2/20/24 | HW 1 | Walter Livingston

1) a) D_a: 1 2 3 4 5 6

$f_{D_a}(d_a) = [1/6 \;\; 1/6 \;\; 1/6 \;\; 1/6 \;\; 1/6 \;\; 1/6]$ for 1 roll $\quad 6 \cdot d_a = d_{a6}$ Σ of 6 rolls

$f_{D_{a6}} = \mathrm{conv}(f_{D_a}, f_{D_a})$ Repeat 5 times $\Rightarrow$ 6 convolutions total

$E[D_{a6}] = \sum_{i=1}^{N} d_{a6_i} f_{D_{a6}}(d_{a_i}) = \boxed{21 = \mu_{D_{a6}}}$

$E[(D_{a6} - \mu_{D_{a6}})^2] = \sum_{i=1}^{N} (d_{a6_i} - \mu_{D_{a6}})^2 f_{D_{a6}}(d_{a6_i}) = \boxed{17.4724 = \sigma_{D_{a6}}^2} \quad \boxed{\sigma_{D_{a6}} = 4.18}$

b) d_b: 4 5 6 7 8 9

$f_{D_b}(d_b) = [1/6 \;\; 1/6 \;\; 1/6 \;\; 1/6 \;\; 1/6 \;\; 1/6] \quad 6 \cdot d_b = d_{b6}$

$E[D_{b6}] = \boxed{39 = \mu_{D_{b6}}} \quad E[(D_{b6} - \mu_{D_{b6}})^2] = \boxed{17.4724 = \sigma_{D_{b6}}^2} \quad \boxed{\sigma_{D_{b6}} = 4.18}$

c) d_c: 1 1 3 3 3 5

$f_{D_c}(d_c) = [1/3 \;\; 1/3 \;\; 1/2 \;\; 1/2 \;\; 1/2 \;\; 1/6] \quad 6 \cdot d_c = d_{c6}$

$E[D_{c6}] = \boxed{16 = \mu_{D_{c6}}} \quad E[(D_{c6} - \mu_{D_{c6}})^2 = \boxed{11.3569 = \sigma_{D_{c6}}^2} \quad \boxed{\sigma_{D_{c6}} = 3.37}$

d) d_1: 1 2 3 4 5 6 $\quad d_2$: 1 1 3 3 3 5

$f_{D_1} = [1/6 \;\; 1/6 \;\; 1/6 \;\; 1/6 \;\; 1/6 \;\; 1/6] \quad f_{D_2} = [1/3 \;\; 1/3 \;\; 1/2 \;\; 1/2 \;\; 1/2 \;\; 1/6]$

$3 \cdot d_1 + 3 \cdot d_2 = d_d \quad E[D_d] = \boxed{18.5 = \mu_{D_d}} \quad E[(D_d - \mu_{D_d})^2] = \boxed{14.440 = \sigma_{D_d}^2}$

$\boxed{\sigma_{D_d} = 3.8}$

2) x_1: 1 2 3 4 5 6

x_2: 1 2 3 4 5 6

a)
$$f_{x_1 x_2} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ \\ \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1/36 & 1/36 & 1/36 & 1/36 & 1/36 & 1/36 \\ 1/36 & 1/36 & 1/36 & 1/36 & 1/36 & 1/36 \\ 1/36 & 1/36 & 1/36 & 1/36 & 1/36 & 1/36 \\ 1/36 & 1/36 & 1/36 & 1/36 & 1/36 & 1/36 \\ 1/36 & 1/36 & 1/36 & 1/36 & 1/36 & 1/36 \\ 1/36 & 1/36 & 1/36 & 1/36 & 1/36 & 1/36 \end{bmatrix} \\ \begin{array}{cccccc} 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \end{array} \end{array} \begin{array}{c} \\ 1/6 \\ 1/6 \\ 1/6 \\ 1/6 \\ 1/6 \\ 1/6 \\ \\ \end{array}$$

$$E[X_1] = [1\ 2\ 3\ 4\ 5\ 6]\overbrace{f_{x_1 x_2}[1\ 1\ 1\ 1\ 1\ 1]^T}^{f_{x_1}} = \boxed{3.5 = \mu_{x_1}}$$

$$E[X_1 - E[X_1]] = (X_1 - \mu_{x_1}) f_{x_1 x_2}[1\ 1\ 1\ 1\ 1\ 1] = \boxed{0}$$

$$E[X_1^2] = X_1 f_{x_1 x_2}[1\ 1\ 1\ 1\ 1\ 1] = \boxed{15.1667}$$

$$E[(X_1 - E[X_1])^2] = (X_1 - \mu_{x_1})^2 f_{x_1 x_2}[1\ 1\ 1\ 1\ 1\ 1] = \boxed{2.9167}$$

$$E[(X_1 - E[X_1])(X_2 - E[X_2])^T] = (X_1 - \mu_{x_1})(X_2 - \mu_{x_2})^T f_{x_1 x_2} = \text{In MATLAB}$$

b) $P_{x_1 x_2} = E[(X_1 - E[X_1])(X_2 - E[X_2])^T] = (X_1 - \mu_{x_1})(X_2 - \mu_{x_2})^T f_{x_1 x_2} = \text{In MATLAB}$

c) $v_1 = x_1 \quad f_{v_1} = f_{x_1} \qquad v_2 = x_1 + x_2 \qquad f_{v_2} = \text{conv}(x_1, x_2)$

