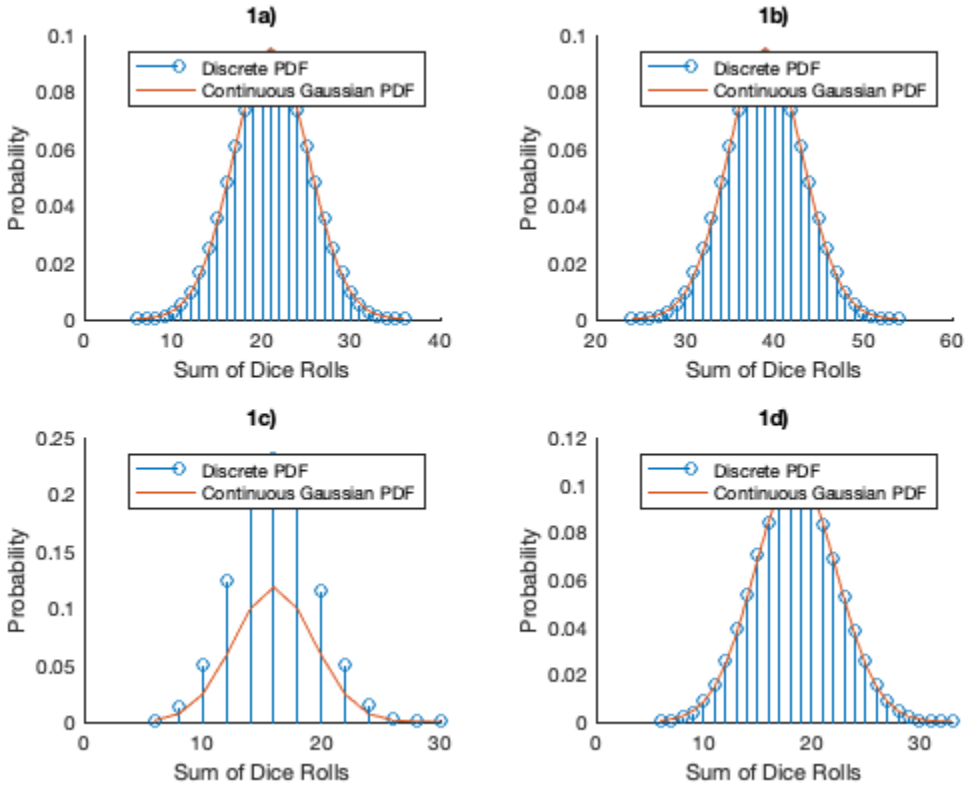
Table of Contents

QUESTION 1

```
clear; close all; clc;

N = 6;
a = (1:6)';
b = (4:9)';
c = [1 1 3 3 3 5]';
d1 = a;
d2 = c;
d = [d1 d2];

% a)
[pmf_a, shift_a, mu_a, sigma_a] = simDice(a, N);
% b)
[pmf_b, shift_b, mu_b, sigma_b] = simDice(b, N);
% c)
[pmf_c, shift_c, mu_c, sigma_c] = simDice(c, N);
% d)
[pmf_d, shift_d, mu_d, sigma_d] = sim2Dice(d1, d2, N/2);

figure();
hold('on');
title('Discrete PDFs vs. Continuous Gaussian PDF');
tiledlayout(2,2);
nexttile();
hold('on');
title('1a)');
stem(shift_a, pmf_a);
plot(shift_a, normpdf(shift_a, mu_a, sigma_a));
xlabel('Sum of Dice Rolls');
ylabel('Probability');
legend('Discrete PDF', 'Continuous Gaussian PDF');

nexttile();
hold('on');
title('1b)');
stem(shift_b, pmf_b);
plot(shift_b, normpdf(shift_b, mu_b, sigma_b));
xlabel('Sum of Dice Rolls');
ylabel('Probability');
```

```
legend('Discrete PDF', 'Continuous Gaussian PDF');

nexttile();
hold('on');
title('1c)');
stem(shift_c, pmf_c);
plot(shift_c, normpdf(shift_c, mu_c, sigma_c));
xlabel('Sum of Dice Rolls');
ylabel('Probability');
legend('Discrete PDF', 'Continuous Gaussian PDF');

nexttile();
hold('on');
title('1d)');
stem(shift_d, pmf_d);
plot(shift_d, normpdf(shift_d, mu_d, sigma_d));
xlabel('Sum of Dice Rolls');
ylabel('Probability');
legend('Discrete PDF', 'Continuous Gaussian PDF');

fprintf('1a) Mean & Standard Deviation: [%0.3g %0.3g]\n', mu_a, sigma_a)
fprintf('1a) Sum of the PDF: %0.3g\n', sum(pmf_a))
