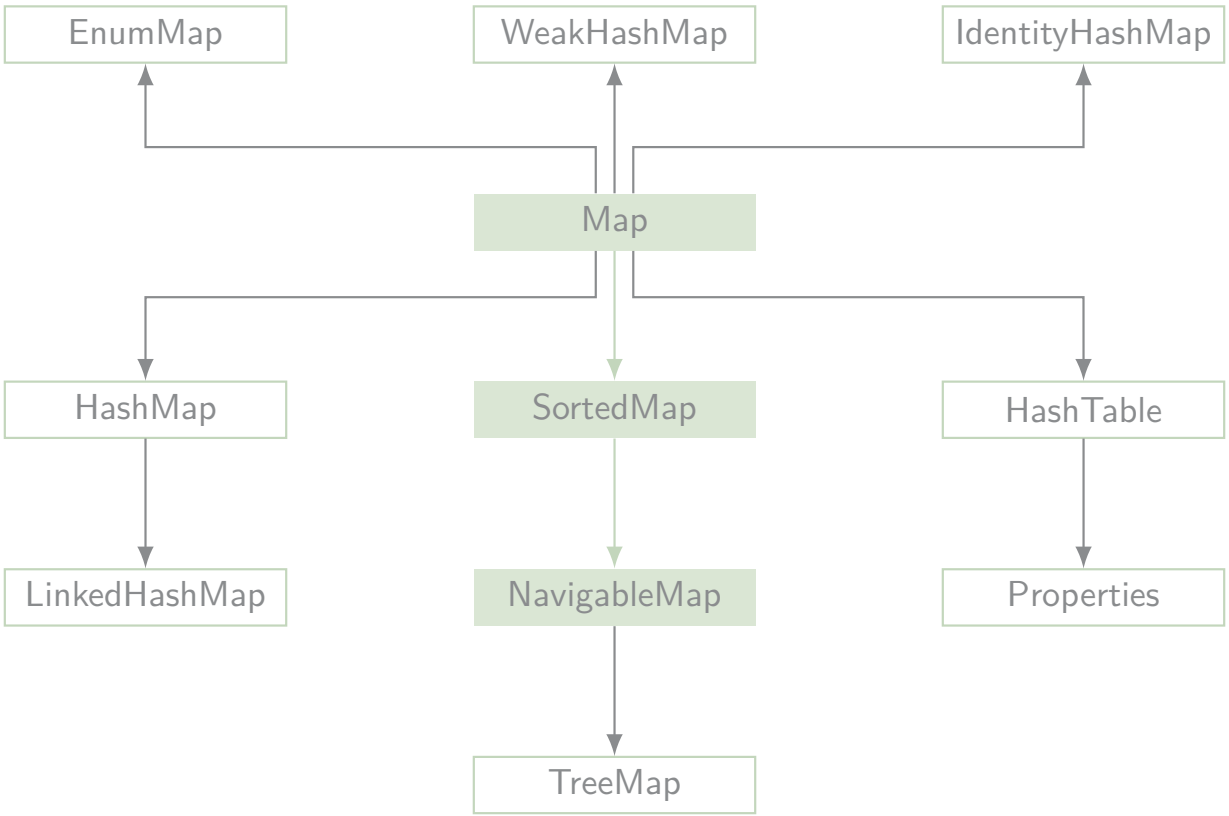
	Impl	`offer`	`peak`	`poll`
PriorityQueue	binary heap	$O(\log n)$	$O(1)$	$O(\log n)$
ArrayDeque	array	$O(1)$	$O(1)$	$O(1)$
LinkedList	linked list	$O(1)$	$O(1)$	$O(1)$

Outline

Java Map API



Java Map API


EnumMap
WeakHashMap
IdentityHashMap
Map
HashMap
SortedMap
HashTable
LinkedHashMap
NavigableMap
Properties
TreeMap

Map Interface

	Impl	`get`	`containsKey`	`next`
HashTable	hash table			
Properties	hash table			
HashMap	hash table			
LinkedHashMap	hash table linked list			
TreeMap	red-black tree			
IdendityHashMap	array			
WeekHashMap	hash table			
EnumMap	array			

Map Interface

	Impl	`get`	`containsKey`	`next`
HashTable	hash table	$O(1)$		
Properties	hash table	$O(1)$		
HashMap	hash table	$O(1)$		
LinkedHashMap	hash table linked list			
TreeMap	red-black tree			
IdendityHashMap	array			
WeekHashMap	hash table			
EnumMap	array			

Map Interface

	Impl	`get`	`containsKey`	`next`
HashTable	hash table	$O(1)$	$O(1)$	
Properties	hash table	$O(1)$	$O(1)$	
HashMap	hash table	$O(1)$	$O(1)$	
LinkedHashMap	hash table linked list			
TreeMap	red-black tree			
IdendityHashMap	array			
WeekHashMap	hash table			
EnumMap	array			

Map Interface

	Impl	`get`	`containsKey`	`next`
HashTable	hash table	$O(1)$	$O(1)$	$O(h/n)$
Properties	hash table	$O(1)$	$O(1)$	$O(h/n)$
HashMap	hash table	$O(1)$	$O(1)$	$O(h/n)$
LinkedHashMap	hash table linked list			
TreeMap	red-black tree			
IdendityHashMap	array			
WeekHashMap	hash table			
EnumMap	array			

Map Interface

	Impl	`get`	`containsKey`	`next`
HashTable	hash table	$O(1)$	$O(1)$	$O(h/n)$
Properties	hash table	$O(1)$	$O(1)$	$O(h/n)$
HashMap	hash table	$O(1)$	$O(1)$	$O(h/n)$
LinkedHashMap	hash table linked list	$O(1)$		
TreeMap	red-black tree			
IdendityHashMap	array			
WeekHashMap	hash table			
EnumMap	array			

Map Interface

	Impl	`get`	`containsKey`	`next`
HashTable	hash table	$O(1)$	$O(1)$	$O(h/n)$
Properties	hash table	$O(1)$	$O(1)$	$O(h/n)$
HashMap	hash table	$O(1)$	$O(1)$	$O(h/n)$
LinkedHashMap	hash table linked list	$O(1)$	$O(1)$	
TreeMap	red-black tree			
IdendityHashMap	array			
WeekHashMap	hash table			
EnumMap	array			

Map Interface

	Impl	`get`	`containsKey`	`next`
HashTable	hash table	$O(1)$	$O(1)$	$O(h/n)$
Properties	hash table	$O(1)$	$O(1)$	$O(h/n)$
HashMap	hash table	$O(1)$	$O(1)$	$O(h/n)$
LinkedHashMap	hash table linked list	$O(1)$	$O(1)$	$O(1)$
TreeMap	red-black tree			
IdendityHashMap	array			
WeekHashMap	hash table			
EnumMap	array			

Map Interface

	Impl	`get`	`containsKey`	`next`
HashTable	hash table	$O(1)$	$O(1)$	$O(h/n)$
Properties	hash table	$O(1)$	$O(1)$	$O(h/n)$
HashMap	hash table	$O(1)$	$O(1)$	$O(h/n)$
LinkedHashMap	hash table linked list	$O(1)$	$O(1)$	$O(1)$
TreeMap	red-black tree	$O(\log n)$		
IdendityHashMap	array			
WeekHashMap	hash table			
EnumMap	array			

Map Interface

	Impl	`get`	`containsKey`	`next`
HashTable	hash table	$O(1)$	$O(1)$	$O(h/n)$
Properties	hash table	$O(1)$	$O(1)$	$O(h/n)$
HashMap	hash table	$O(1)$	$O(1)$	$O(h/n)$
LinkedHashMap	hash table linked list	$O(1)$	$O(1)$	$O(1)$
TreeMap	red-black tree	$O(\log n)$	$O(\log n)$	
IdendityHashMap	array			
WeekHashMap	hash table			
EnumMap	array			

Map Interface

	Impl	`get`	`containsKey`	`next`
HashTable	hash table	$O(1)$	$O(1)$	$O(h/n)$
Properties	hash table	$O(1)$	$O(1)$	$O(h/n)$
HashMap	hash table	$O(1)$	$O(1)$	$O(h/n)$
LinkedHashMap	hash table linked list	$O(1)$	$O(1)$	$O(1)$
TreeMap	red-black tree	$O(\log n)$	$O(\log n)$	$O(\log n)$
IdendityHashMap	array			
WeekHashMap	hash table			
EnumMap	array			

Map Interface

	Impl	`get`	`containsKey`	`next`
HashTable	hash table	$O(1)$	$O(1)$	$O(h/n)$
Properties	hash table	$O(1)$	$O(1)$	$O(h/n)$
HashMap	hash table	$O(1)$	$O(1)$	$O(h/n)$
LinkedHashMap	hash table linked list	$O(1)$	$O(1)$	$O(1)$
TreeMap	red-black tree	$O(\log n)$	$O(\log n)$	$O(\log n)$
IdendityHashMap	array	$O(1)$	$O(1)$	$O(h/n)$
WeekHashMap	hash table	$O(1)$	$O(1)$	$O(h/n)$
EnumMap	array	$O(1)$	$O(1)$	$O(1)$

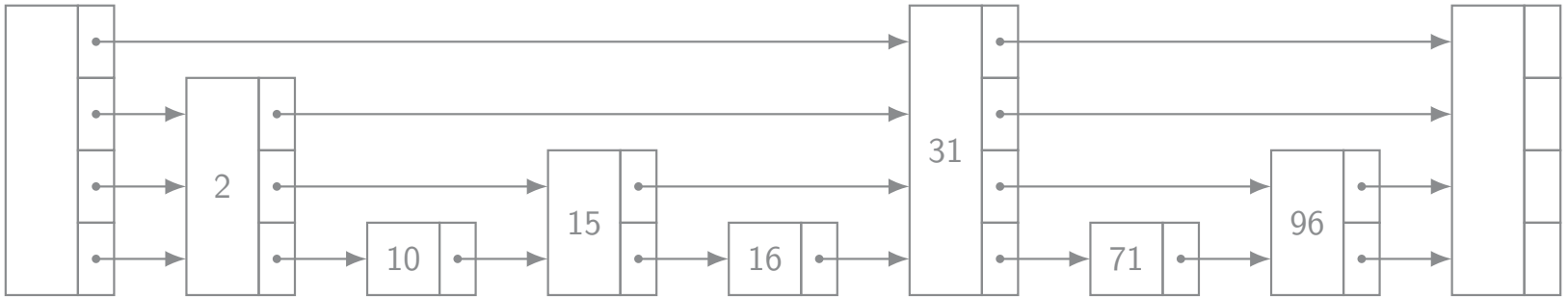
Outline

Outline

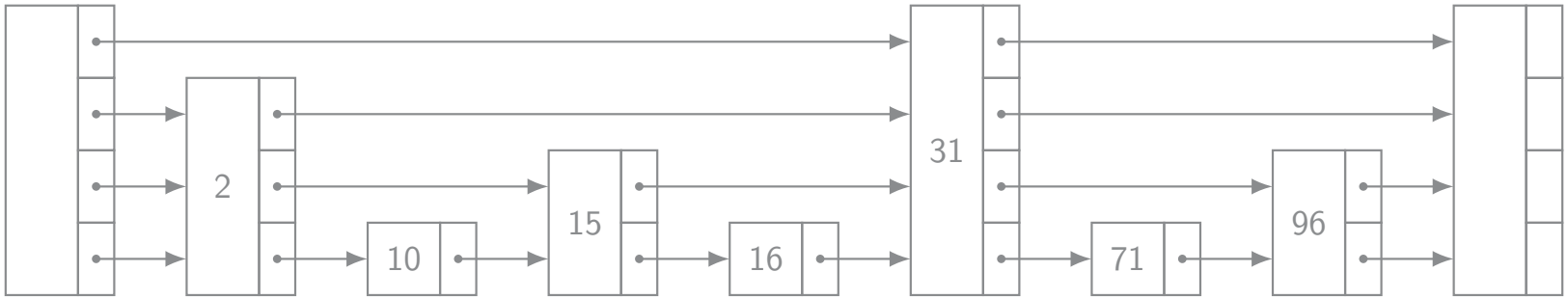
Skip List

- Balanced binary tree alternative
  - `insert`, `delete` & `search` in $O(\log n)$
- Probabilistic balancing rather than strictly enforced balancing
- Insertion and deletion $\rightarrow$ simpler and faster
- Linked hierarchy of subsequences, with each successive subsequence skipping over fewer elements than the previous one
  - Hierarchy has $\approx \log n$ levels