$$f_{v_1 v_2} = f_{x_1} f_{x_2}' = \text{In MATLAB}$$

d) $E[v_1] = E[x_1] = \boxed{3.5} \qquad E[(v_1 - E[v_1])^2] = E[v_1^2] - \mu_{v_1}^2 = \boxed{15.1667}$

$$\sqrt{E[v_1^2]} = \sqrt{\sum_{i=1}^{n} v_{1i} f_{v_1}(v_{1i})} = \boxed{3.8944}$$

e) $E[v_2] = E[x_1 + x_2] = E[x_1] + E[x_2] = \boxed{7}$

$$E[(v_2 - E[v_2])^2] = (v_2 - E[v_2])^2 f_{v_2} = \boxed{5.83}$$

$$\sqrt{E[v_2^2]} = \sqrt{\sum_{i=1}^{n} v_{2i} f_{v_2}(v_{2i})} = \boxed{7.4}$$

f) $P_{v_1 v_2} = E[(v_1 - E[v_1])(v_2 - E[v_2])] = \text{In MATLAB}$

3) Independent: $E[g(x_1)h(x_2)] = E[g(x_1)]E[h(x_2)] \quad \forall\, g(\cdot), h(\cdot)$

a) $g(x_1) = x_1 \quad h(x_2) = x_2 \Rightarrow E[g(x_1)h(x_2)] = E[x_1 x_2] = E[x_1]E[x_2]$

$$E[(x_1 - \bar{x}_1)(x_2 - \bar{x}_2)] = E[x_1x_2 - x_1\bar{x}_2 - \bar{x}_1x_2 + \bar{x}_1\bar{x}_2] = E[x_1x_2] - \bar{x}_2 \underbrace{E[x_1]}_{\bar{x}_1} - \bar{x}_1 \underbrace{E[x_2]}_{\bar{x}_2} + \bar{x}_1\bar{x}_2$$

$$= \underbrace{E[x_1x_2]}_{\bar{x}_1\bar{x}_2} - \bar{x}_1\bar{x}_2$$ due to independence $E[x_1x_2] = E[x_1]E[x_2]$

$$E[(x_1 - \bar{x}_1)(x_2 - \bar{x}_2)] = 0$$ Uncorrelated

b) Uncorrelated: $E[(x_1 - \bar{x}_1)(x_2 - \bar{x}_2)] = 0$

$$f_{x_1x_2}(x_1, x_2) = \left((2\pi)^{\frac{2}{2}}|P|^{\frac{1}{2}}\right)^{-1}\exp\left(\frac{-1}{2}x^TP^{-1}x\right) = \left(2\pi\sigma_1\sigma_2(1-\cancelto{0}{\rho_{12}^2})^{\frac{1}{2}}\right)^{-1}\exp\left(\frac{-1}{2}\left(\frac{x_1^2}{\sigma_1^2} - \cancelto{0}{2\rho_{12}\frac{x_1x_2}{\sigma_1\sigma_2}} - \frac{x_2^2}{\sigma_2^2}\right)\right)$$

$$\rho_{12} = 0 \Rightarrow (2\pi\sigma_1\sigma_2)^{-1}\exp\left(\frac{-1}{2}\left(\frac{x_1^2}{\sigma_1^2} + \frac{x_2^2}{\sigma_2^2}\right)\right) = (2\pi\sigma_1)\exp\left(\frac{-1}{2}\frac{x_1^2}{\sigma_1^2}\right)(2\pi\sigma_2)\exp\left(\frac{-1}{2}\frac{x_2^2}{\sigma_2^2}\right)$$

$$= f_{x_1} \cdot f_{x_2}$$ $\Rightarrow$ Independence

4) b) $V_o(k) = \{-2.5, -1.5, -0.5, 0.5, 1.5, 2.5\}$

$$E[V_o(k)] = \frac{1}{N}\sum_{k=1}^{N} V_o(k) = 0 = \mu_o \qquad E[(V_o(k) - \bar{V}_o)^2] = \frac{1}{N-1}\sum_{k=1}^{N}(V_o(k) - \bar{V}_o)^2 = 3.5 = \sigma_o^2$$

$\sigma_o = 1.87$

c) $V_N(k+1) = (1-R)V_N(k) + RV_o(k)$

$$E[V_N(k+1)] = E[(1-R)V_N(k) + RV_o(k)] = (1-R)E[V_N(k)] + R\mu_o$$

$$\mu_N(k+1) = (1-R)\mu_N(k) + R\mu_o \qquad @\ ss \qquad \mu_N(k+1) = \mu_N(k)$$

$$\mu_N(k) = (1-R)\mu_N(k) + R\mu_o \qquad \mu_N(k)(1-(1-R)) = R\mu_o \qquad \mu_N(k) = \frac{\cancel{R}\mu_o}{\cancel{R}}$$

$$\boxed{\mu_N(k) = \mu_o = 0}$$ @ ss

$$E[(V_N(k+1) - \mu_N(k+1))^2] = E[((1-R)V_N(k) + RV_o(k) - (1-R)\mu_N(k) - R\mu_o)^2]$$

$$E[(1-R)^2V_N(k)^2 + (1+R)\cancelto{0}{V_N(k)RV_o(k)} - (1-R)^2V_N(k)\mu_N(k) - (1-R)\cancelto{0}{RV_N(k)}\mu_o + (1-R)\cancelto{0}{V_N(k)RV_o(k)}$$

$$+ R^2V_o(k)^2 - (1-R)\cancelto{0}{R\mu_N(k)}V_o(k) - \cancelto{0}{R^2V_o(k)}\mu_o - \cancel{(1-R)^2V_N(k)\mu_N(k)} - (1-R)\cancelto{0}{RV_o(k)}\mu_N(k) + \cancel{(1-R)^2\mu_N(k)^2}$$

