Exercice 3 : Monopoly with network effects

Consider a monopoly market in which consumers' opportunity cost x is uniformly distributed on the unit interval; i.e., $x \in [0; 1]$. A consumer's utility of consuming one unit of the good offered by the monopolist relative to the outside option is $1/4 + n^e/2 - p - x$, where p is the price set by the monopolist and n^e is the expected number of fellow buyers of the good. There is mass one of consumers. The monopolist's marginal costs are zero.

1 Suppose that consumers believe that $n^e = 1$. Determine the monopoly solution under these consumer believes.

2 Suppose that consumers form beliefs n^e before observing price p and that these beliefs are confirmed in equilibrium; i.e., $n^e = n^*$. Determine the monopoly solution under these consumer beliefs.

3 Suppose that consumers first observe price and then form beliefs $n^e(p)$ and that beliefs are self-fulfilling; i.e., they are confirmed in the monopoly solution. Determine the monopoly solution under these consumer beliefs.

4 Compare your findings in parts 2 and 3. Explain what is going on.

5 Suppose that there is a second group of consumers (of mass 1) who also have an opportunity cost x that is uniformly distributed on the unit interval. The utility of consumer x in this second group is $1 = 4 + n_1^e(p_1) = 2 - p_2 - x$, where $n_1^e(p1)$ is the expected number of consumers in the first group buying the product and price p_2 is the price charged to the second group of consumers. Suppose that, similar to part 3, consumers of both groups first observe prices and then form beliefs. Thus, a consumer x in the first group has net utility $1 = 4 + n_1^e(p_1) = 2 - p_1 - x$. Suppose that consumers from both groups hold self-fulfilling beliefs. Determine the monopoly solution (prices and quantities) under these consumer beliefs.

6 Within the setting of part 5, suppose that the monopolist is forced to charge the same price to both groups of consumers; i.e., $p = p_1 = p_2$. Determine the monopoly solution.

7 Explain your findings in parts 5 and 6. Does the monopolist have an incentive to charge prices $p_1 \neq p_2$? Why or why not is this the case?

Solution :

1) $n^e = 1$, thus, demand $x = 1/4 + n^e/2 - p$ becomes $x = 3/4 - p$. We can deduce the monopoly's program.

$$\mathscr{P}_{Monopoly} \left| \quad \underset{\{p\}}{\text{Max}} \left(\frac{3}{4} - p\right) p \right. \tag{1}$$

The first order condition (FOC) of the monopoly's program, $\dfrac{\partial \pi}{\partial p} = 3/4 - 2p$, gives us $p^m = 3/8$ the price set by the monopolist, $n^m = 3/8$ the number of buyers of the good at this price and $\pi^m = \frac{27}{256}$ the monopoly's profit. The second order condition (SOC) is satisfied $\dfrac{\partial^2 \pi}{\partial p^2} = -2 < 0$.

2) Demand is $x = 1/4 + n^e/2 - p$. The new monopoly's program is

$$\mathscr{P}_{Monopolist} \left| \quad \underset{\{p\}}{\text{Max}} \left(\frac{1}{4} + \frac{n^e}{2} - p\right) p \right. \tag{2}$$

Adapting the FOC from the previous answer, we get $p^m = 1/8$ (as the SOC is still verified) and demand $n = 1/4 + n^e/2 - \underbrace{(1/8 + n^e/4)}_{p^m} = 1/8 + n^e/4$. Since we suppose that n^e (the expected demand) is equal to n^* (the realised demand), we get

$$
\begin{aligned}
n^e &= \frac{1}{8} + \frac{n^e}{4} \\
n^e &= \frac{1}{6}
\end{aligned}
$$

Therefore, given that the consumers expect that 1/6 of all consumers will participate, the monopolist program becomes

$$
\mathscr{P}_{Monopolist} \left| \quad \underset{\{p\}}{\text{Max}} \left(\frac{1}{3} - p\right) p \right. \tag{3}
$$

The SOC is once again verified and the FOC gives us an expected price of 1/6, hence, all consumers with an opportunity cost that is less that 1/6 expect to buy and the monopoly profit is 1/36.

3) Consumers first observe the price p set by the monopolist. Given this price p, there is a consumer with an opportunity cost x that will be indifferent between buying and not buying only if $x = 1/4 + n^e/2 - p$. Demand is $n = x$. As the consumers' beliefs are confirmed in the monopoly solution (self-fulfilling), $n = n^e$ and $n = 1/4 + n/2 - p \Leftrightarrow n = 1/2 - 2p$. Therefore, the new monopoly's program is

$$
\mathscr{P}_{Monopolist} \left| \quad \underset{\{p\}}{\text{Max}} \left(\frac{1}{4} + \frac{\frac{1}{2} - 2p}{2} - p\right) p \right. \tag{4}
$$

The FOC is $\dfrac{\partial \pi}{\partial p} = 1/2 - 4p$ and the SOC $\dfrac{\partial^2 \pi}{\partial p^2} = -4 < 0$ is verified. Equalising the FOC to zero gives us $p^m = 1/8$. Replacing the monopoly price in the demand and in the profit functions we get, respectively, $n = n^e = 1/4$ and $\pi^m = 1/32$.

