MECH 325, Assignment 3

Group C2

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Total price: \$ 195.23

Total Mass: 2.58 kg

Performance metric: $1.985 \times 10^{-3} \frac{1}{kg \cdot \$}$

1 Summary – Approach and Solution

The aim of this report is to design the bearings and shaft accessories for a drivetrain that effectively transmits power from a motor to an output shaft for a candy polishing machine. The output shaft is required to turn a drum weighing 100 lbs and containing 50 lbf of candy for 4 hours, 4 times a day, 250 days a year for a total of 4 years. The drivetrain imposes additional loads on the bearings which must be accounted for. In addition the design of mounting bolts for the motor of the drive train is included.

To design the various components, we split the problem into three components, the design of the upper shaft,the design of the motor shaft, and the design of the mounting bolts.
The cost per component is, 100$ for bearings, 4$ for bolts, 17.52$ for machining, 15.00$ for the key-way, 20.01$ for the grove, and 38.69$ for the shaft.
The total cost of all components is: $195.23
The total mass of the system is: 2.58 kg

2 Assumptions and methods

- Overall reliability for roller bearings >0.95
- Shock loading on drum due to early stages of Jelly-Bean handled by use of significant safety-factor
- Shaft material: G10450 cold-drawn steel with specific density of 7.87 $\frac{g}{cm^3}$ but we neglect the bending of the shaft.
- Ignore gravity loading of drum and motor mount mass for bolt calculations.
- Design costs for each component follows a model provided in the assignment
- No calculations done for bearings on motor shaft
- The mass of the shaft and sprocket is negligible when calculating the force on the bearings.
- A negligible amount of power from the motor is not transferred to the shaft.
- The bolts are at a distance of 50cm from the middle of the motor mount base.
- The tension force can be conservatively modeled as cyclic to account for start-up acceleration vibrations, load irregularities, clumping of the beans.
- The drum is mounted to the shaft by use of a screw going into the shaft, but specking this falls outside the scope of this assignment.

3 Shaft bearings on drum-shaft

See next page and free body diagram in appendix 7.4 for our positioning of the components on the drum-shaft and the force distribution on the bearings. We use a pair of direct-mount tapered roller bearings located at A (right) and B (left). B is as close to the drum as possible and A is as close to the free end as possible. The sprocket is located close to bearing A. The positioning of the sprocket at this end results in the force of tension in the chain working to counter the torque created by the load in the drum, thus minimizing the net forces present on the bearings.

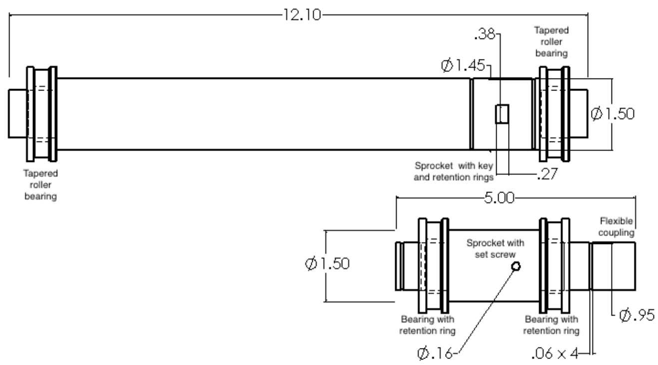
Having the tapered roller pair in a direct mounting configuration will make bearing A carry the external thrust load. As bearing B will experience the greatest radial loads, this will more evenly distribute the load carried between the two bearings. Having a pair will also make it impossible for the shaft to fall out or get dislodged during operation where forces do not significantly exceed the specifications.

The chosen bearings were taken from Shigley's Figure 11-15 with part numbers of cone 3189 cup of 3120 for both bearing A and B.

4 Shaft Accessories and Machining

Drum shaft Starting with a 1.5 inch shaft that is 12 inches long, one inch shoulders are machined into each side of the shaft for the tapered roller bearings to rest against. To secure the drum sprocket against axial loads, two retaining rings are used, one on each side. To secure the drum sprocket against torsional loads, a 3/8 inch by 3/8 inch by 0.28 inch long key made of UNS G10060 hot-rolled steel was chosen.