$$+ (1-R)\cancelto{0}{R\mu_N(k)}\mu_o - (1-R)\cancelto{0}{RV_N(k)}\mu_o - \cancelto{0}{R^2V_o(k)}\mu_o + (1-R)\cancelto{0}{R\mu_N(k)}\mu_o + \cancelto{0}{R^2\mu_o^2}]$$

$$\sigma_N(k+1)^2 = (1-R)^2(\sigma(k)^2 - \mu_N(k)^2) + R^2\sigma_o^2 \qquad \sigma_N(k)^2 = (1-R)^2\sigma_N(k)^2 - \cancelto{0}{(1-R)^2\mu_N(k)^2} - R^2\sigma^2$$

$$\sigma_N(k)^2(\cancel{1} - \cancel{1} + 2R - R^2) = R^2\sigma_o^2 \qquad \boxed{\sigma_N(k) = \frac{R\sigma_o^2}{2-R}}$$

d) $R(k) = E[V_N(k)V_N(k-L)] = E[((1-r)V_N(k-1) + rV_0(k-1))((1-r)V_N(k-1-L) + rV_0(k-1-L))]$

$E[(1-r)^2 V_N(k-1)V_N(k-1-L) + (1-r)rV_N(k-1)V_0(k-1-L) + (1-r)rV_0(k-1)V_N(k-1-L)$
$+ r^2 V_0(k-1)V_0(k-1-L)]$

$(1-r)^2 E[V_N(k-1)V_N(k-1-L)] + (1+r)rE[V_N(k-1)]\cancel{\mu_0}^{0} + (1-r)r\cancel{\mu_0}^{0}E[V_N(k-1-L)] + r^2\sigma_0^2$

$(1-r)^2 E[V_N(k-1)V_N(k-1-L)] + r^2\sigma^2$

$R(k) = E[V_N(k)V_N(k-L)] \Rightarrow \boxed{\sigma_N^2(k-L) = (1-r)^2\sigma_N(k-L) + r^2\sigma^2}$

e) $0 < r < 1$

5) $f_x(x) = \begin{cases} 0 & x < 0 \\ \frac{x}{2} & 0 < x \le 2 \\ 0 & x > 2 \end{cases}$

$E[x] = \int_0^2 x \frac{x}{2}\,dx = \frac{x^3}{6}\Big|_0^2 = \left(\frac{2^3}{6} - 0\right) = \frac{8}{6} = \boxed{\frac{4}{3}}$ (a)

$E[(x-\bar{x})^2] = E[x^2 - 2x\bar{x} + \bar{x}^2] = E[x^2] - \cancel{2\bar{x}E[x]}^{\bar{x}} + \cancel{\bar{x}^2} = E[x^2] - \bar{x}^2$

$E[x^2] = \int_0^2 x^2 \frac{x}{2}\,dx = \frac{x^4}{8}\Big|_0^2 = \frac{16}{8} - 0 = 2$

(b)

$E[(x-\bar{x})^2] = E[x^2] - \bar{x}^2 = 2 - \left(\frac{4}{3}\right)^2 = 2 - \frac{8}{9} = \boxed{1.1111}$

6) $P_x = \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}$ $\qquad x \sim N(0, P_x)$

a) $\det(A - \lambda I) = 0 \rightarrow \lambda = 4.4142, 1.5858$

b) $x^T P_x^{-1} x = c^2 \qquad PCA = \frac{1}{\sqrt{c^2\lambda}} \frac{V}{\|V\|}$

c) IN MATLAB

d) $f_x(x) = \frac{1}{(2\pi)^{n/2}|P|^{1/2}} \exp\left(\frac{-1}{2} \underbrace{x^T P^{-1} x}_{c^2}\right) \qquad f_x(c) = \left((2\pi)^{\frac{n}{2}}|P_x|^{1/2}\right)^{-1} \exp\left(\frac{-1}{2}c^2\right)$

$\boxed{f(0.25) = 0.0503 \qquad f_x(1) = 0.0365 \qquad f_x(1.5) = 0.0195}$

7) $x \sim N(0, \sigma_x^2)$ $\quad y = 2x^2$

a) $\mu_y = E[y] = E[2x^2] = 2E[x^2] = \boxed{2\sigma_x^2}$

$\sigma_y = E[(y - E[y])^2] = E[y^2 - \overset{\mu_y}{\cancel{2y}}\mu_y + \cancel{\mu_y}^2] = E[y^2] - \mu_y^2 = E[(2x^2)^2] - \mu_y^2$

$\sigma_y = 4E[x^4] - \mu_y^2$

$M_x(s) = e^{\sigma s^2/2}$ $\quad M_x^{(4)} = E[x^4]$ $\quad M_x' = \sigma s M_x$ $\quad M_x'' = \sigma M_x + \sigma s M_x' = \sigma M_x + \sigma^2 s^2 M_x$

$M_x'' = M_x(\sigma + \sigma^3 s^2)$ $\quad M_x''' = 2\sigma^2 s M_x + (\sigma + \sigma^3 s^2) M_x' = 2\sigma^2 s M_x + (\sigma^3 s + \sigma^4 s^3) M_x$

$M_x''' = M_x(3\sigma^3 s + \sigma^4 s^3)$ $\quad M_x^{(4)} = M_x(3\sigma^3 + 3\sigma^4 s^2) + M_x'(3\sigma^3 s + \sigma^4 s^3) = M_x(3\sigma^4 + \underline{3\sigma^4 s^2} + \underline{3\sigma^4 s^2} + \sigma^5 s^4)$

$M_x^{(4)} = M_x(3\sigma^4 + 6\sigma^4 s^2 + \sigma^5 s^4)$ $\quad M^{(4)}(0) = 3\sigma^4$

$\boxed{\sigma_y = 3\sigma_x^4 - 2\sigma_x^2}$

c) The mean of y is equal to $2\sigma_x^2$. The standard deviation of y is greatly expanded.

d) Y is a Gaussian Random Variable.

