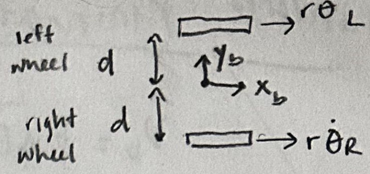
① Foward Kinematics

#: DRB



$$\mathcal{V}_b = \begin{bmatrix} \omega_{bz} \\ v_{bx} \\ v_{by} \end{bmatrix} = F\Delta\theta = r \begin{bmatrix} -1/(2d) & 1/(2d) \\ 1/2 & 1/2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\theta_L \\ \Delta\theta_R \end{bmatrix}$$

$$= r \begin{bmatrix} (-1/(2d))\Delta\theta_L + (1/(2d))\Delta\theta_R \\ \Delta\theta_L/2 + \frac{\Delta\theta_R}{2} \\ 0 + 0 \end{bmatrix} = r \begin{bmatrix} -\frac{\Delta\theta_L}{2d} + \frac{\Delta\theta_R}{2d} \\ \frac{\Delta\theta_L + \Delta\theta_R}{2} \\ 0 \end{bmatrix}$$

$$= r \begin{bmatrix} \frac{-\Delta\theta_L + \Delta\theta_R}{2d} \\ \frac{\Delta\theta_L + \Delta\theta_R}{2} \\ 0 \end{bmatrix}$$

$$\mathcal{V}_b = \begin{bmatrix} \omega_{bz} \\ v_{bx} \\ v_{by} \end{bmatrix} = \begin{bmatrix} \frac{r(-\Delta\theta_L + \Delta\theta_R)}{2d} \\ \frac{r(\Delta\theta_L + \Delta\theta_R)}{2} \\ 0 \end{bmatrix}$$

$$v_{by} = 0 \qquad \omega_{bz} = \frac{r(-\Delta\theta_L + \Delta\theta_R)}{2d} \qquad v_{bx} = \frac{r(\Delta\theta_L + \Delta\theta_R)}{2}$$

if $\omega_{bz} = 0$:

$$\Delta q_b = \begin{bmatrix} \Delta\phi_b \\ \Delta x_b \\ \Delta y_b \end{bmatrix} = \begin{bmatrix} 0 \\ v_{bx} \\ v_{by} \end{bmatrix}$$

if $\omega_{bz} \neq 0$:

$$\Delta q_b = \begin{bmatrix} \Delta\phi_b \\ \Delta x_b \\ \Delta y_b \end{bmatrix} = \begin{bmatrix} \omega_{bz} \\ (v_{bx}\sin(\omega_{bz}) + v_{by}(\cos(\omega_{bz} - 1))/\omega_{bz} \\ (v_{by}\sin(\omega_{bz}) + v_{bx}(1 - \cos(\omega_{bz}))/\omega_{bz} \end{bmatrix}$$

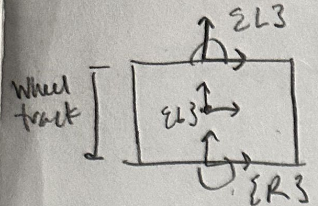
Then
$$\Delta q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi_k) & -\sin(\phi_k) \\ 0 & \sin(\phi_k) & \cos(\phi_k) \end{bmatrix} \Delta q_b \quad \begin{bmatrix} \Delta\phi_b \\ \Delta x_b \\ \Delta y_b \end{bmatrix}$$

$$\Delta q = \begin{bmatrix} \Delta\phi_b \\ \Delta x_b \cdot \cos(\phi_k) - \Delta y_b \sin(\phi_k) \\ \Delta x_b \sin(\phi_k) + \Delta y_b \cos(\phi_k) \end{bmatrix}$$

$$q_{k+1} = q_k + \Delta q$$

(2) Inverse Kinematics

DRB



$T(\theta, x, y)$

$T_{bL}\left(0, 0, \frac{\text{wheel-track}}{2}\right)$

$T_{bR}\left(0, 0, -\frac{\text{wheel-track}}{2}\right)$

$$V_b = \begin{bmatrix} \dot{\theta} \\ v_x \\ v_y \end{bmatrix}$$

Adjoints:

$$A_{Lb} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{\text{wheel-track}}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad A_{Rb} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{\text{wheel-track}}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Then $V_i = A_{ib} V_b$

$$\begin{bmatrix} \dot{\theta} \\ v_{xL} \\ v_{yL} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{\text{wheel-track}}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ v_x \\ v_y \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ -\frac{\text{wheel-track}}{2} \cdot \dot{\theta} + v_x \\ v_y \end{bmatrix}$$

$$\begin{bmatrix} \dot{\theta} \\ v_{xR} \\ v_{yR} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{\text{wheel-track}}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ v_x \\ v_y \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ \frac{\text{wheel-track}}{2} \dot{\theta} + v_x \\ v_y \end{bmatrix}$$

conventional wheel: → assume wheel does not slip

$$\begin{bmatrix} v_{xi} \\ v_{yi} \end{bmatrix} = \begin{bmatrix} r\dot{\phi}_i \\ 0 \end{bmatrix}$$

Then:

$$\begin{bmatrix} \dot{\theta} \\ v_{xL} \\ v_{yL} \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ r\dot{\phi}_L \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ -\frac{\text{wheel-track}}{2} \dot{\theta} + v_x \\ v_y \end{bmatrix} \Rightarrow \dot{\phi}_L = \frac{1}{r}\left(-\frac{\text{wheel-track}}{2} \dot{\theta} + v_x\right)$$

$$\begin{bmatrix} \dot{\theta} \\ v_{xR} \\ v_{yR} \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ r\dot{\phi}_R \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ \frac{\text{wheel-track}}{2} \dot{\theta} + v_x \\ v_y \end{bmatrix} \Rightarrow \dot{\phi}_R = \frac{1}{r}\left(\frac{\text{wheel-track}}{2} \dot{\theta} + v_x\right)$$