fprintf('1b) Mean & Standard Deviation: [%0.3g %0.3g]\n', mu_b, sigma_b)
fprintf('1b) Sum of the PDF: %0.3g\n', sum(pmf_b))
fprintf('1c) Mean & Standard Deviation: [%0.3g %0.3g]\n', mu_c, sigma_c)
fprintf('1c) Sum of the PDF: %0.3g\n', sum(pmf_c))
fprintf('1d) Mean & Standard Deviation: [%0.3g %0.3g]\n', mu_d, sigma_d)
fprintf('1d) Sum of the PDF: %0.3g\n\n', sum(pmf_d))
```



QUESTION 2

```
clear;

% a)
x1 = 1:6;
x2 = 1:6;
fx1 = simDice(x1', 1)';
fx2 = simDice(x2', 1)';
fx1x2 = fx1*fx2';
fprintf('2a) fx1x2:\n');
fprintf('\t %0.3g %0.3g %0.3g %0.3g %0.3g %0.3g\n', fx1x2);
Ex1 = x1*fx1x2*ones(1,length(x1))'; % E[x1]
fprintf('\n2a) E[x1]: %0.3g\n', Ex1);
Ex2 = x2*fx1x2*ones(1,length(x2))'; % E[x2]
fprintf('2a) E[x2]: %0.3g\n', Ex2);
Ex1_Ex1 = round((x1 - Ex1)*fx1);    % E[(x1-E[x1])]
fprintf('2a) E[(x1 - E[x1])]: %0.3g\n', Ex1_Ex1);
Ex12 = (x1.^2)*fx1;                 % E[x1^2]
fprintf('2a) E[x1^2]: %0.3g\n', Ex12);
Px1 = ((x1 - Ex1).^2)*fx1;          % E[(x1 - E[x1])^2]
fprintf('2a) E[(x1 - E[x1])^2]: %0.3g\n', Px1);
Px1x2 = ((x1 - Ex1)*(x2 - Ex2)')*fx1x2; % E[(x1 - E[x1])(x2 - E[x2])]
fprintf('2a) Px1x2:\n')
fprintf('\t %0.3g %0.3g %0.3g %0.3g %0.3g %0.3g\n', Px1x2');
```

```
% b)
Px1x2 = ((x1 - Ex1)*(x2 - Ex2)')*fx1x2;
fprintf('\n2b) Px1x2:\n')
fprintf('\t %0.3g %0.3g %0.3g %0.3g %0.3g %0.3g\n', Px1x2');

% c)
v1 = x1;
fv1 = fx1;
[fv2, v2] = simDice(x1', 2);
fv2 = fv2';
fv1v2 = fv1*fv2';
fprintf('\n2c) fv1v2:\n');
fprintf(['\t %0.3g %0.3g %0.3g %0.3g %0.3g %0.3g %0.3g %0.3g ' ...
