MFCS

FUNCTIONS BASIC ABSTRACT

SET THEORY AND RELATIONS

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WHOA!

I know the worth of your time and Patience into this lecture. I’m Sure, by the end of this session, you’ll be Confident about this topic and feel free to take a quick notes and write down your queries!

01

INTRODUCTION TO FUNCTIONS

Q : Why we need Functions?
Don’t stress out. I’m here to explain ☺

AN INTRODUCTION TO FUNCTION

All the scientists use mathematics essentially to study relationships. Physicists, Chemists, Engineers and even Social Scientists, all seek to discern connection among the various elements of their choosen fields and so to arrive at a clear understanding in

"WHY THESE ELEMENTS BEHAVE THE WAY THEY DO?"

Simple!, This Question was answered in 1694 by latin matemetician "Lejeune Peter Gustav Dirichlet"

"A Function is a Special Case of a Relation"

Wait! Some of us don't even know what even a relation is!. Don't worry, I'll cover that too ☺

RELATIONS

1. Cartesian Product :

Let A and B be Two sets. Then **$\{(a, b) : a \in A, b \in B\}$** is called the cartesian product of A and B, and it is denoted by A x B (to be read as A cross B).

If $A = \{1, 2, 3\}$ The Cartesian product of A and B = $A \times B = \{1, 2, 3\} \times \{3, 4\}$

$B = \{3, 4\}$ $= \{(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\}$

2. Relation :

If A and B are non-empty set, Then any subset of **A x B** is called a **Relation** from A to B.
In particular, any relation from A to A is called a binary relation on A.

If $A = \{1, 2, 3\}$ $A \times B = \{(1, x), (1, y), (2, x), (2, y), (3, x), (3, y)\}$

$B = \{x, y\}$ (i) $f = \{(1, x), (1, y), (3, x)\}$ is a relation from A to B

(ii) $g = \{(1, x), (1, y)\}$ is a relation from A to B

FUNCTIONS

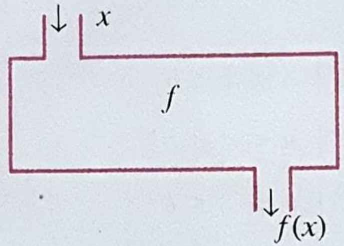
Let A and B be two non-empty sets and f be a relation from A to B.

If for each element **a ∈ A**, there exists a unique **b ∈ B** such that **(a, b) ∈ f,** then f is called a "Function" (or mapping) from **A** to **B** (or **A into B**).

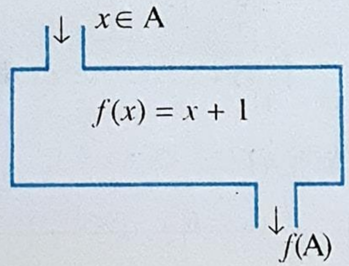
It is denoted by **f : A -> B**. The set A is called the **domain** of f and B is called the **co-domain** of f.

Wait! It's such a big statement consisting of many weird symbols. TBH, We actually don't require this definition. Let's break it down!