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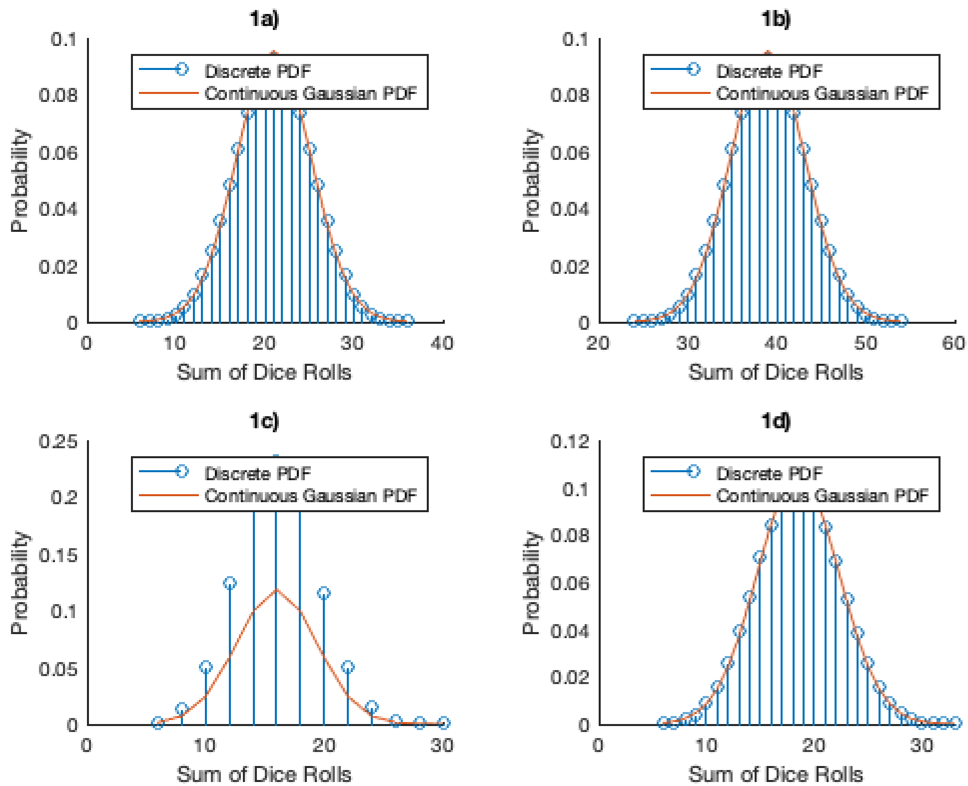
QUESTION 1

```
clear; close all; clc;

N = 6;
a = (1:6)';
b = (4:9)';
c = [1 1 3 3 3 5]';
d1 = a;
d2 = c;
d = [d1 d2];

% a)
[pmf_a, shift_a, mu_a, sigma_a] = simDice(a, N);
fprintf('1a) Mean & Standard Deviation: [%0.3g %0.3g]\n', mu_a, sigma_a);
fprintf('1a) Sum of the PDF: %0.3g\n', sum(pmf_a));
% b)
[pmf_b, shift_b, mu_b, sigma_b] = simDice(b, N);
fprintf('1b) Mean & Standard Deviation: [%0.3g %0.3g]\n', mu_b, sigma_b);
fprintf('1b) Sum of the PDF: %0.3g\n', sum(pmf_b));
% c)
[pmf_c, shift_c, mu_c, sigma_c] = simDice(c, N);
fprintf('1c) Mean & Standard Deviation: [%0.3g %0.3g]\n', mu_c, sigma_c);
fprintf('1c) Sum of the PDF: %0.3g\n', sum(pmf_c));
% d)
[pmf_d, shift_d, mu_d, sigma_d] = sim2Dice(d1, d2, N/2);
fprintf('1d) Mean & Standard Deviation: [%0.3g %0.3g]\n', mu_d, sigma_d);
fprintf('1d) Sum of the PDF: %0.3g\n\n', sum(pmf_d));

figure();
hold('on');
title('Discrete PDFs vs. Continuous Gaussian PDF');
tiledlayout(2,2);
nexttile();
hold('on');
title('1a)');
stem(shift_a, pmf_a);
plot(shift_a, normpdf(shift_a, mu_a, sigma_a));
xlabel('Sum of Dice Rolls');
ylabel('Probability');
legend('Discrete PDF', 'Continuous Gaussian PDF');
```

```
nexttile();
hold('on');
title('1b)');
stem(shift_b, pmf_b);
plot(shift_b, normpdf(shift_b, mu_b, sigma_b));
xlabel('Sum of Dice Rolls');
ylabel('Probability');
legend('Discrete PDF', 'Continuous Gaussian PDF');

nexttile();
hold('on');
title('1c)');
stem(shift_c, pmf_c);
plot(shift_c, normpdf(shift_c, mu_c, sigma_c));
xlabel('Sum of Dice Rolls');
ylabel('Probability');
legend('Discrete PDF', 'Continuous Gaussian PDF');

nexttile();
hold('on');
title('1d)');
stem(shift_d, pmf_d);
plot(shift_d, normpdf(shift_d, mu_d, sigma_d));
xlabel('Sum of Dice Rolls');
ylabel('Probability');
legend('Discrete PDF', 'Continuous Gaussian PDF');
```