    '%0.3g %0.3g %0.3g\n'], fv1v2');

% d)
Ev1 = v1*fv1;
fprintf('\n2d) E[v1]: %0.3g\n', Ev1);
v1RMS = sqrt((v1.^2)*fv1);
fprintf('2d) RMS(v1): %0.3g\n', v1RMS);
Pv1 = ((v1 - Ev1).^2) * fv1;
fprintf('2d) E[(v1 - E[v1])^2): %0.3g\n', Pv1);

% e)
Ev2 = v2*fv2;
fprintf('2e) E[v2]: %0.3g\n', Ev2);
v2RMS = sqrt((v2.^2)*fv2);
fprintf('2e) RMS(v2): %0.3g\n', v2RMS);
Pv2 = ((v2 - Ev2).^2) * fv2;
fprintf('2e) E[(v2 - E[v2])^2): %0.3g\n', Pv2);

% f)
Pv1v2 = ((v1 - Ev1)*(v2(1:6) - Ev2)')*fv1*fv2';
fprintf('2f) Pv1v2:\n');
fprintf(['\t %0.3g %0.3g %0.3g %0.3g %0.3g %0.3g %0.3g %0.3g ' ...
    '%0.3g %0.3g %0.3g\n'], Pv1v2');
```

```
2a) fx1x2:
     0.0278 0.0278 0.0278 0.0278 0.0278 0.0278
     0.0278 0.0278 0.0278 0.0278 0.0278 0.0278
     0.0278 0.0278 0.0278 0.0278 0.0278 0.0278
     0.0278 0.0278 0.0278 0.0278 0.0278 0.0278
     0.0278 0.0278 0.0278 0.0278 0.0278 0.0278
     0.0278 0.0278 0.0278 0.0278 0.0278 0.0278

2a) E[x1]: 3.5
2a) E[x2]: 3.5
2a) E[(x1 - E[x1])]: 0
2a) E[x1^2]: 15.2
2a) E[(x1 - E[x1])^2]: 2.92
2a) Px1x2:
     0.486 0.486 0.486 0.486 0.486 0.486
     0.486 0.486 0.486 0.486 0.486 0.486
```

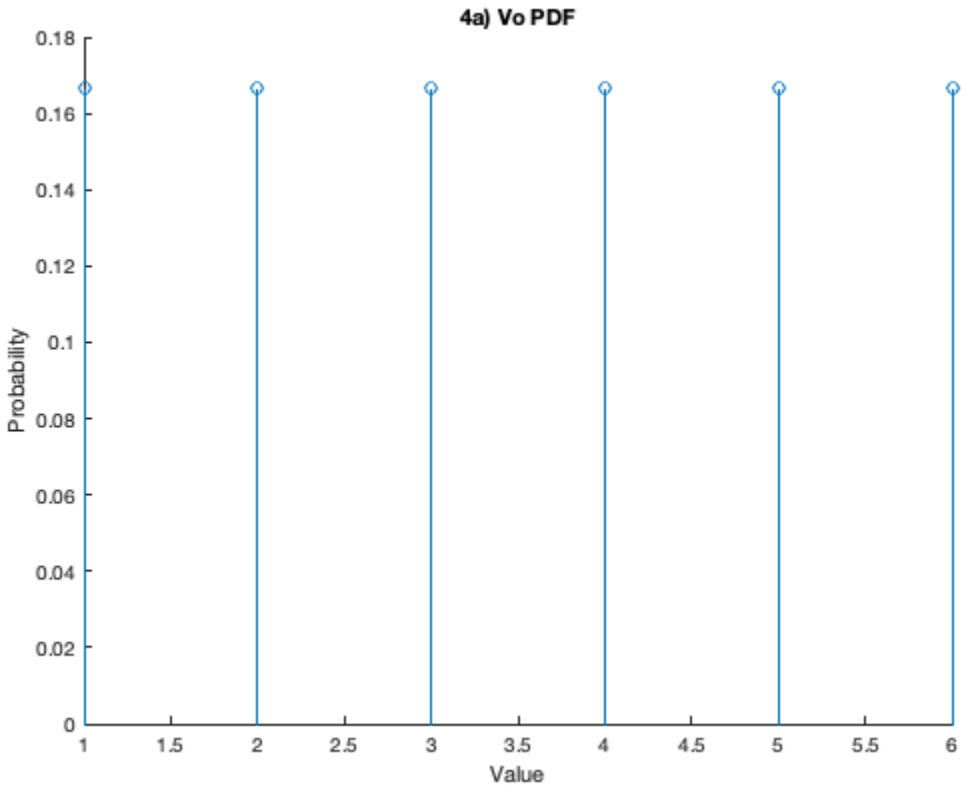
```
     0.486 0.486 0.486 0.486 0.486 0.486