Motor shaft A #8 cup point socket set screw is used to secure the motor sprocket for axial and radial loads. In order to keep strain off of the motor, the motor shaft is split into two parts with a flexible coupling between them. To hold the second shaft in place, a pair of roller bearings identical to the ones used for the drum shaft are put in place. In order to make room for the bearings, one end is machined down to a diameter of 1 inch over a length of 2 inches on one side and the other side is machined down to a diameter of 1 inch over a length of 1 inch.



5 Mounting Bolts for Motor Mount

To secure the motor mount and motor shaft to the base, four SAE Grade 2 1.5in steel bolts were used. These were place in a square configuration at a distance of 0.5m from the center of the motor mount. Due to the cyclic loading on the drum, the magnitude of the tension in the chain oscillates during operation, creating cyclic loading on the motor mount and bolts. As a result, fatigue and static loading failure modes needed to be addressed.
By determining the maximum bending moment and shear stress acting on the bolts, the static tension load was analyzed and found to have a safety factor of 1411.6. Assuming the worst case scenario of fully cyclic loading, the alternating and mid-range stress were calculated. It was found using the Goodman criteria that the bolts achieved infinite life with a factor of safety of 12.12.

6 References

Calculations based on equations used from:
Shigley's Mechanical Engineering Design 10^{th} Ed. by Richard Budynas and Keith Nisbett
Specific weight of stainless steel: $www.engineeringtoolbox.com/density-specific-weight-gravity-d\_290.html$
Retaining ring selection from Machinery's Handbook, 29^{th} edition by Erik Oberg

7 Appendix: Force calculations

Bearing on our design from assignment 2, the following calculations outline the forces present in the system.

7.1 Power

Outer diameter of sprocket on driven shaft $d_d = \frac{9}{60} \cdot 89 = 14.60in = 35.56cm$ due to linear scaling of largest sprocket available in selection. As the motor is delivering 1hp and the angular velocity of the drum is 120rev/min, we see that

$$P = 1hp \cdot 745.7W/hp = 745.7W$$

7.2 Tension in drum-shaft (driven)

Angular velocity, torque and tension is the chain is determined as follows:

$$\omega_{\mathrm{d}} = 120rev/min \cdot \frac{2\pi}{60} = 12.57rad/sec$$

$$\tau_{\mathrm{d}} = \frac{P}{\omega_{\mathrm{d}}} = \frac{745.7W}{12.57rad/sec} = 59.35Nm$$

$$T_{\mathrm{d}} = \frac{\tau_{\mathrm{d}}}{d_{\mathrm{d}}/2} = \frac{59.35Nm}{35.56cm/2} = 333.8N = 75.04lbs$$

where the subscript d denotes the drum-shaft. We assume the slack-side tension in the chain to be negligible, as there is no pretension on the roller-chain.

7.3 Tension in motor-shaft (driving)

We know the overall train value of the system to be $n = 9.972$ and $d_{\mathrm{m}} = 3.62in = 7.62cm$, so that