QUESTION 2

```
clear;

% a)
x1 = 1:6;
x2 = 1:6;
fx1 = simDice(x1', 1)';
fx2 = simDice(x2', 1)';
fx1x2 = fx1*fx2';
fprintf('2a) fx1x2:\n');
fprintf('\t %0.3g %0.3g %0.3g %0.3g %0.3g %0.3g\n', fx1x2);
Ex1 = x1*fx1x2*ones(1,length(x1))'; % E[x1]
fprintf('\n2a) E[x1]: %0.3g\n', Ex1);
Ex2 = x2*fx1x2*ones(1,length(x2))'; % E[x2]
fprintf('2a) E[x2]: %0.3g\n', Ex2);
Ex1_Ex1 = round((x1 - Ex1)*fx1);    % E[(x1-E[x1])]
fprintf('2a) E[(x1 - E[x1])]: %0.3g\n', Ex1_Ex1);
Ex12 = (x1.^2)*fx1;                 % E[x1^2]
fprintf('2a) E[x1^2]: %0.3g\n', Ex12);
Px1 = ((x1 - Ex1).^2)*fx1;          % E[(x1 - E[x1])^2]
fprintf('2a) E[(x1 - E[x1])^2]: %0.3g\n', Px1);
Px1x2 = ((x1 - Ex1)*(x2 - Ex2)')*fx1x2; % E[(x1 - E[x1])(x2 - E[x2])]
fprintf('2a) Px1x2:\n')
fprintf('\t %0.3g %0.3g %0.3g %0.3g %0.3g %0.3g\n', Px1x2');
```

```
% b)
Px1x2 = ((x1 - Ex1)*(x2 - Ex2)')*fx1x2;
fprintf('\n2b) Px1x2:\n')
fprintf('\t %0.3g %0.3g %0.3g %0.3g %0.3g %0.3g\n', Px1x2');

% c)
v1 = x1;
fv1 = fx1;
[fv2, v2] = simDice(x1', 2);
fv2 = fv2';
fv1v2 = fv1*fv2';
fprintf('\n2c) fv1v2:\n');
fprintf(['\t %0.3g %0.3g %0.3g %0.3g %0.3g %0.3g %0.3g %0.3g ' ...
    '%0.3g %0.3g %0.3g\n'], fv1v2');

% d)
Ev1 = v1*fv1;
fprintf('\n2d) E[v1]: %0.3g\n', Ev1);
v1RMS = sqrt((v1.^2)*fv1);
fprintf('2d) RMS(v1): %0.3g\n', v1RMS);
Pv1 = ((v1 - Ev1).^2) * fv1;
fprintf('2d) E[(v1 - E[v1])^2): %0.3g\n', Pv1);

% e)
Ev2 = v2*fv2;
fprintf('2e) E[v2]: %0.3g\n', Ev2);
v2RMS = sqrt((v2.^2)*fv2);
fprintf('2e) RMS(v2): %0.3g\n', v2RMS);
Pv2 = ((v2 - Ev2).^2) * fv2;
fprintf('2e) E[(v2 - E[v2])^2): %0.3g\n', Pv2);

% f)
Pv1v2 = ((v1 - Ev1)*(v2(1:6) - Ev2)')*fv1*fv2';
fprintf('2f) Pv1v2:\n');
fprintf(['\t %0.3g %0.3g %0.3g %0.3g %0.3g %0.3g %0.3g %0.3g ' ...
    '%0.3g %0.3g %0.3g\n'], Pv1v2');
```

```
2a) fx1x2:
     0.0278 0.0278 0.0278 0.0278 0.0278 0.0278
     0.0278 0.0278 0.0278 0.0278 0.0278 0.0278
     0.0278 0.0278 0.0278 0.0278 0.0278 0.0278
     0.0278 0.0278 0.0278 0.0278 0.0278 0.0278
     0.0278 0.0278 0.0278 0.0278 0.0278 0.0278
     0.0278 0.0278 0.0278 0.0278 0.0278 0.0278

2a) E[x1]: 3.5
2a) E[x2]: 3.5
2a) E[(x1 - E[x1])]: 0
2a) E[x1^2]: 15.2
2a) E[(x1 - E[x1])^2]: 2.92
2a) Px1x2:
     0.486 0.486 0.486 0.486 0.486 0.486
     0.486 0.486 0.486 0.486 0.486 0.486
```