     0.486 0.486 0.486 0.486 0.486 0.486
     0.486 0.486 0.486 0.486 0.486 0.486
     0.486 0.486 0.486 0.486 0.486 0.486

2b) Px1x2:
     0.486 0.486 0.486 0.486 0.486 0.486
     0.486 0.486 0.486 0.486 0.486 0.486
     0.486 0.486 0.486 0.486 0.486 0.486
     0.486 0.486 0.486 0.486 0.486 0.486
     0.486 0.486 0.486 0.486 0.486 0.486
     0.486 0.486 0.486 0.486 0.486 0.486

2c) fv1v2:
     0.00463 0.00926 0.0139 0.0185 0.0231 0.0278 0.0231 0.0185 0.0139 0.00926
0.00463
     0.00463 0.00926 0.0139 0.0185 0.0231 0.0278 0.0231 0.0185 0.0139 0.00926
0.00463
     0.00463 0.00926 0.0139 0.0185 0.0231 0.0278 0.0231 0.0185 0.0139 0.00926
0.00463
     0.00463 0.00926 0.0139 0.0185 0.0231 0.0278 0.0231 0.0185 0.0139 0.00926
0.00463
     0.00463 0.00926 0.0139 0.0185 0.0231 0.0278 0.0231 0.0185 0.0139 0.00926
0.00463
     0.00463 0.00926 0.0139 0.0185 0.0231 0.0278 0.0231 0.0185 0.0139 0.00926
0.00463

2d) E[v1]: 3.5
2d) RMS(v1): 3.89
2d) E[(v1 - E[v1])^2): 2.92
2e) E[v2]: 7
2e) RMS(v2): 7.4
2e) E[(v2 - E[v2])^2): 5.83
2f) Pv1v2:
     0.081 0.162 0.243 0.324 0.405 0.486 0.405 0.324 0.243 0.162 0.081
     0.081 0.162 0.243 0.324 0.405 0.486 0.405 0.324 0.243 0.162 0.081