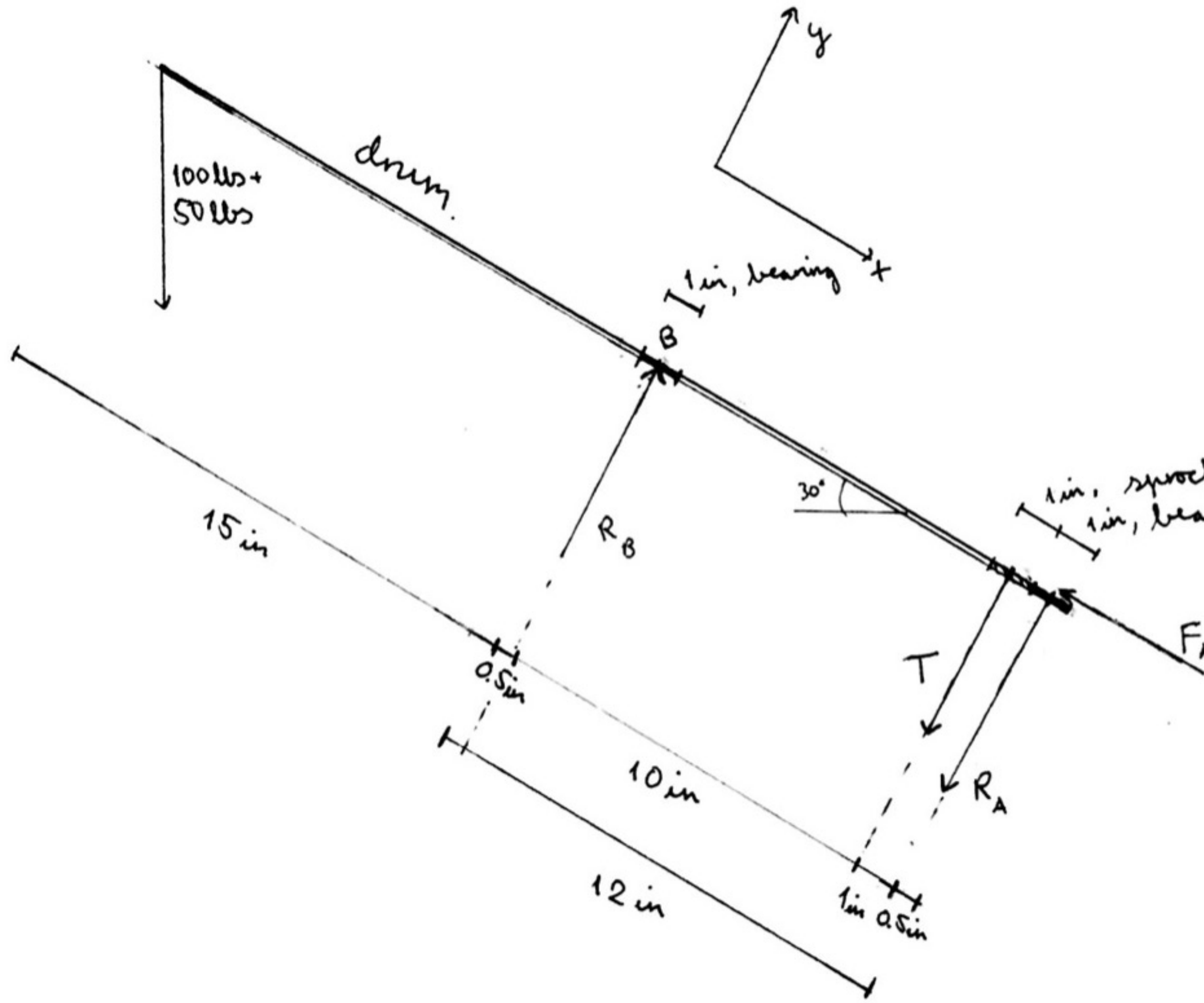
$$\tau_{\mathrm{m}} = \frac{\tau_{\mathrm{d}}}{n} = \frac{59.35Nm}{9.972} = 5.952Nm$$

$$T_{\mathrm{m}} = \frac{\tau_{\mathrm{m}}}{d_{\mathrm{m}}/2} = \frac{5.952Nm}{7.62cm/2} = 156.2N = 35.12lbs$$

where m denotes quantities for the motor shaft.

7.4 Solve for static equilibrium

Free body diagram for drum-shaft:



From static equilibrium of drum-shaft:

$$\sum F_x = 0, \qquad F_A = (100 + 50)\sin(30) = 75lbs$$
$$\sum F_y = 0, \qquad R_B = T + R_A + (100 + 50)\cos(30)$$
$$\sum (M_z)_B = 0, \quad 10T + 11R_A = (100 + 50) \cdot 15.5 \cdot \cos(30)$$

As the loading is going to vary depending on whether the machine is active and we have tension in the chain or whether it is inactive, we determine the forces present for these two extreme cases and perform our calculations based on the situation in which the largest forces are present.