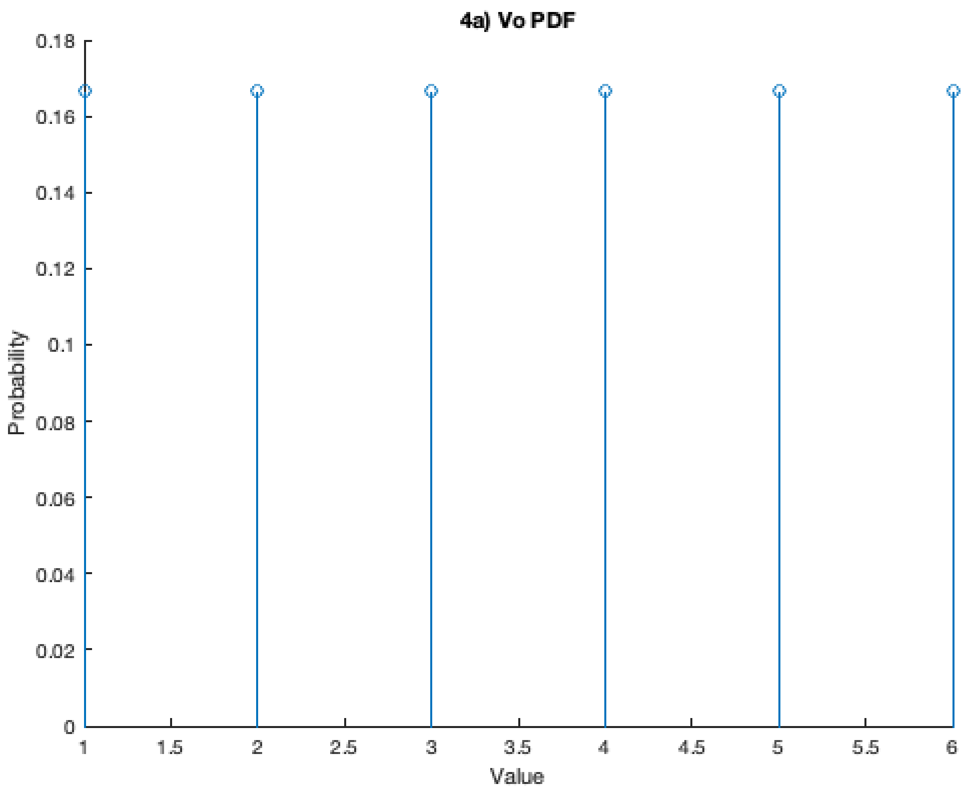
```
     0.486 0.486 0.486 0.486 0.486 0.486
     0.486 0.486 0.486 0.486 0.486 0.486
     0.486 0.486 0.486 0.486 0.486 0.486
     0.486 0.486 0.486 0.486 0.486 0.486

2b) Px1x2:
     0.486 0.486 0.486 0.486 0.486 0.486
     0.486 0.486 0.486 0.486 0.486 0.486
     0.486 0.486 0.486 0.486 0.486 0.486
     0.486 0.486 0.486 0.486 0.486 0.486
     0.486 0.486 0.486 0.486 0.486 0.486
     0.486 0.486 0.486 0.486 0.486 0.486

2c) fv1v2:
     0.00463 0.00926 0.0139 0.0185 0.0231 0.0278 0.0231 0.0185 0.0139 0.00926
0.00463
     0.00463 0.00926 0.0139 0.0185 0.0231 0.0278 0.0231 0.0185 0.0139 0.00926
0.00463
     0.00463 0.00926 0.0139 0.0185 0.0231 0.0278 0.0231 0.0185 0.0139 0.00926
0.00463
     0.00463 0.00926 0.0139 0.0185 0.0231 0.0278 0.0231 0.0185 0.0139 0.00926
0.00463
     0.00463 0.00926 0.0139 0.0185 0.0231 0.0278 0.0231 0.0185 0.0139 0.00926
0.00463
     0.00463 0.00926 0.0139 0.0185 0.0231 0.0278 0.0231 0.0185 0.0139 0.00926
0.00463

2d) E[v1]: 3.5
2d) RMS(v1): 3.89
2d) E[(v1 - E[v1])^2): 2.92
2e) E[v2]: 7
2e) RMS(v2): 7.4
2e) E[(v2 - E[v2])^2): 5.83
2f) Pv1v2:
     0.081 0.162 0.243 0.324 0.405 0.486 0.405 0.324 0.243 0.162 0.081
     0.081 0.162 0.243 0.324 0.405 0.486 0.405 0.324 0.243 0.162 0.081
     0.081 0.162 0.243 0.324 0.405 0.486 0.405 0.324 0.243 0.162 0.081
     0.081 0.162 0.243 0.324 0.405 0.486 0.405 0.324 0.243 0.162 0.081
     0.081 0.162 0.243 0.324 0.405 0.486 0.405 0.324 0.243 0.162 0.081
     0.081 0.162 0.243 0.324 0.405 0.486 0.405 0.324 0.243 0.162 0.081
```

QUESTION 4

```
clear;

Vo = [-2.5 -1.5 -0.5 0.5 1.5 2.5];
fx = groupcounts(Vo')./length(Vo);

figure();
hold('on');
title('4a) Vo PDF')
stem(fx);
xlabel('Value');
```