4) In part 3, the monopolist can commit to a price. By setting a low price, it can convince more consumers that participation is worthwhile. The price reduction increases demand not only through the increase in demand for given beliefs, but also through the positive direct network effect embedded in the change of beliefs.

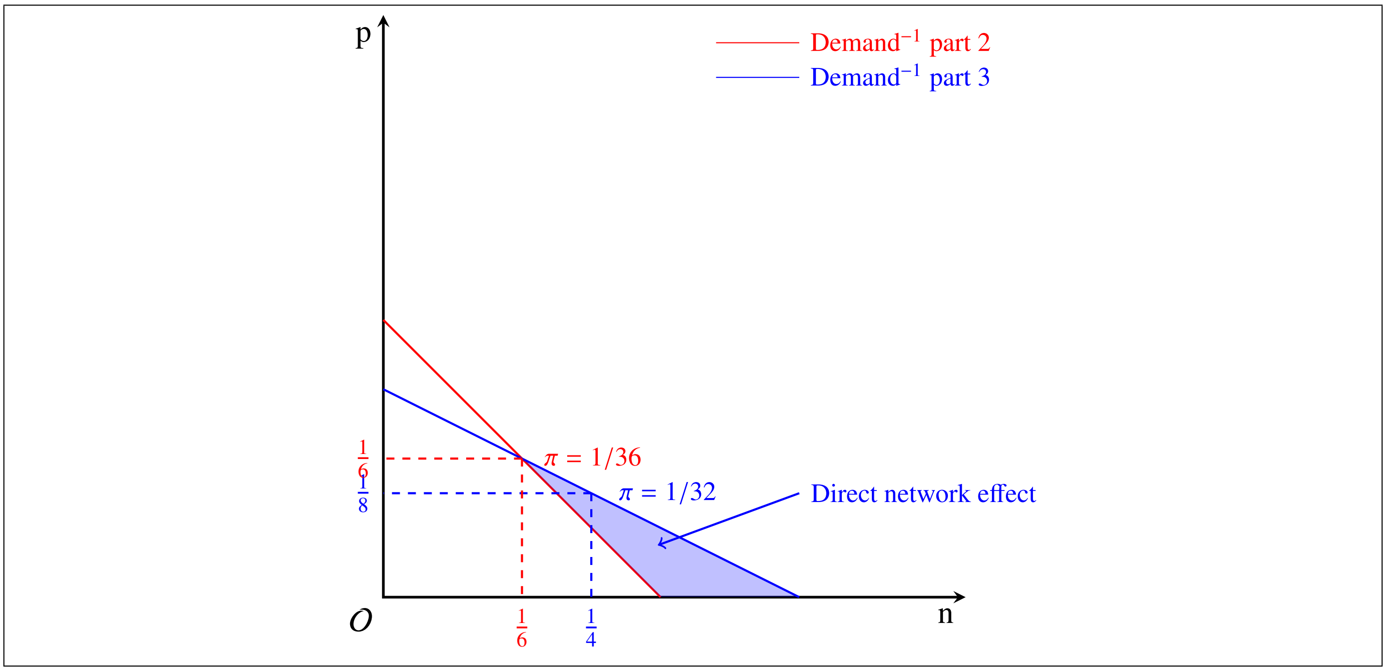

Figure 3 : Demand part 2 vs. Demand part 3

5) As both groups of consumers first observe prices and then form beliefs, the monopolist sets two prices, p_1 and p_2, in order to maximise its profits. Its program then becomes

$$\mathscr{P}_{Monopolist} \left| \; \underset{\{p_1,p_2\}}{\text{Max}} \left(\frac{1}{4} + \frac{n_1^e(p_1)}{2} - p_1\right) p_1 + \left(\frac{1}{4} + \frac{n_1^e(p_1)}{2} - p_2\right) p_2. \right. \tag{5}$$

- If we calculate the partial derivative from the function to maximise with respect to p_2, we get one FOC $\frac{\partial \pi}{\partial p_2} = 1/4 + n_1^e(p_1)/2 - 2p_2 = 0$ when $p_2 = 1/8 + n_1^e(p_1)/4$ (the SOC is verified as $\frac{\partial^2 \pi}{\partial p_2^2} = -2 < 0$).
- If we calculate the partial derivative from the function to maximise with respect to p_1, we get one FOC $\frac{\partial \pi}{\partial p_1} = 1/4 + n^e(p_1)' p_1/2 + n_1^e(p_1)/2 - 2p_1 + n_1^e(p_1)'(p_2/2) = 0$.

