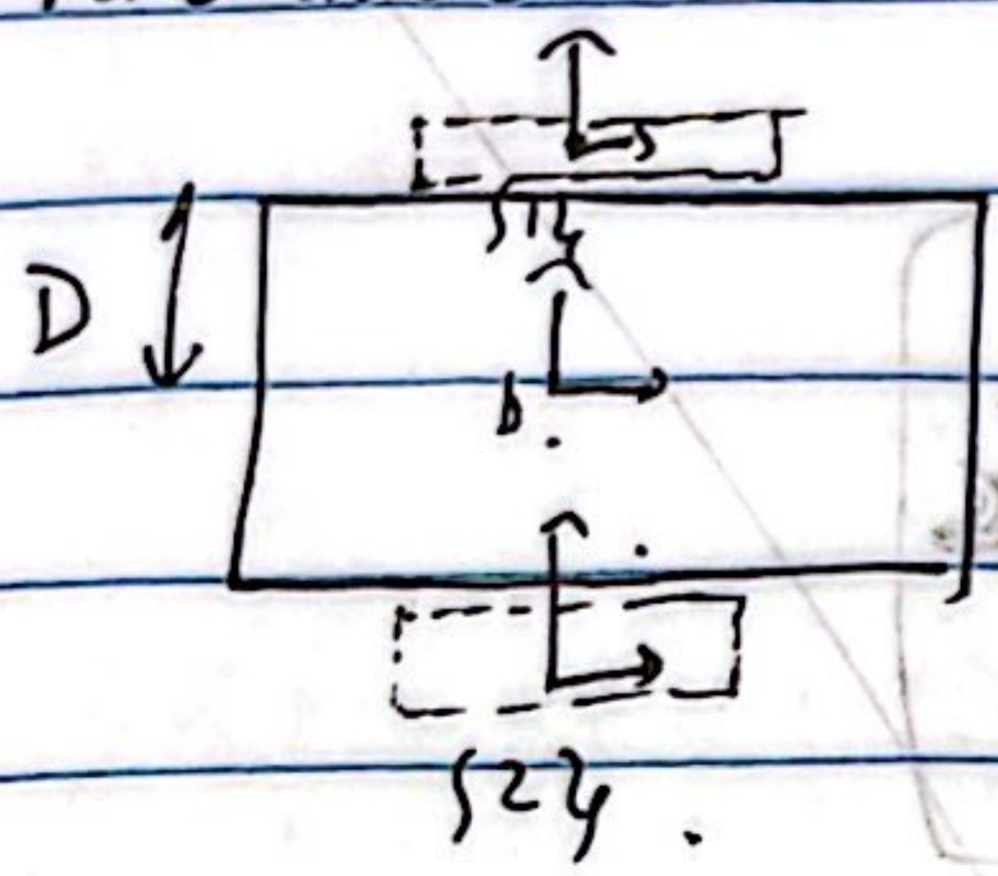
Kinematics :- (Two wheeled diff drive robot)



$T_{b1} = (0, 0, D)$ $\quad T_{b2} = (0, 0, -D)$

$A_{b1} = \begin{bmatrix} 1 & 0 & 0 \\ D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad A_{b2} = \begin{bmatrix} 1 & 0 & 0 \\ -D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A_{1b} = \begin{bmatrix} 1 & 0 & 0 \\ -D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad A_{2b} = \begin{bmatrix} 1 & 0 & 0 \\ D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$V_b = \begin{bmatrix} \dot{\theta} \\ \dot{x} \\ \dot{y} \end{bmatrix}$

$\therefore V_1 = A_{1b} V_b$ $\quad V_2 = A_{2b} V_b$

$V_1 = \begin{bmatrix} 1 & 0 & 0 \\ -D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{x} \\ \dot{y} \end{bmatrix}$ $\quad V_2 = \begin{bmatrix} 1 & 0 & 0 \\ D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{x} \\ \dot{y} \end{bmatrix}$

$V_1 = \begin{bmatrix} \dot{\theta} \\ -D\dot{\theta} + \dot{x} \\ \dot{y} \end{bmatrix}$ $\quad V_2 = \begin{bmatrix} \dot{\theta} \\ D\dot{\theta} + \dot{x} \\ \dot{y} \end{bmatrix}$

For Conventional Wheels :- Calculating I.K.

Wheel left - . wheel Angles.

$$\begin{bmatrix} \dot{\theta} \\ r\dot{\phi}_L \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ -D\dot{\theta} + \dot{x} \\ \dot{y} \end{bmatrix} \Rightarrow r\dot{\phi}_L = -D\dot{\theta} + \dot{x}$$

$$\dot{\phi}_L = \left(\frac{\dot{x} - D\dot{\theta}}{r} \right) \quad \text{— (1)}$$

Wheel right

$$\begin{bmatrix} \dot{\theta} \\ r\dot{\phi}_R \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ D\dot{\theta} + \dot{x} \\ \dot{y} \end{bmatrix} \Rightarrow r\dot{\phi}_R = D\dot{\theta} + \dot{x}$$

$$\therefore \dot{\phi}_R = \left(\frac{\dot{x} + D\dot{\theta}}{r} \right) \quad \text{— (2)}$$

→ From Modern Robotics. Textbook.