Skip List

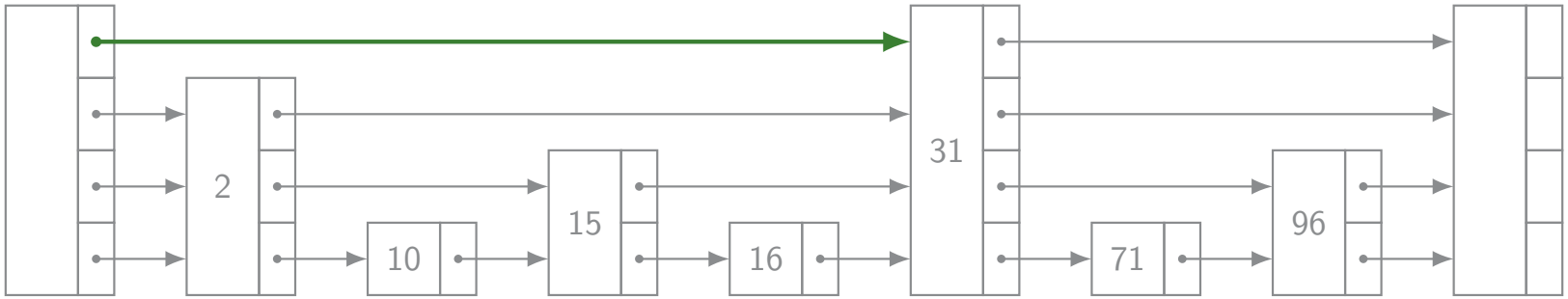


Skip List



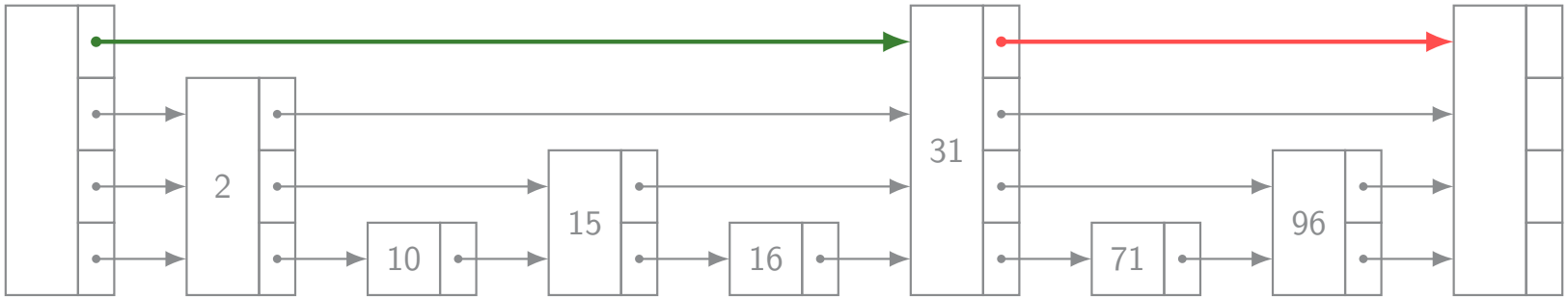
`find 71`

Skip List



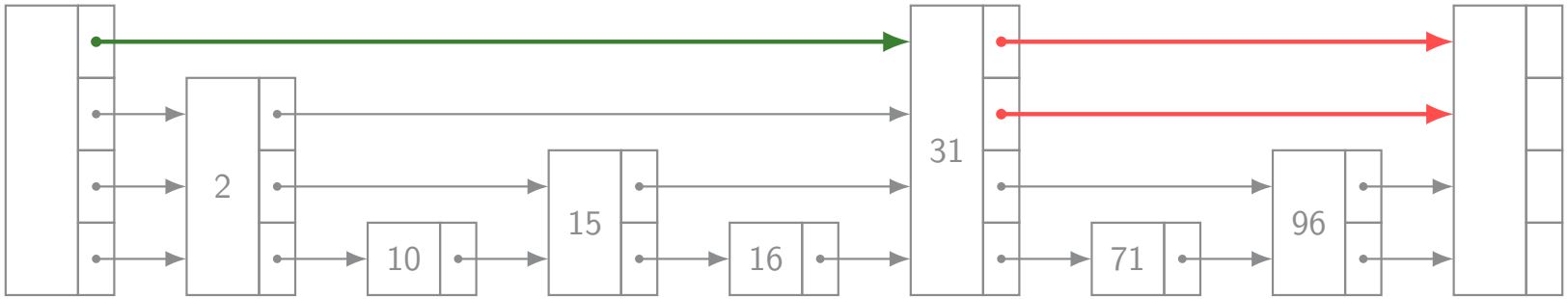
`find 71`

Skip List



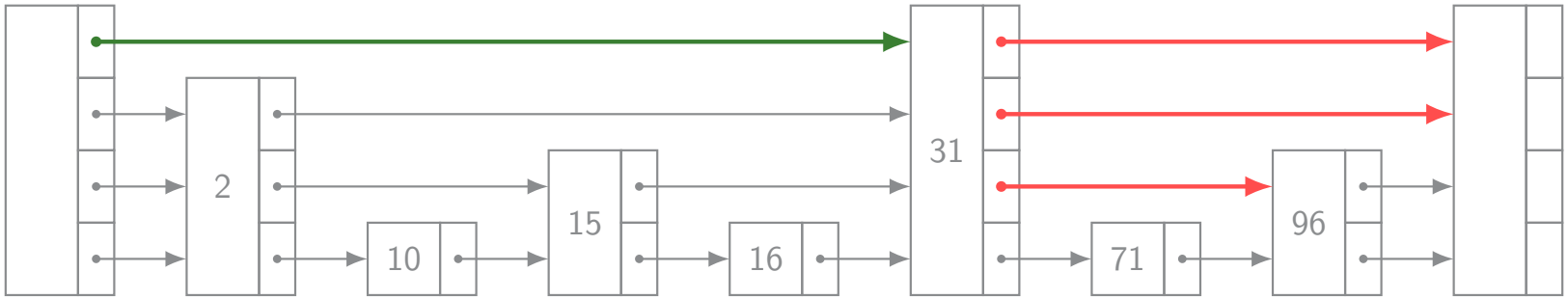
`find 71`

Skip List



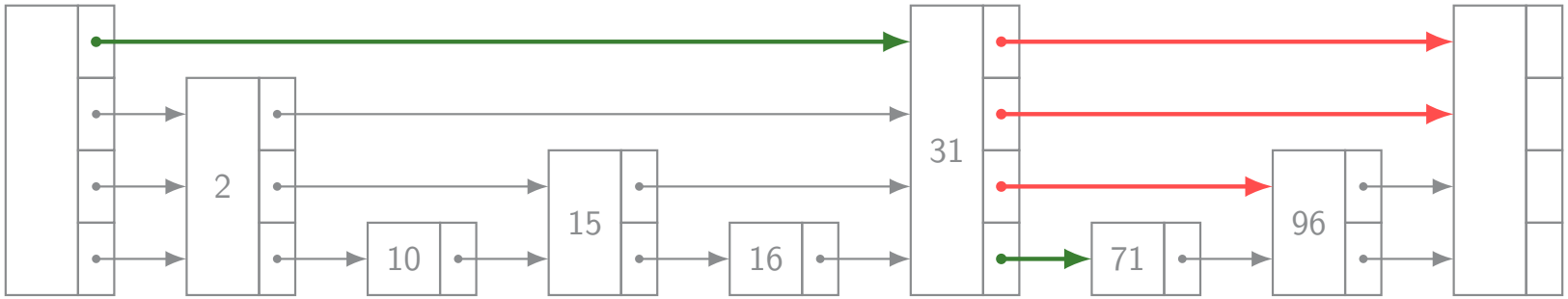
`find 71`

Skip List



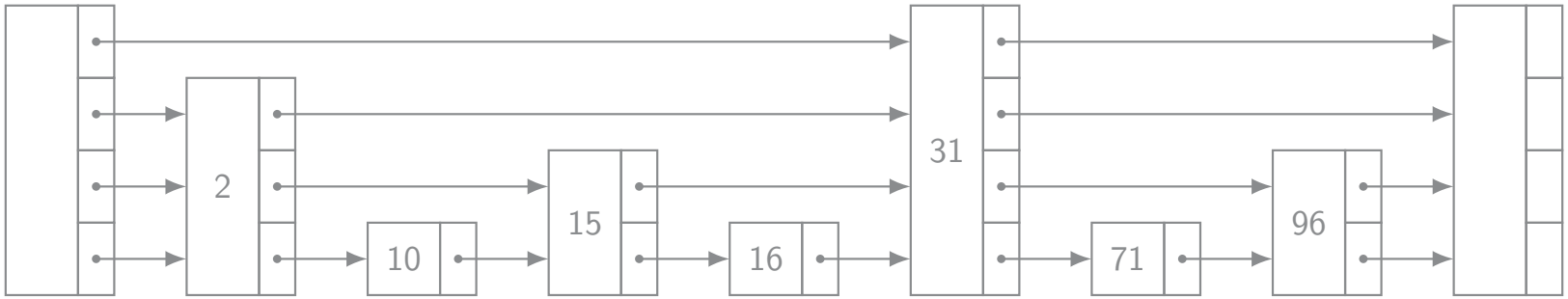
`find 71`

Skip List



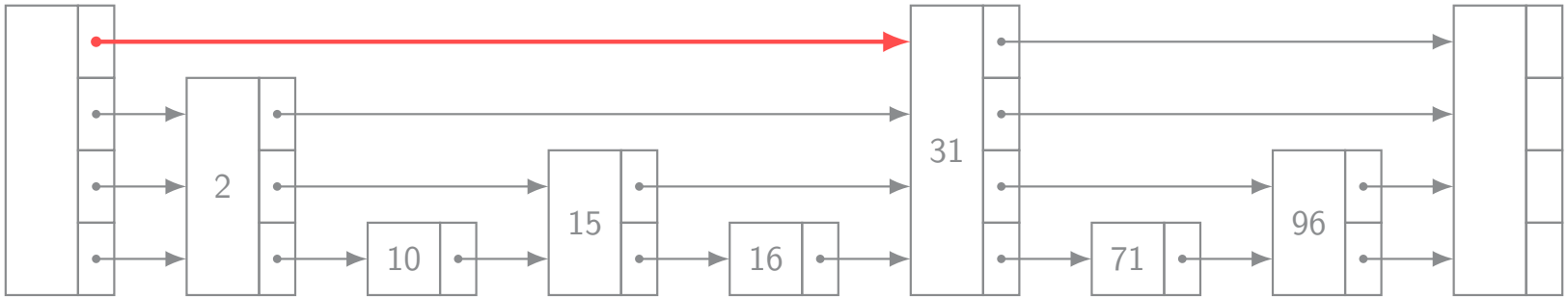
`find 71`

Skip List



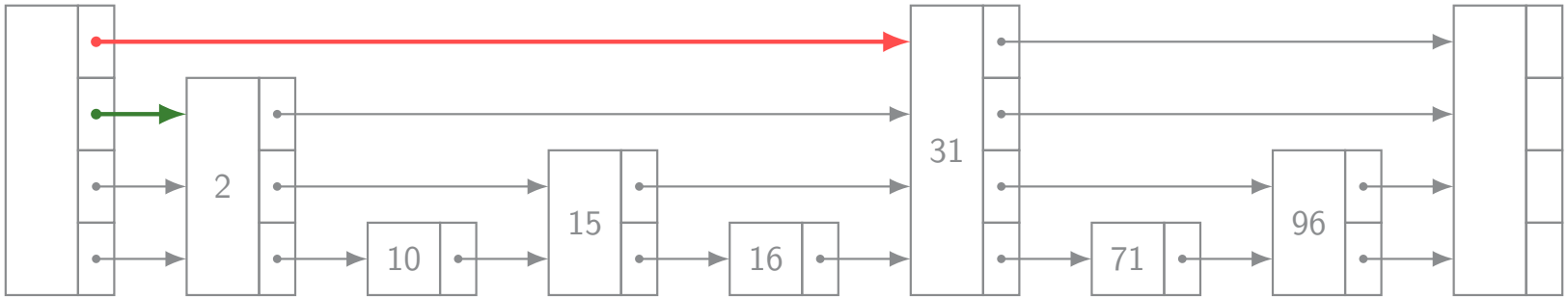
`find 12`

Skip List



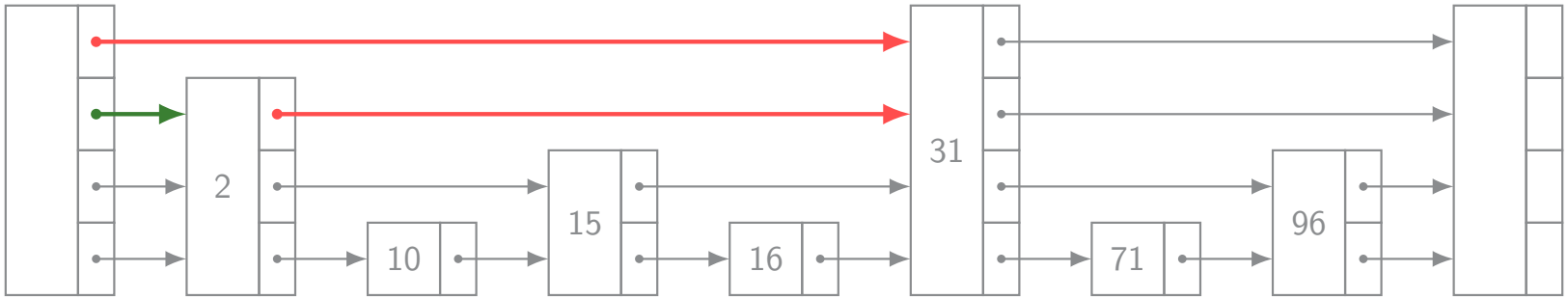

```
find 12
```

Skip List



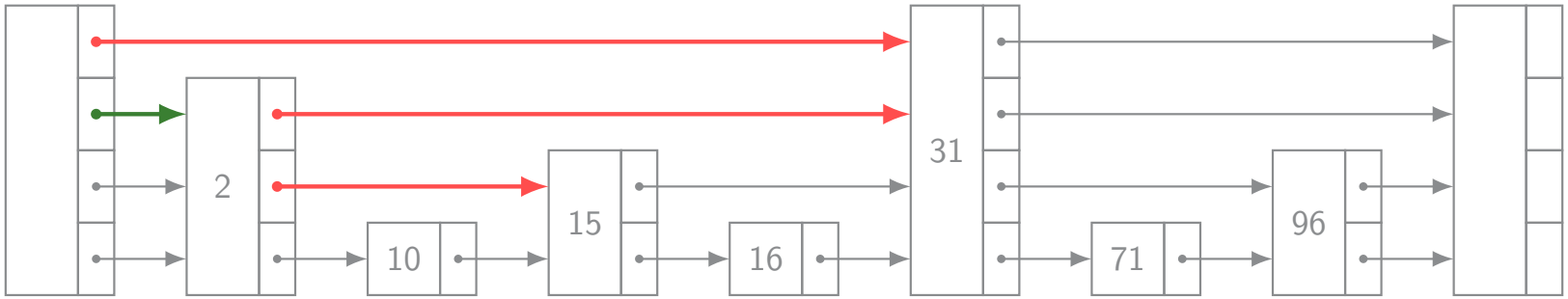
`find 12`

Skip List



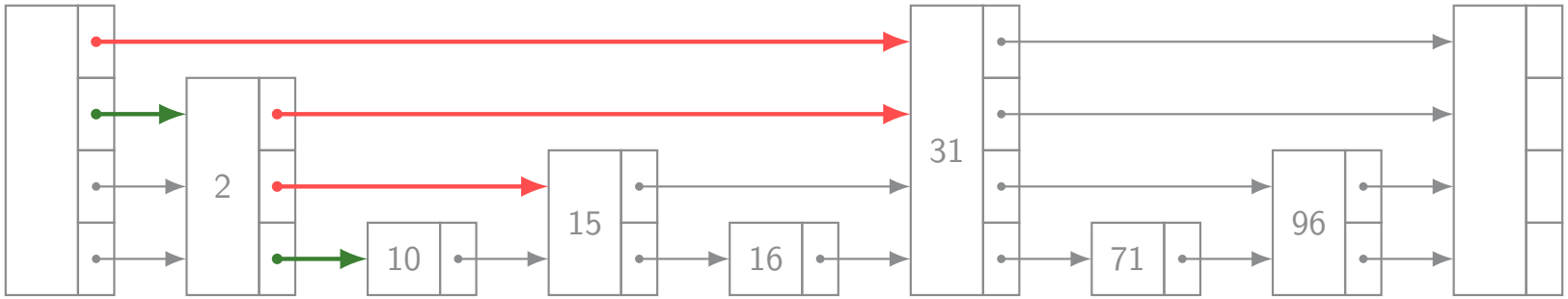
`find 12`

Skip List



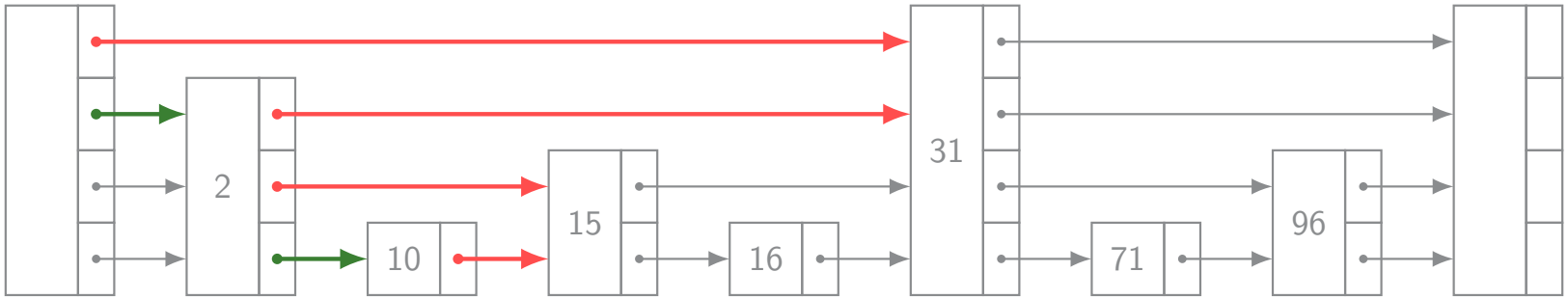
`find 12`

Skip List



`find 12`

Skip List

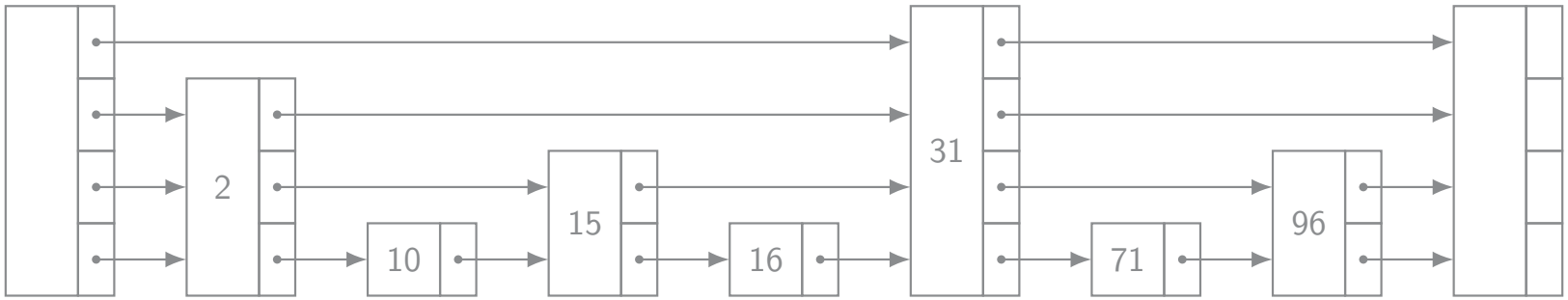


`find 12`

Skip List

Insertion

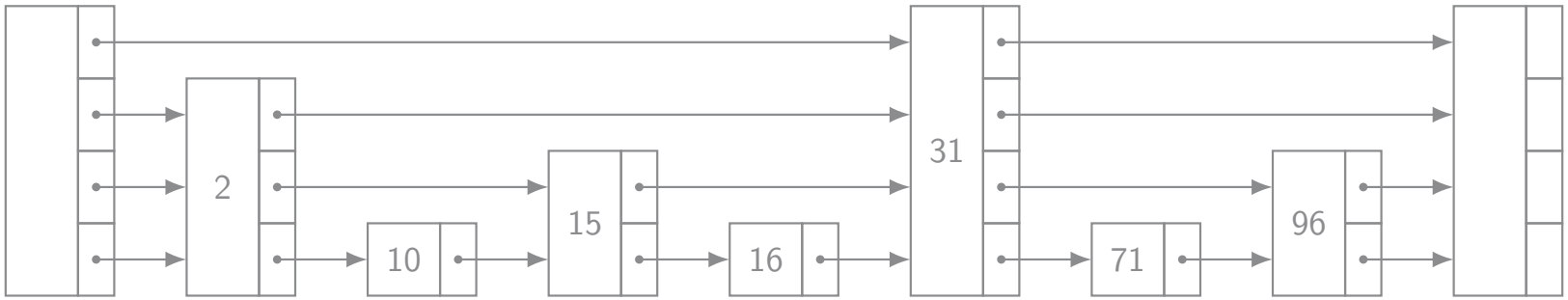
- `insert at level=1`
- `while (coinflip() == HEADS)`
  `insert at ++level`



Skip List

Insertion

- `insert at level=1`
- `while (coinflip() == HEADS)`
  `insert at ++level`

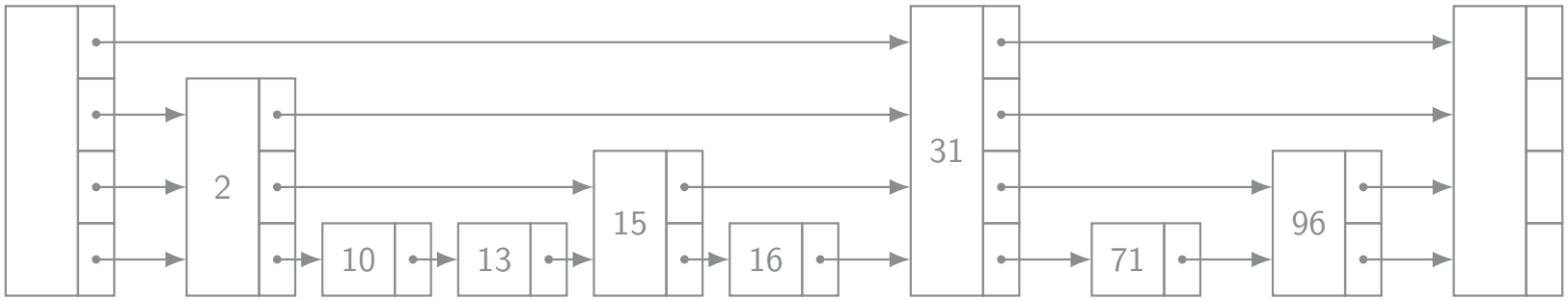


`insert 13`

Skip List

Insertion

- `insert at level=1`
- `while (coinflip() == HEADS)`
  `insert at ++level`

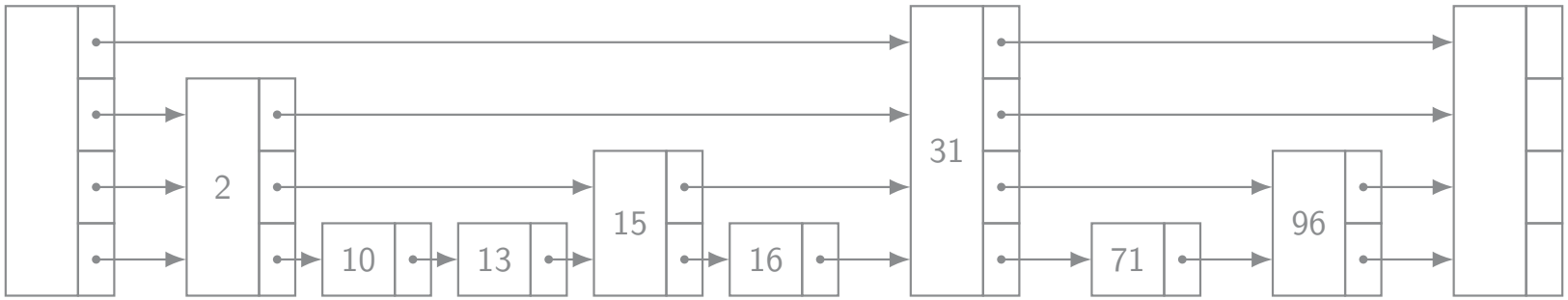


`insert 13`

Skip List

Insertion

- `insert at level=1`
- `while (coinflip() == HEADS)`
  `insert at ++level`

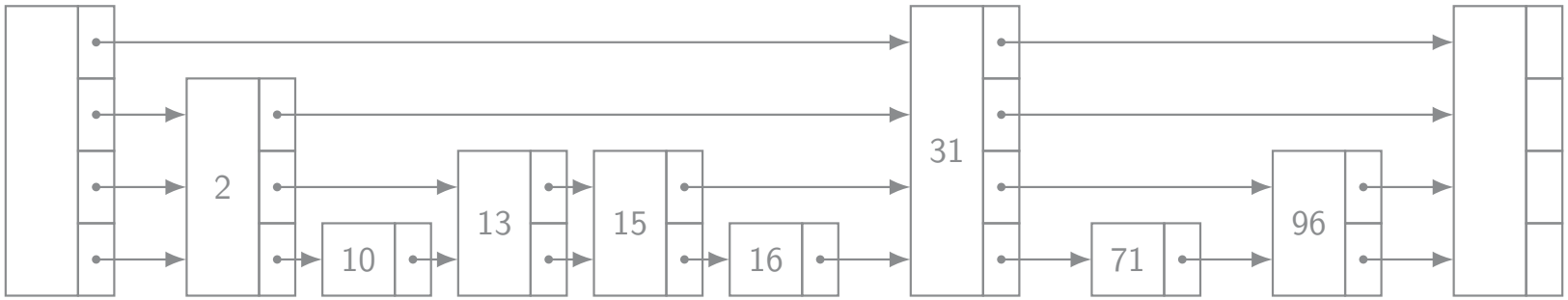


`insert 13` — Coinflips: H

Skip List

Insertion

- `insert at level=1`
- `while (coinflip() == HEADS)`
  `insert at ++level`

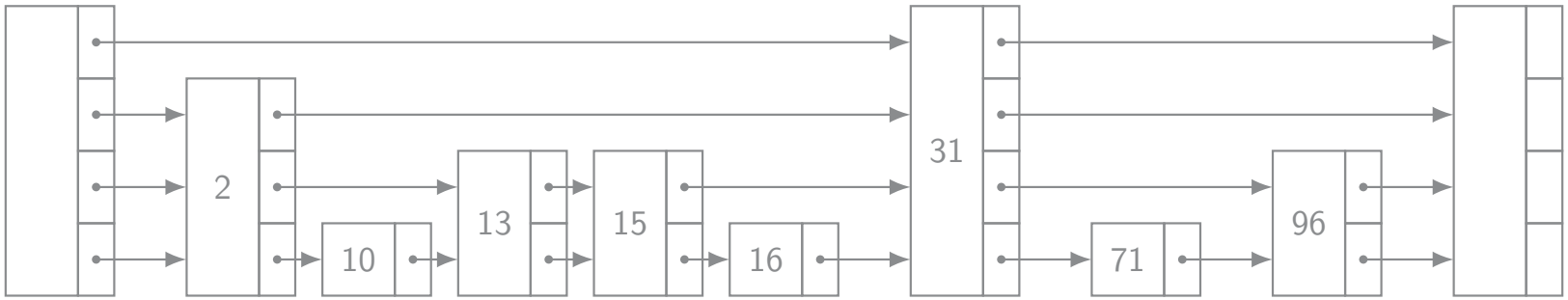


`insert 13` — Coinflips: H

Skip List

Insertion

- `insert at level=1`
- `while (coinflip() == HEADS)`
  `insert at ++level`

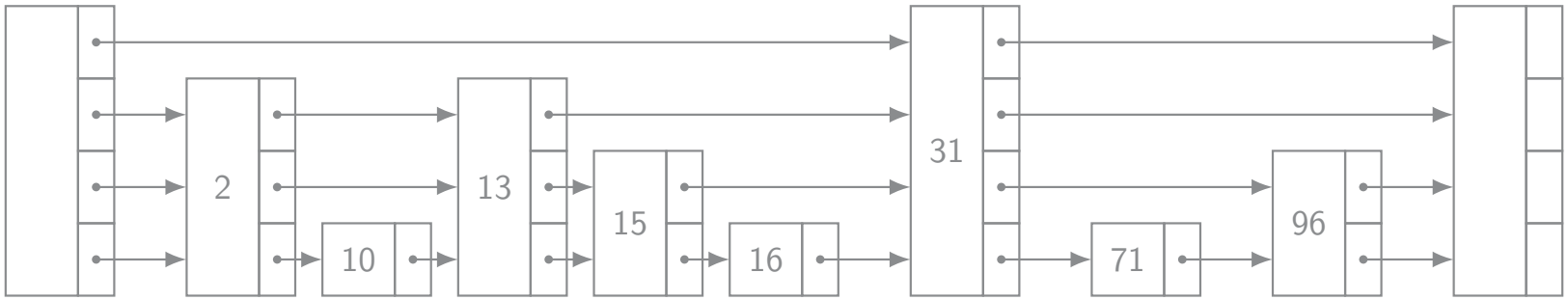


`insert 13` — Coinflips: HH

Skip List

Insertion

- `insert at level=1`
- `while (coinflip() == HEADS)`
  `insert at ++level`

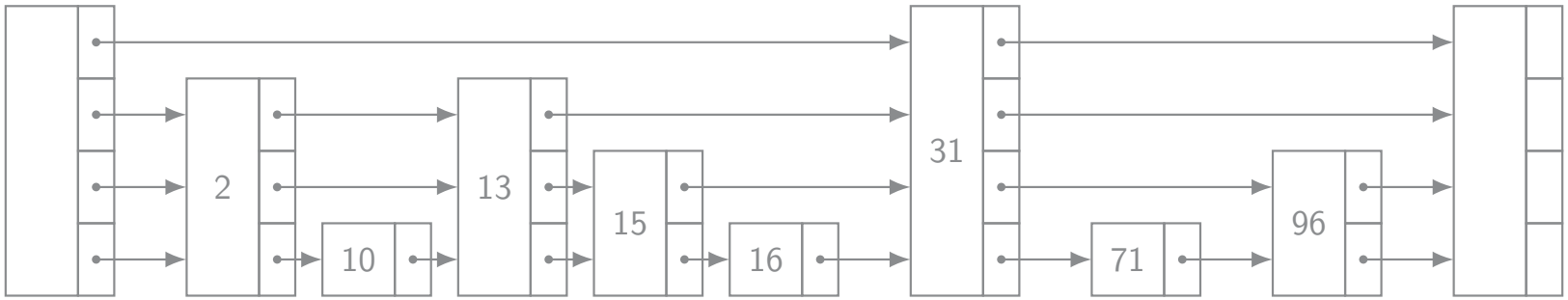


`insert 13` — Coinflips: HH

Skip List

Insertion

- `insert at level=1`
- `while (coinflip() == HEADS)`
  `insert at ++level`



`insert 13` — Coinflips: HHT

Skip List

Operations

- `insert`
- `delete`
- `search`

Skip List

Operations

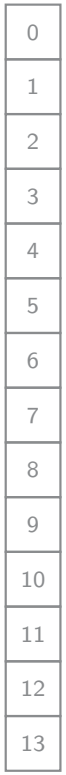
- `insert` $O(\log n)$ / $O(n)$
- `delete` $O(\log n)$ / $O(n)$
- `search` $O(\log n)$ / $O(n)$