We are going to assume that during normal operation, the torque corresponding to 1hp is going to be the largest torque transmitted in the system, and that the tension in the chain is thus going to be $T_d = 75.04lbs$ as calculated in section 8.1. Using a large safety-factor of 2.5 will account for any momentary increase in the load caused by uneven jolting of the load.
The minimum force in the chain is zero

$$T_{\min} = 0lbs$$

. Using these two extreme values alternately, we solve the system for static equilibrium to find that

$$\begin{aligned} \text{For } T_{\max}: \quad & R_A = 114.8lbs \\ & R_B = 319.8lbs \\ \text{For } T_{\min}: \quad & R_A = 183.0lbs \\ & R_B = 313.0lbs \end{aligned}$$

We see that the largest force exerted on bearing B is in the situation of no tension in the chain, whereas the largest force on bearing A is when there is tension in the chain. As an over-estimate, the largest values are used in the following calculations.

8 Tapered roller bearing calculation

8.1 Application-Specific variables

From statics calculations (see relevant appendix), we find that the design forces (including safety-factors) will be

$$F_{rA} = 183.0 * 4.45 * 2.5 = 2036N$$

$$F_{rB} = 319.8 * 4.45 * 2.5 = 3558N$$

$$F_{ae} = 75 * 4.45 * 2.5 = 834.4N$$

Other application-specific variables required for this calculation includes

- Reliability-factor is assumed to be $R_D = 0.95$
- Design Life in hours $L = 2.5 * 4 * 250 * 16 = 40000$
- Rotational speed $\omega = 120rpm$
- Application factor is found to be $a_f = 3$ (see table 11-5), where we assume our system can be described as machinery with moderate impact.

