IWCM 2013

International Workshop and Tutorial on Computational Mathematics - Advances in Computational PDEs

Tutorial

March 25 - 26 (10AM-3PM with lunch break)

Workshop

March 27 - 28

Where

Yonsei-Samsung Library,
Seminar Room / 7F
Yonsei University, Seoul, 120-749, Korea

http://cse.yonsei.ac.kr/iwcm2013/

Organizing Committee

Carsten Carstensen (Co-Chair)
Eun-Jae Park (Co-Chair)

Local Organizers

Jeehyun Lee
Eunjung Lee

Computational Science & Engineering

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International Workshop and Tutorial on Computational Mathematics -Advances in Computational PDEs

March 25-28, 2013

Department of Computational Science and Engineering, Yonsei University

IWCM 2013

International Workshop and Tutorial on Computational Mathematics - Advances in Computational PDEs

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International Workshop and Tutorial on Computational Mathematics -Advances in Computational PDEs

March 25-28, 2013

Department of Computational Science and Engineering, Yonsei University

Welcoming Message

Welcome to Yonsei University for the International Workshop and Tutorial on Computational Mathematics - Advances in computational PDEs.

The workshop and tutorial aim to provide a platform to disseminate recent developments in finite element methods and related methods for various PDEs, enable in-depth technical discussions on a wide variety of major computational efforts in science and engineering, and foster the interdisciplinary culture required to meet recent challenges in CSE. Further, it is planned to provide a good exposure for students and young researchers and encourage them to pursue their research career in these areas.

The 4-day program consists of 2-day tutorial and 2-day workshop, it brings together local and overseas researchers with diverse backgrounds. We believe you will find many talks of interests. We also hope that you will take time to explore the city of Seoul, one of the most exciting metropolises in the world. Wish you a most enjoyable stay!
Lastly but not least, we gratefully acknowledge the generous support of the National Research Foundation of Korea through the Department of Computational Science & Engineering, Yonsei University.