Outline

Bloom Filter

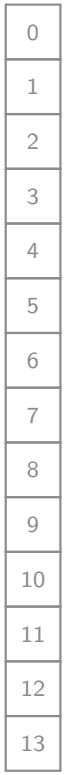
- Data structure to indicate if an element is contained in a set
- Bit array $\rightarrow$ space efficient
- Probabilistic
  - False positives possible
  - False negatives impossible
- Elements can be added to the set, but not removed

Bloom Filter

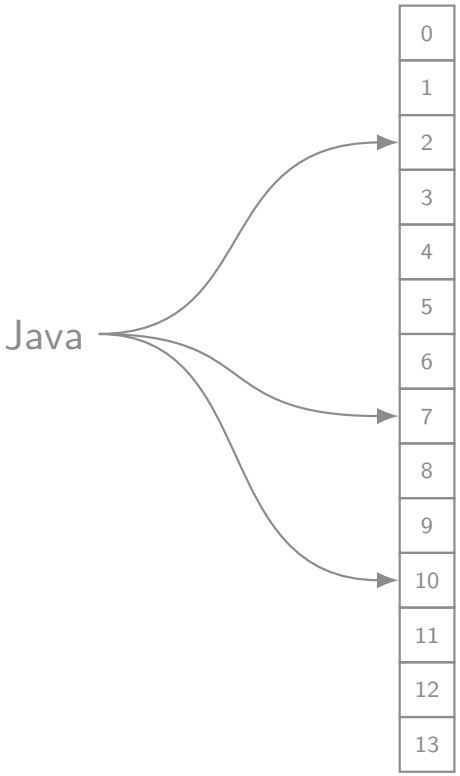


Bloom Filter

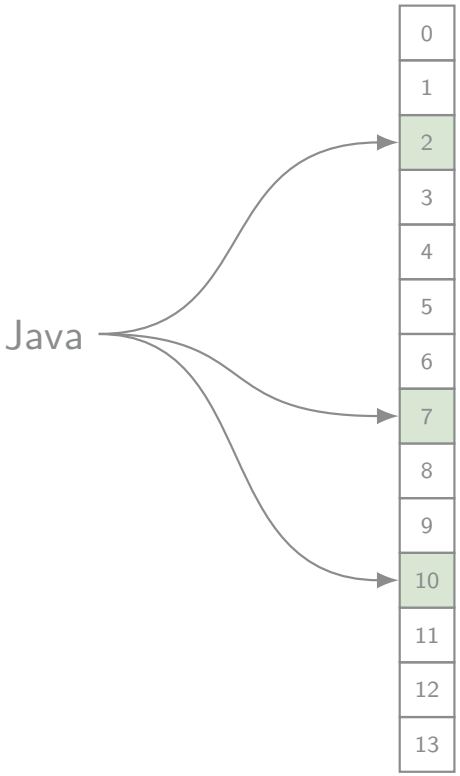
Java



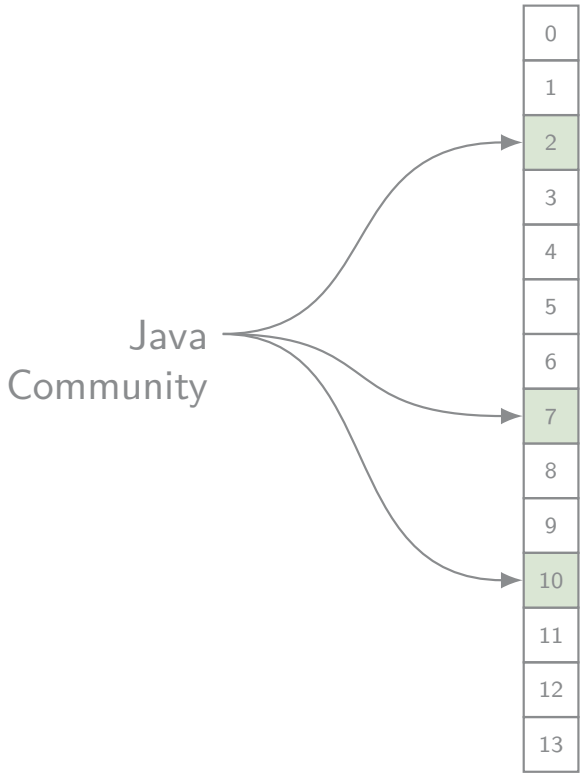
Bloom Filter



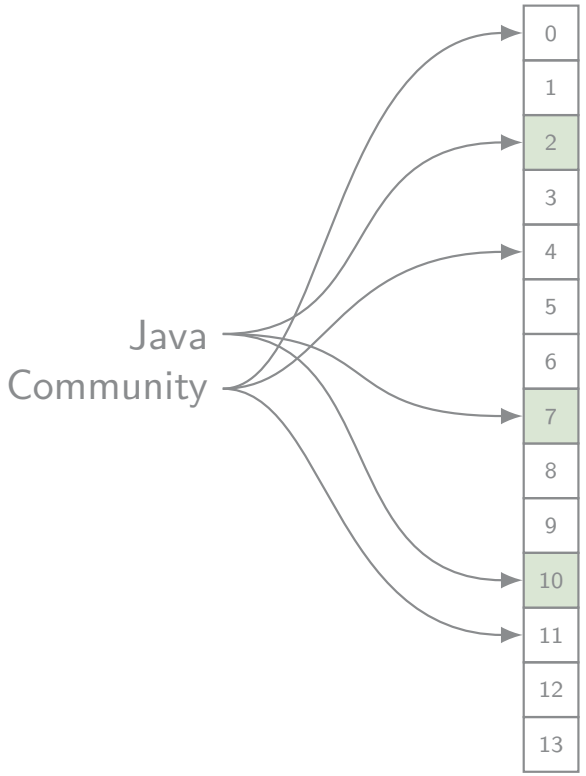
Bloom Filter



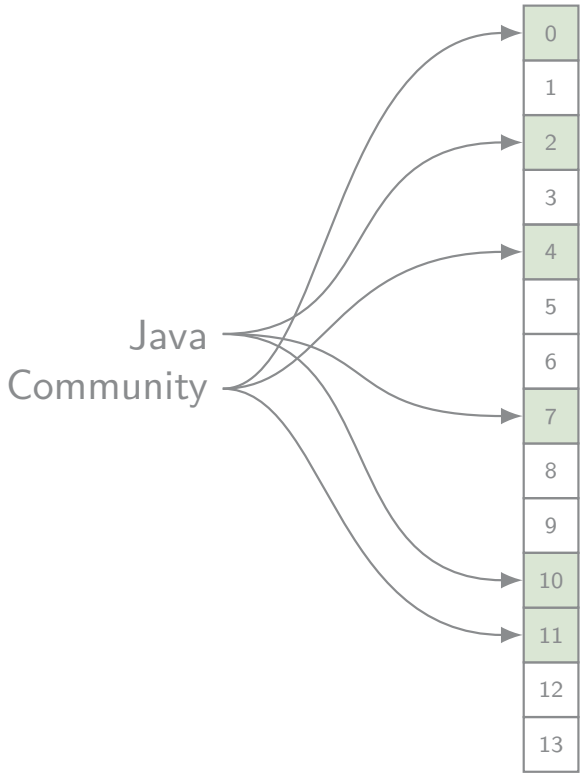
Bloom Filter



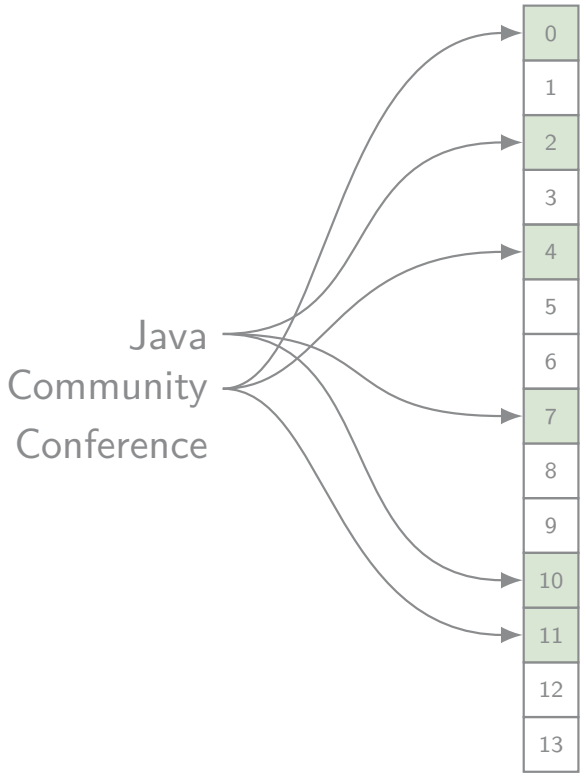
Bloom Filter



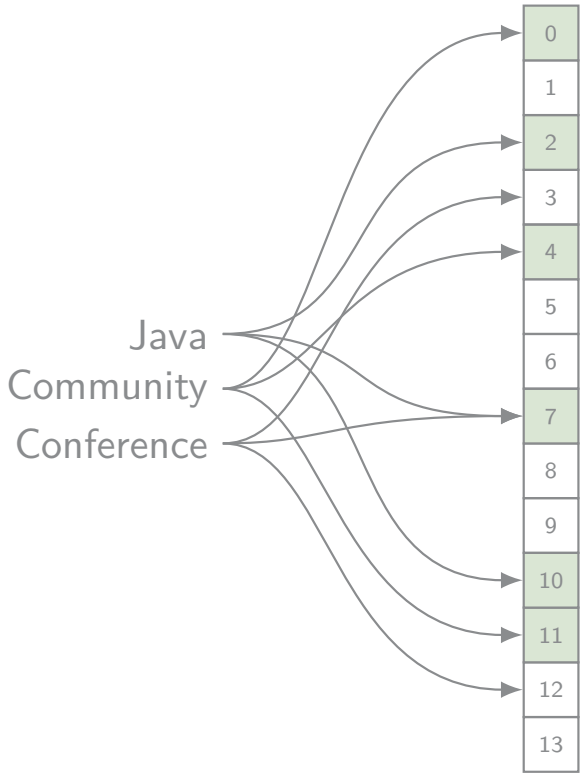
Bloom Filter



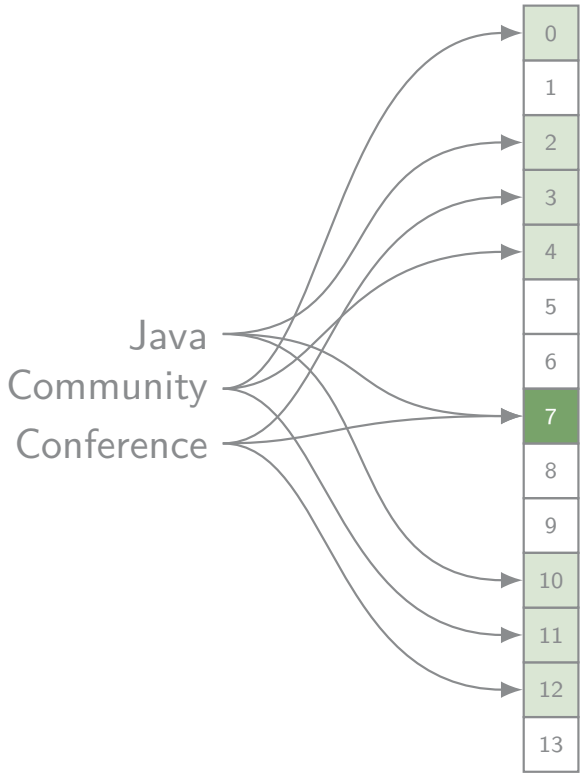
Bloom Filter



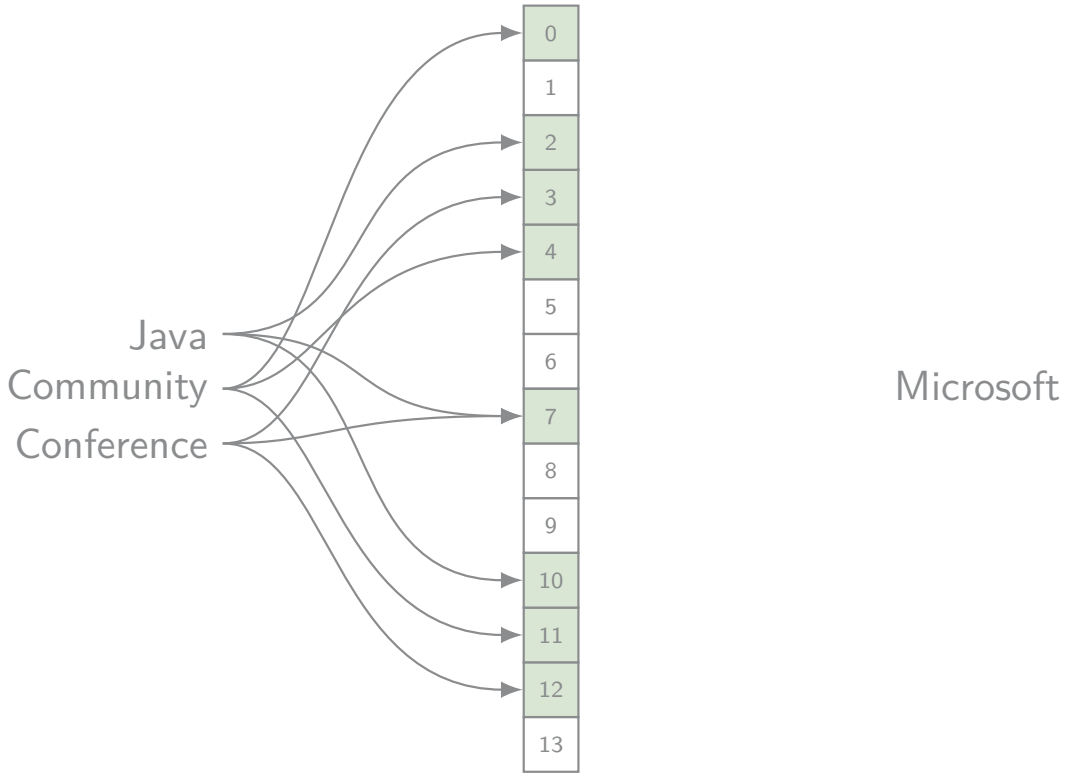
Bloom Filter



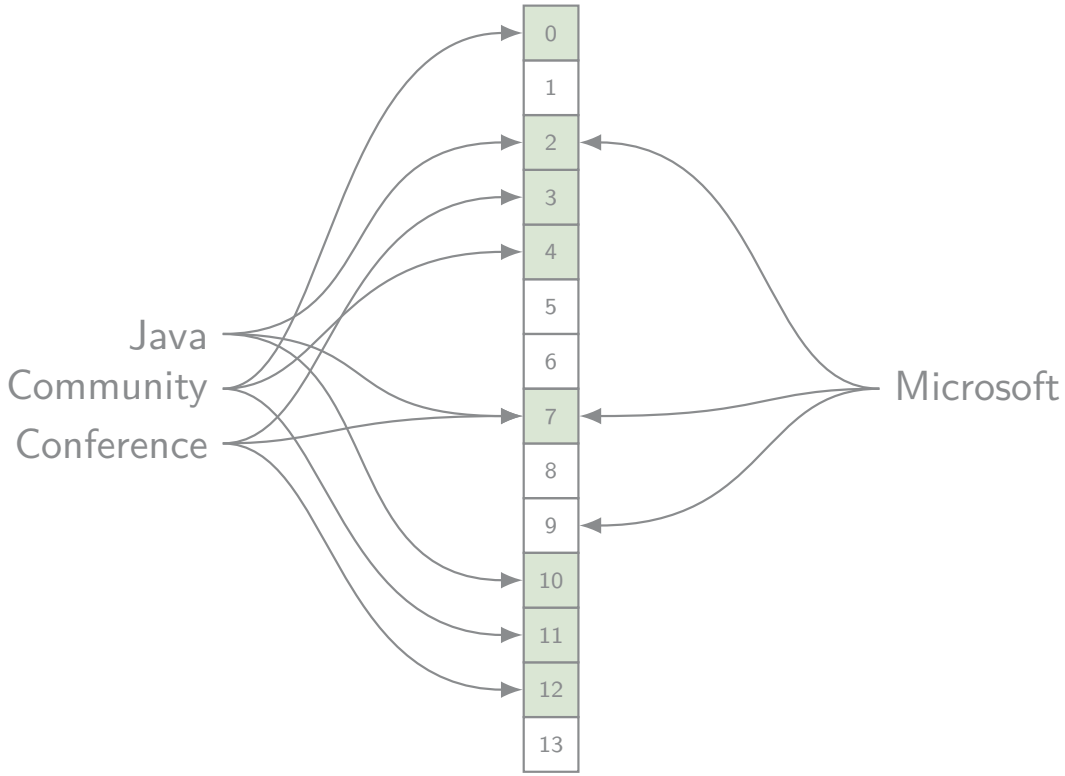
Bloom Filter



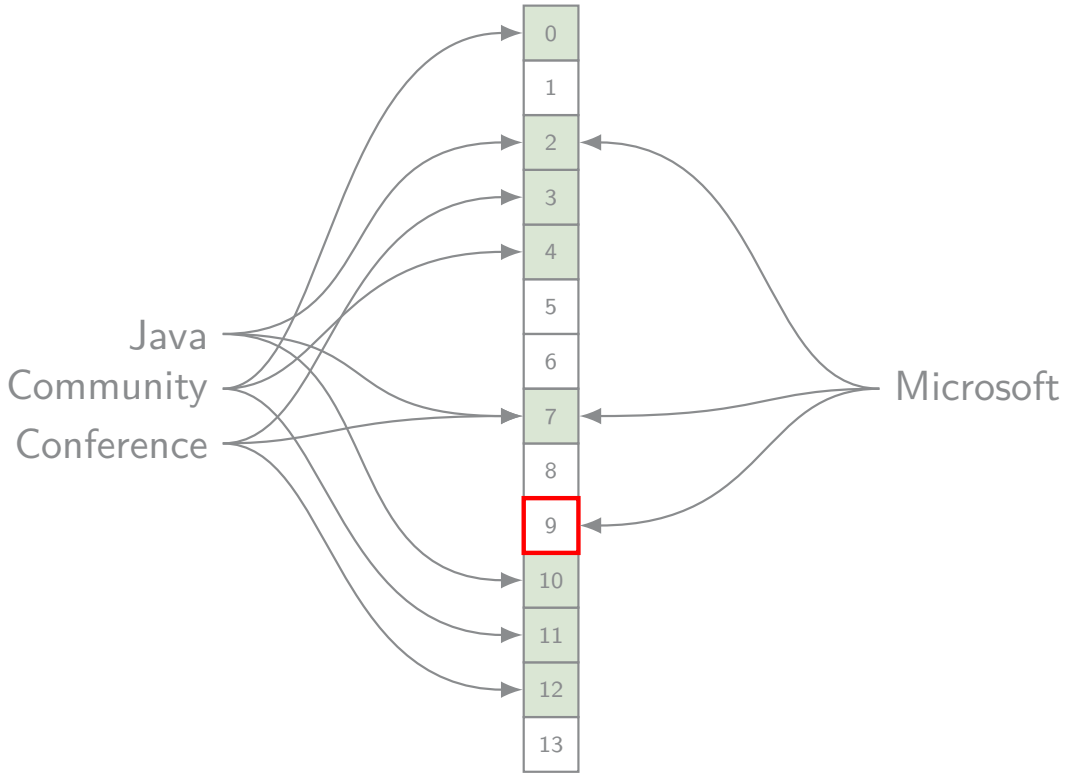
Bloom Filter



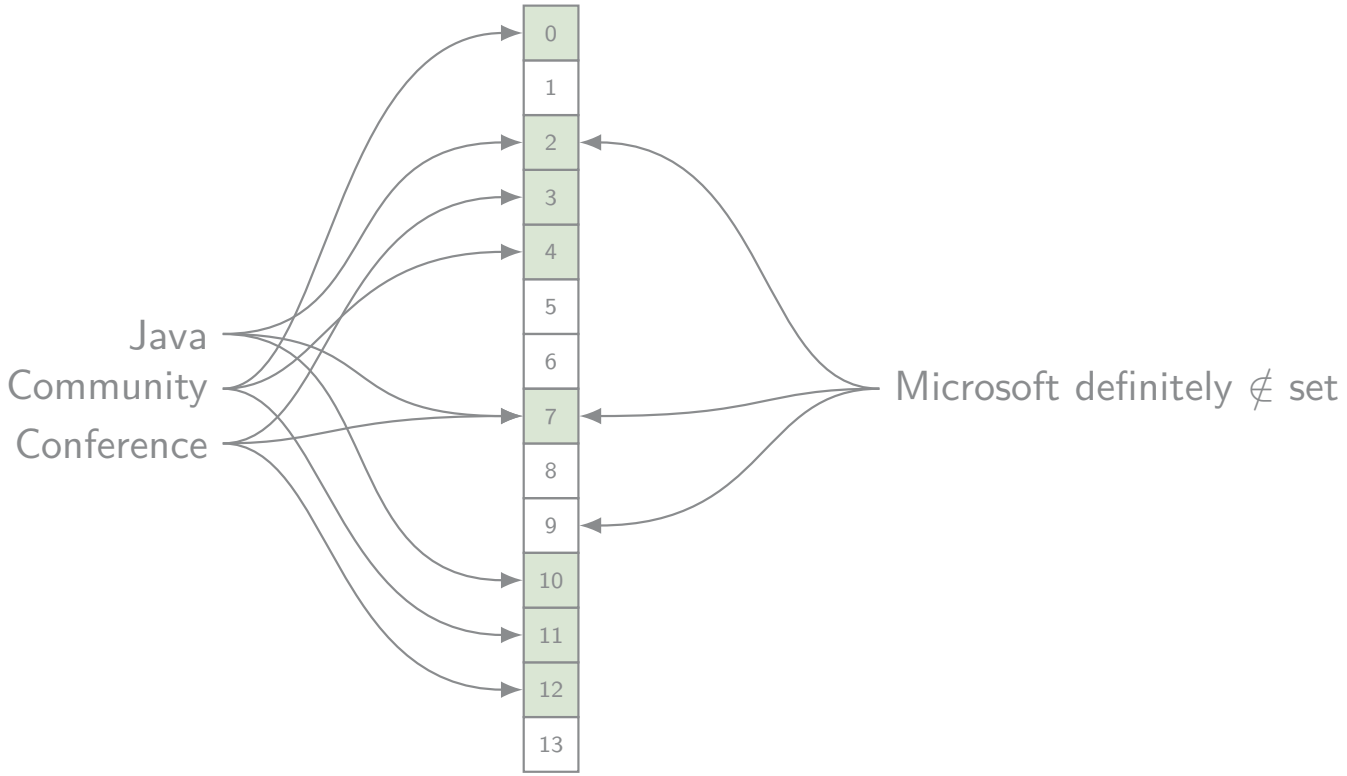
Bloom Filter