As consumers from both groups hold self-fulfilling beliefs, $n = x = n^e$, we have $n_1^e(p_1) = 1/2 - 2p$ and $n_1^e(p_1)' = -2$. Replacing these terms in the previous FOC, we get

$$\begin{aligned} \frac{1}{4} - \frac{2p_1}{2} + \frac{\frac{1}{2} - 2p_1}{2} - 2p_1 - \frac{2p_2}{2} &= 0 \\ \frac{1}{2} - 4p_1 - p_2 &= 0 \\ \text{SOC: } -4 &< 0. \end{aligned}$$

We can also rewrite p_2, found in the first FOC, $p_2 = 1/8 + (1/2 - 2p_1)/4 = 1/4 - p_1/2$ and use it in the second FOC. Hence,

$$\begin{aligned} \frac{1}{2} - 4p_1 - \frac{1}{4} + \frac{p_1}{2} &= 0 \\ \frac{7p_1}{2} &= \frac{1}{4} \\ p_1^m &= \frac{1}{14}. \end{aligned}$$

Consequently, $n^m = 1/2 - 2p_1^m = 5/14$ and consumers from the second group pay $p_2^m = 1/4 - p_1^m/2 = 3/14$. Finally the monopoly profit is

$$\pi^m = \left(\frac{1}{4} + \frac{\frac{5}{14}}{2} - \frac{1}{14}\right)\frac{1}{14} + \left(\frac{1}{4} + \frac{\frac{5}{14}}{2} - \frac{3}{14}\right)\frac{3}{14} = \frac{1}{14}$$

6) The monopolist is now forced to charge the same prices to both group. Taking the previous setting (question 5), the problem of the monopolist becomes

$$\mathscr{P}_{Monopolist} \left| \; \underset{\{p\}}{\text{Max}} \left(\frac{1}{4} + \frac{n_1^e(p)}{2} - p\right) p + \left(\frac{1}{4} + \frac{n_1^e(p)}{2} - p\right) p. \right. \tag{6}$$

The corresponding FOC is obviously very similar to the one question 5. We have

$$\frac{\partial \pi}{\partial p} = 2\left(\frac{1}{4} + \frac{n_1^e(p)' p}{2} + \frac{n_1^e(p)}{2} - 2p\right) = 0$$

As consumers from both groups, once again, hold self-fulfilling beliefs, $n = x = n^e$, we have $n_1^e(p_1) = 1/2 - 2p$ and $n_1^e(p_1)' = -2$, therefore we get

$$\frac{\partial \pi}{\partial p} = \frac{1}{2} - 2p + \frac{1}{2} - 2p - 4p = 0 \text{ when } p = \frac{1}{8}.$$

The SOC is easily verified ($\dfrac{\partial^2\pi}{\partial p^2} = -8 < 0$) and the monopolist solution to its program is $p^m = 1/8$, $n^m = 1/4$ and $\pi^m = 1/16$.

7) Let's take a fixed level of demand for group 2, how does the price for the second group vary in function of the price set for group 1 ?

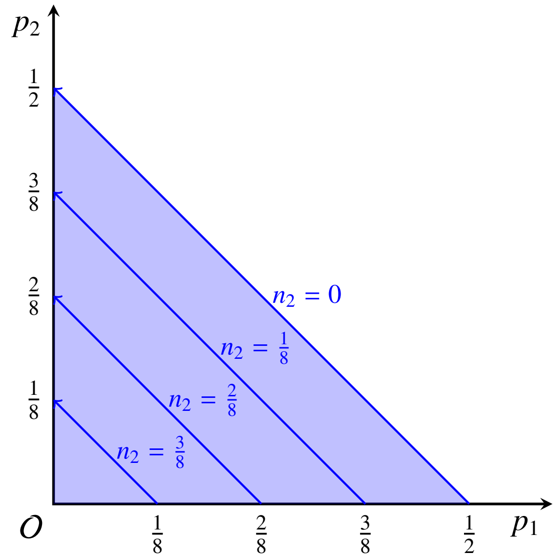

Figure 4 : Relationship between prices p_1 and p_2

When the monopolist is allowed to set different prices to the distinct consumer groups, he obtains higher profit. Because they can charge a low price to consumers of group 1, more consumers from group 1 will buy and these consumers exert a positive externality on consumers of group 2 allowing the monopolist to charge higher prices to more consumers of group 2 and maximise its profits.

Pythonisation: We could rewrite more complex and less symmetric demands for both groups and observe multiple equilibria graphically and analytically. The final result would be an algorythm that will automatically find the equilibria and compute some interactive graphs illustrating the solutions, it would take as inputs the chosen demands for both groups.