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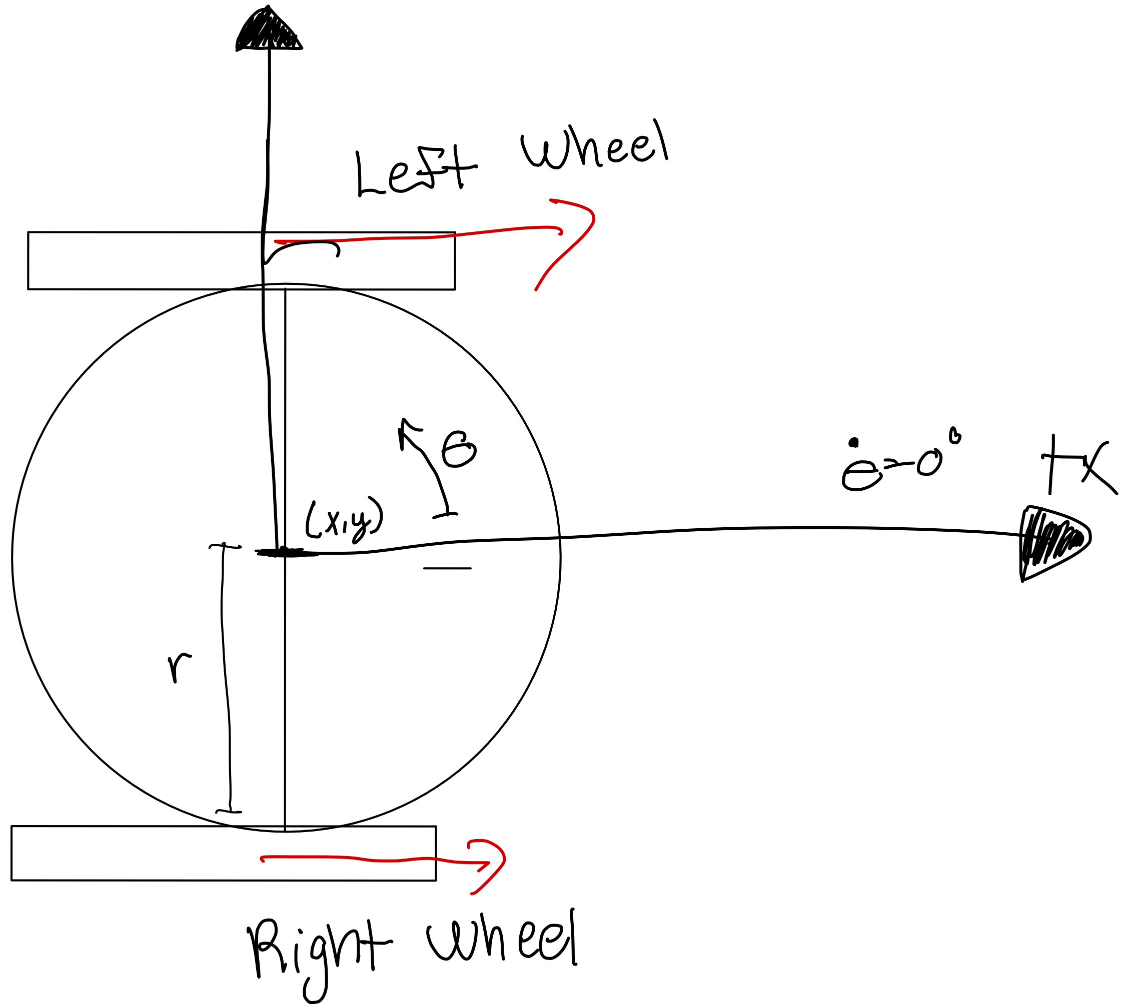