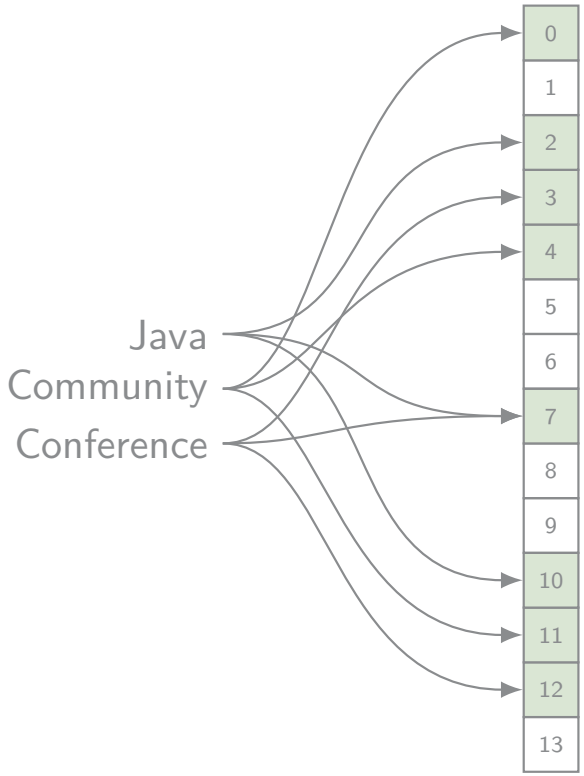
Bloom Filter



Bloom Filter

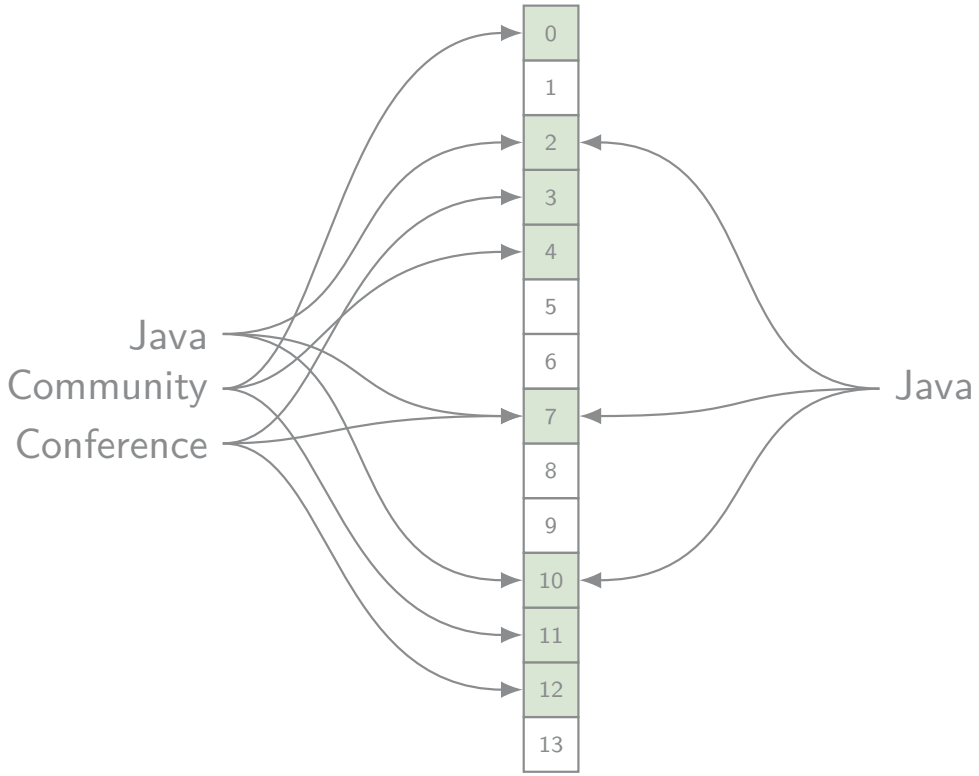


Bloom Filter

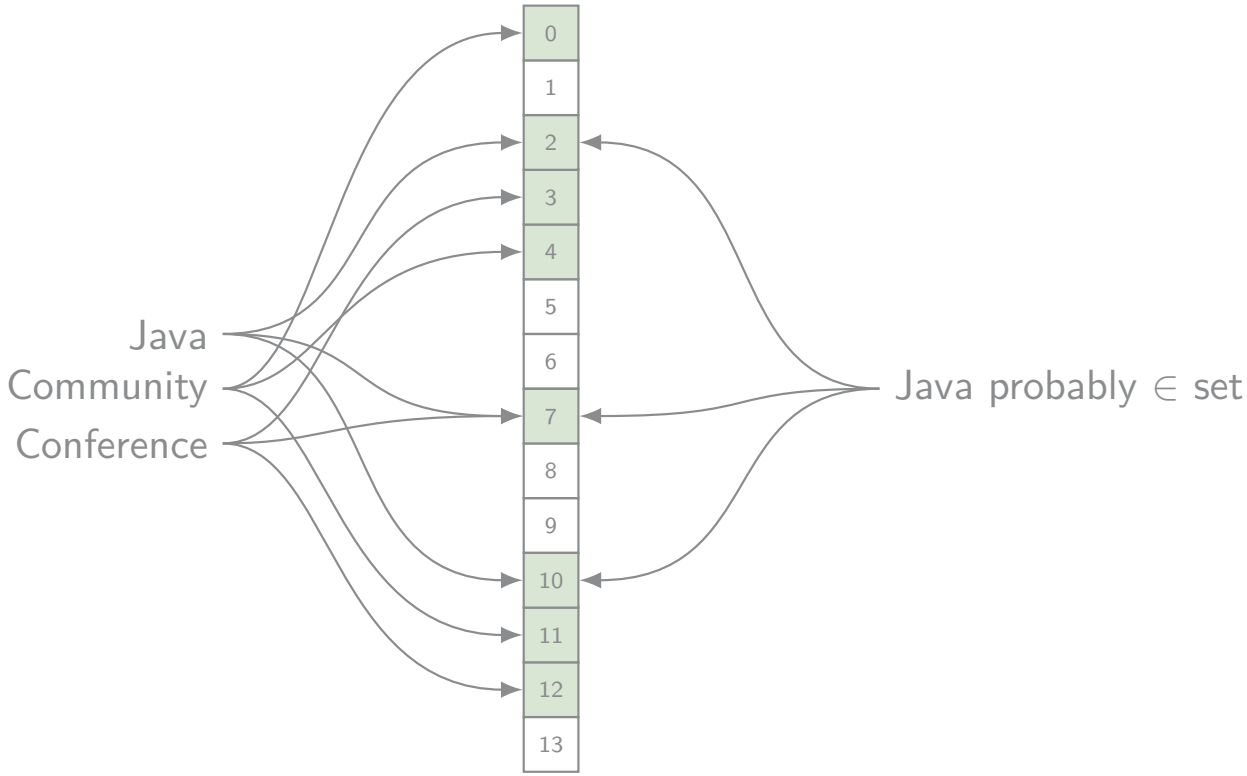


Java

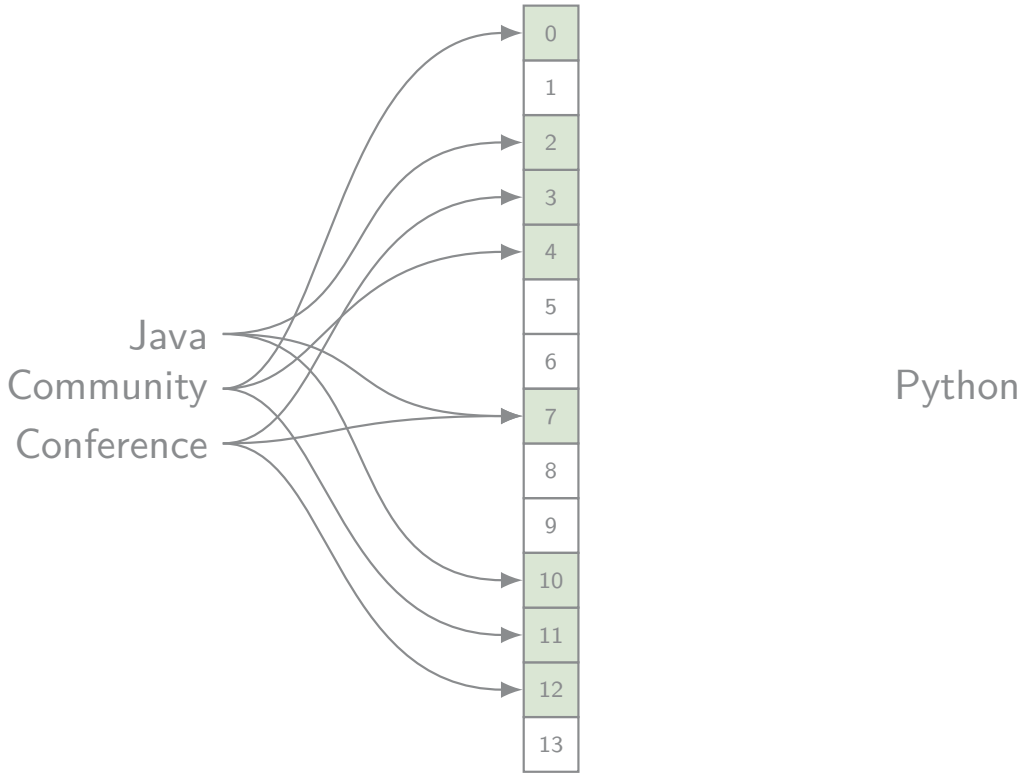
Bloom Filter



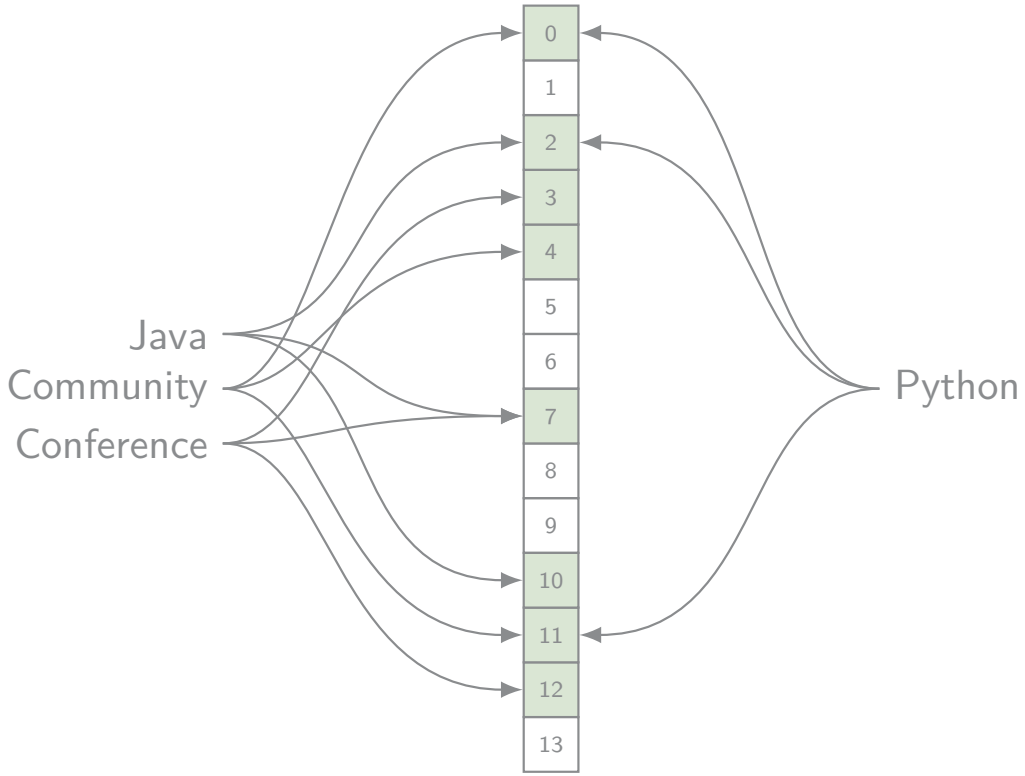
Bloom Filter



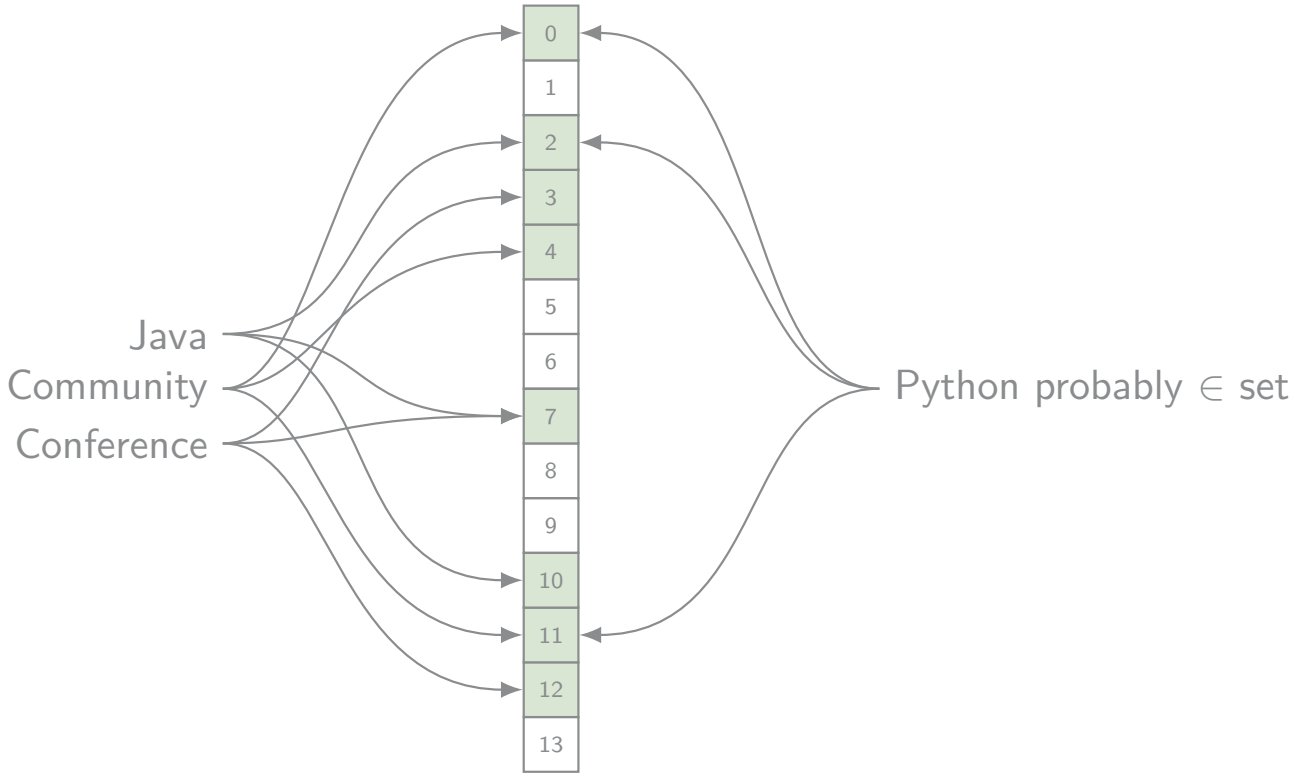
Bloom Filter



Bloom Filter



Bloom Filter



Bloom filter

Math

- m: array length
- k: # hash functions
- n: # of inserted elements
- p: false positive probability
- # bits needed: $m = \frac{-n \ln p}{(\ln 2)^2}$
- Optimal # of hash functions: $k = \frac{m}{n} \ln(2)$

Bloom filter

Math

- m: array length
- k: # hash functions
- n: # of inserted elements
- p: false positive probability
- # bits needed: $m = \frac{-n \ln p}{(\ln 2)^2}$
- Optimal # of hash functions: $k = \frac{m}{n} \ln(2)$

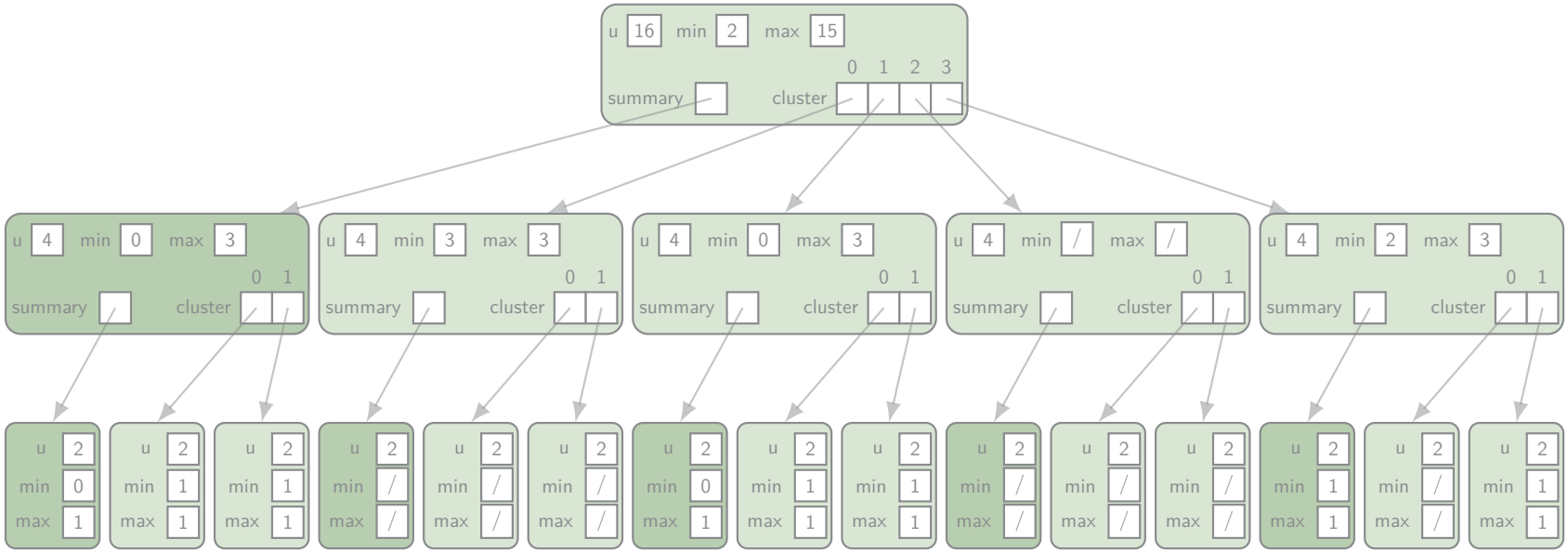
- Supports unions and intersections
- Fits arbitrarily # of elements (false positive rate increases)
- Common application: caching

