Visualizing covariance and variance

The covariance of two random variables, X and Y, is a number that quantifies how the random variables "move" together. In this section we will visualize how this metric describes the relationship between X and Y.

Visualizing covariance

Recall that covariance is defined as

$$\mathrm{Cov}(X, Y) = E\left[(X - E(X))(Y - E(Y))\right]$$

. When X and Y take on values x and y respectively, we can form a rectangle with sides of length $x - E(X)$ and $y - E(Y)$. The covariance of X and Y is then the expected area of the rectangle. Note, however, that in our discussion, a "length" can have a negative value and thus some rectangles have a negative "area". Positive-area rectangles correspond to points in which the values of the random variables are either both above or both below their respective means (they "agree") and negative area rectangles correspond to the situation in which one variable is above it's mean and the other is below (they "disagree").

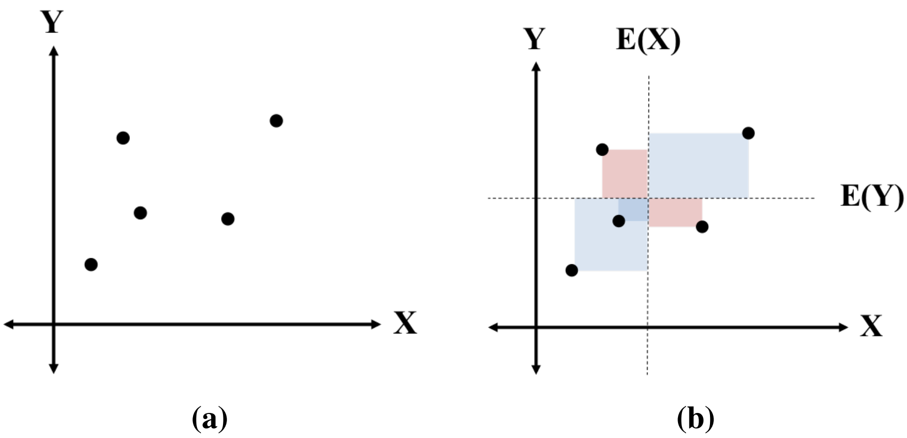

Figure 1: (a) A plot of the values of two discrete random variables X and Y. (b) The covariance of X and Y is the expected area of the rectangle formed by sides of length $x - E(X)$ and $y - E(Y)$.

Visualizing variance and standard deviation

Such a visualization can help in gaining intuition for variance and standard deviation as well. Recall that $\mathrm{Var}(X) = \mathrm{Cov}(X, X)$.

From Figure 2 we see that the variance is always positive because the values of X, when plotted on two axes, will always sit on the line $f(x) = x$. We also note that the rectangles will always be squares because the sides are equal. Furthermore, when the values of X are more spread out, the areas of the rectangles will be larger and thus, the average size of the rectangles will be a larger number. Lastly, we note that the standard deviation is the length of the sides of the average square.

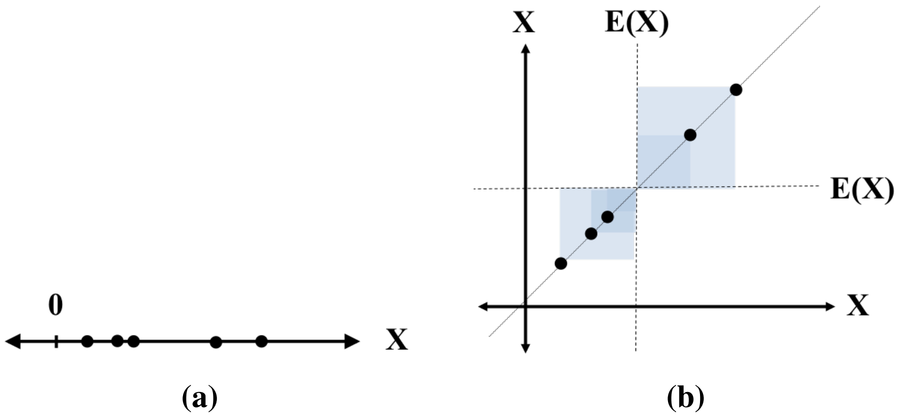

Figure 2: (a) A random variable X plotted on the real-number line. (b) X plotted on two axes and the same visualization strategy is used as in Figure 1.