     0.081 0.162 0.243 0.324 0.405 0.486 0.405 0.324 0.243 0.162 0.081
     0.081 0.162 0.243 0.324 0.405 0.486 0.405 0.324 0.243 0.162 0.081
     0.081 0.162 0.243 0.324 0.405 0.486 0.405 0.324 0.243 0.162 0.081
     0.081 0.162 0.243 0.324 0.405 0.486 0.405 0.324 0.243 0.162 0.081
```

QUESTION 4

```
clear;

Vo = [-2.5 -1.5 -0.5 0.5 1.5 2.5];
fx = groupcounts(Vo')./length(Vo);

figure();
hold('on');
title('4a) Vo PDF')
stem(fx);
xlabel('Value');
```

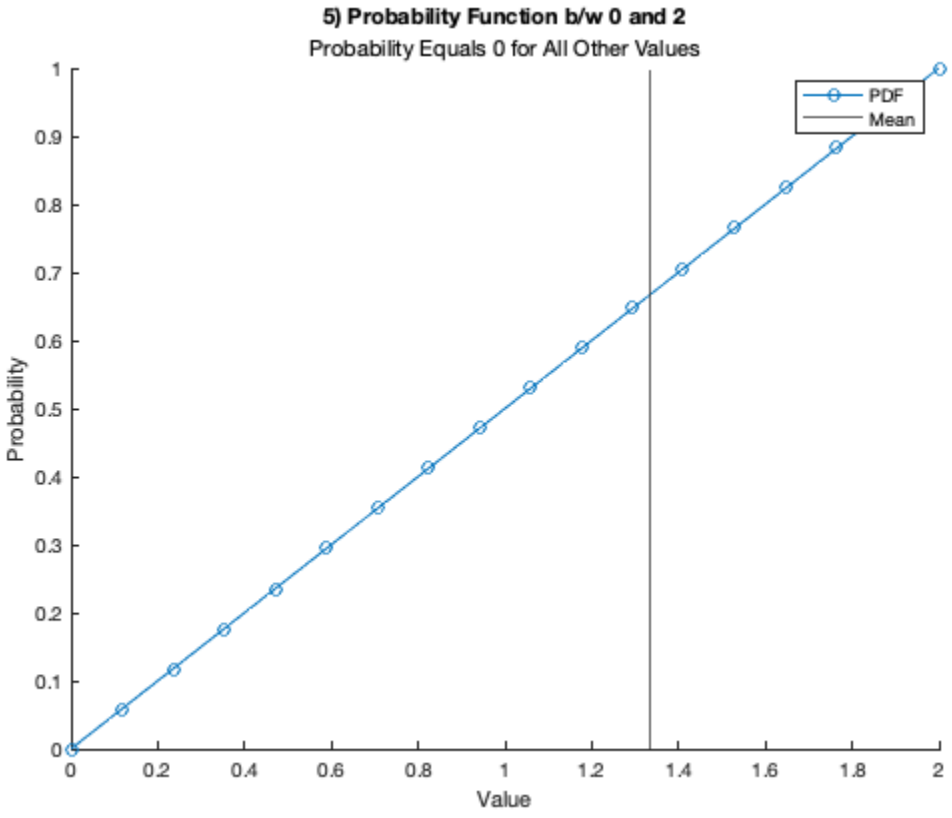
```
ylabel('Probability');

muVo = mean(Vo);
fprintf('\n4b) The mean of Vo: %0.3g\n', muVo);
sigmaVo = std(Vo);
fprintf('4b) The standard deviation of Vo: %0.3g\n', sigmaVo);
varVo = var(Vo);
fprintf('4b) The variance of Vo: %0.3g\n\n', varVo);
```

```
4b) The mean of Vo: 0
4b) The standard deviation of Vo: 1.87
4b) The variance of Vo: 3.5
```



QUESTION 5

```
figure();
hold('on');
title('5) Probability Function b/w 0 and 2');
subtitle('Probability Equals 0 for All Other Values');
fplot(@(x) x/2, [0 2], '-o');
xline(4/3);
xlabel('Value');
ylabel('Probability');
legend('PDF', 'Mean');
```

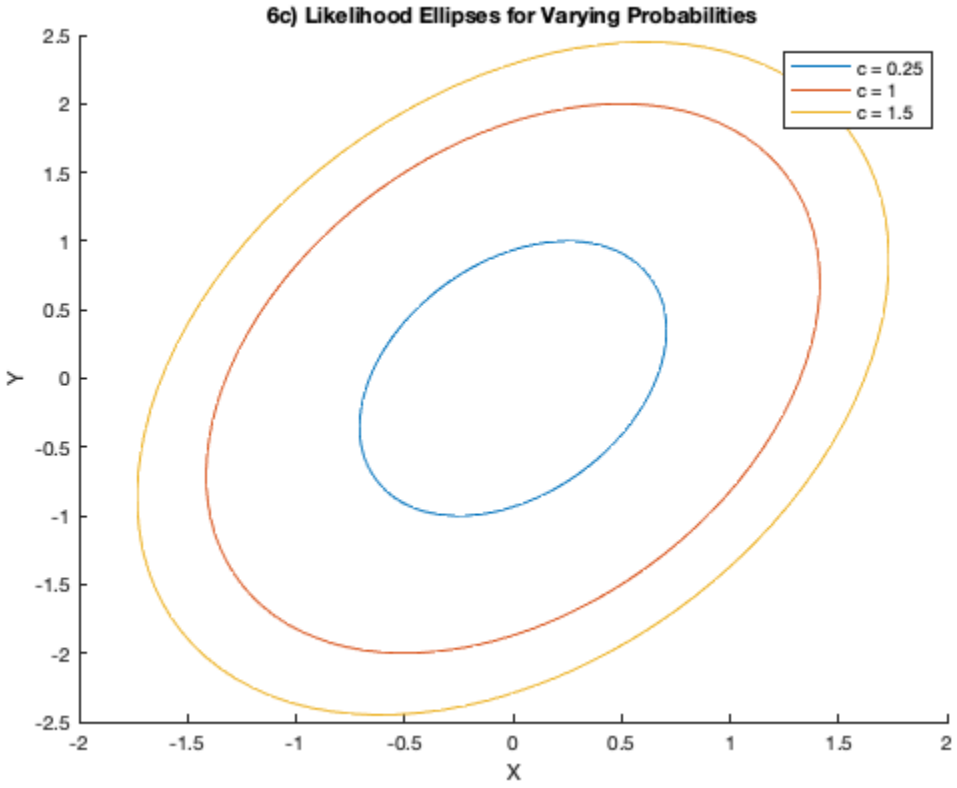


QUESTION 6

```
clear;

Px = [2 1;
      1 4];
[V, D] = eigs(Px);
