MECH 7710 Homework Assignment #1
Due: February 19, 2024

1. Use the MATLAB convolve function to produce discrete probability functions (PDF's) for throws of six dice as follows (note: this is effectively the sum of 6 random variables)
   a) 6 numbered 1,2,3,4,5,6
   b) 6 numbered 4,5,6,7,8,9
   c) 6 numbered 1,1,3,3,3,5
   d) 3 numbered 1,2,3,4,5,6 and 3 numbered 1,1,3,3,3,5

Check that the ΣPDF = 1.0
Plot each PDF with a normal distribution plot of same average and sigma.

Note that even peculiar random distributions, taken in aggregate, tend to produce "normal" error distributions

2. What is the joint PDF for 2 fair dice (x_1, x_2) (make this a 6x6 matrix with the indices equal to the values of the random variables). Note each row should add to the probability of the index for x_1 and each column to the probability of the index for x_2
   a) What are $E(X_1)$, $E(X_1-E(X_1))$, $E(X_1^2)$, $E((X_1-E(X_1))^2)$, and $E(((X_1-E(X_1))*(X_2-E(X_2)))$
   b) Form the covariance matrix for x_1 and x_2
   c) Now find the PDF matrix for the variables $v_1=x_1$ and $v_2=x_1+x_2$.
   d) Now what is the mean, $E(v_1-E(v_1))$, rms, and variance of v_1
   e) What is the mean, $E(v_2-E(v_2))$, rms and variance of v_2
   f) What is the new covariance matrix P.

3. Two random vectors X_1 and X_2 are called uncorrelated if

$$E\{(X_1-\bar{X}_1)(X_2-\bar{X}_2)\} = 0$$

Show that:
   a) Independent random vectors are uncorrelated
   b) Uncorrelated Gaussian random vectors are independent

4. Consider a sequence created by throwing a pair of dice and summing the numbers which are {-2.5, -1.5, -0.5, 0.5, 1.5, 2.5}. Call this $V_o(k)$.
   a) What is the PDF?
   b) What are the mean and variance of this sequence?

   If we generate a new random sequence – $V_N(k+1) = (1-r)V_N(k) + rV_o(k)$,
   $V_N(k)$ is serially-correlated (not white).
   c) In steady state, what are the mean and variance of this new sequence (V_N)?
   d) What is the covariance function: $R(k) = E\{V_N(k)V_N(k-L)\}$
      (Hint: $V_N(k)$ and $V_o(k)$ are uncorrelated).
   e) Are there any practical constraints on r?

5. A random variable x has a PDF given by:

$$f_X(x) = \begin{cases} 0, & x < 0 \\ \frac{x}{2}, & 0 \le x \le 2 \\ 0, & x \ge 2 \end{cases}$$

a) what is the mean of x?
b) what is the variance of x?

6. Consider a normally distributed two-dimensional vector X, with mean value zero and

$$P_X = \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}$$

a) Find the eigenvalues of Px
b) The likelihood ellipses are given by an equation of the form: $x^T P_x^{-1} x = c^2$. What are the principle axes in this case?
c) Plot the likelihood ellipses for c = 0.25, 1, 1.5
d) What is the probability of finding X inside each of these ellipses?

7. Given x ~ $N(0, \sigma_x^2)$ and $y = 2x^2$
a) Find the PDF of y
b) Draw the PDFs of x and y on the same plot for σ_x=2.0
c) How has the density function changed by this transformation
d) Is y a normal random variable?