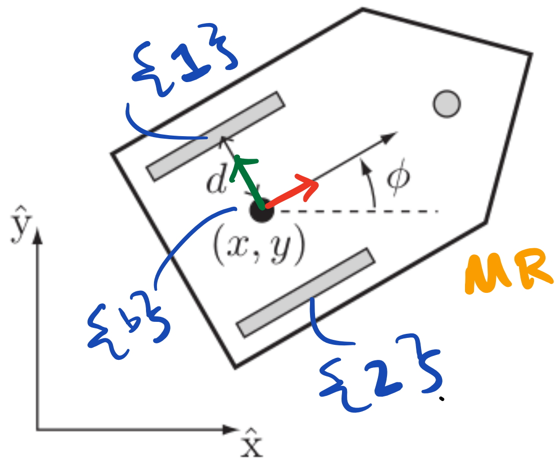
Kinematics Derivations, Differential Drive Robot

Body frame twist $\mathcal{V}_b = \begin{bmatrix} \dot{\theta} \\ \dot{x} \\ \dot{y} \end{bmatrix}$



Transformation Matrices

$$T_{bw_1} = \begin{array}{ccc} \theta & t_x & t_y \\ (0, & 0, & d) \end{array}$$

$$T_{bw_2} = \begin{array}{ccc} \theta & t_x & t_y \\ (0, & 0, & -d) \end{array}$$

Adjoints

- map twists to other frames
- 1,2 refer to wheel frames

$$A_{b1} = \begin{bmatrix} 1 & 0 & 0 \\ d & 1 & 0 \\ \text{L=0} & 0 & 1 \end{bmatrix} \qquad A_{b2} = \begin{bmatrix} 1 & 0 & 0 \\ -d & 1 & 0 \\ \text{L=0} & 0 & 1 \end{bmatrix}$$

$$A_{1b} = \begin{bmatrix} 1 & 0 & 0 \\ -d & 1 & 0 \\ \text{L=0} & 0 & 1 \end{bmatrix} \qquad A_{2b} = \begin{bmatrix} 1 & 0 & 0 \\ d & 1 & 0 \\ \text{L=0} & 0 & 1 \end{bmatrix}$$

Mapping body twist $\mathcal{V}_b$ to wheel velocities

$$V_i = A_{ib} V_b$$

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{x}_1 \\ \dot{y}_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -d & 1 & 0 \\ \text{L=0} & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_b \\ \dot{x}_b \\ \dot{y}_b \end{bmatrix}$$

$$\dot{\theta}_1 = \dot{\theta}_b$$

$$\Rightarrow \quad \dot{x}_1 = -d\dot{\theta}_b + \dot{x}_b$$

$$\dot{y}_1 = \dot{y}_b$$

Inverse Kinematics

- finding wheel velocities for achieving body twists

$$\begin{bmatrix} r\dot{\phi}_1 \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \end{bmatrix} = \begin{bmatrix} -d\dot{\theta}_b + \dot{x}_b \\ \dot{y}_b \end{bmatrix}$$

$$r\dot{\phi}_1 = -d\dot{\theta}_b + \dot{x}_b$$

$$\boxed{\begin{aligned} \dot{\phi}_1 &= \frac{1}{r}\left(-d\dot{\theta}_b + \dot{x}_b\right) \\ \dot{\phi}_2 &= \frac{1}{r}\left(d\dot{\theta}_b + \dot{x}_b\right) \end{aligned}}$$

By symmetry

Forward Kinematics

- updating the robot's configuration given wheel movement (ϕ_1', ϕ_2')

- Convert the wheel configuration to a body twist

$$\mathcal{V}_b = \text{computeBodyTwist}(\phi_1', \phi_2')$$

see below

- Integrate $\mathcal{V}_b$ to get $T_{bb'}$

$$T_{bb'} = \text{integrateTwist}(\dot{\theta}, \dot{x}, \dot{y})$$

- Get the location in the world frame

$$\underbrace{T_{wb'}}_{\text{update the current configuration with this transform's parameters}} = T_{wb} T_{bb'}$$

current robot pos from object state

Converting from wheel velocity commands to a body twist (computeBodyTwist)

$$\dot{q} = \begin{bmatrix} \dot{\theta} \\ \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -r/2d & r/2d \\ \frac{r}{2}\cos\theta & \frac{r}{2}\cos\theta \\ \frac{r}{2}\sin\theta & \frac{r}{2}\sin\theta \end{bmatrix} \begin{bmatrix} u_L \\ u_R \end{bmatrix}$$

EQN 13.15 from Modern Robotics

$$\boxed{\dot{q} = \begin{bmatrix} \dot{\theta} = -\frac{r}{2d} u_L + \frac{r}{2d} u_R \\ \dot{x} = \frac{r}{2} u_L + \frac{r}{2} u_R \\ \dot{y} = 0 \end{bmatrix}}$$