Using Equation 13.14.

$$\begin{bmatrix} \dot{\phi} \\ \dot{x} \\ \dot{y} \\ \dot{\theta}_L \\ \dot{\theta}_R \end{bmatrix} = \dot{q} = \begin{bmatrix} -r/2d & r/2d \\ r/2 \cos\phi & r/2 \cos\phi \\ r/2 \sin\phi & r/2 \sin\phi \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_L \\ u_R \end{bmatrix}.$$

$$\therefore \dot{\phi} = \frac{+r}{2d}\left(u_R - u_L\right) \rightarrow ③$$

$$\dot{x}_c = \frac{r}{2}\cos\phi\left(u_L + u_R\right) - ④$$

We consider $\dot{y} = 0$ as 2 wheel diff drive

$$\dot{\theta}_L = u_L$$

$$\Rightarrow \theta_L = u_L + \theta_{L,old} - ⑤$$

$$\dot{\theta}_R = u_R$$

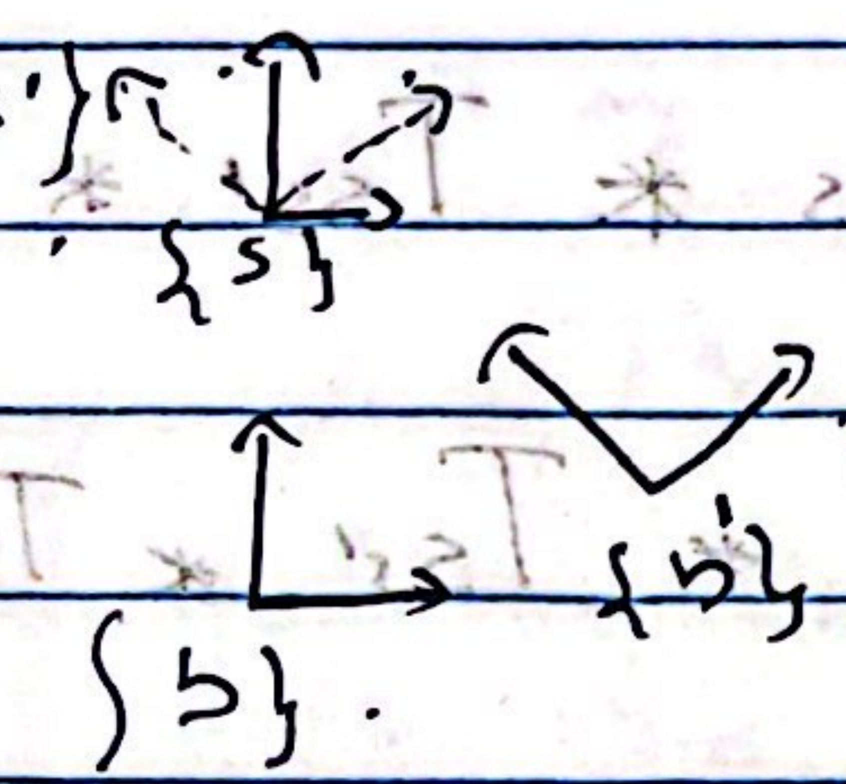
$$\theta_R = u_R + \theta_{R,old} - ⑥$$

Nowe we consider the body twis

$$V = \begin{bmatrix} \cancel{\dot{\theta}_{b}} \; \dot{\phi} \\ \dot{x}_c \\ 0.0 \end{bmatrix}$$

We apply integrate twist to convert the twist into Tbbprime

⇒ . For integrate Twist



$T_{sb} = (0, x_{sb}, y_{sb})$.

$V_s = \begin{bmatrix} \omega \\ 0 \\ 0 \end{bmatrix}$

∴ $V_s = A_{sb} V_b$.

As space frame will only observe rotation

$$\begin{bmatrix} \omega \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ y_s & 1 & 0 \\ -x_s & 0 & 1 \end{bmatrix} \begin{bmatrix} \omega \\ \dot{x}_b \\ \dot{y}_b \end{bmatrix}$$

$V_b = \begin{bmatrix} \omega \\ \dot{x}_b \\ \dot{y}_b \end{bmatrix}$

(can experience both Rotation & translation.)

$$\Rightarrow \begin{bmatrix} \omega \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \omega \\ y_s \omega + \dot{x}_b \\ -x_s \omega + \dot{y}_b \end{bmatrix}$$

$$\therefore \; y_s = -\frac{\dot{x}_b}{\omega} \quad \& \quad x_s = \frac{+\dot{y}_b}{\omega}$$

$$\therefore \; T_{sb} = \left(0, \dot{y}_b/\omega, -\dot{x}_b/\omega\right).$$

Now $T_{s'b'} = T_{sb}$.

$$T_{ss'} = (\omega, 0, 0)$$

$$T_{bb'} = T_{bs} * T_{ss'} * T_{s'b'}$$
$$= T_{sb}^{-1} * T_{ss'} * T_{s'b'}$$

$$= \begin{bmatrix} 1 & 0 & -\dot{y}_{b}/\omega \\ 0 & 1 & \dot{x}_{b}/\omega \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\omega) & -\sin(\omega) & 0 \\ \sin(\omega) & \cos(\omega) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & \dot{y}_{b}/\omega \\ 0 & 1 & -\dot{x}_{b}/\omega \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_{bb'} = \begin{bmatrix} \cos(\omega) & -\sin(\omega) & \frac{\dot{y}_b * \cos(\omega) + \dot{x}_b \sin(\omega) - \dot{y}_b}{\omega} \\ \sin(\omega) & \cos(\omega) & \frac{\dot{y}_b \sin(\omega) - \dot{x}_b \cos(\omega) + \dot{x}_b}{\omega} \\ 0 & 0 & 1 \end{bmatrix}$$