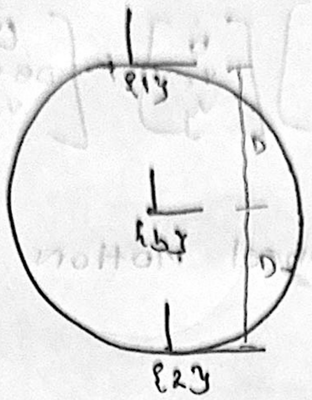
RAHOLROY

KINEMATICS PDF



D is wheel track

FROM DERIVE KINEMATICS NOTES

TRANSFORMATION MATRICES

$T_{b1}(0,0,D)$

$T_{b2}(0,0,-D)$

ADJOINTS

$A_{b1} = \begin{bmatrix} 1 & 0 & 0 \\ D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad A_{b2} = \begin{bmatrix} 1 & 0 & 0 \\ -D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

INVERSE ADJOINTS

$A_{1b} = \begin{bmatrix} 1 & 0 & 0 \\ -D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad A_{2b} = \begin{bmatrix} 1 & 0 & 0 \\ D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

BODY TWIST IN THE WHEEL FRAMES

Let $V_b = \begin{bmatrix} \dot{\theta} \\ v_x \\ v_y \end{bmatrix} \Rightarrow$ body frame twist

Let $V_i = \begin{bmatrix} \dot{\theta} \\ v_{xi} \\ v_{yi} \end{bmatrix}$ = twist in frame @ wheel i

Then $V_i = A_{ib} V_b$

$V_1 = A_{1b} V_b$

$$\begin{bmatrix} \dot{\theta} \\ v_{x1} \\ v_{y1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ v_x \\ v_y \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ -D\dot{\theta} + v_x \\ v_y \end{bmatrix}$$

$v_a = A_{ab} v_b$

$$\begin{bmatrix} \dot\theta \\ v_{xa} \\ v_{ya} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot\theta \\ v_x \\ v_y \end{bmatrix} = \begin{bmatrix} \dot\theta \\ D\dot\theta + v_x \\ v_y \end{bmatrix}$$

Body Twist to Wheel Motion

Left wheel

$$\begin{bmatrix} \dot\theta \\ r\dot\phi_1 \\ 0 \end{bmatrix} = \begin{bmatrix} \dot\theta \\ -D\dot\theta + v_x \\ v_y \end{bmatrix}$$

$$\begin{bmatrix} \dot\phi_1 \end{bmatrix} = \begin{bmatrix} -\frac{D}{r} & \frac{1}{r} & 0 \end{bmatrix} \begin{bmatrix} \dot\theta \\ v_x \\ v_y \end{bmatrix}$$

Right wheel

$$\begin{bmatrix} \dot\theta \\ r\dot\phi_2 \\ 0 \end{bmatrix} = \begin{bmatrix} \dot\theta \\ D\dot\theta + v_x \\ v_y \end{bmatrix}$$

$$\begin{bmatrix} \dot\phi_2 \end{bmatrix} = \begin{bmatrix} \frac{D}{r} & \frac{1}{r} & 0 \end{bmatrix} \begin{bmatrix} \dot\theta \\ v_x \\ v_y \end{bmatrix}$$

Inverse Kinematics

$$\dot\phi = H V_b$$

$$\begin{bmatrix} \dot\phi_1 \\ \dot\phi_2 \end{bmatrix} = \frac{1}{r} \begin{bmatrix} -D & 1 & 0 \\ D & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot\theta \\ v_x \\ v_y \end{bmatrix}$$

$$\dot\phi_1 = \frac{1}{r}(-D\dot\theta + v_x) \quad - (1)$$

$$\dot\phi_2 = \frac{1}{r}(D\dot\theta + v_x) \quad \rightarrow (2)$$

FORWARD KINEMATICS

FROM MODERN ROBOTICS pg 549

$$V_b = H^{+}(0)\Delta\theta = F\Delta\theta$$

(3) $$V_b = F\Delta\theta = r\begin{bmatrix} -1/2d & 1/2d \\ 1/2 & 1/2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \Delta\theta_L \\ \Delta\theta_L \end{bmatrix}$$

Taking the twist

$$T_{bb'} = e^{[V_{b6}]} \quad (4)$$

Extracting change in coordinates relative to body frame

if $\omega_{bz} = 0$ $$\Delta q_b = \begin{bmatrix} \Delta\phi_b \\ \Delta x_b \\ \Delta y_b \end{bmatrix} = \begin{bmatrix} 0 \\ v_{bx} \\ v_{by} \end{bmatrix}$$

if $\omega_{bz} \neq 0$ $$\Delta q_b = \begin{bmatrix} \Delta\phi_b \\ \Delta x_b \\ \Delta y_b \end{bmatrix} = \cancel{\begin{bmatrix} \omega_{bz} \\ v_{bx}\sin\omega_{bz} + \\ \ \end{bmatrix}}$$

$$= \begin{bmatrix} \omega_{bz} \\ (v_{bx}\sin\omega_{bz} + v_{by}(\cos\omega_b - 1))/\omega_{bz} \\ (v_{by}\sin\omega_{bz} + v_{bx}(1-\cos\omega_{bz}))/\omega_{bz} \end{bmatrix}$$

TRANSFORMING Δq_b in {b} to Δq in fixed frame {s}

$$\Delta q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_k & -\sin\phi_k \\ 0 & \sin\phi_k & \cos\phi_k \end{bmatrix}\Delta q_b \quad - (5)$$

update the odometry → chasis config

$$q_{k+1} = q_k + \Delta q \quad - (6)$$