Outline

van Emde Boas Tree

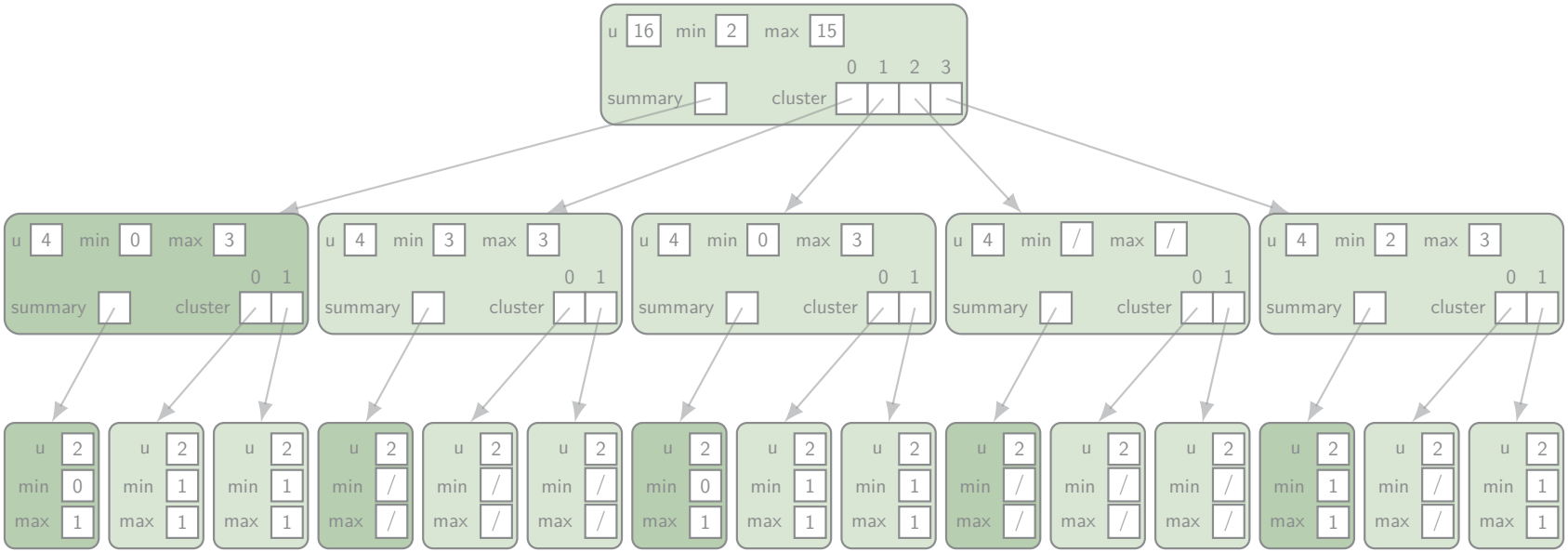
- Dynamic ordered set/map
  - `insert`, `delete`, `contains`, `minimum`, `maximum`, `successor`, `predecessor` operations in $O(\log \log U)$
- Integer keys
  - No duplicates
- $O(U)$ space $\rightarrow$ only for large collections

van Emde Boas Tree



{2,3,4,5,7,14,15}

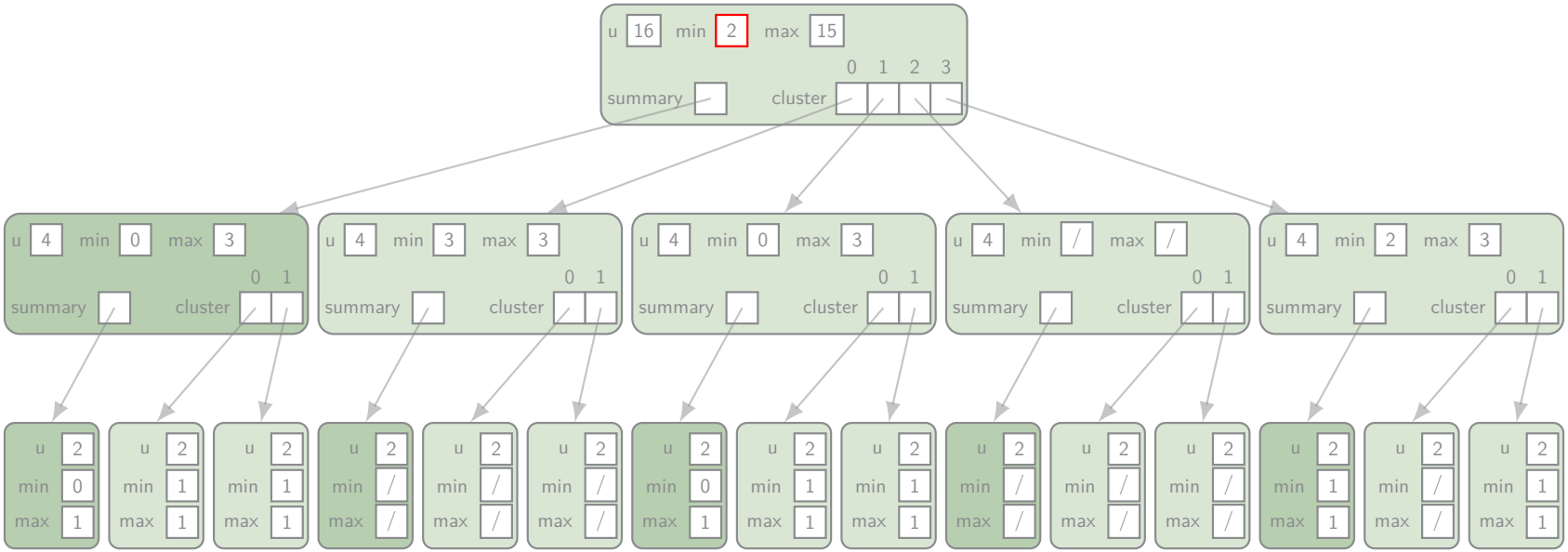
van Emde Boas Tree



{2,3,4,5,7,14,15}

minimum

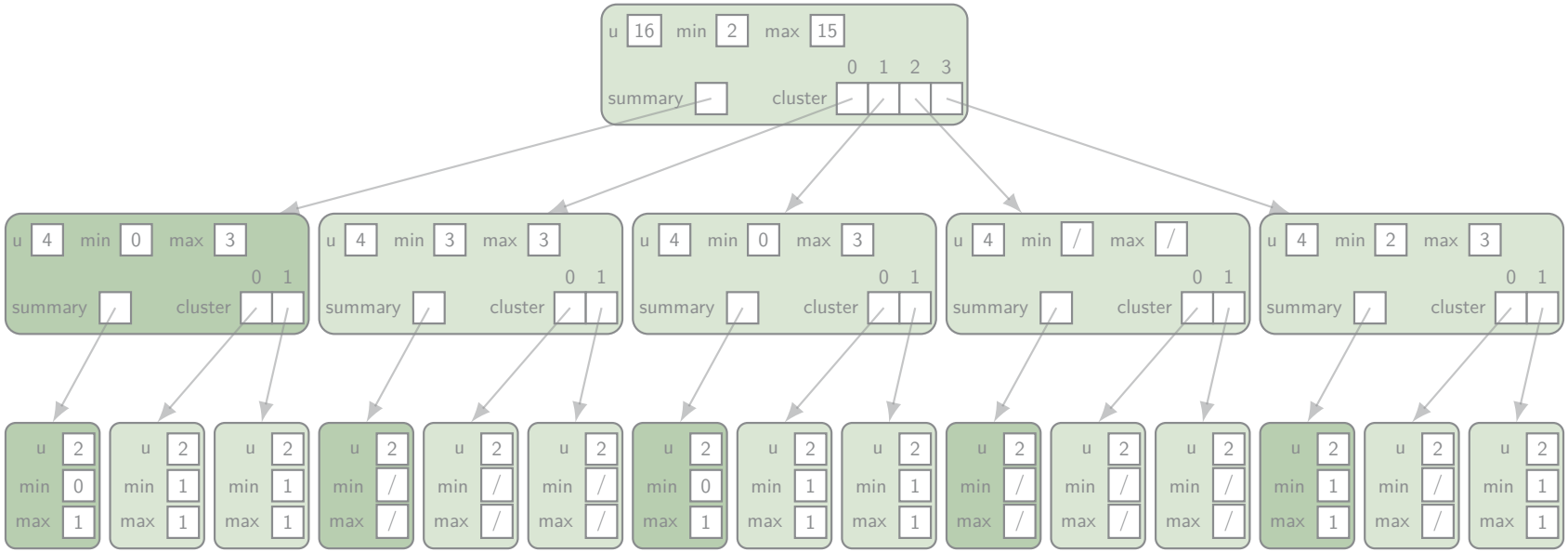
van Emde Boas Tree



$$\{2,3,4,5,7,14,15\}$$

`minimum`

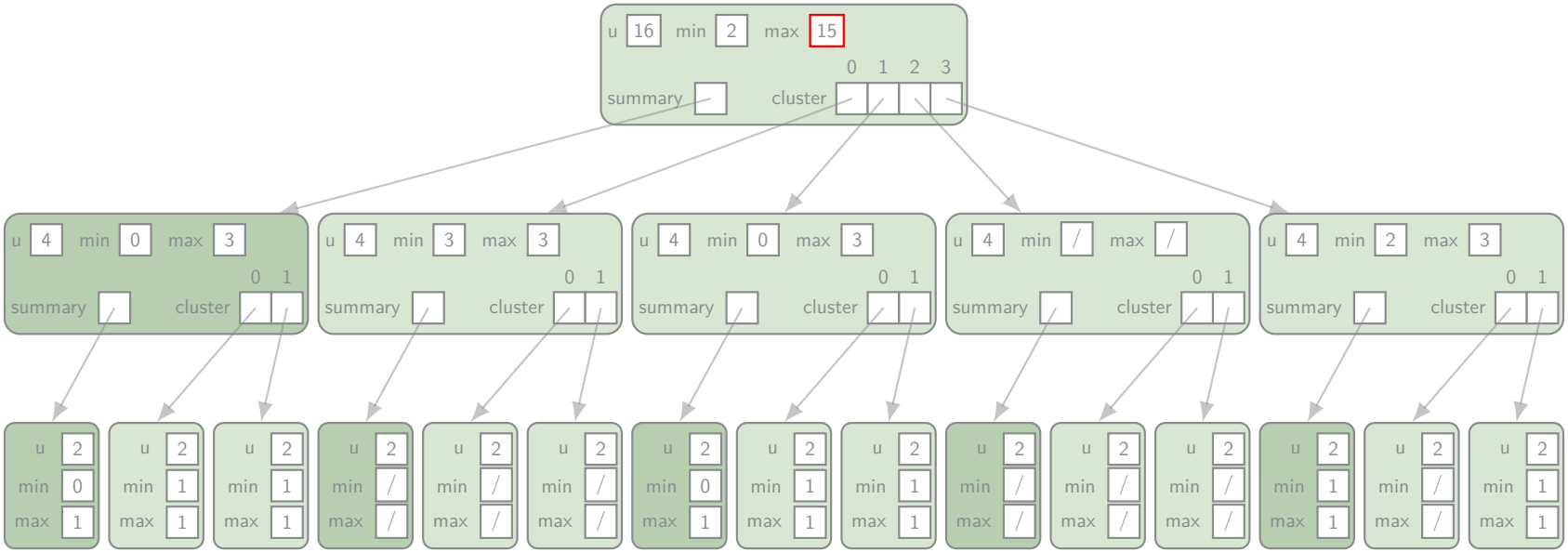
van Emde Boas Tree



$$\{2,3,4,5,7,14,15\}$$

maximum

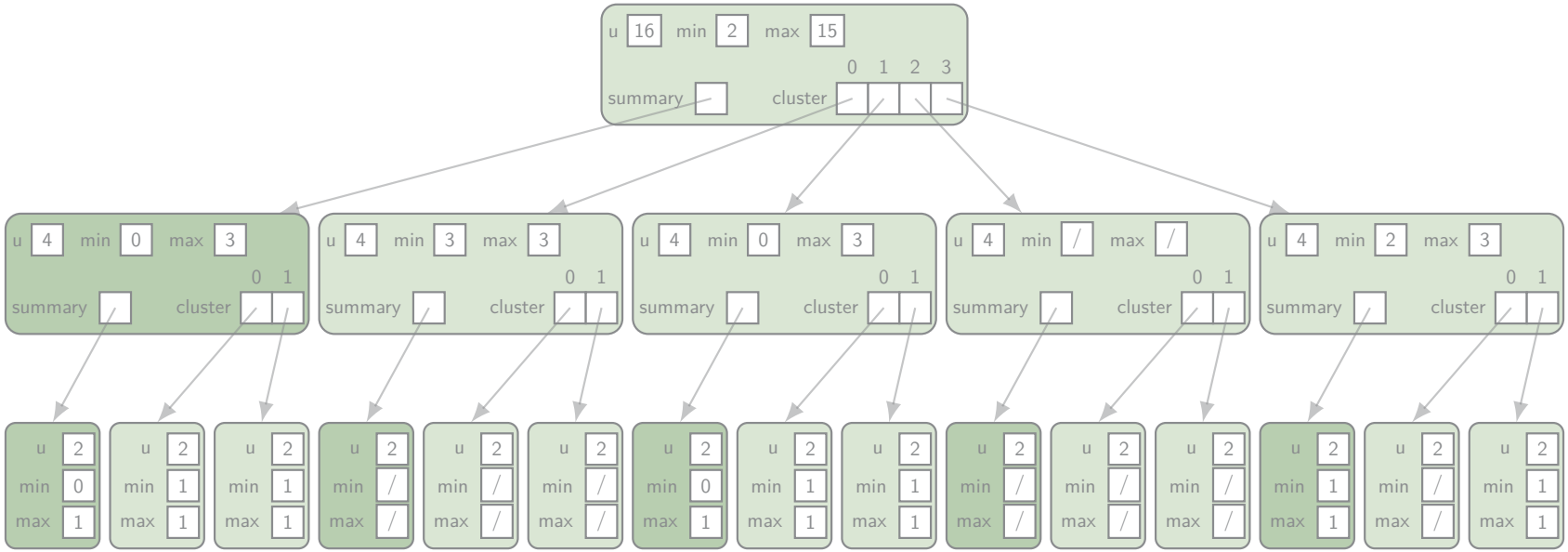
van Emde Boas Tree



$$\{2,3,4,5,7,14,15\}$$

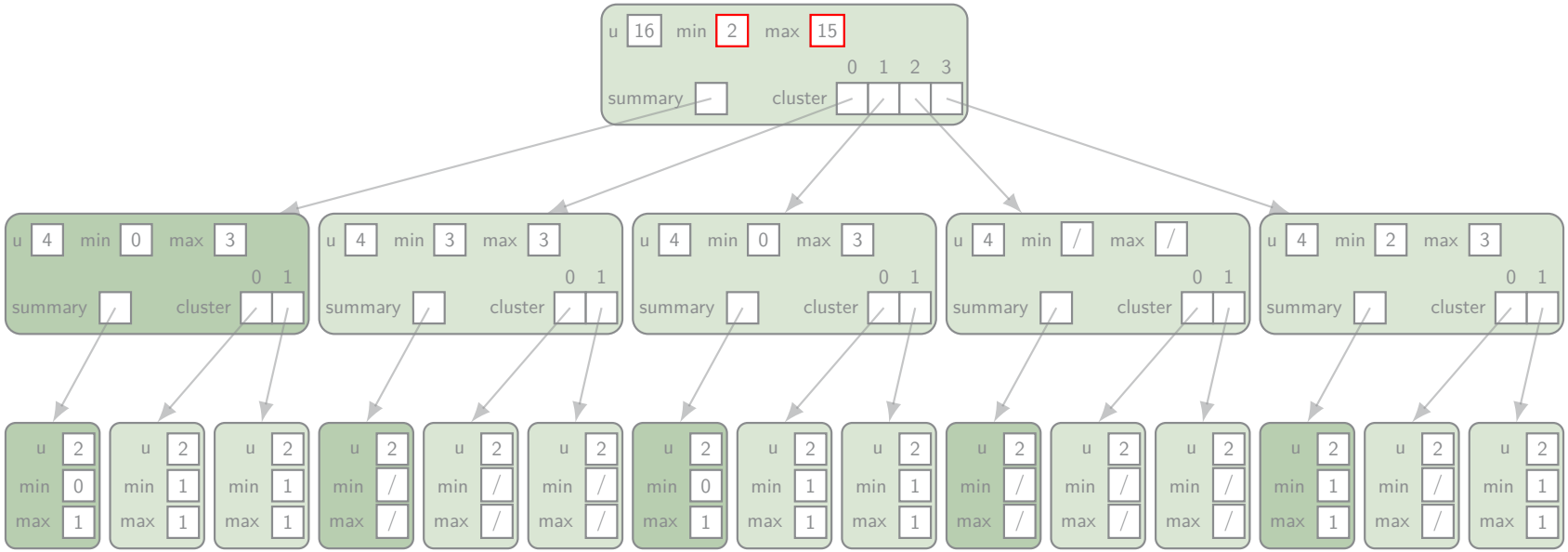
maximum

van Emde Boas Tree



{2,3,4,5,7,14,15}
contains(5)

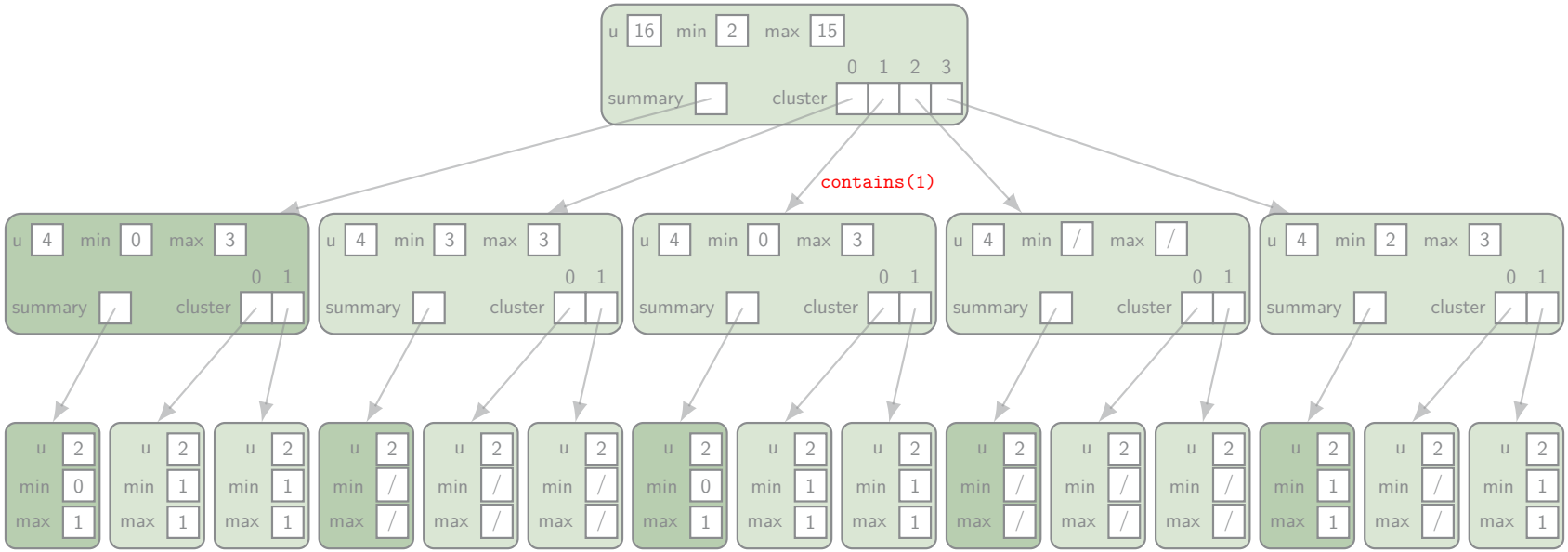
van Emde Boas Tree



{2,3,4,5,7,14,15}

contains(5)

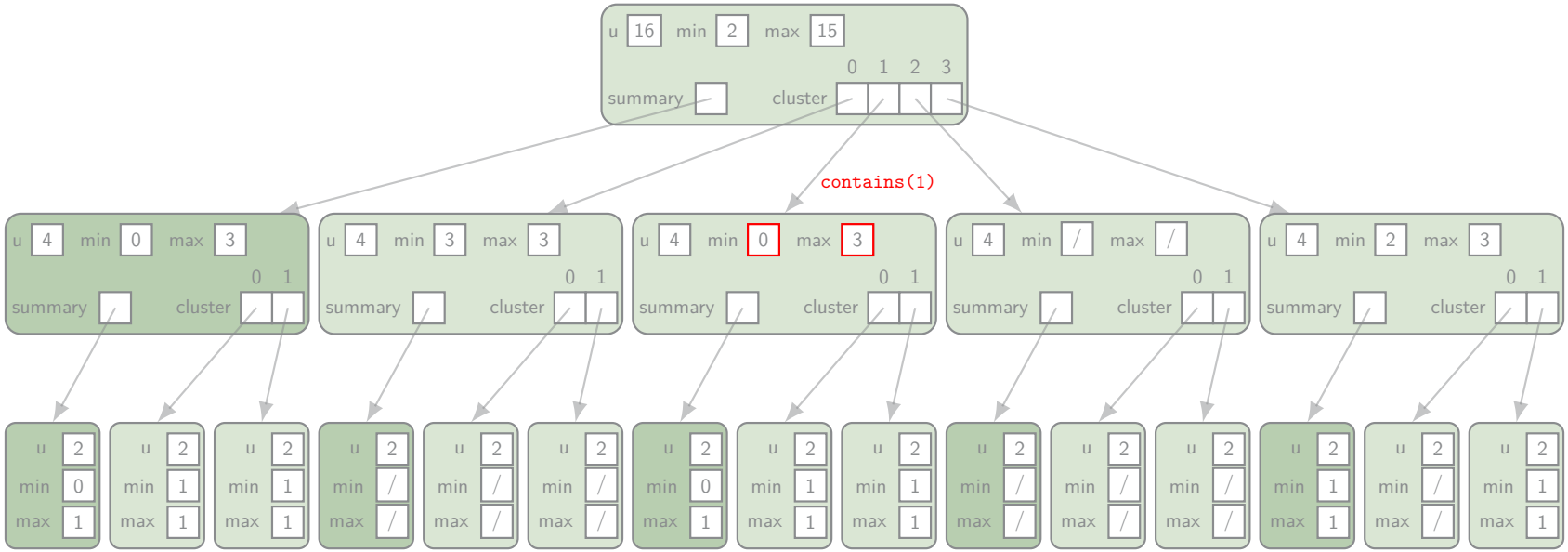
van Emde Boas Tree



{2,3,4,5,7,14,15}

contains(5)