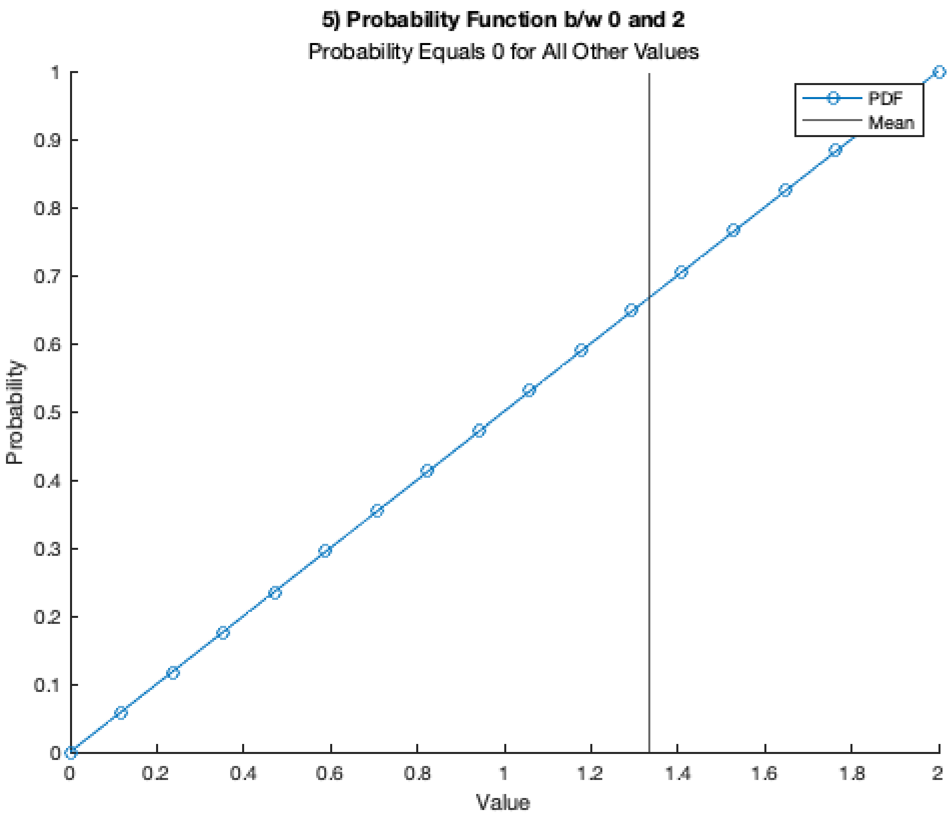
```
ylabel('Probability');

muVo = mean(Vo);
fprintf('\n4b) The mean of Vo: %0.3g\n', muVo);
sigmaVo = std(Vo);
fprintf('4b) The standard deviation of Vo: %0.3g\n', sigmaVo);
varVo = var(Vo);
fprintf('4b) The variance of Vo: %0.3g\n\n', varVo);
```

```
4b) The mean of Vo: 0
4b) The standard deviation of Vo: 1.87
4b) The variance of Vo: 3.5
```



QUESTION 5

```
figure();
hold('on');
title('5) Probability Function b/w 0 and 2');
subtitle('Probability Equals 0 for All Other Values');
fplot(@(x) x/2, [0 2], '-o');
xline(4/3);
xlabel('Value');
ylabel('Probability');
legend('PDF', 'Mean');
```

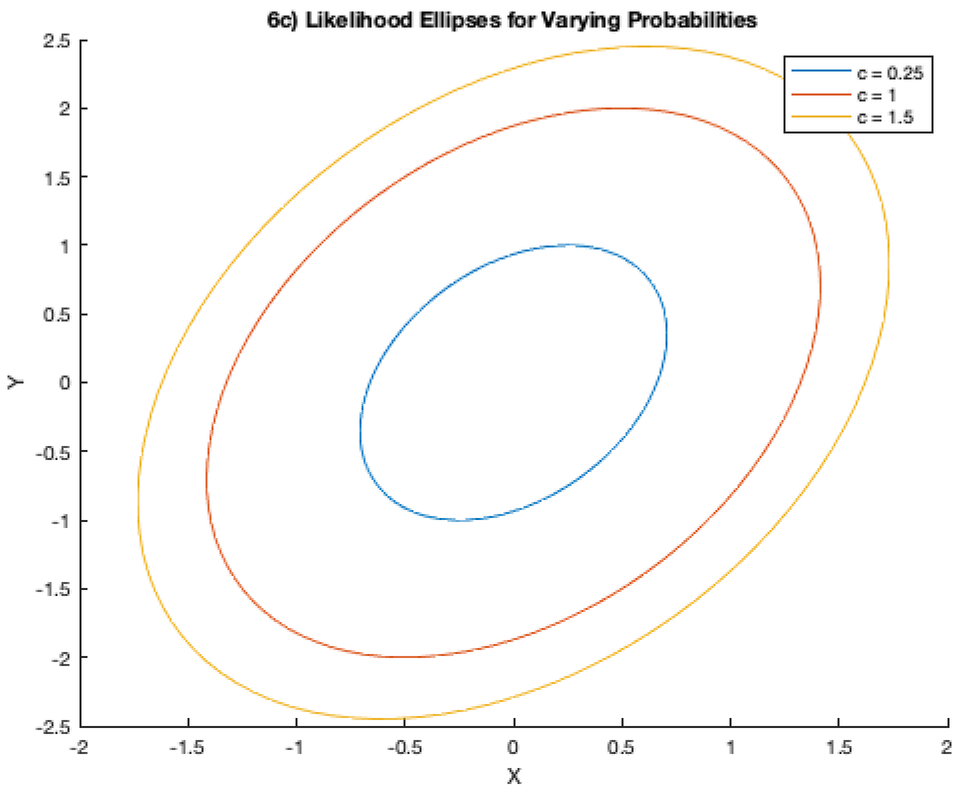


QUESTION 6

```
clear;

Px = [2 1;
      1 4];
[V, D] = eigs(Px);
fprintf('6a) Eigenvalues of Px: [%0.3g %0.3g]\n', diag(D));

c = [0.25, 1, 1.5];
t = linspace(0, 2 * pi);
figure();
hold("on");
title("6c) Likelihood Ellipses for Varying Probabilities")
for k = 1:length(c)
    a = (V * sqrt(c(k)*D)) * [cos(t); sin(t)];
    plot(a(1, :), a(2, :));
end
xlabel("X");
ylabel("Y");
legend('c = 0.25', 'c = 1', 'c = 1.5');

fx = @(c) ((2*pi)^(size(Px,1)/2) * det(Px)^(1/2))^(-1) .* exp(-1/2.*(c.^2));
probs = fx(c);
fprintf('6d) The probability for c = 0.25: %0.3g\n', probs(1));
```

```
fprintf('6d) The probability for c = 1: %0.3g\n', probs(2));
fprintf('6d) The probability for c = 1.5: %0.3g\n\n', probs(3));
```