FUNCTIONS



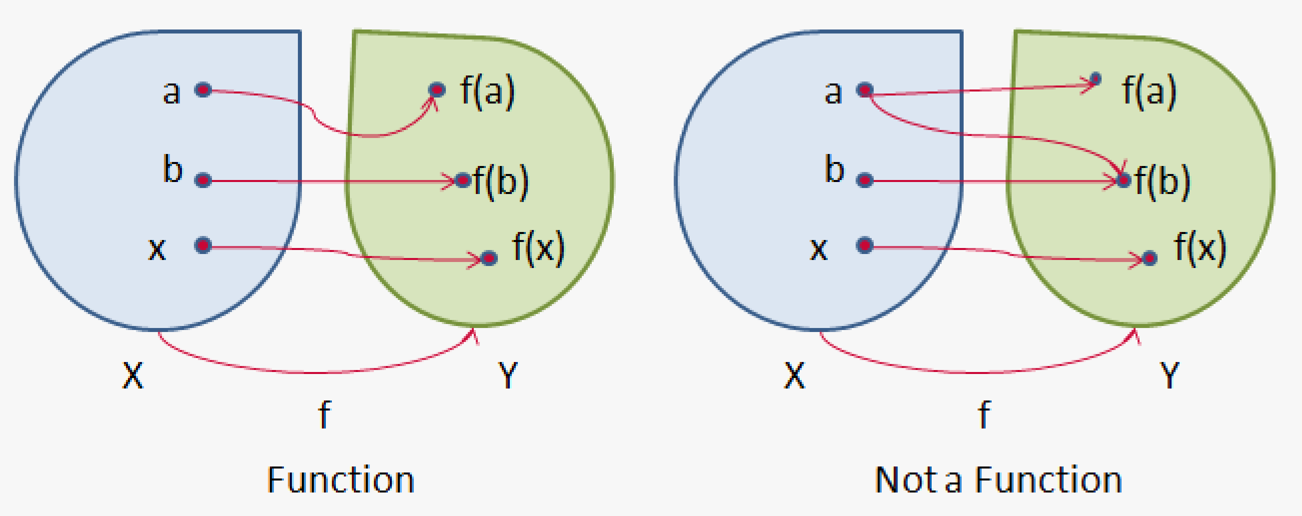
For example, if $f : A \to B$ is a function defined as $f(x) = x + 1$ and $A = \{1, 2, 3\}$, then $f(A) = \{2, 3, 4\}$.



1.1.2 Note

A relation f from A to B (*i.e.* $f \subseteq A \times B$) is a function from A to B if for each $a \in A$, there exists exactly one $b \in B$ such that $(a, b) \in f$ and this 'b' will be denoted by $f(a)$. In other words, for each $a \in A$, there exists a unique element $f(a) \in B$ such that $\left(a, f(a)\right) \in f$.

FUNCTIONS



If for each element **a ∈ X**, there exists a unique **b ∈ Y** such that **(a, b)** ∈ **f,** then f is called a "Function" (or mapping) from **X** to **Y** (or **X into Y**). It is denoted by **f : X –› Y**. The set **X** is called the **domain** of **f** and **Y** is called the **co-domain** of **f**.

02

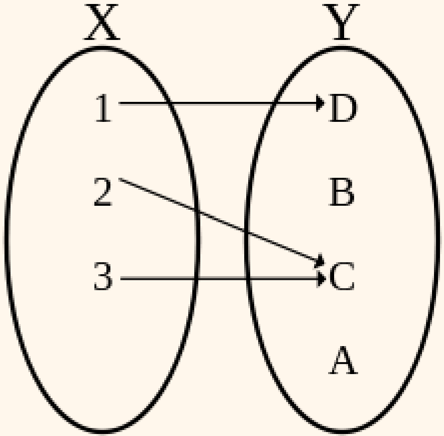
IMAGE, PRE-IMAGE

Image, Pre-Image, Domain, Co-Domain

IMAGE AND PRE-IMAGE

X = { 1, 2, 3 }
Y = { D, B, C, A }

F = { (1,D), (2,C), (3,C) }



Here, X is Called **Domain** of F
Y is Called **Co-Domain** of F

{ (1 , D) }
domain_| |_co-domain

From the above Venn-Diagram ; **The Image of element '1' is 'D'**
and ; **The Pre-Image of element 'D' is '1'**

03

TYPES OF FUNCTIONS

08 Basic Types of Functions

Types of Functions

- One-one Function (**Injective Function**)
- Onto Function (**Surjective Function**)
- **Bijective** Function
- **Into** Function
- **Inverse** Function
- **Modulus** Function
- **Greatest Integer** Function
- **Even** and **Odd** Function

One-One (Injective) Function :