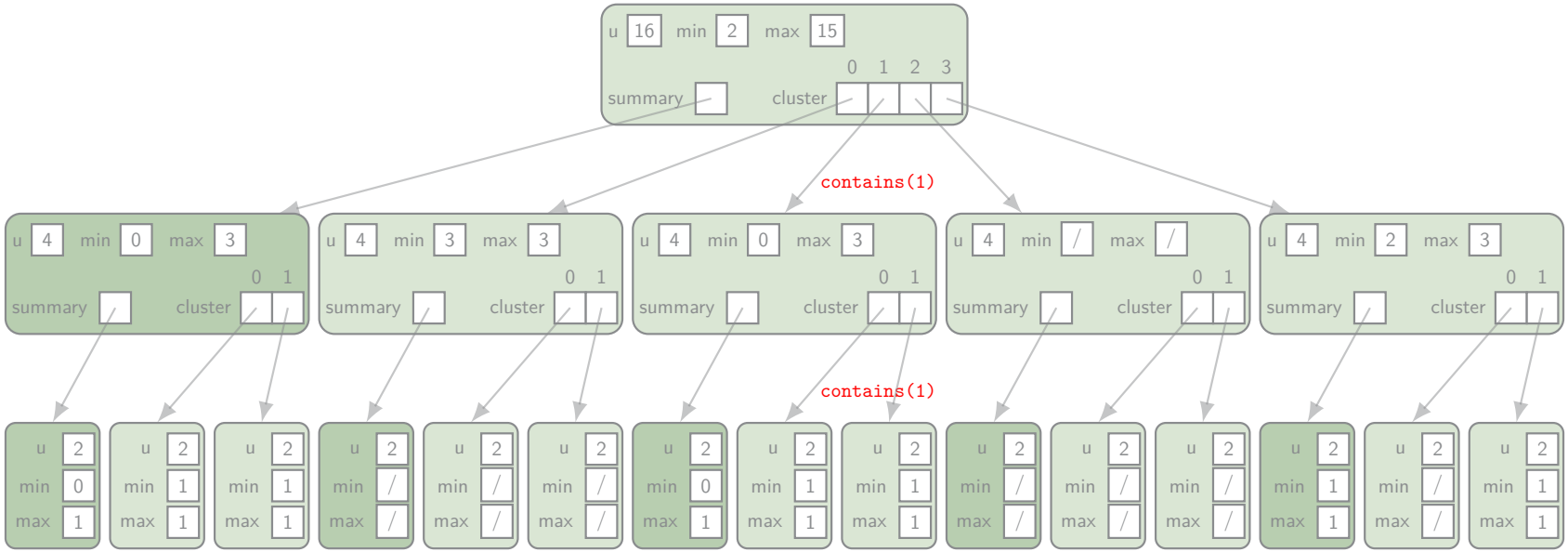
van Emde Boas Tree



{2,3,4,5,7,14,15}

contains(5)

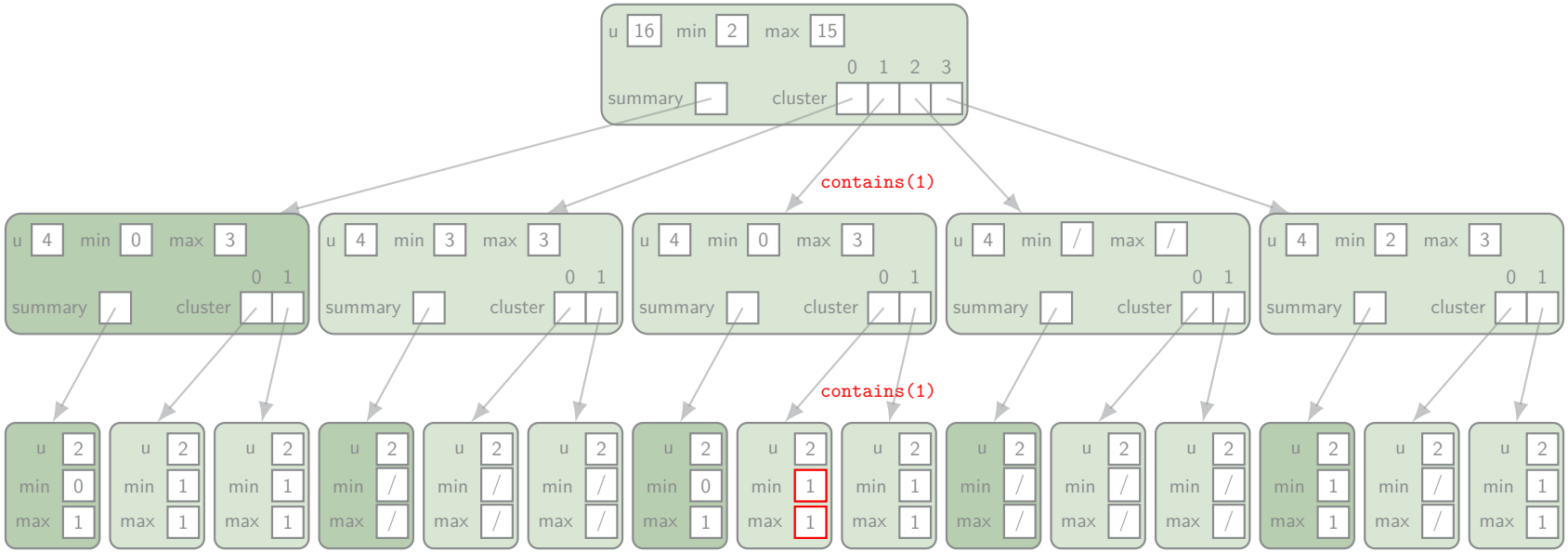
van Emde Boas Tree



{2,3,4,5,7,14,15}

contains(5)

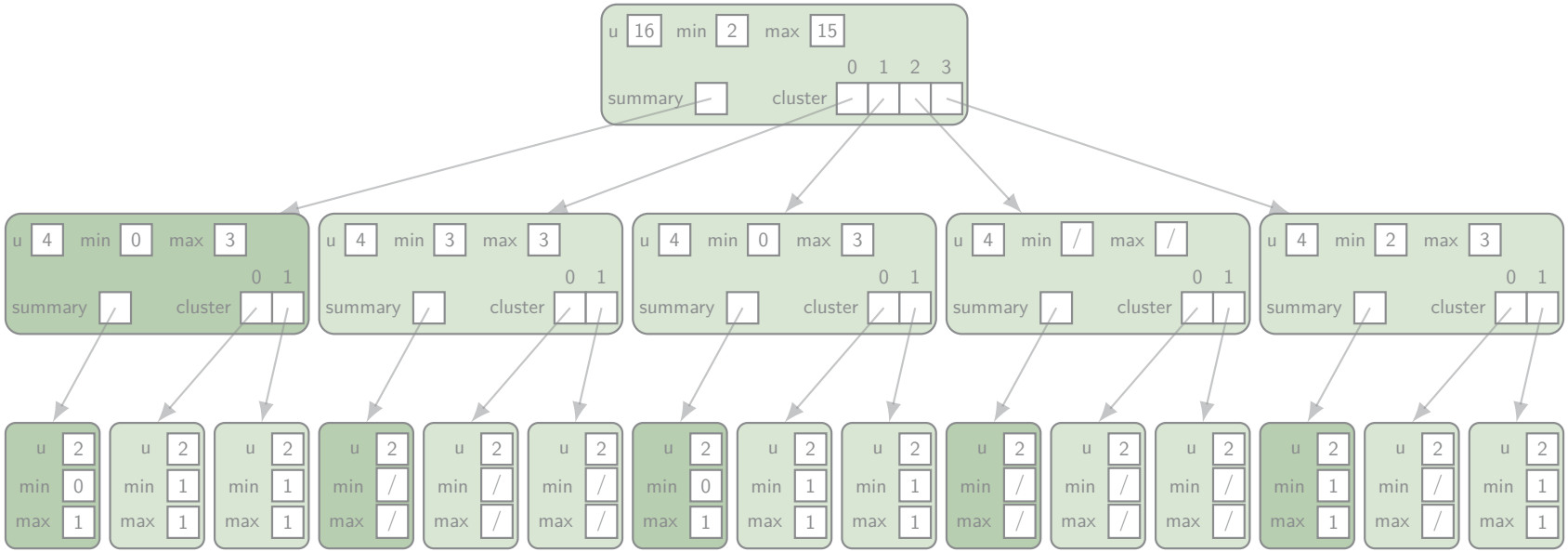
van Emde Boas Tree



{2,3,4,5,7,14,15}

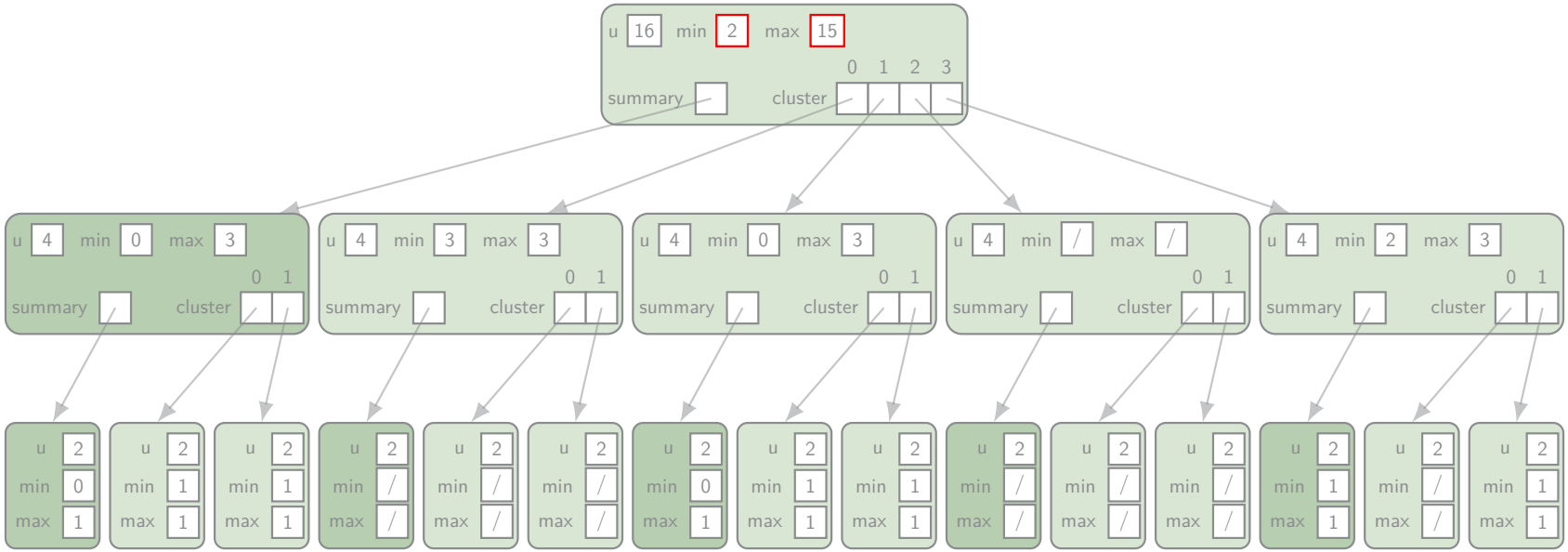
contains(5)

van Emde Boas Tree



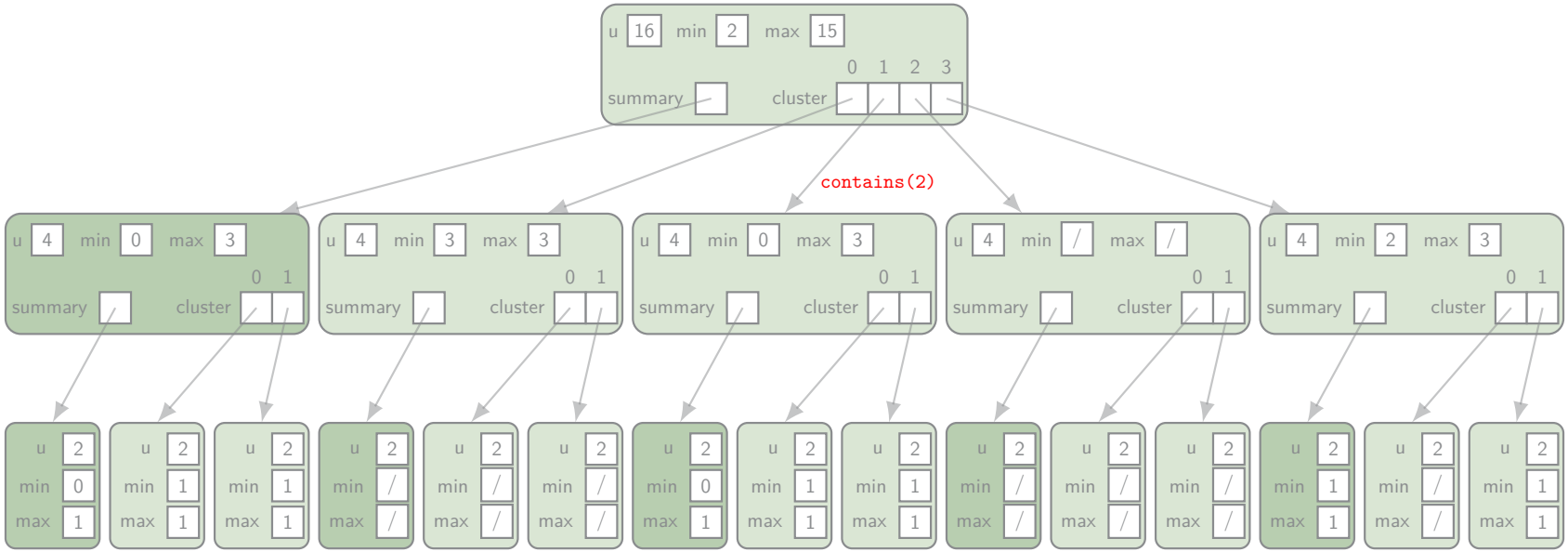
{2,3,4,5,7,14,15}
contains(6)

van Emde Boas Tree



{2,3,4,5,7,14,15}
contains(6)

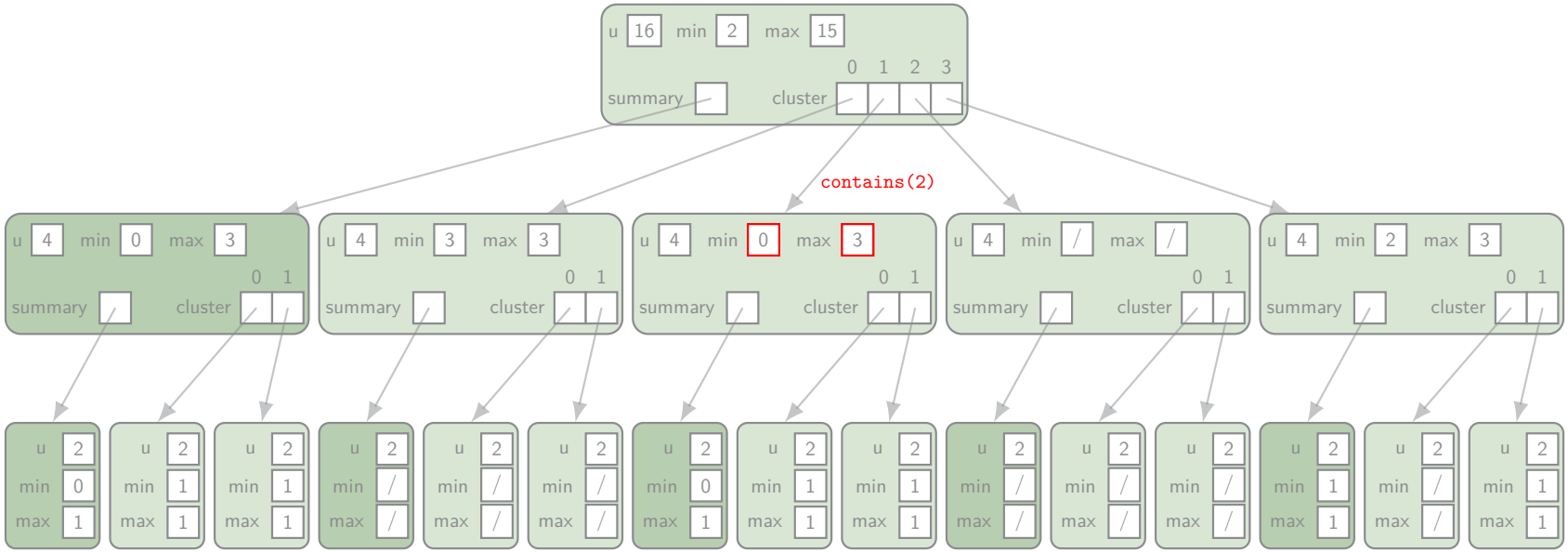
van Emde Boas Tree



{2,3,4,5,7,14,15}

contains(6)

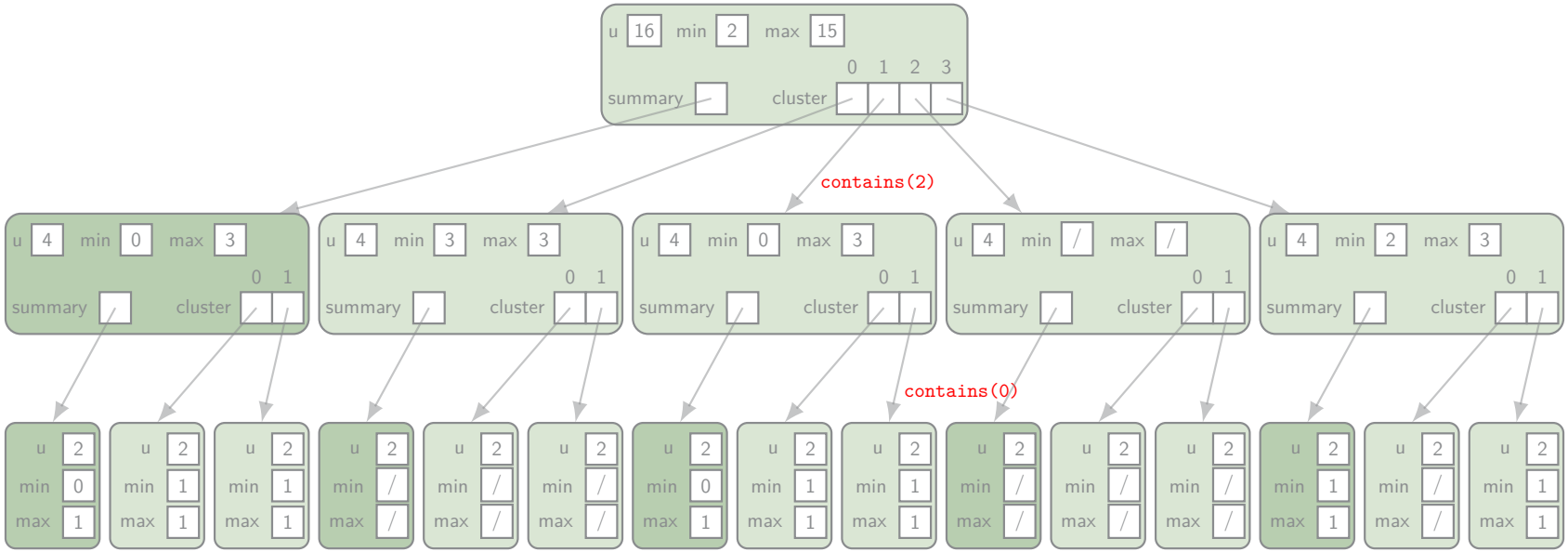
van Emde Boas Tree



{2,3,4,5,7,14,15}

contains(6)

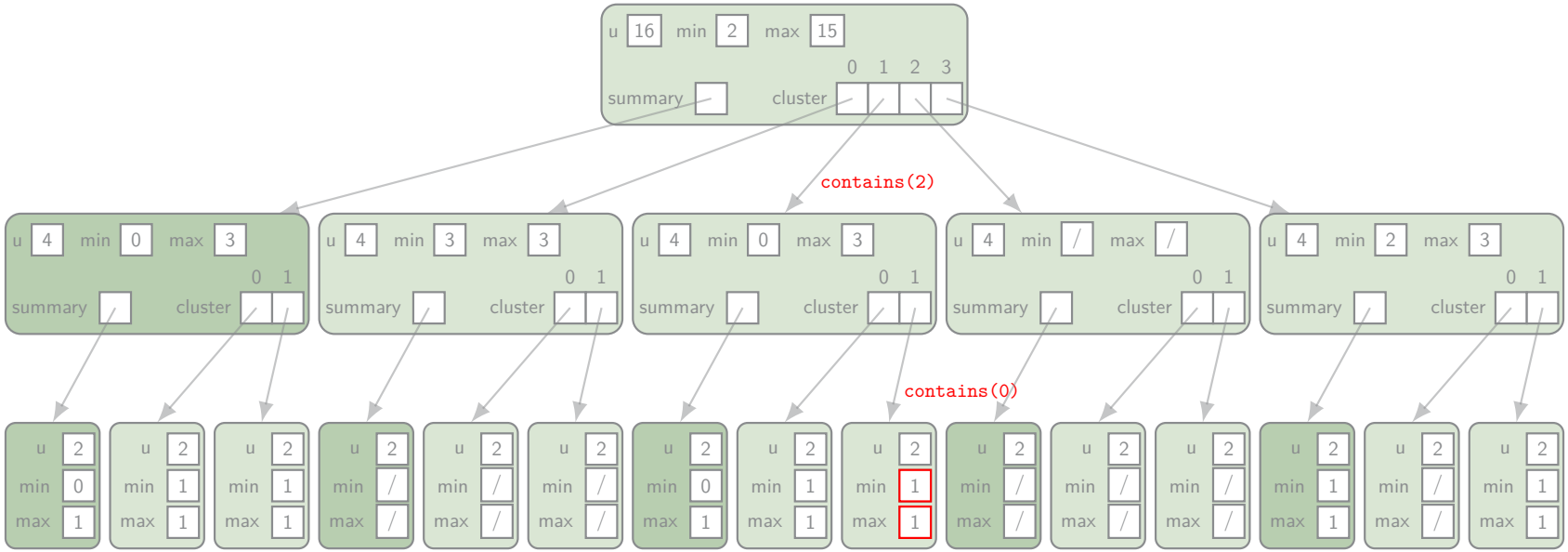
van Emde Boas Tree



{2,3,4,5,7,14,15}

contains(6)