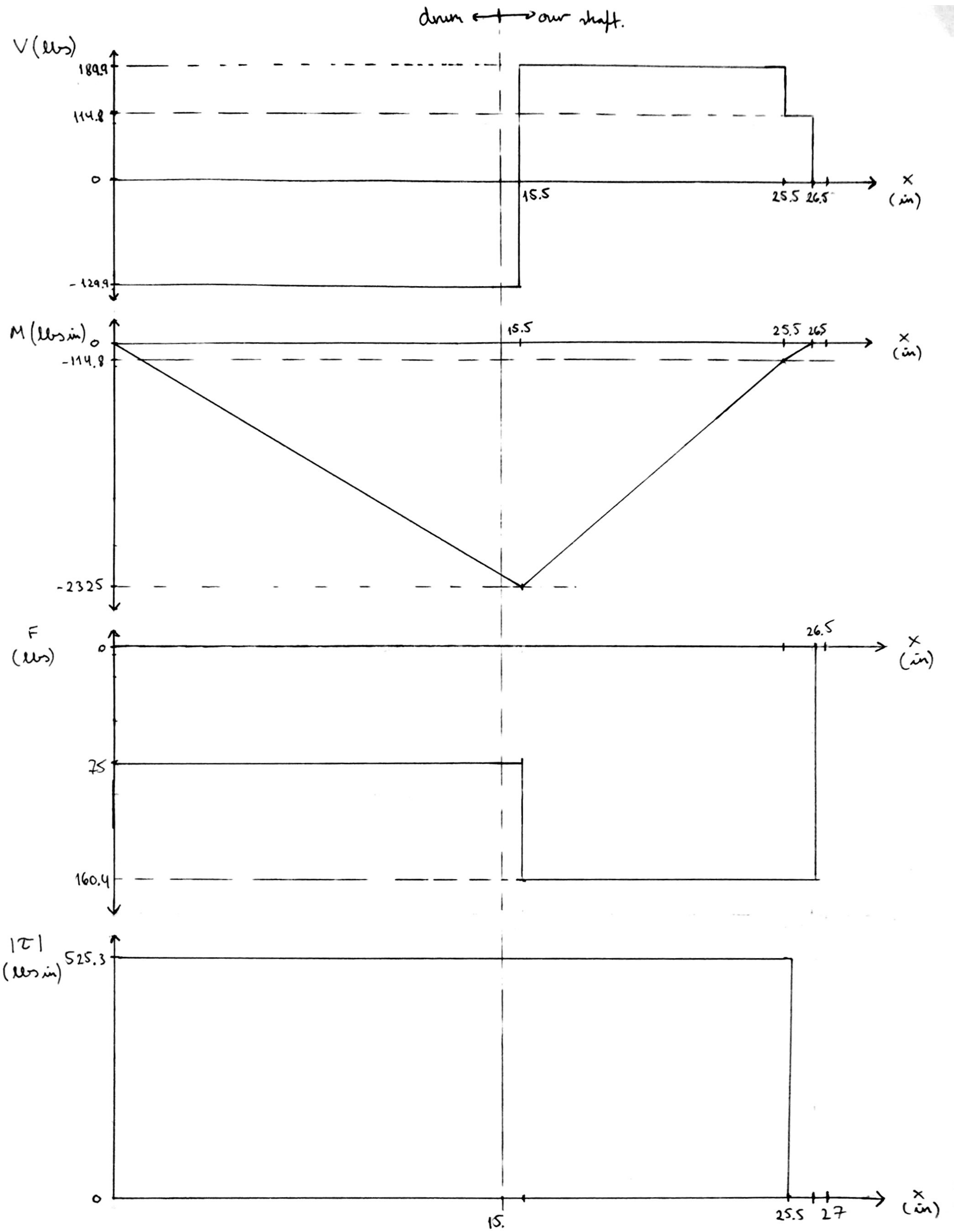
8.2 Constants

- for roller-bearing in general we have that $a = 10/3$.
- $V = 1$ as inner race rotates.
- Desired life is $L_D = L * \omega * 60 = 288000000$ in revolutions.
- Rating life is constant $L_R = 90 * 10^6$ revolutions.
- $x_D = \frac{L_D}{L_R} = 3.2000$ is a dimensionless multiple of rating life (for convenience)

8.3 Values used by Timken manufacturer

For details, see top of page 590.

- guaranteed life: $x_0 = 0$
- shape parameter $b = 3/2$
- scale parameter $\theta = 4.48$



drum ⟷ our shaft.
V (lbs)
189.9
114.8
0
-129.9
15.5
25.5
26.5
x
(in)
M (lbs in)
0
-114.8
-2325
15.5
25.5
26.5
x
(in)
F
(lbs)
0
75
160.4
26.5
x
(in)
|τ|
(lbs in)
525.3
0
15.
25.5
27
x
(in)

8.4 Bearing calculations, using tapered roller bearings

As an initial guess, we use the following values for geometry-factors and an assumption of equal distributions of the reliability-factor between the two tapered roller bearings:

- geometry factor for A $K_A = 1.5$
- geometry factor for B $K_B = 1.5$
- $R_{DA} = \sqrt{R_D} = 0.975$ as an estimate of the distribution between the two bearings
- $R_{DB} = \sqrt{R_D} = 0.975$

The induced load from the bearings is found to be

$$F_{iA} = \frac{0.47F_{rA}}{K_A} = 637.9N$$

$$F_{iB} = \frac{0.47F_{rB}}{K_B} = 1115N$$

Since F_{iA} is clearly less than $F_{iB} + F_{ae}$, we see that bearing A carries the net thrust load, and the equation numbered 11-19 is applicable as follows for the dynamic equivalent loads

$$F_{eA} = 0.4F_rA + K_A(F_{iB} + F_{ae}) = 3738N$$

$$F_{eB} = F_{rB} = 3558N$$

For bearing A Using eq. 11-10 we find

$$C_{10} = a_f F_{eA} \left(\frac{x_D}{(x_0 + (\theta - x_0)(1 - R_{DA})^{\frac{1}{b}}} \right)^{\frac{1}{a}} = 21146N$$

The catalog entry for C_{10} should equal or exceed this value.
We see that this correspond to part numbers of 3189 for cone and 3120 for cup, so $K_A = 1.76$ to be used in next iteration of calculation.