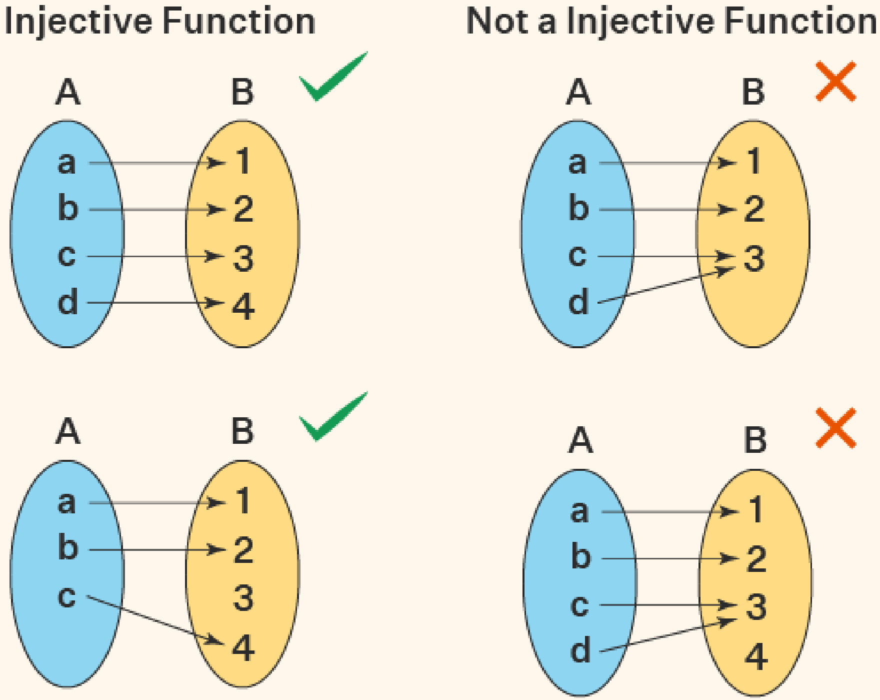
- An injective function or one-to-one function is a function that maps **distinct elements of its domain** to **distinct elements of its codomain**.

- In brief, let us consider '**f**' is a function whose domain is set A. The function is said to be injective **if for all x and y in A**,

- Whenever **$f(x)=f(y)$,** then **$x=y$**
- And equivalently, if **$x \neq y$**, then **$f(x) \neq f(y)$**

- **$f : N \rightarrow N$, $f(x) = 5x$, f is injective.**

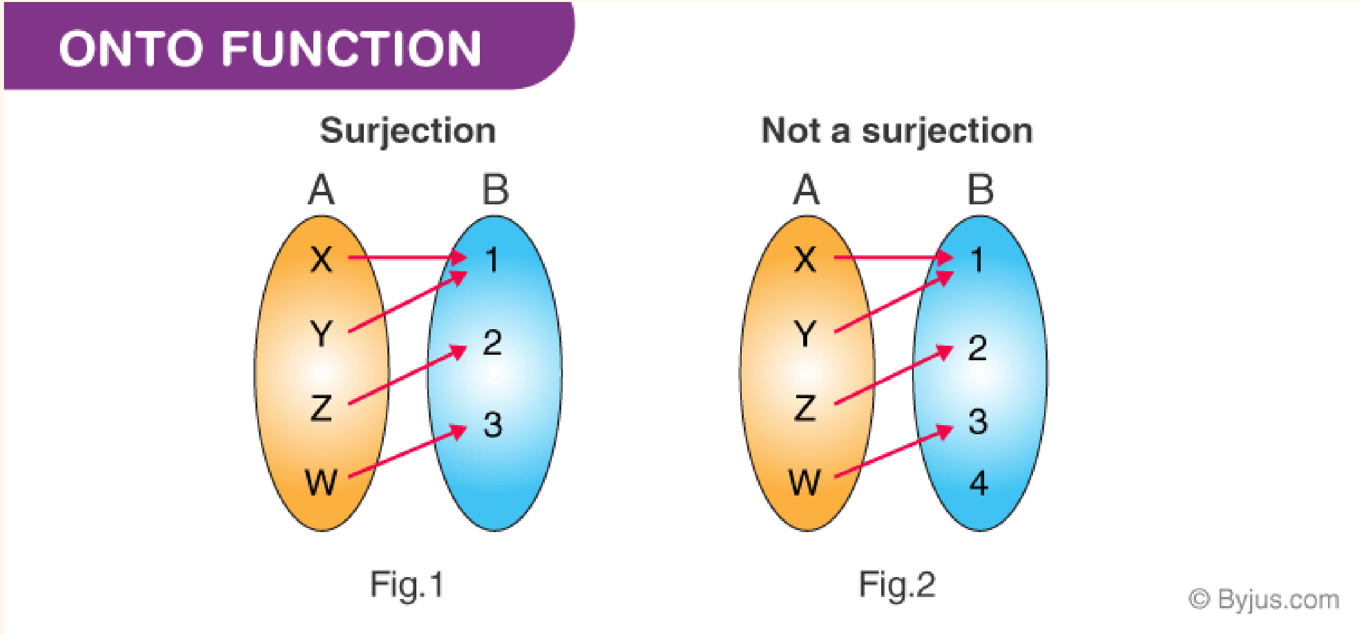
Injective Function - One-to-One Function