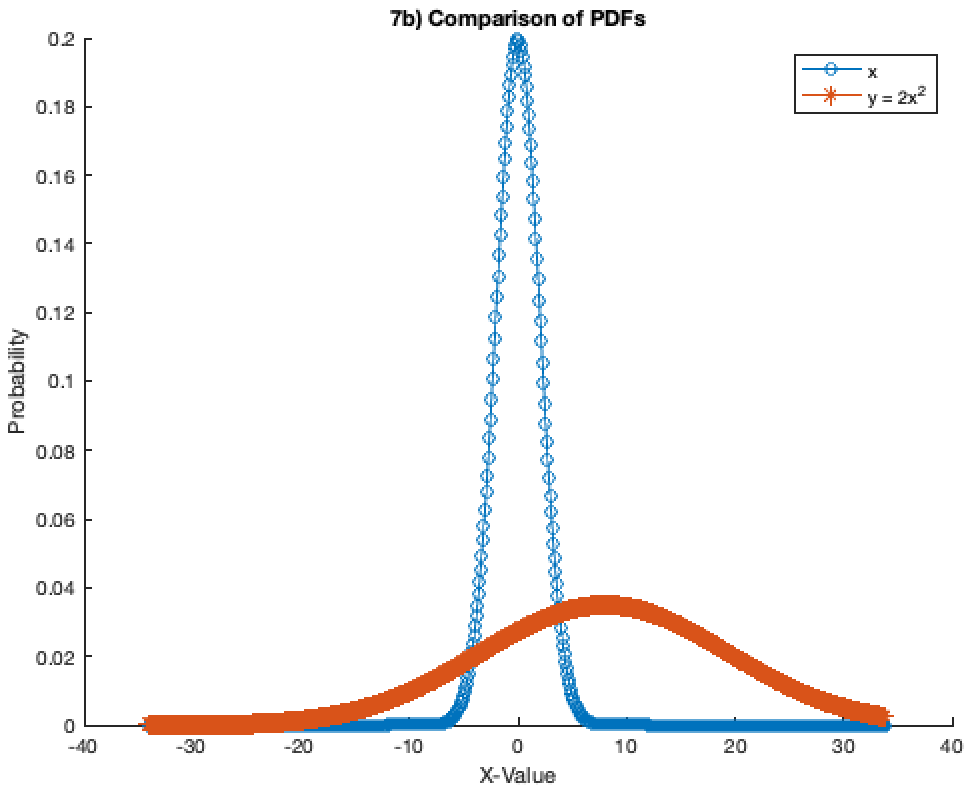
```
6a) Eigenvalues of Px: [4.41 1.59]
6d) The probability for c = 0.25: 0.0583
6d) The probability for c = 1: 0.0365
6d) The probability for c = 1.5: 0.0195
```



QUESTION 7

```
clear;
sigmax = 2.0;
varx = sigmax^2;
muy = 2*varx;
vary = 4*3*varx^2 - muy^2;
sigmay = sqrt(vary);

figure();
hold("on");
title('7b) Comparison of PDFs');
plot(-3*sigmay:0.1:3*sigmay, normpdf(-3*sigmay:0.1:3*sigmay, 0, sigmax), '-
o');
plot(-3*sigmay:0.1:3*sigmay, normpdf(-3*sigmay:0.1:3*sigmay, muy, sigmay), '-
*');
xlabel('X-Value');
```

```
ylabel('Probability');
legend('x', 'y = 2x^2');
```



FUNCTIONS

```
function [pmf, shift, mu, sigma] = simDice(die, N)
    pmf = genPMF(die, N);
    shift = linspace(min(die*N), max(die*N), length(pmf));
    mu = sum(shift.*pmf);
    sigma = sqrt(sum(((shift - mu).^2).*pmf));
end

function [pmf, shift, mu, sigma] = sim2Dice(d1, d2, N)
    pmf = genPMF(d1, N);
    [~, fx] = genPMF(d2, N);
    for i = 1:N
        pmf = conv(pmf, fx);
    end
    pmf(pmf == 0) = [];
    maxs = d1*N + d2*N;
    shift = linspace(min(d1*N + d2*N), max(d1*N + d2*N), length(pmf));
    mu = sum(shift.*pmf);
    sigma = sqrt(sum(((shift - mu).^2).*pmf));
end

function [pmf, fx] = genPMF(die, N)
```

```
    [probs, vals] = groupcounts(die);
    fx = zeros(length(die),1);
    fx(vals) = probs./length(die);
    pmf = fx';
    for i = 1:N-1
        pmf = conv(pmf, fx');
    end
    pmf(pmf == 0) = [];
end
```

```
1a) Mean & Standard Deviation: [21 4.18]
1a) Sum of the PDF: 1
1b) Mean & Standard Deviation: [39 4.18]
1b) Sum of the PDF: 1
1c) Mean & Standard Deviation: [16 3.37]
1c) Sum of the PDF: 1
1d) Mean & Standard Deviation: [18.5 3.8]
1d) Sum of the PDF: 1
```

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