For bearing B

$$C_{10} = a_f F_{eB} \left(\frac{x_D}{(x_0 + (\theta - x_0)(1 - R_{DB})^{\frac{1}{b}}} \right)^{\frac{1}{a}} = 20126N$$

which corresponds to the same bearing as for A, so that $K_B = K_A = 1.76$.

Performing the above calculations again with the updated values for K_A and K_B we find that for bearing A

$$C_{10} = 22373N$$

and for B

$$C_{10} = 20126N$$

and our existing bearing selection still works for these new values. Therefore, we have confirmed that we can use parts with number 3189 for cone and 3120 for cup on both bearings and maintaining a 2.5 overall safety-factor and an overall bearing reliability exceeding 0.95.

9 Mounting Bolts Design

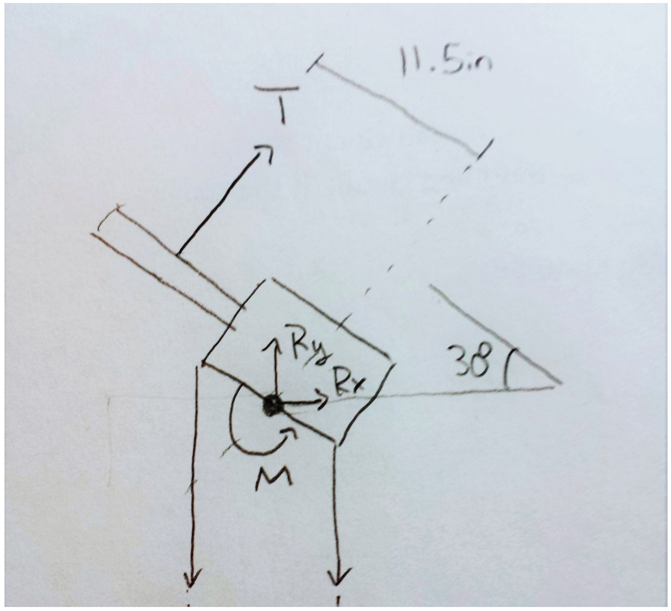
9.1 Assumptions

- The bolts are at a distance of 50cm from the middle of the motor mount base.
- The tension force can be conservatively modeled as cyclic to account for start-up acceleration vibrations, load irregularities, clumping of the beans.
- Assuming that Ad is close enough to At for the calculations needed to be performed

9.2 Static Tension Failure Check

From Appendix 6.3, it was found that the maximum tension in the motor shaft was found to be 35.12lbs. From the FBD below, we are then able to calculate the maximum moment experience by the motor mount.

FBD of Motor:



From static equilibrium of motor:

$$\sum R_x = 0, \qquad R_x = (-T)cos(30) = -17.56lbs$$
$$\sum R_y = 0, \qquad R_y = (-T)sin(30) = -30.41lbs$$
$$\sum (M_z) = 0, \quad M = T * 11.5 = 403.88lbs * in$$

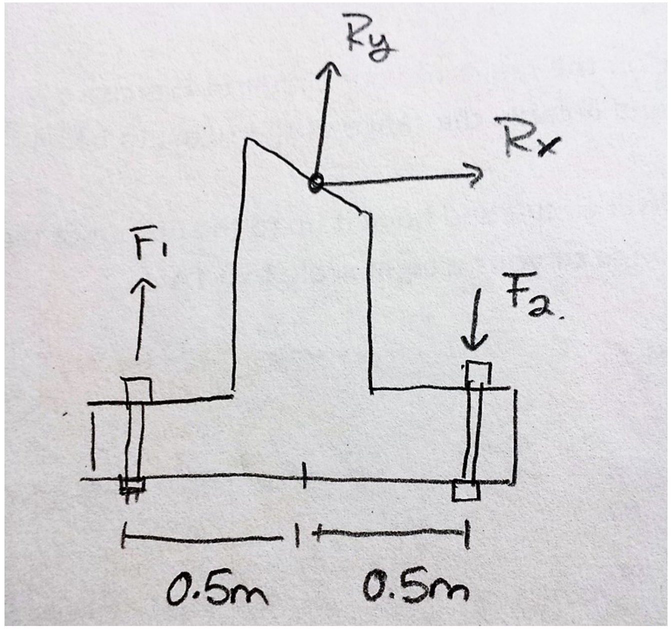
FBD of motor mount:

From static equilibrium of the motor mount:

$$\sum F_y = 0, \qquad 17.6lbs + 2(F_1) - 2(F_2), \qquad F_2 = F_1 + 8.8lbs$$
$$\sum (M_z) = 0, \qquad 404lbs * in - 2(F_2)x - 2(F_1)x = 0, \qquad 202/x - F_2 - F_1 = 0$$

Using the force balance of the y-components:

$$202/x - (F_1 + 8.8lbs0 - F_1 + 2(F_1) - 2(F_2)$$
$$F_1 = 4.4lbs + 101/x$$
$$F_2 = 13.2lbs + 101/x$$
$$Assuming\ a\ base\ length\ of\ 0.5m$$
$$F_1 = 105.4lbs$$
$$F_2 = 114.2lbs$$



Selecting the bolt SAE 2 (1-1/2in) from Shigley's, it was found that the specifications were as follows:

$$S_y = 36kpsi$$
$$S_ut = 60kpsi$$
$$S_p = 33kpsi$$
$$E = 29900kpsi$$

Finding C To find the fraction of the load taken by the bolt, the equivalent spring constants need to be found for the bolt, as well as the material within the fastener grip

The tensile area A_t needs to be found to compute the spring constants:

$$A_t = \pi/4 * (0.9 * d) = \pi/4 * (0.9 * 1.5) = 1.431 \text{ in}^2$$

We can now find the spring constants k_b and k_m:

$$k_b = A_d * A_t * E/(A_d * l_d + A_t * l_d)$$
$$k_b = A_t^2 * E/A_t * l = (1.432^2 * 29.9x10^6)/(1.432 * 1.5)$$
$$k_b = 28.5x10^6 lb/in$$
$$k_m = E * d * A * e^{(B*d/l)}$$
$$A = 1.5 \text{ and } B = 0.62873 \text{ (from Shigley's)}$$
$$k_m = 66.2x10^6 lb/in$$
$$C = k_b/(k_m + k_b) = 0.301$$

To find the proof stress on the bolt:

$$o_b = F_b/A_b$$
$$S_p = F_p/A_t$$
$$F_p = 33kpsi * 1.432in^2 = 47.26kip$$
$$F_i = 0.15 * F_p = 7.089kip$$
$$o_i = 7.089kip/1.432in^2 = 4.95kpsi$$
$$n = (S_p - o_i)/(C * P/A_t) = (36kpsi - 4.726kpsi)/(0.301 * 105.4/1.432) = 1411.6$$

9.3 Finding the Safety Factor for Infinite Life

To be conservative:

$$\sigma_a = \frac{CP}{2At} = \frac{(0.301)(105.4lbs)}{2(1.431in^2)} = 11.09psi$$

$$\sigma_m = \frac{CP}{2At} + \frac{F_i}{A_t} = 11.09psi + \frac{7.089kip}{1.431in^2} = 4.95kpsi$$

Using $S_e = 16.3kpsi$ from table 8-17. Using Goodman Criteria:

$$\frac{S_a}{S_e} + \frac{S_m}{S_ut} = \frac{1}{n}$$

$$n = \frac{11.09}{16.3 \times 10^3} + \frac{15.71 \times 10^3}{60 \times 10^3} = 12.12$$