Onto (Surjective) Function :

- Onto function could be explained by considering two sets, Set A and Set B, which consist of elements.

- If **for every element of B**, there is **at least one or more than one element matching with A**, then the function is said to be onto function or surjective function.



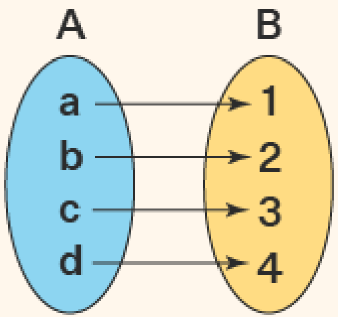
- In the first figure, you can see that for each element of B, there is a pre-image or a matching element in Set A. Therefore, it is an onto function. But if you see in the second figure, one element in Set B is not mapped with any element of set A, so it's not an onto or surjective function.

Bijective Function :

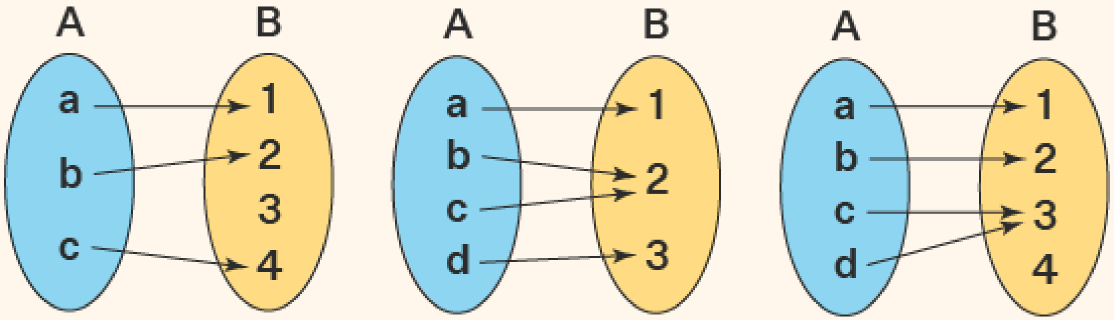
- A function **f : A→B** is said to be a bijective function.
- if f is both **one-one** and **onto**, that is, **every element in A has a unique image in B** and **every element of B has a pre-image in set A**.
- In simple words, we can say that a function f is a bijection **if it is both injection and surjection.**