fprintf('6a) Eigenvalues of Px: [%0.3g %0.3g]\n', diag(D));

c = [0.25, 1, 1.5];
t = linspace(0, 2 * pi);
figure();
hold("on");
title("6c) Likelihood Ellipses for Varying Probabilities")
for k = 1:length(c)
    a = (V * sqrt(c(k)*D)) * [cos(t); sin(t)];
    plot(a(1, :), a(2, :));
end
xlabel("X");
ylabel("Y");
legend('c = 0.25', 'c = 1', 'c = 1.5');

fx = @(c) ((2*pi)^(size(Px,1)/2) * det(Px)^(1/2))^(-1) .* exp(-1/2.*(c.^2));
probs = fx(c);
fprintf('6d) The probability for c = 0.25: %0.3g\n', probs(1));
```

```
fprintf('6d) The probability for c = 1: %0.3g\n', probs(2));
fprintf('6d) The probability for c = 1.5: %0.3g\n\n', probs(3));
```

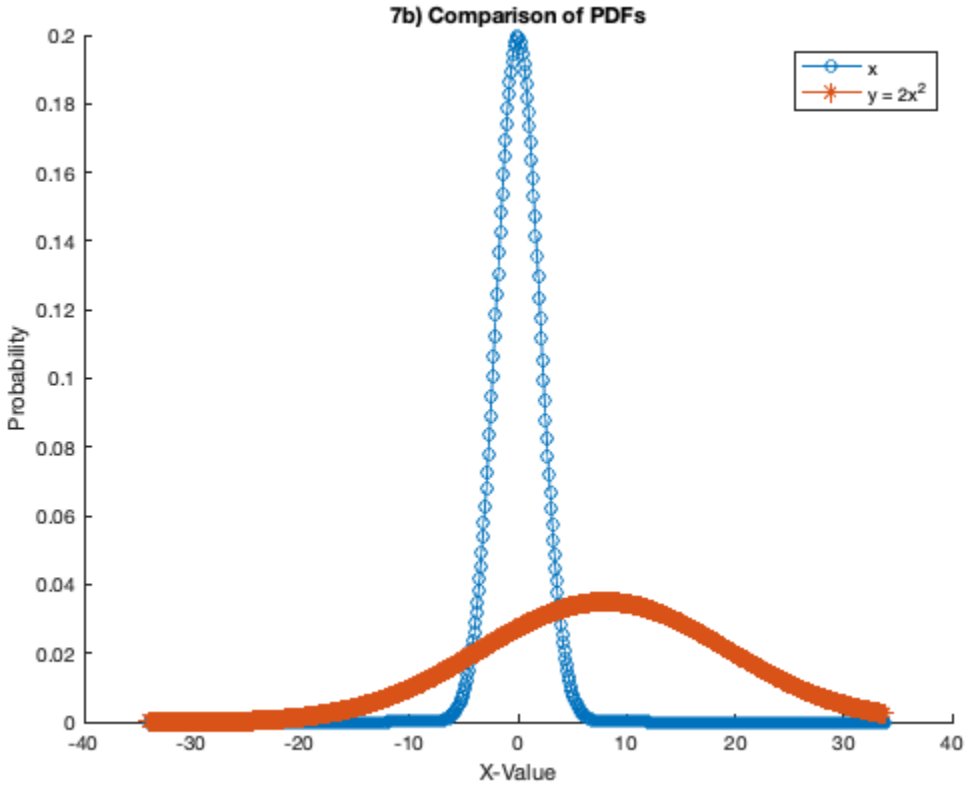
```
6a) Eigenvalues of Px: [4.41 1.59]
6d) The probability for c = 0.25: 0.0583
6d) The probability for c = 1: 0.0365
6d) The probability for c = 1.5: 0.0195
```



QUESTION 7

```
clear;
sigmax = 2.0;
varx = sigmax^2;
muy = 2*varx;
vary = 4*3*varx^2 - muy^2;
sigmay = sqrt(vary);

figure();
hold("on");
title('7b) Comparison of PDFs');
plot(-3*sigmay:0.1:3*sigmay, normpdf(-3*sigmay:0.1:3*sigmay, 0, sigmax), '-
o');
plot(-3*sigmay:0.1:3*sigmay, normpdf(-3*sigmay:0.1:3*sigmay, muy, sigmay), '-
*');
xlabel('X-Value');
```

```
