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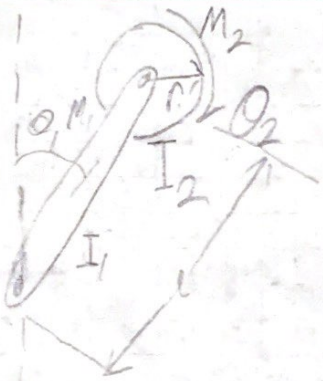

M_1 = arm & motor h

M_2 = Wheel & rotating mass of motor

$L = T - V \Rightarrow L = K - P$

$K = K_1 + K_2$

$K_1 = \frac{1}{2} M_1 \left(\frac{l}{2}\right)^2 (\dot{\theta}_1)^2 + \frac{1}{2} I_1 (\dot{\theta}_1)^2$

$K_2 = \frac{1}{2} M_2 (l)^2 (\dot{\theta}_1)^2 + \frac{1}{2} I_2 (\dot{\theta}_1 + \dot{\theta}_2)^2$

$P = P_1 + P_2$

$P_1 = g M_1 \left(\frac{l}{2}\right)(1 + \cos(\theta_1))$

$P_2 = g M_2 (l)(1 + \cos(\theta_1))$

$L = \underbrace{\frac{1}{2} M_1 \left(\frac{l}{2}\right)^2 (\dot{\theta}_1)^2 + \frac{1}{2} I_1 (\dot{\theta}_1)^2 + \frac{1}{2} M_2 (l)^2 (\dot{\theta}_1)^2 + \frac{1}{2} I_2 (\dot{\theta}_1 + \dot{\theta}_2)^2}_{K} \cdots$

$\cdots \underbrace{-\left(g M_1 \left(\frac{l}{2}\right)(-1 + \cos(\theta_1)) + g M_2 (l)(-1 + \cos(\theta_1))\right)}_{P}$

(1) $\frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}_1} - \frac{\partial L}{\partial \theta_1} = 0$

(2) $\frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}_2} - \frac{\partial L}{\partial \theta_2} = \tau$

① $M_1 l_c^2 + M_2 l^2 + J_1 + J_2)\ddot{\theta}_1 + J_2\ddot{\theta}_2 + (M_1 l_c + M_2 l_1) g\theta_1 = 0$

② $\bar{J}_2\ddot{\theta}_1 + J_2\ddot{\theta}_2 = \bar{T}(t)$

$\bar{M} = m_1 l_{c_1} + m_2 l_1$

$\ddot{\theta}_2 = -\dfrac{(M_1 l_c^2 + M_2 l^2 + J_1 + J_2)}{J_2}\ddot{\theta}_1 - \dfrac{\bar{M}}{J_2} g\theta_1$

$\ddot{\theta}_1 = \dfrac{\bar{T}(t)}{\bar{J}_2} - \dfrac{J_2}{J_2}\ddot{\theta}_2 \Rightarrow \dfrac{t(t)}{\bar{J}_2} - \ddot{\theta}_2$

$\ddot{\theta}_2 = (M_1 l_c^2 + M_2 l^2 + \bar{J}_1 + \bar{J}_2)\left(-\dfrac{t(t)}{\bar{J}_2} + \ddot{\theta}_2\right) - \dfrac{\bar{M}}{\bar{J}_2} g\theta_1$

$\ddot{\theta}_2 - \ddot{\theta}_2 (M_1 l_c^2 + M_2 l^2 + \bar{J}_1 + \bar{J}_2) = -\dfrac{t(t)}{\bar{J}_2}(M_1 l_c^2 + M_2 l^2 + \bar{J}_1 + \bar{J}_2) - \dfrac{\bar{M}}{\bar{J}_2} g\theta_1$

$$\ddot{\theta}_2\left(1 - M_1 l_c^2 + M_2 l^2 + J_1 + J_2\right) = -\frac{\tau(t)}{J_2}\left(M_1 l_c^2 + M_2 l^2 + J_1 + J_2\right) - \frac{\overline{M}}{J_2} g \theta_1$$

$$\ddot{\theta}_2 = -\frac{\tau(t)}{J_2}\frac{\left(M_1 l_c^2 + M_2 l^2 + J_1 + J_2\right)}{\left(1 - M_1 l_c^2 + M_2 l^2 + J_1 + J_2\right)} - \frac{\overline{M}}{J_2}\frac{g\theta_1}{\left(1 - M_1 l_c^2 + M_2 l^2 + J_1 + J_2\right)}$$

$$\ddot{\theta}_2 = \frac{\tau(t)}{J_2} - \frac{J_2}{J_2}\ddot{\theta}_1 \Rightarrow \ddot{\theta}_2 = \frac{\tau(t)}{J_2} - \ddot{\theta}_1$$

$$\frac{\tau(t)}{J_2} - \ddot{\theta}_1 = -\frac{\tau(t)}{J_2}\frac{\left(M_1 l_c^2 + M_2 l^2 + J_1 + J_2\right)}{\left(1 - M_1 l_c^2 + M_2 l^2 + J_1 + J_2\right)} - \frac{\overline{M}}{J_2}\frac{g\theta_1}{\left(1 - M_1 l_c^2 + M_2 l^2 + J_1 + J_2\right)}$$

$$\ddot{\theta}_1 = \frac{\tau(t)}{J_2} + \frac{\tau(t)}{J_2}\frac{\left(M_1 l_c^2 + M_2 l^2 + J_1 + J_2\right)}{\left(1 - M_1 l_c^2 + M_2 l^2 + J_1 + J_2\right)} + \frac{\overline{M}}{J_2}\frac{g\theta_1}{\left(1 - M_1 l_c^2 + M_2 l^2 + J_1 + J_2\right)}$$

$$\ddot{\theta}_1 = \tau(t)\left(\frac{1}{J_2} + \frac{1}{J_2}\frac{M_1 l_c^2 + M_2 l^2 + J_1 + J_2}{\left(1 - M_1 l_c^2 + M_2 l^2 + J_1 + J_2\right)} + \frac{\overline{M}}{J_2}\frac{g\theta_1}{\left(1 - M_1 l_c^2 + M_2 l^2 + J_1 + J_2\right)}\right)$$

$$\dot{x}_1 = \dot{\theta}_1 = \theta_2$$

$$\dot{\theta}_2 = \ddot{\theta}_1$$

$$\dot{\theta}_3 = \ddot{\theta}_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{MG}{J_2(1-M_1 l_c^2 + M_2 l^2 + J_1 + J_2} & 0 & 0 \\ -\frac{MG}{J_2(1-M_1 l_c^2 + M_2 l^2 + J_1 + J_2)} & 0 & 0 \end{bmatrix} + \cdots$$

$$\begin{bmatrix} 0 \\ \frac{1}{J_2} + \frac{1}{J_2}\frac{M_1 l_c^2 + M_2 l^2 + J_1 + J_2}{(1 - M_1 l_c^2 + M_2 l^2 + J_1 + J_2)} \\ -\frac{1}{J_2}\frac{(M_1 l_c^2 + M_2 l^2 + J_1 + J_2)}{(1 - M_1 l_c^2 + M_2 l^2 + J_1 + J_2} \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$