Bijective Function - One-to-One & Onto Function

Bijective Function



Not a Bijective Function



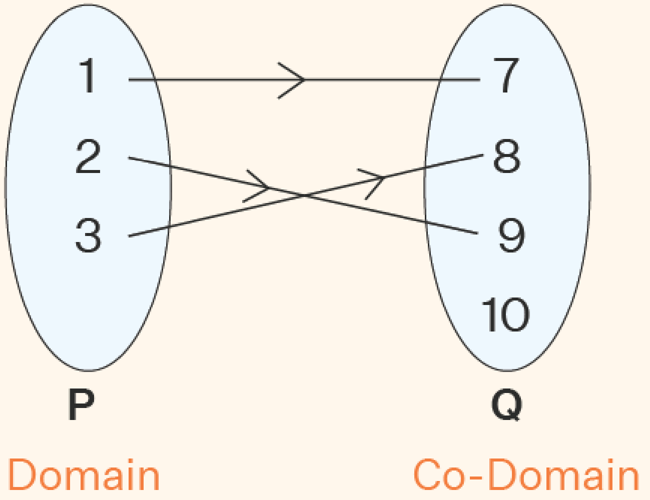
Into Function :

- For a function **f : A → B** to be an into function there will **be one or more elements in set B that do not have a pre-image in set A.**

- In other words, **in an into function, not all elements of the codomain will be mapped to the elements of the domain**.

- As a consequence, the range of an into function will be a subset of the codomain however, the range and codomain will never be equal.

Into Function Arrow Diagram



- Let set P = {1, 2, 3} and set Q = {7, 8, 9, 10} be defined by the function
- f = {(1, 7), (2, 9), (3, 8)}.

- As element 10 of set Q does not have a pre-image in set P thus, this function is an into function.

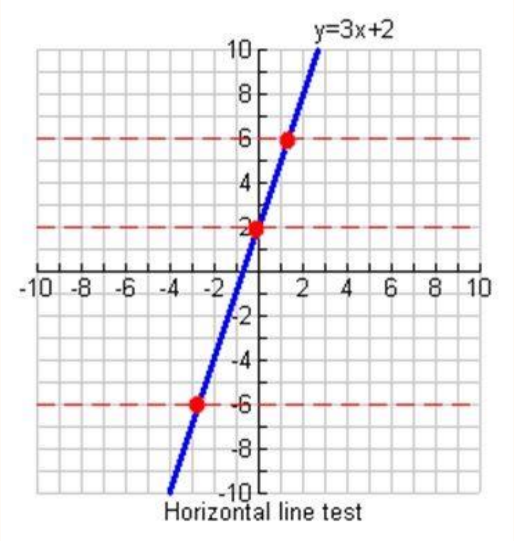
Inverse Function :

The inverse function returns the original value for which a function gave the output.

- If you consider functions, f and g are inverse, **$f(g(x)) = g(f(x)) = x$**. A function that consists of its inverse fetches the original value.
- Example: $f(x) = 2x + 5 = y$
- Then, $g(y) = (y-5)/2 = x$ **is the inverse of $f(x)$**.

Graph – Inverse Function

- The graph of the inverse of a function reflects two things, one is the function and second is the inverse of the function, over the line $y = x$.
- This line in the graph passes through the origin and has slope value 1. It can be represented as;
- $y = f^{-1}(x)$
- which is equal to;
- $x = f(y)$



Modulus Function :

- A relation ‘f’ is called a function, if each element of a non-empty set X, has only one image or range to a non-empty set Y. The modulus function f(x) of x is defined as :

