

MECH3750 - Tutorial 3

Question 1.

Suppose p_1, p_2, p_3 are three vectors (or functions). Suppose we wish to find another vector (or function) $y = a_1 p_1 + a_2 p_2 + a_3 p_3$ which minimizes the distance squared:

$$d^2(y, f) = \|(y - f, y - f)\|$$

Where d is the “distance” as introduced in lectures, and (\cdot, \cdot) is a well defined inner product.

Obtain a_1, a_2, a_3 (to find the best approximation) for the following cases:

(a) $p_1 = (1, 0, -1, 0), p_2 = (1, 1, 1, 0), p_3 = (1, -2, 1, 0)$. With inner product: $(u, v) = \sum_i u_i v_i$ and:

$$(i) f = (4, 0, 2, 0) \quad (ii) f = (0, 0, 0, 1)$$

(b) $p_1 = \sin x, p_2 = \sin 2x, p_3 = \sin 3x$. With inner product: $(u, v) = \int_0^\pi u(x)v(x) dx$ and:

$$(i) f(x) = \sin 2x + \sin 3x \quad (ii) f(x) = \sin 4x$$

Hint: Use the result established in lectures that:

$$(\sin nx, \sin mx) = \begin{cases} \frac{\pi}{2} & m = n \\ 0 & m \neq n \end{cases}$$

(c) $p_1 = 1, p_2 = x, p_3 = \frac{1}{2}(3x^2 - 1)$. With inner product: $(u, v) = \int_{-1}^1 u(x)v(x) dx$ and:

$$(i) f(x) = x^2 \quad (ii) f(x) = 5x^3 - 3x$$

(d) Comment on why $d^2(y, f) = (y - f, y - f) = 0$ in all (i) cases. Comment on (ii) results and why all these cases are non zero.

(e) There is an alternative way of solving all (i) cases. Why?

Question 2.

(a) Find the best approximation to the function $f(x) = 1$ on the interval $[0, \pi]$ using the set $\sin nx, n = 1, 2, \dots, N$. On the worksheet, you are asked to plot your answer for different values of N .

(b) Find the best approximation to the function $f(x) = x$ on the interval $[0, \pi]$ using the set $\sin nx, n = 1, 2, \dots, N$. On the worksheet, you are asked to plot your answer for different values of N .

(c) Find the best approximation to the function $f(x) = x$ on the interval $[0, \pi]$ using the set $1, \cos nx, n = 1, 2, \dots, N$.

Note, if we take $N \rightarrow \infty$, we call our solutions the Fourier sine, or cosine series for $f(x)$. These are important functions in many partial differential equations useful in engineering applications such as heat transfer (MECH3400), and vibration of strings and beams (MECH3200).