D = body radius

r = wheel radius

$$U = \begin{bmatrix} U_L \\ U_R \end{bmatrix} = \frac{1}{r}\begin{bmatrix} -D & 1 & 0 \\ D & 1 & 0 \end{bmatrix}\begin{bmatrix} \dot{\theta} \\ V_x \\ V_y \end{bmatrix}$$

$$L = \frac{1}{r}(-D\dot{\theta} + V_x)$$

$$R = \frac{1}{r}(D\dot{\theta} + V_x)$$

$$T_{BL} = (0, 0, D)$$

$$T_{BR} = (0, 0, -D)$$

$$A_{BL} = \begin{bmatrix} 1 & 0 & 0 \\ D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad A_{BR} = \begin{bmatrix} 1 & 0 & 0 \\ -D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_{LB} = \begin{bmatrix} 1 & 0 & 0 \\ -D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad A_{RB} = \begin{bmatrix} 1 & 0 & 0 \\ D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$u = \begin{bmatrix} u_L \\ u_R \end{bmatrix} = \frac{1}{r} \begin{bmatrix} -D & 1 & 0 \\ D & 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ V_x \\ V_y \end{bmatrix}$$

$$u = \begin{bmatrix} u_L \\ u_R \end{bmatrix} = \begin{bmatrix} \dot{\phi}_L \\ \dot{\phi}_R \end{bmatrix} = \frac{1}{r} \begin{bmatrix} -D & 1 & 0 \\ D & 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ V_x \\ V_y \end{bmatrix}$$

Left

$$\begin{bmatrix} \dot\theta \\ V_{x_1} \\ V_{y_2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot\theta \\ V_x \\ V_y \end{bmatrix} = \begin{bmatrix} \dot\theta \\ -D\dot\theta + V_x \\ V_y \end{bmatrix} = V_{Left}$$

Right

$$\begin{bmatrix} \dot\theta \\ V_{x_2} \\ V_{y_2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot\theta \\ V_x \\ V_y \end{bmatrix} = \begin{bmatrix} \dot\theta \\ D\dot\theta + V_x \\ V_y \end{bmatrix} = V_{Right}$$

Left

$$\begin{bmatrix} \dot\theta \\ r\dot\phi_L \\ 0 \end{bmatrix} = \begin{bmatrix} \dot\theta \\ -D\dot\theta + V_x \\ V_y \end{bmatrix} \Rightarrow \begin{bmatrix} \dot\theta = \dot\theta \\ r\dot\phi_L = -D\dot\theta + V_x \\ 0 = V_y \end{bmatrix}$$

$$\dot\phi_L = \frac{-D\dot\theta + V_x}{r}$$

right

$$\begin{bmatrix} \dot\theta \\ r\dot\phi_R \\ 0 \end{bmatrix} = \begin{bmatrix} \dot\theta \\ D\dot\theta + V_x \\ V_y \end{bmatrix} \Rightarrow \begin{bmatrix} \dot\theta = \dot\theta \\ r\dot\phi_R = D\dot\theta + V_x \\ 0 = V_y \end{bmatrix}$$

$$\dot\phi_R = \frac{D\dot\theta + V_x}{r}$$

$$\dot\phi_L = -\frac{D}{r}\dot\theta + \frac{V_x}{r}$$

$$\dot\phi_R = \frac{D}{r}\dot\theta + \frac{V_x}{r}$$

solve for $\dot\theta$ & V_x

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Right Wheel Inverse kinematics

$$\dot{\phi}_L - \frac{V_x}{r} = \frac{-D}{r}\dot{\theta}$$ Left wheel Inverse kinematics

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$$\underbrace{\frac{-r}{D}\left(\dot{\phi}_L - \frac{V_x}{r}\right) = \dot{\theta}}$$

$$\dot{\phi}_R = \frac{D}{r}\left(\frac{-r}{D}\left(\dot{\phi}_L - \frac{V_x}{r}\right)\right) + \frac{V_x}{r}$$

$$\dot{\phi}_R = \left(\frac{V_x}{r} - \dot{\phi}_L\right) + \frac{V_x}{r}$$

$$\dot{\phi}_R = \frac{2V_x}{r} - \dot{\phi}_L$$ # Vx in terms of rates

$$r\left(\frac{\dot{\phi}_R + \dot{\phi}_L}{2}\right) = V_x$$ => twist.x_dot

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$$\dot{\theta} = \frac{-r}{D}\left(\dot{\phi}_L - \left(\frac{\dot{\phi}_R + \dot{\phi}_L}{2}\right)\right)$$

$$\dot{\theta} = \frac{r}{D}\left(\frac{\dot{\phi}_R}{2} - \frac{\dot{\phi}_L}{2}\right)$$

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twist.thetadot

Forward Kinematics

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