- $f(x) = |x|$

  Or

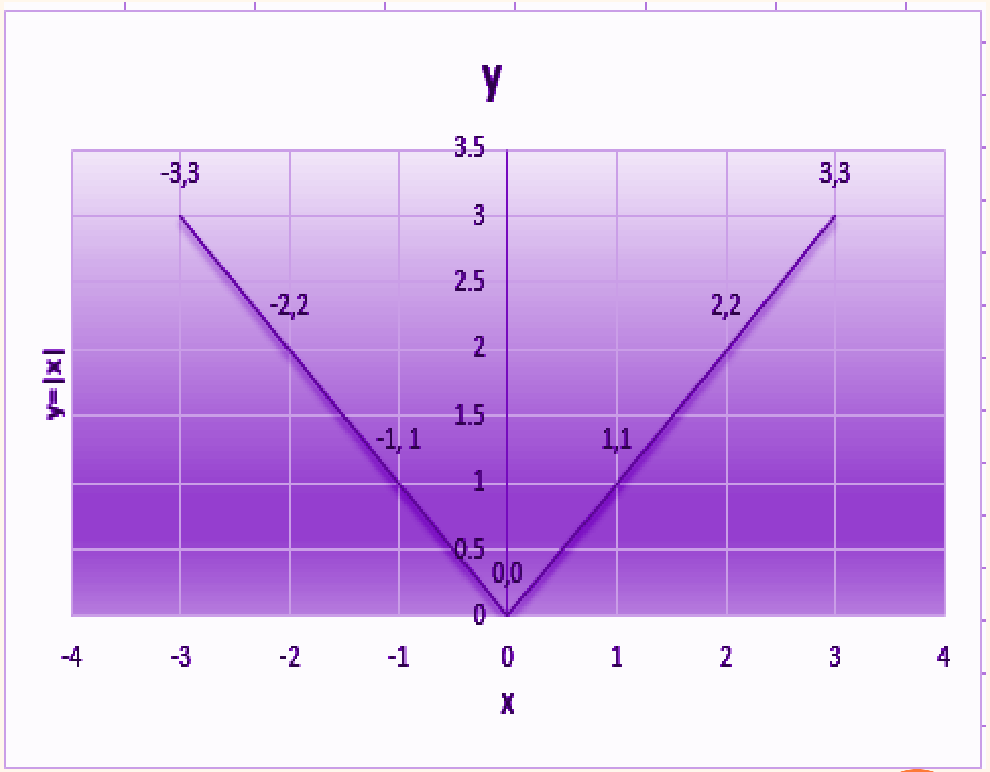
- $y = |x|$

- Where $f:R \rightarrow R$ and $x \in R$

- And $|x|$ states modulus or mod of x.

Graph – Modulus Function

x < 0			x >= 0				
x	-3	-2	-1	0	1	2	3
y	3	2	1	0	1	2	3



Greatest Integer Function :

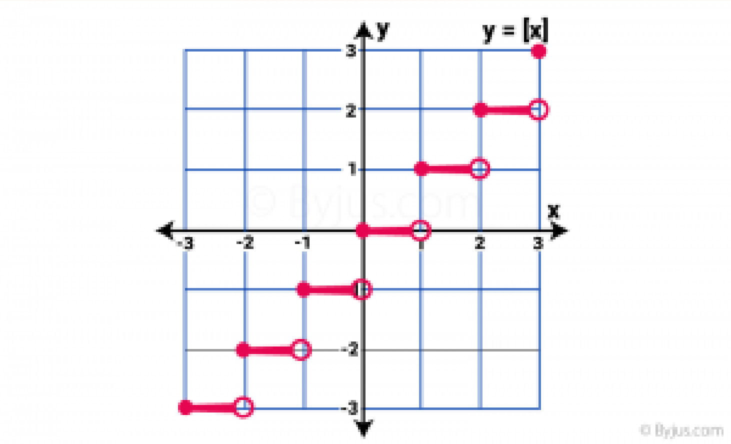
- The greatest integer function is denoted by $\lfloor x \rfloor$, for any real function. The function rounds – off the real number down to the integer less than the number.

- This function is also known as the Floor Function.

- **For example:**
- $\lfloor 1.15 \rfloor = 1$
- $\lfloor 4.56567 \rfloor = 4$
- $\lfloor 50 \rfloor = 50$
- $\lfloor -3.010 \rfloor = -4$

Graph – GIF

- the greatest integer function of the interval [3,4) will be 3.
- The graph is not continuous. For instance, below is the graph of the function f(x) = ⌊ x ⌋.
- The above graph is viewed as a group of steps and hence the integer function is also called a **Step function**.



