Diff Drive kinematics

Assumptions

We assume a diff drive robot is described as following:

- Have only 2 drive wheel (other supporting wheels are not modeled and considered as perfectly frictionless.)
- Two drive wheels are in parallel, drive direction line-up with front of robot.
- Body frame centered between two wheels. Body Frame x axis points to front of robot (and drive direction of wheel)

Figure 1: image showing diff drive robot wheel and frame layout

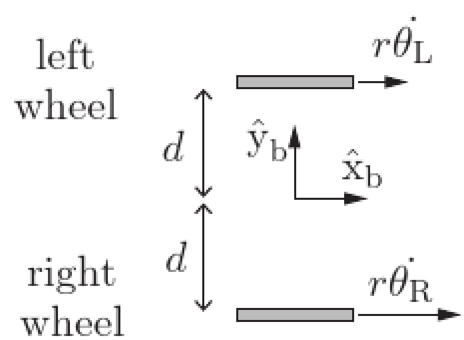

Figure 1: Diff-drive-layout

Each diff drive robot's kinematic is characteristic by the two parameters * wheel track to body width (variable d in image above) * wheel radius

Source of the math

Most of the math are taken from ***Lynch, Park Modern Robotics*** Chapter 13

Common Kinematics equation

We define the jacobian between wheel velocity and body velocity as $H(\phi)$

Specifically for diff drive robot, This matrix is:

$$H(\phi) = r\begin{bmatrix} -1/(2d) & cos(\phi)/2 & sin(\phi)/2 \\ 1/(2d) & cos(\phi)/2 & sin(\phi)/2 \end{bmatrix} H(0) = r\begin{bmatrix} -1/(2d) & 1/2 & 0 \\ 1/(2d) & 1/2 & 0 \end{bmatrix}$$

equation 1

where r is wheel radius and d is body to wheel distance (as in figure 1 above)

Forward Kinematics

This is the math to go from wheel velocity to body displacement.

Get the body twist

From Modern Robotics chapter 13.4 equation 13.34

$$\mathcal{V}_b = H(0)u$$

equation 2

Which evaluate to

$$\mathcal{V}_b\omega = -1/(2d)u_L + 1/(2d)u_R\mathcal{V}_b x = \frac{1}{2}u_L + \frac{1}{2}u_R\mathcal{V}_b y = 0$$

equation 3

Integrate twist

Updating of body configuration is done using twist integration and frame transformation.

We integrate the body twist $\mathcal{V}_b$ into $T_{bb'}$ using center of rotation trick.

The equations are given in the class notes: https://nu-msr.github.io/navigation_site/lectures/odometry.html

The center of rotation (CoR) frame {s} from body frame {b} is found using:

$$s_x = \mathcal{V}_b y/\mathcal{V}_b\omega s_y = \mathcal{V}_b x/\mathcal{V}_b\omega$$

equation 4

Then we rotate frame {s} by amount of twist to get frame {s'}

Finally, we transform the inverse amount of T_{bs} to get frame {b'}

The overall transformation stackup is

$$T_{bb'} = T_{bs}T_{ss'}T_{s'b'}$$

equation 5

Inverse Transformation

The problem here is to go from command twist to desire wheel velocity.

This is basically the inverse of the equation above.

From Modern robotics Chapter 13.2 Equation 13.8

$$u = H(0)\mathcal{V}_b$$

Which expands to

$$

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