ylabel('Probability');
legend('x', 'y = 2x^2');
```



FUNCTIONS

```
function [pmf, shift, mu, sigma] = simDice(die, N)
    pmf = genPMF(die, N);
    shift = linspace(min(die*N), max(die*N), length(pmf));
    mu = sum(shift.*pmf);
    sigma = sqrt(sum(((shift - mu).^2).*pmf));
end

function [pmf, shift, mu, sigma] = sim2Dice(d1, d2, N)
    pmf = genPMF(d1, N);
    [~, fx] = genPMF(d2, N);
    for i = 1:N
        pmf = conv(pmf, fx);
    end
    pmf(pmf == 0) = [];
    maxs = d1*N + d2*N;
    shift = linspace(min(d1*N + d2*N), max(d1*N + d2*N), length(pmf));
    mu = sum(shift.*pmf);
    sigma = sqrt(sum(((shift - mu).^2).*pmf));
end

function [pmf, fx] = genPMF(die, N)
```

```
    [probs, vals] = groupcounts(die);
    fx = zeros(length(die),1);
    fx(vals) = probs./length(die);
    pmf = fx';
    for i = 1:N-1
        pmf = conv(pmf, fx');
    end
    pmf(pmf == 0) = [];
end
```

```
1a) Mean & Standard Deviation: [21 4.18]
1a) Sum of the PDF: 1
1b) Mean & Standard Deviation: [39 4.18]
1b) Sum of the PDF: 1
1c) Mean & Standard Deviation: [16 3.37]
1c) Sum of the PDF: 1
1d) Mean & Standard Deviation: [18.5 3.8]
1d) Sum of the PDF: 1
```

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