Even and Odd Functions :

- If a given function is symmetric about the y-axis, it is known as an even function.
- A function is even if **$f(x) = f(-x)$ for all values of x**

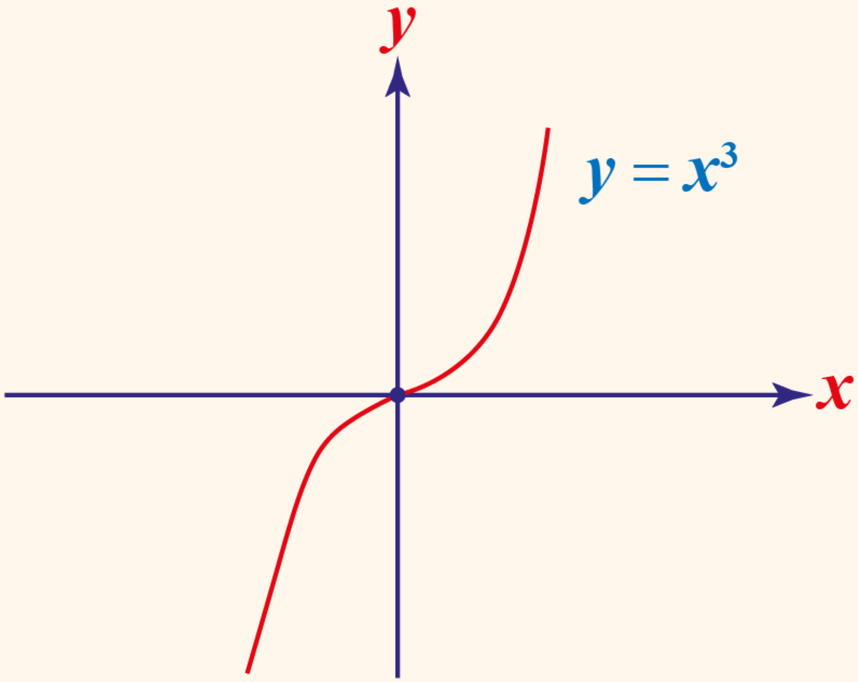
- For an even function $f(x)$, if we plug in $-x$ in place of x, then the value of $f(-x)$ is equal to the value of $f(x)$.
- Thus, the formula to check if the function is even is given as: **$f(x) = f(-x)$**

- A function in which one side of the x-axis is sign-inverted with respect to the other side or graphically, symmetric about the origin is known as an odd function.
- A function is **odd if $f(-x) = - f(x)$** for all values of x

- For an odd function $f(x)$, if we plug in $-x$ in place of x, then the value of $f(-x)$ is equal to the value of $- f(x)$.
- Thus, the formula to check if the function is odd is given as: **$f(-x) = - f(x)$**

Graph – even and odd functions

- $f(x) = x^3$



Domain and Range of a Function :

- The set of all possible values which qualify as inputs to a function is known as the **domain** of the function,
  or
  it can also be defined as the entire set of values possible for independent variables.

- The domain can be found in – the denominator of the fraction is not equal to zero and the digit under the square root bracket is positive.

Domain of most used functions :

- Domain of $\sqrt{a^2 - x^2}$ is [-a, a]
- Domain of $\frac{1}{\sqrt{a^2-x^2}}$ is (-a, a)
- Domain of $\sqrt{x^2 - a^2}$ is (-∞, -a] U [a, ∞)
- Domain of $\frac{1}{\sqrt{x^2-a^2}}$ is (-∞, -a) U (a, ∞)
- Domain of log(x-a)(x-b) is (-∞, -a) U (b, ∞)

Range of a Function :

- The set of all the outputs of a function is known as the range of the function or after substituting the domain, the entire set of all values possible as out comes of the dependent variable.

- **How to Find the Range of a Function**

- Consider a function y = f(x).
- The spread of all the y values from minimum to maximum is the range of the function.
- In the given expression of y, substitute all the values of x to check whether it is positive, negative or equal to other values.
- Find the minimum and maximum values for y.