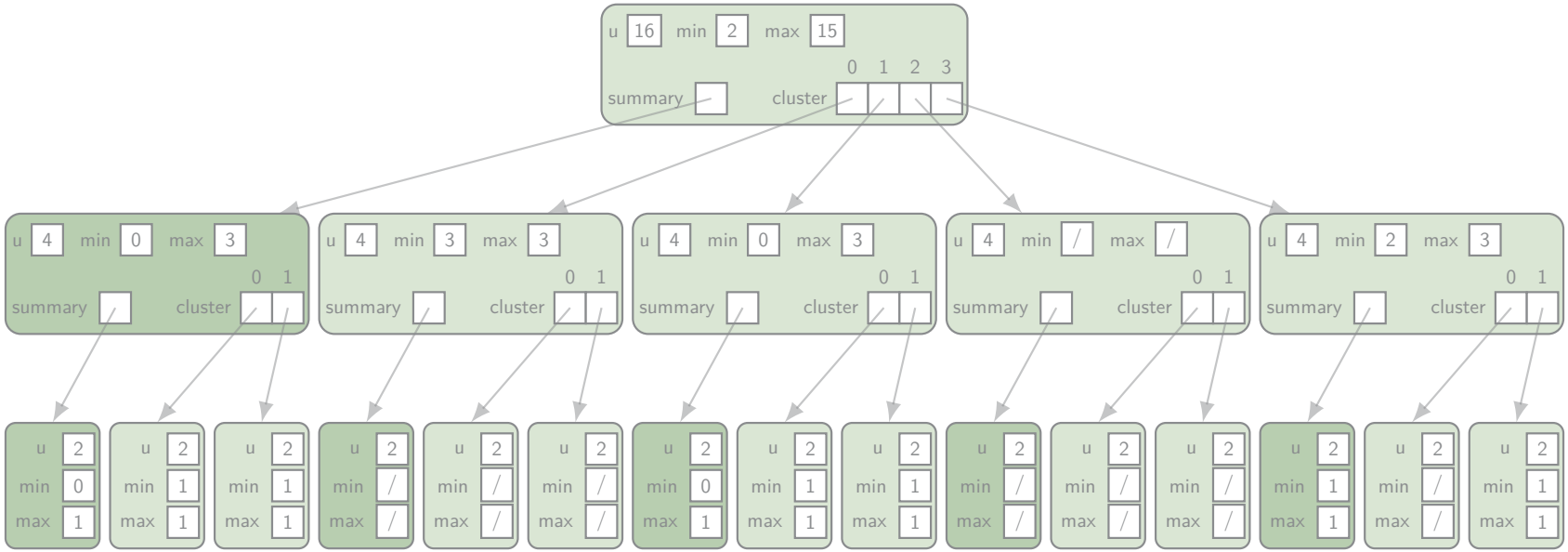
van Emde Boas Tree



{2,3,4,5,7,14,15}

contains(6)

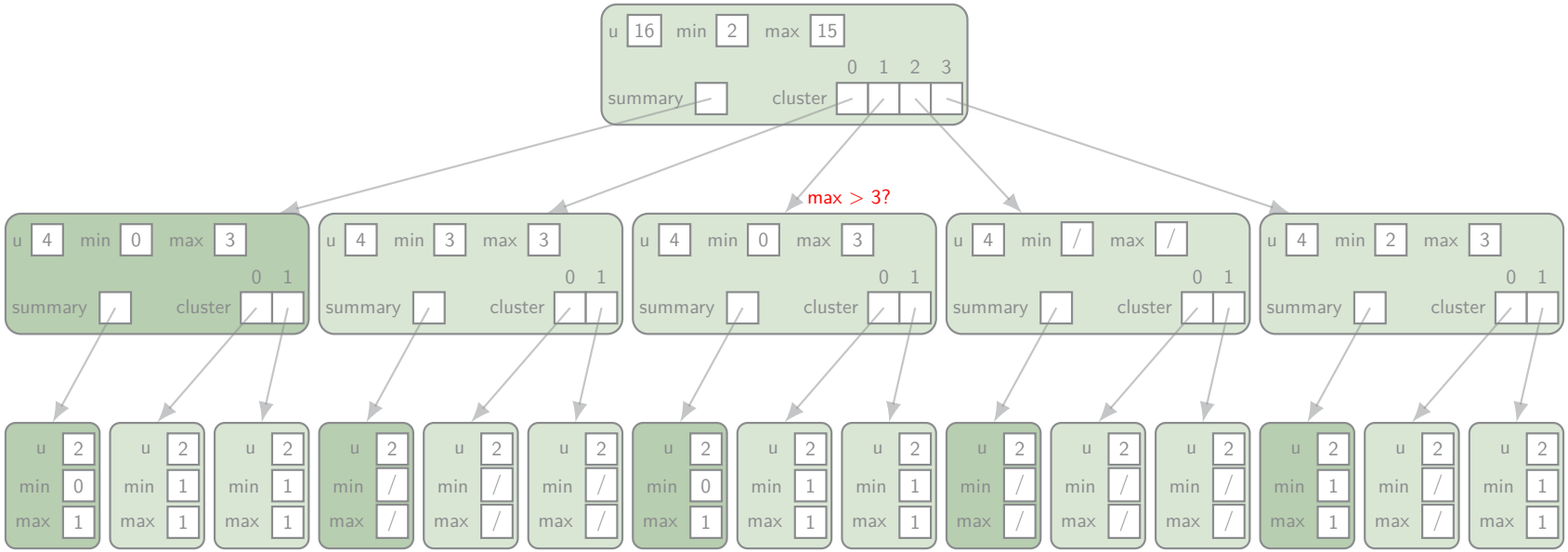
van Emde Boas Tree



{2,3,4,5,7,14,15}

successor(7)

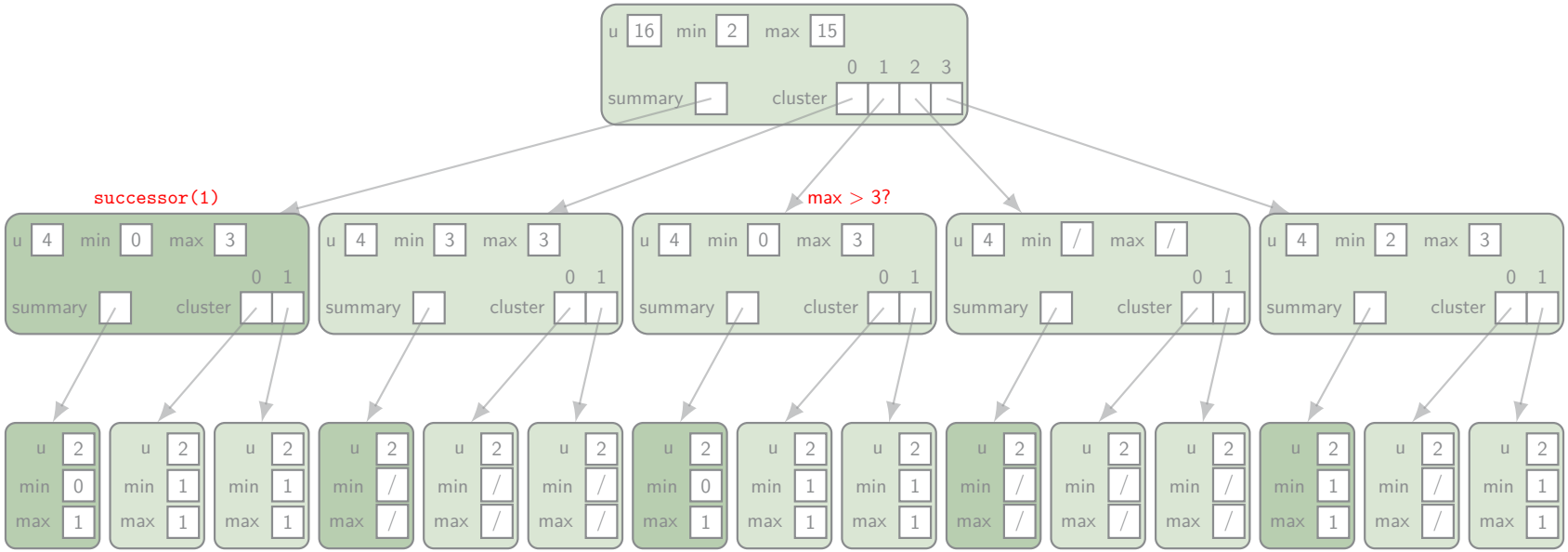
van Emde Boas Tree



{2,3,4,5,7,14,15}

successor(7)

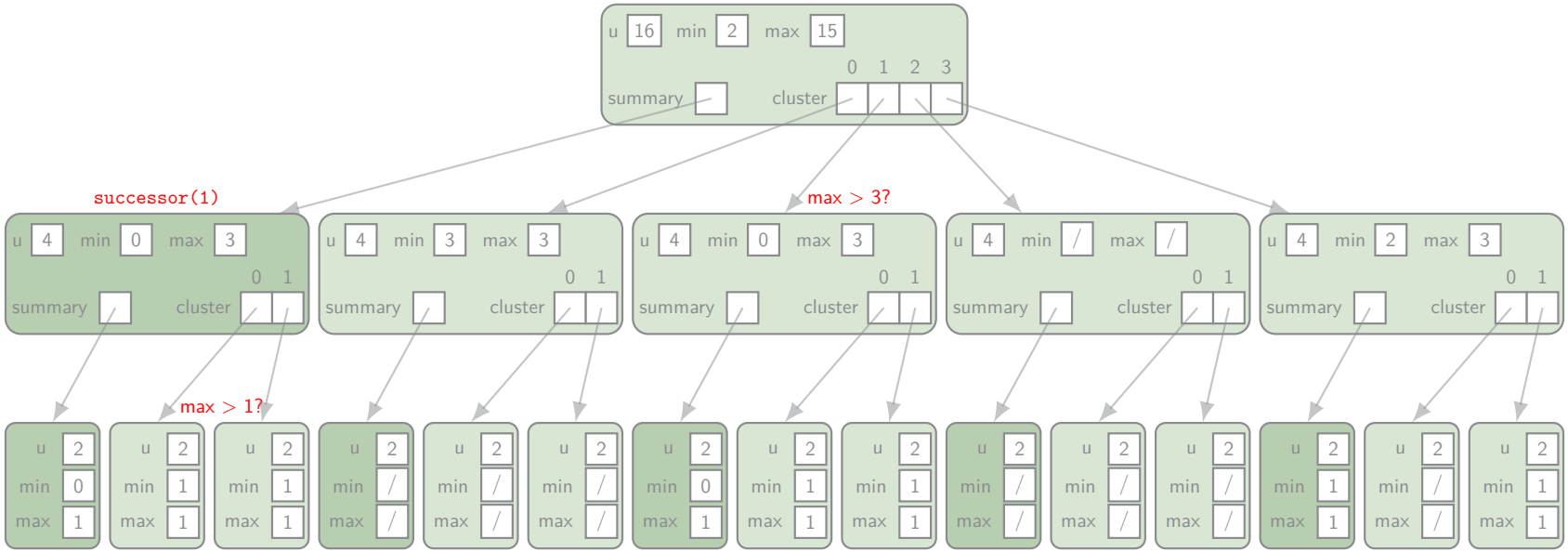
van Emde Boas Tree



{2,3,4,5,7,14,15}

successor(7)

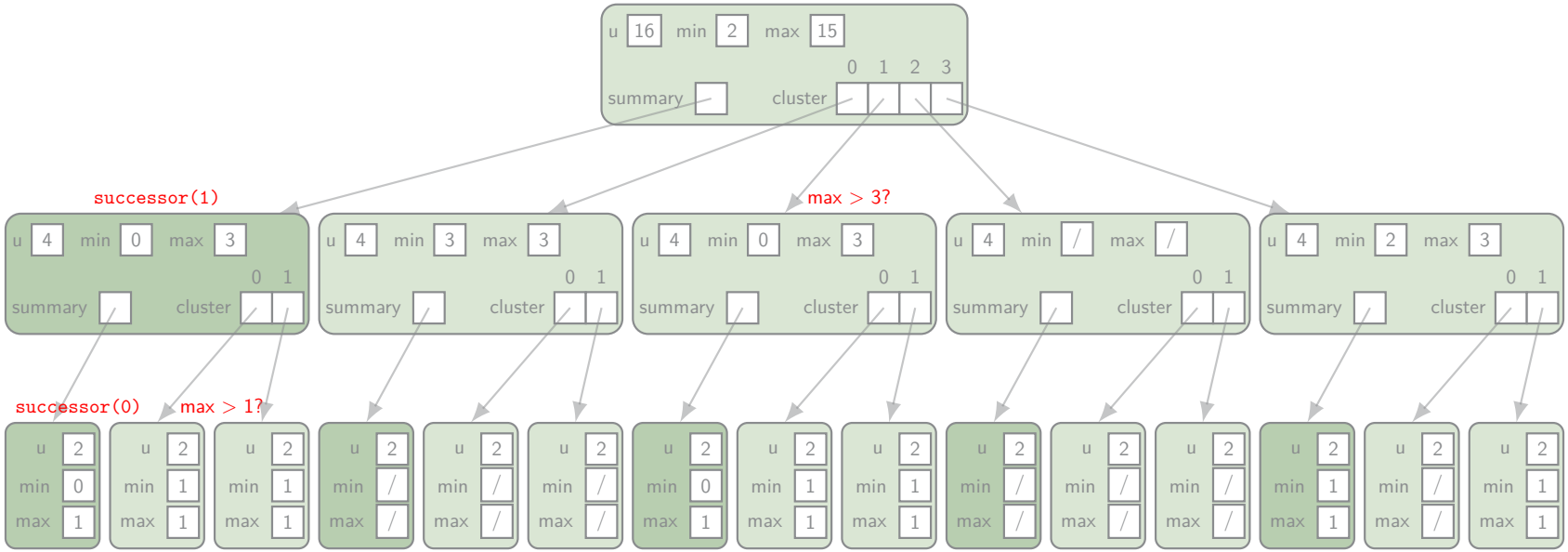
van Emde Boas Tree



{2,3,4,5,7,14,15}

successor(7)

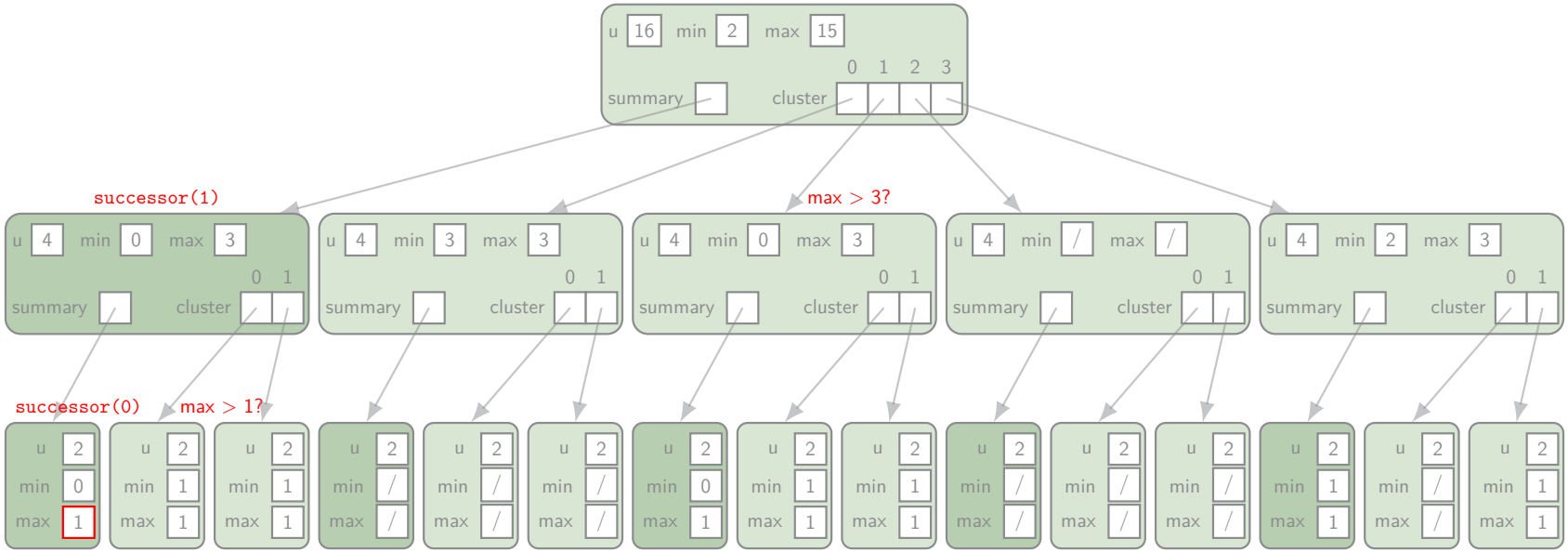
van Emde Boas Tree



{2,3,4,5,7,14,15}

successor(7)

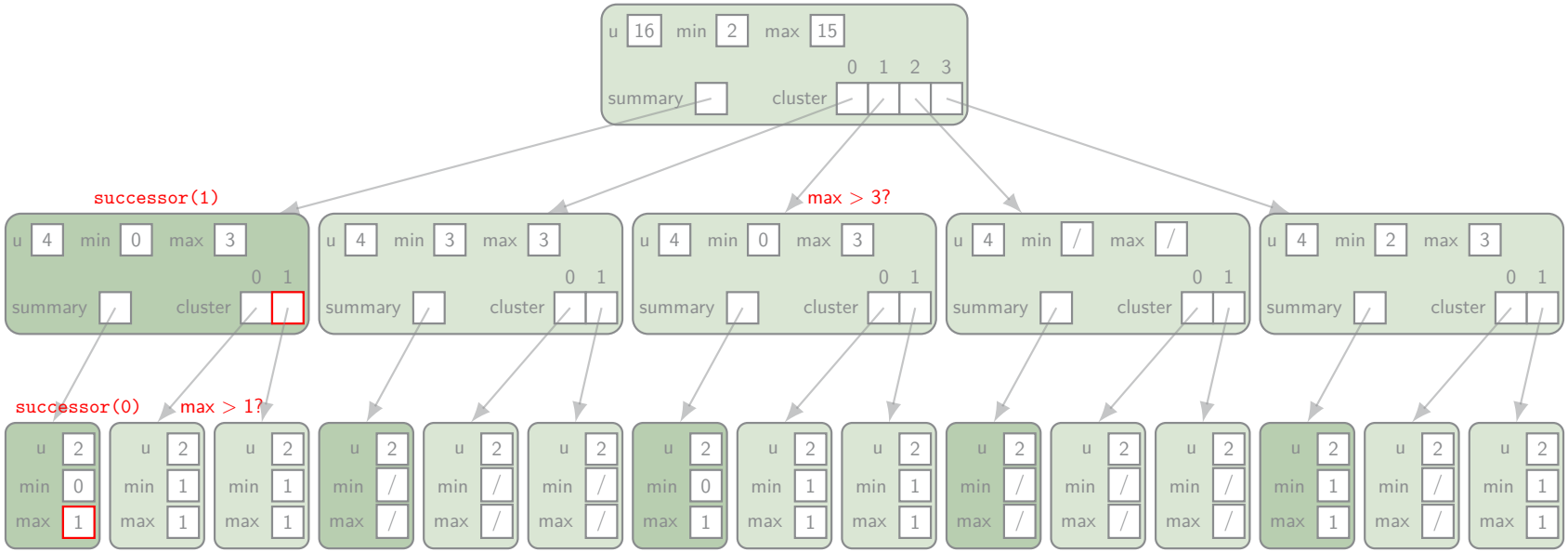
van Emde Boas Tree



{2,3,4,5,7,14,15}

successor(7)