10 Shaft Accessory Design

10.1 Key (Drum Shaft)

Shigley's Table 7-6 gives a table for typical rectangular key sizes for a given shaft diameter. In this case, with a shaft diameter of 1.5 inches, two acceptable key sizes are $3/8 \times 3/8$ and $3/8 \times 1/4$ (width by height) inches. Since the cost of machining the key-way is dependent on length only, we will choose the $3/8 \times 3/8$ key size.
To optimize for length, we have to consider the failure of key in two ways: shear and compression. The length of the key governed by shear failure is given by the equation,

$$L \geq \frac{2T}{DwS_{sy}} n_f$$

where T is torque at the surface of the shaft, D is shaft diameter, w is key width, S_{sy} is shear yield strength of the key material, and n_f is the safety factor. The length of the key governed by compression failure is:

$$L \geq \frac{4T}{DHS_{yc}} n_f$$

where H is the key height, and S_{yc} is the yield strength of the key material. Thus, the design length of the key is the maximum length from these two equations. From the previous assignment, we know that the torque at the surface of the driven shaft is 525.3 lb-in. From table A-20 in Shigley's, we will choose UNS G10060 hot-rolled steel with a yield strength of 24 ksi as the key material, and we will use a factor of safety of 2. We then find that we require a key length of 0.28 inches for the upper shaft.

10.2 Setscrew (Motor Shaft)

Using the above equations, and knowing that the torque on the lower shaft is 52.68 lb-in, we find that the force at the surface of the shaft is $2T/D = 70.24$ lb. With a factor of safety of 5 (as suggested by Shigley's for dynamic loading), we find from Shigley's table 7-4 that a size #8 cup-point socket setscrew with length of 0.75 inches (half the shaft diameter) can be used. Note that no retaining rings are required for this sprocket, as there is minimal axial load and the setscrew can take both torsional and axial loading.

10.3 Retaining Rings

Since there is little to no axial load exerted by the sprocket, we can use two thin retaining rings to axially locate each. The following two equations are used to determine the maximum load for a retaining ring, with the actual P_{max} being the minimum from the two equations:

$$P_{max} = S_{ssy(ring)} \pi Dt/n$$

$$P_{max} = S_{y(shaft)} \pi Dd/n$$

where D is shaft diameter, t is ring thickness, d is groove depth,$S_{ssy(ring)}$ is shear yield strength of the ring material, and $S_{y(shaft)}$ is yield strength of the shaft material. From the Machinery's Handbook (29th edition), two 3AM1-38 retaining rings can be used to axially locate the sprocket on the drum shaft. The specification is based on the shaft diameter being used, with 1.5 inches being equal to 38 mm; the ring has a thickness of 1.3 mm, with a groove width of 1.4 mm and depth of 1.1 mm. This type of retaining ring can take up to 20.5 kN (4610 lb) with a safety factor of 2, which is enough for this application.

11 Cost calculations

The performance metric is calculated based on all selected components as specified in the assignment.

Stock shaft

- specific weight of the shafts $s = 7.87e3kg/m^3$
- diameter $d = 37mm$
- length $l = 12 * 25.4cm$
- shaft cost

$$\text{cost} = 15\left(\frac{1}{4}\pi d^2 l\right)s = \$38.69$$

Shaft machining

- length removed $l = (1+1) * 25.4mm$
- outer diameter $d_o = 37mm$
- inner diameter $d_i = 25mm$
- removed volume $V = \frac{\pi}{4}(d_o^2 - d_i^2)l = 2.968e-05m^3$
- machining cost

$$\text{cost} = 75Vs = \$17.52$$

Shaft splines Not used

Key-ways

- key-way length $l = 0.28 * 7.112mm$
- key-way cost

$$\text{cost} = 15 + 0.5 * l = \$15.00$$

Retaining ring grooves

- cost per grove $c = 10 + 0.5 * (d_i - 10) = 10.01$
- total cost

$$\text{cost} = 2c = \$20.02$$

Bearings and shaft collars

- bore diameter $d = 25mm$
- bearing price per unit $c = 20 + 2 * (d - 10)$
- bearing cost

$$\text{cost} = 2 * c = \$100$$

Bolts

- cost of bolts

$$\text{cost} = 4 * (5 - 4) = \$4$$

Total cost

$$\text{cost} = \$195.23$$