Example 1:
Find the domain and range of a function $f(x) = 3x^2 - 5$.

Solution:
Given function:
$f(x) = 3x^2 - 5$

We know that the domain of a function is the set of input values for f, in which the function is real and defined.

The given function has no undefined values of x.
Thus, for the given function, the domain is the set of all real numbers.

Domain = $[-\infty, \infty]$
Also, the range of a function comprises the set of values of a dependent variable for which the given function is defined.

Ley $y = 3x^2 - 5$
$3x^2 = y + 5$
$x^2 = (y + 5)/3$
$x = \sqrt{[(y + 5)/3]}$
Square root function will be defined for non-negative values.

So, $\sqrt{[(y + 5)/3]} \geq 0$
This is possible when y is greater than $y \geq -5$.
Hence, the range of $f(x)$ is $[-5, \infty)$.

Example 2:
Find the domain and range of a function $f(x) = (2x – 1)/(x + 4)$.

Solution:
Given function is:
$f(x) = (2x – 1)/(x + 4)$

We know that the domain of a function is the set of input values for f, in which the function is real and defined.

The given function is not defined when $x + 4 = 0$, i.e. $x = -4$
So, the domain of given function is the set of all real number except -4.

i.e. Domain = $(-\infty, -4) \cup (-4, \infty)$
Also, the range of a function comprises the set of values of a dependent variable for which the given function is defined.

Let $y = (2x – 1)/(x + 4)$
$xy + 4y = 2x – 1$
$2x – xy = 4y + 1$
$x(2 – y) = 4y + 1$
$x = (4y + 1)/(2 – y)$
This is defined only when y is not equal to 2.
Hence, the range of the given function is $(-\infty, 2) \cup (2, \infty)$.

Composition of Functions :

- Let $f : A \to B$ and $g : B \to C$ be two functions. Then the composition of f and g, denoted by $g \circ f$, is defined as the function $g \circ f : A \to C$ given by $g \circ f(x) = g(f(x)), \forall x \in A$.

- The below figure shows the representation of **composite functions**.

- **Symbol:** It is also denoted as $(g \circ f)(x)$, where $\circ$ is a small circle symbol. We cannot replace $\circ$ with a dot (.), because it will show as the product of two functions, such as $(g.f)(x)$.

- **Domain:** $f(g(x))$ is read as f of g of x. In the composition of $(f \circ g)(x)$ the domain of function f becomes $g(x)$. The domain is a set of all values which go into the function.

- <u>**Properties of Function Compositions :**</u>

- **Associative Property:**

  $$f \circ (g \circ h) = (f \circ g) \circ h$$

- **Commutative Property:**

  $$g \circ f = f \circ g$$

- **Few more properties are:**
- The function composition of one-to-one function is always one to one.
- The function composition of two onto function is always onto
- The inverse of the composition of two functions f and g is equal to the composition of the inverse of both the functions, such as $(f \circ g)^{-1} = (g^{-1} \circ f^{-1})$.

Ex : If there are three functions, such as $f(x) = x$, $g(x) = 2x$ and $h(x) = 3x$. Then find the composition of these functions such as $[f \circ (g \circ h)](x)$ for $x = -1$.

Solution:
Given,
$f(x) = x$
$g(x) = 2x$
$h(x) = 3x$

To find: $[f \circ (g \circ h)](x)$

$[f \circ (g \circ h)](x) = f \circ (g(h(x)))$
$= f \circ g(3x)$
$= f(2(3x))$
$= f(6x)$
$= 6x$
If $x = -1$, then;
$[f \circ (g \circ h)](-1) = 6(-1) = -6$

That’s a Lot for Today!

Revise Your understanding a little!

Source : NCERT Class 11 Mathematics Text book

Solve Your Queries

This material will be shared with you for further assistance

THANK YOU!

Hope you had a basic Understanding about this Topic!

22a81a05q0@sves.org.in
+91 8008149866
22A81A05Q0 – CSE_D

V. Ganesh

Have a Nice Day ☺