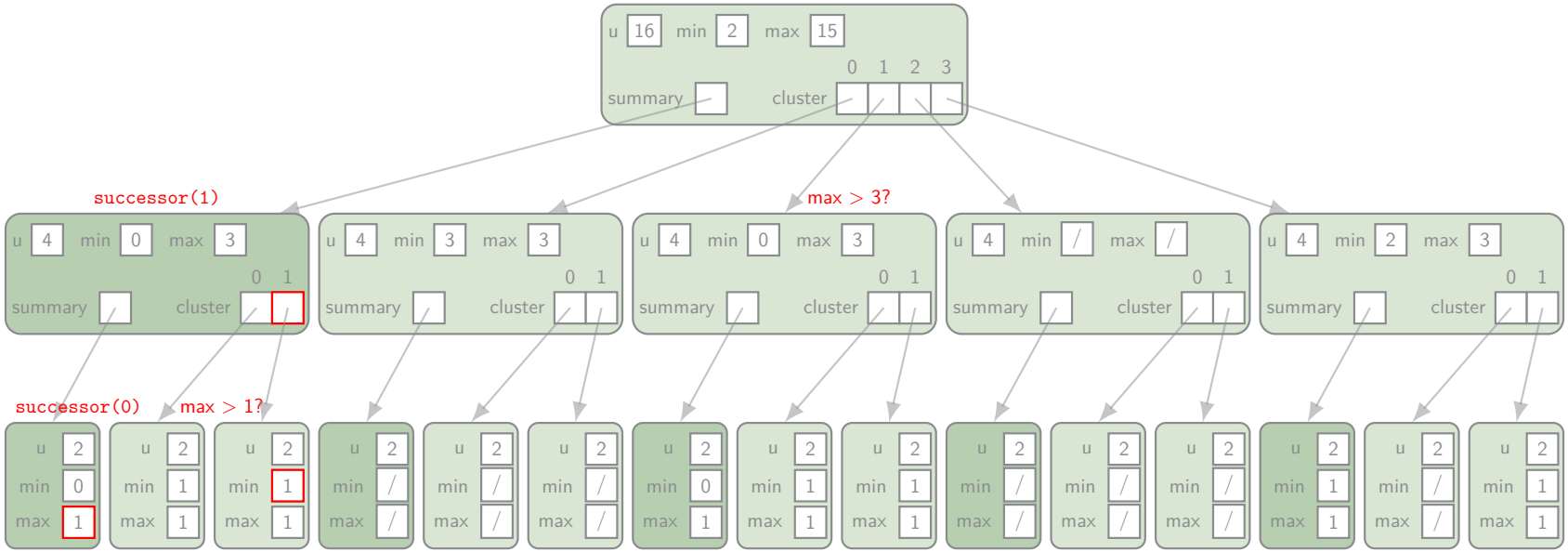
van Emde Boas Tree



{2,3,4,5,7,14,15}

successor(7)

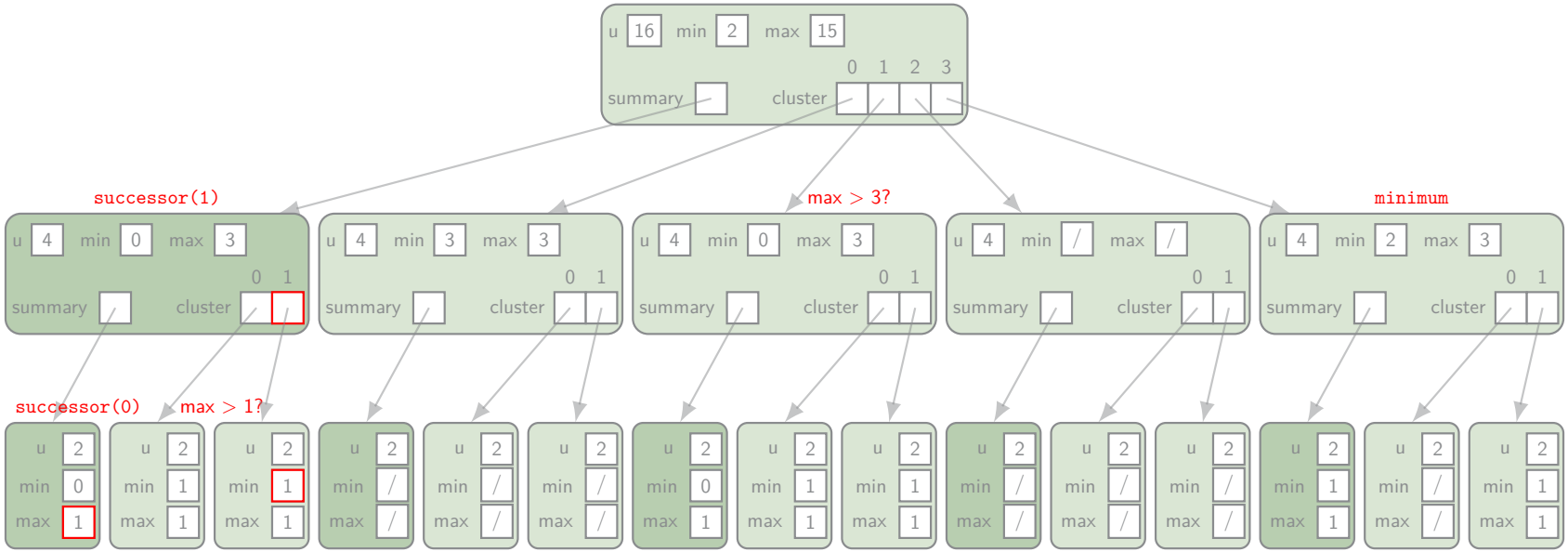
van Emde Boas Tree



{2,3,4,5,7,14,15}

successor(7)

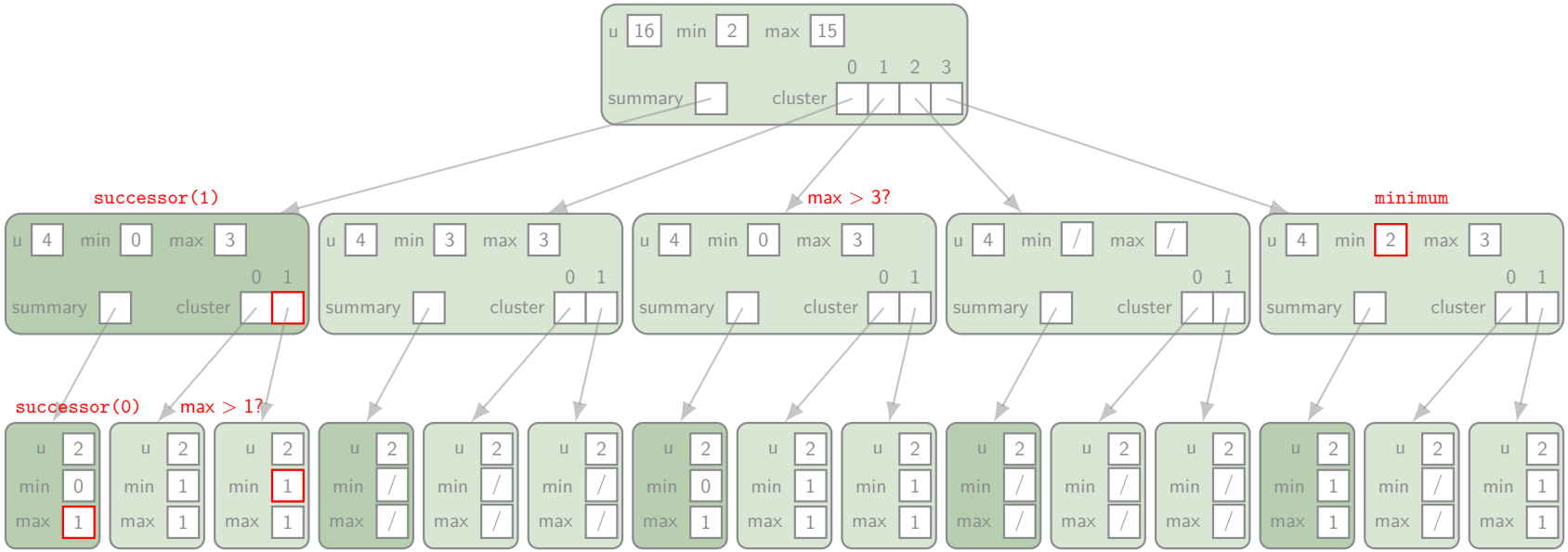
van Emde Boas Tree



{2,3,4,5,7,14,15}

successor(7)

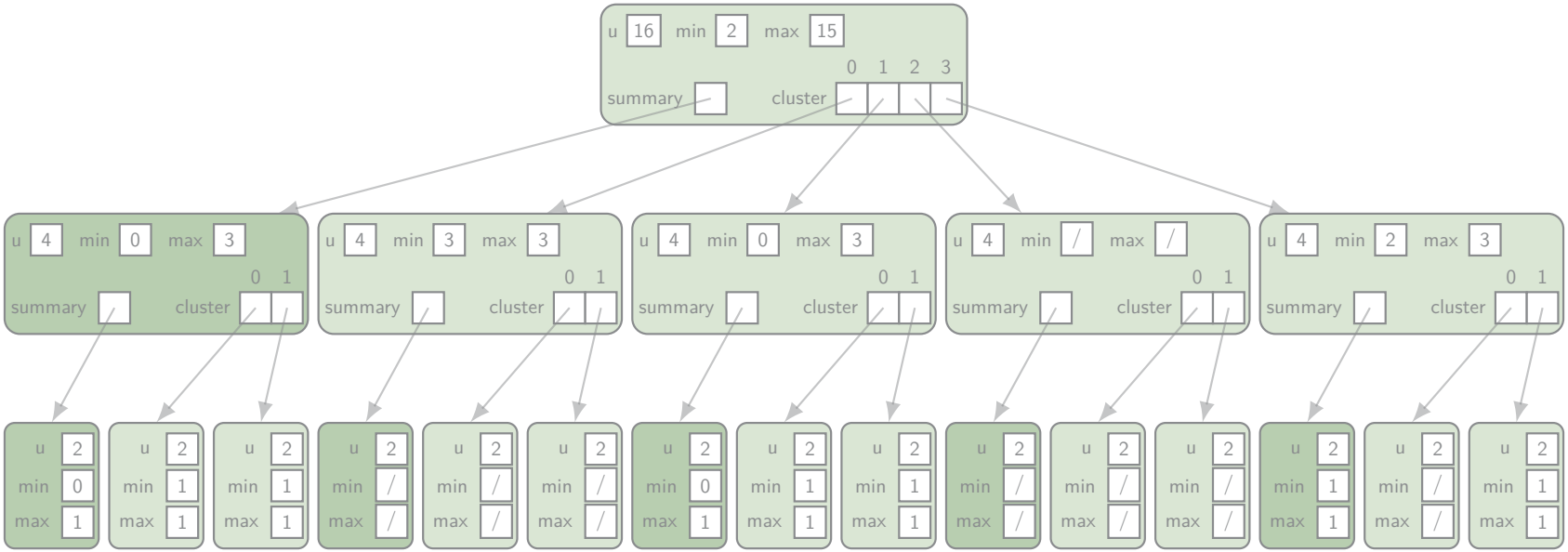
van Emde Boas Tree



{2,3,4,5,7,14,15}

successor(7)

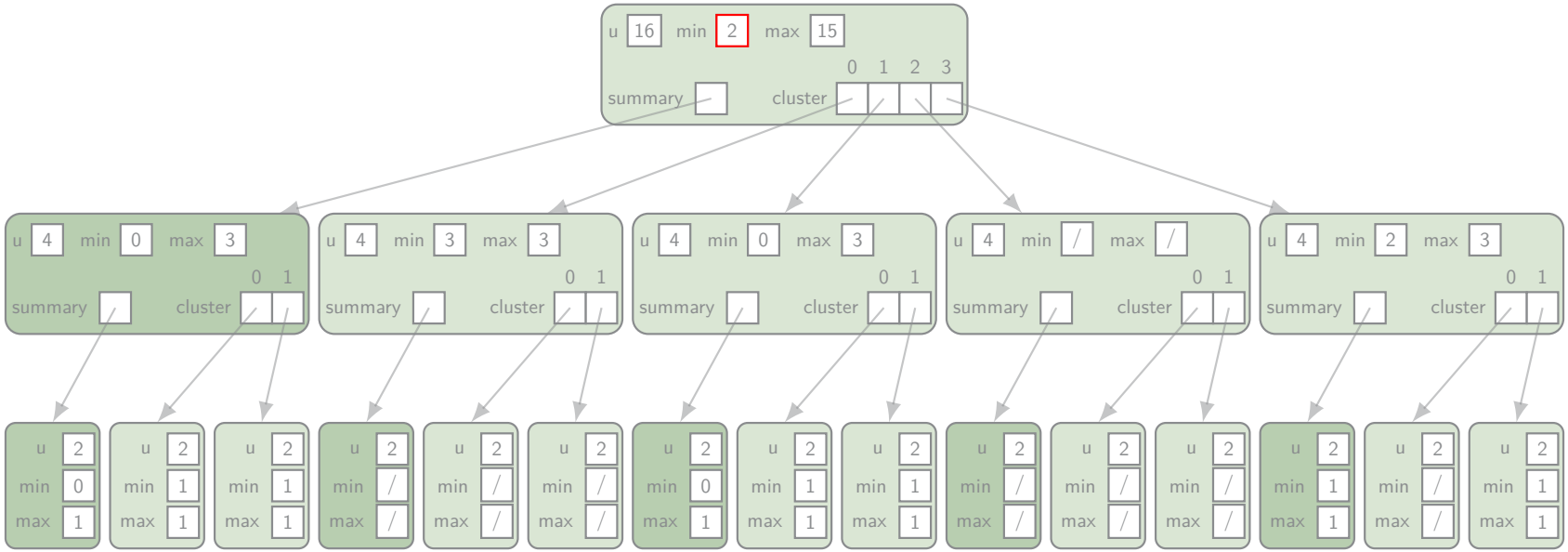
van Emde Boas Tree



{2,3,4,5,7,14,15}

insert(11)

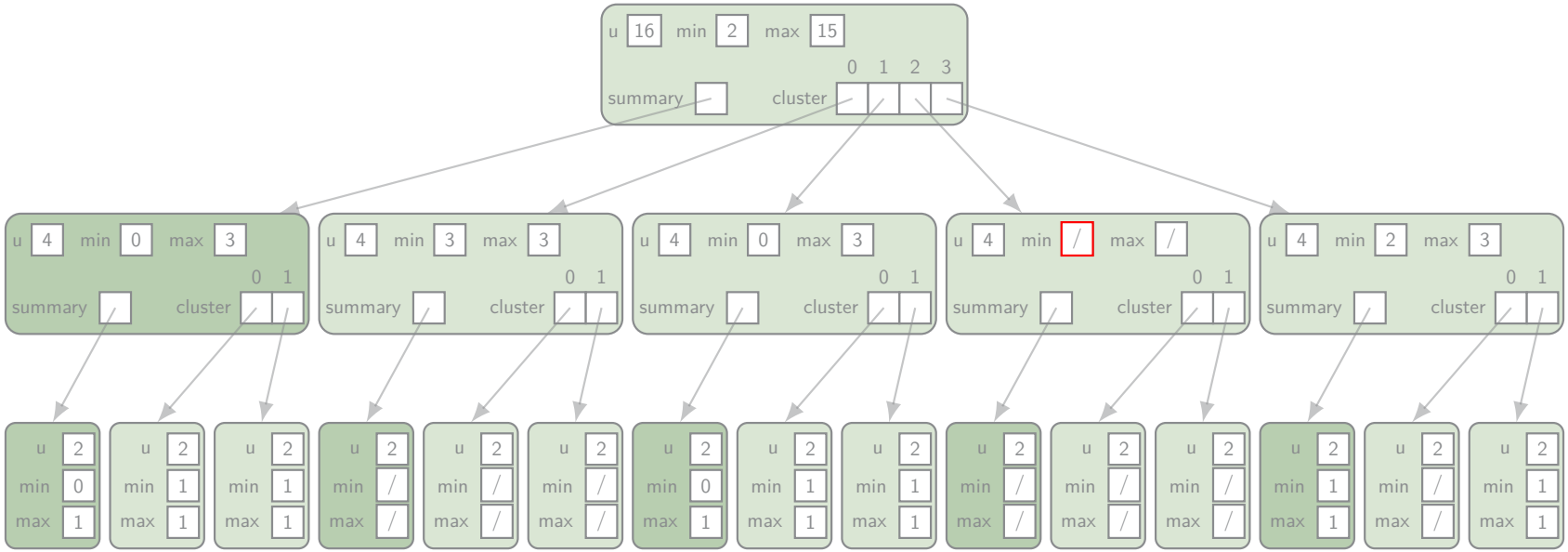
van Emde Boas Tree



{2,3,4,5,7,14,15}

insert(11)

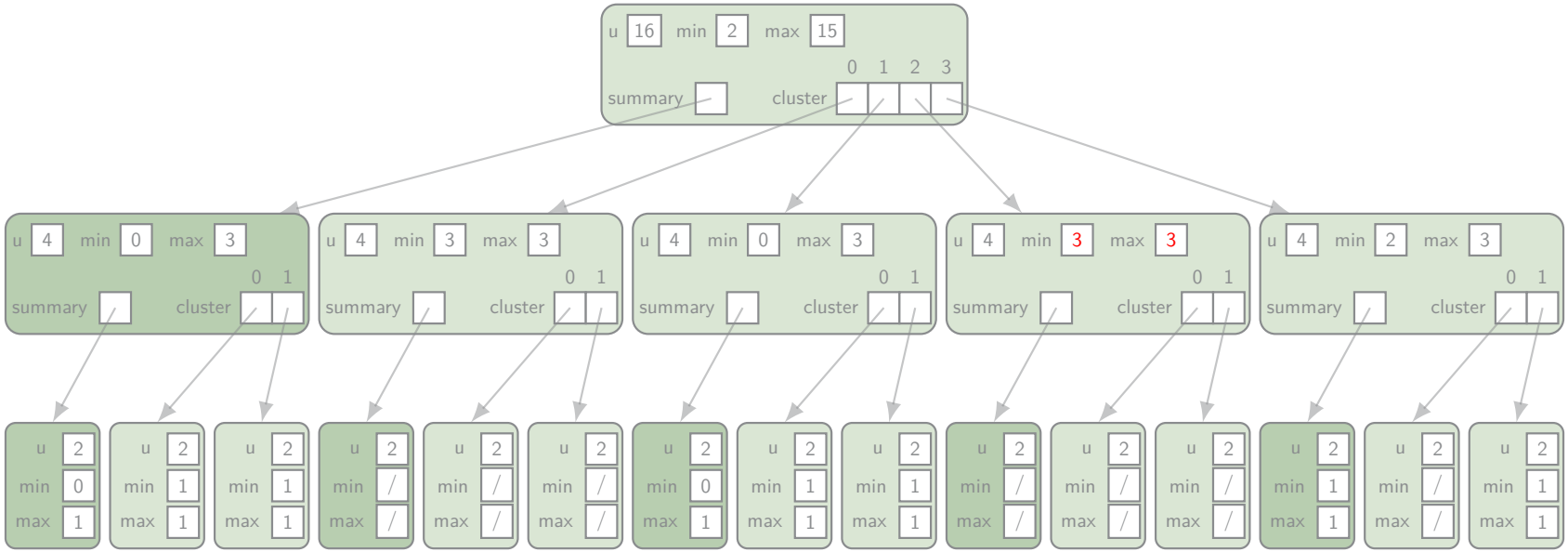
van Emde Boas Tree



{2,3,4,5,7,14,15}

insert(11)

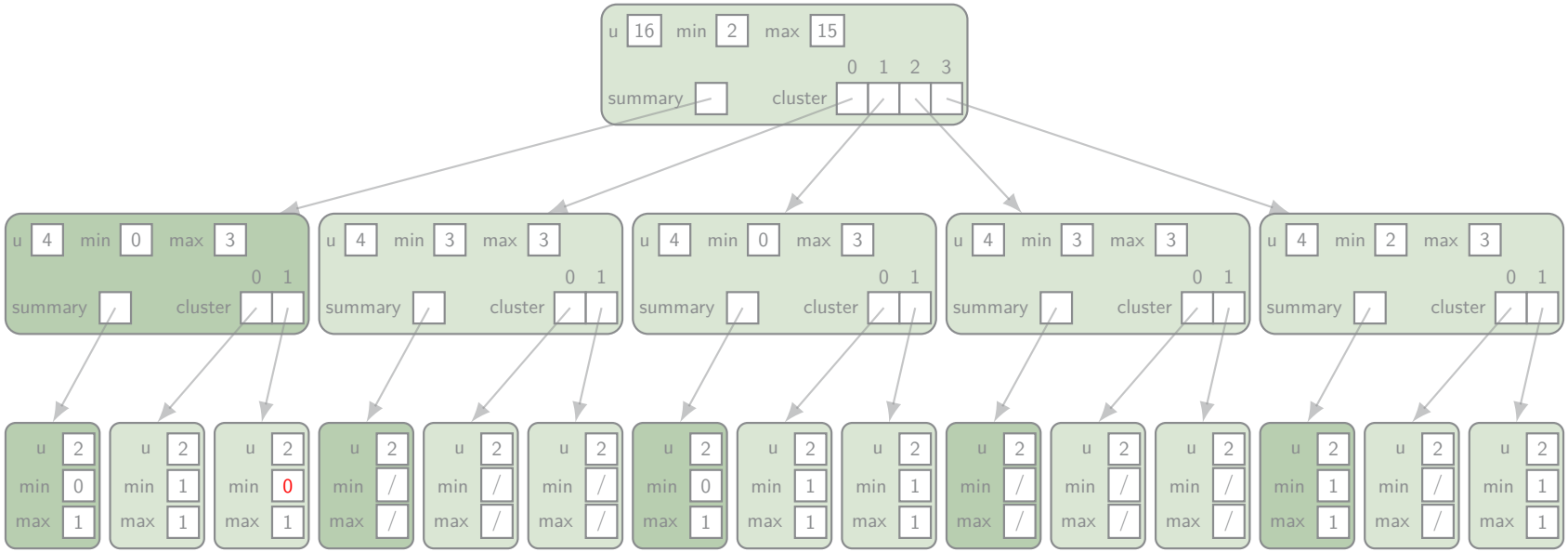
van Emde Boas Tree



{2,3,4,5,7,14,15}

insert(11)

van Emde Boas Tree



{2,3,4,5,7,14,15}

insert(11)

van Emde Boas Tree

Operations

- `min`, `max`, `isEmpty` $O(1)$
- `insert`, `delete`, `contains`, `succ`, `pred` $O(\log \log U)$

- $O(U)$ space
  - X-fast trie: $O(n \log U)$ space
  - Y-fast trie: $O(n)$ space
- $U = 2^k, k \in \mathbb{N}$

Thank you for your attention!

Slides:
https://github.com/MDhondt/Data-Structures-presentation