Organizing Committee

Carsten Carstensen
Humboldt University and Yonsei University

Eun-Jae Park
Yonsei University

Local Organizers

Eunjung Lee
Yonsei University

Jeehyun Lee
Yonsei University

Conference Coordinator

Haeeun Han
Dept. of CSE, Yonsei University

Useful Information

Yonsei Campus Map

Student Union

Food Court

Operating hour : 9:00 - 19:00

Woori Bank

Operating hour : 9:00 - 16:00

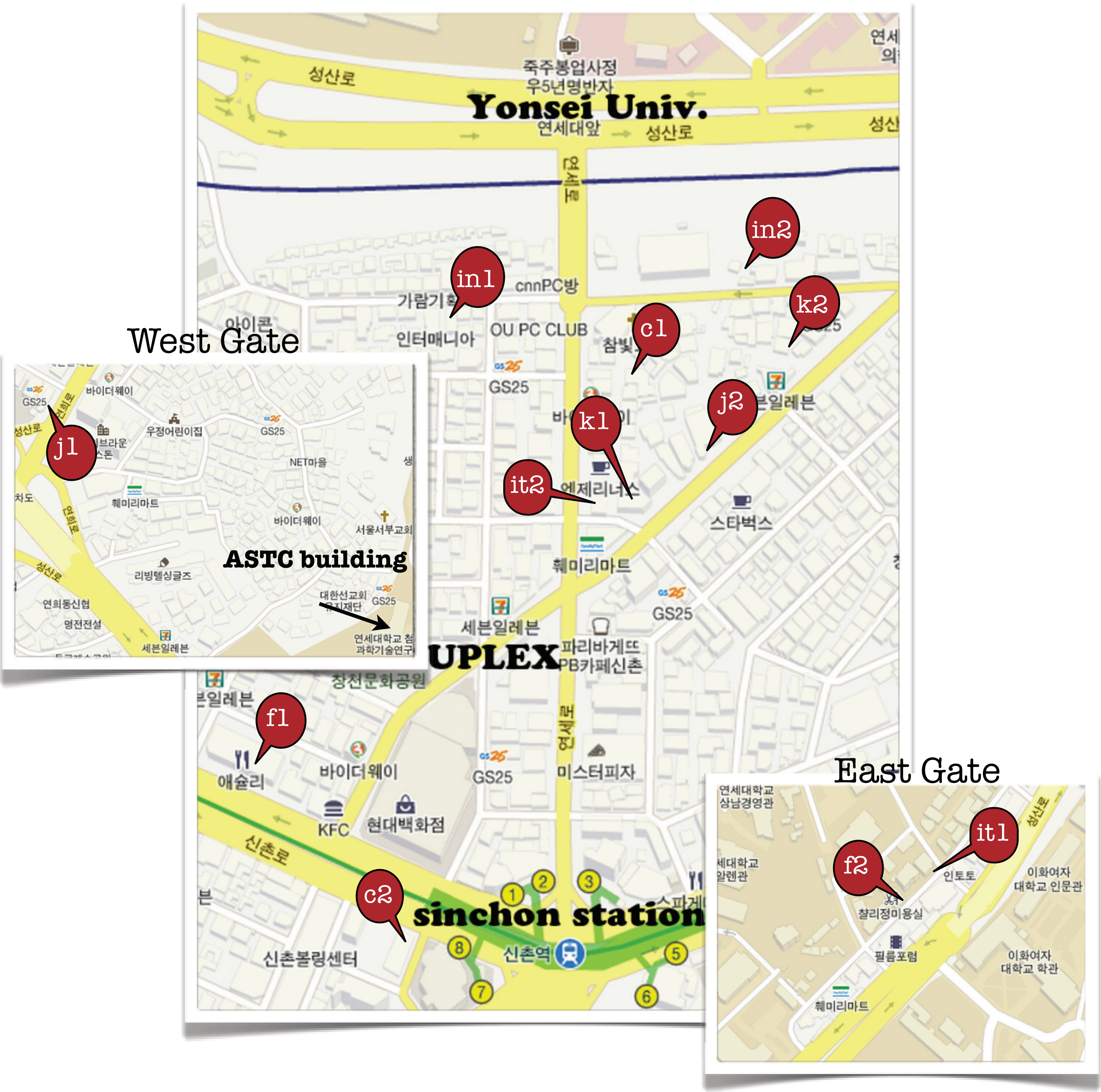
Post office

Operating hour : 9:00 - 18:00

Souvenir shop (Boram saem)

Operating hour : 9:00 - 19:00

Map



Yonsei Univ.
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성산로
cnnPC방
OU PC CLUB
인터매니아
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애슐리
바이더웨이
KFC
현대백화점
미스터피자
신촌로
sinchon station
신촌역
신촌볼링센터
in1
in2
c1
k2
j2
k1
it2
f1
c2
West Gate
j1
우정어린이집
NET마을
서울서부교회
ASTC building
리빙텔싱글즈
연희동신협
East Gate
연세대학교 상남경영관
it1
f2
인토토
이화여자 대학교 인문관
필름포럼
이화여자 대학교 학관

Korean

k1

Hyungjai Kalbi (meat)

Operating hour : 11:30 - 22:00

k2

Bongchu steamed chicken

Operating hour : 11:00 - 22:00

Indian

in1

Manokamana restaurant

Operating hour : 10:30 - 11:00

in2

NaaN

Operating hour : 11:00 - 23:30

Chinese

c1

Bok sung gak

Operating hour : 11:00 - 22:00

c2

Dong chun hong

Operating hour : 11:30 - 21:30

Italian

it1

ZINO

Operating hour : 11:30 - 15:00
17:00 - 23:00

it2

Italiano

Operating hour : 11:30 - 23:00

Japanese

j1

Hujisan (sushi)

Operating hour : 11:30 - 22:30

j2

Sushien

Operating hour : 11:30 - 22:30

Family Restaurant

f1

Ashley

Operating hour : 11:00 - 23:00

f2

Jessica's kitchen

Operating hour : 11:00 - 22:00

International Workshop and Tutorial on Computational Mathematics -Advances in Computational PDEs

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IWCM 2013

International Workshop and Tutorial on Computational Mathematics - Advances in Computational PDEs

March 25 / 26 (Monday / Tuesday)

Tutorial on Finite Element Approximation of Eigenvalue Problems Arising from Partial Differential Equations

By **Daniele Boffi** (University of Pavia, Italy)

Description

This tutorial is devoted to the study of the finite element approximation of eigenvalue problems. In the case of Galerkin discretizations of elliptic partial differential equations, the conditions for the convergence of eigenvalues and eigenfunctions are the same as for the convergence of the solutions to the corresponding source problem. We review the basic arguments of the analysis which is often referred to as Babuska-Osborn theory. In the case of mixed formulations, however, it will be shown that standard conditions for the source problem (like the well-known Babuska-Brezzi inf-sup condition) are not suffcient in general for the good convergence of eigenvalues and eigenfunctions. Finally, we shall discuss the numerical approximation of eigenvalue problems in the setting of differential forms. This includes the study of the Hodge-Laplace eigenvalue problem in the framework of de Rham complex. The abstract theory is applied to the approximation of Maxwell's equations.

Schedule	Day1
10:00-11:30	Lecture 1
11:30-12:00	Discussion
12:00-13:00	Lunch break
13:00-14:30	Lecture 2
14:30-15:00	Discussion

Schedule	Day2
10:00-11:30	Lecture 3
11:30-12:00	Discussion
12:00-13:00	Lunch break
13:00-14:30	Lecture 4
14:30-15:00	Discussion

International Workshop and Tutorial on Computational Mathematics -Advances in Computational PDEs

Chang Ki-Won Seminar room (Yonsei-Samsung Library 7F), Yonsei University
March 25-28, 2013

IWCM 2013
International Workshop and Tutorial on Computational Mathematics - Advances in Computational PDEs

March 25 / 26 (Monday / Tuesday)

Tutorial (Daniele Boffi, University of Pavia, Italy) From 10 am to 3 pm with lunch break

March 27 (Wednesday)

Time	Program
08:30-09:00	Registration
09:00-09:10	Opening Jin Keun Seo
	Session I
09:10-10:00	Arieh Iserles (Cambridge University, UK) Computing the Schrodinger equation with no fear of commutators
10:00-10:50	Dongwoo Sheen (Seoul National University) A class of nonparametric DSSY(Douglas-Santos-Sheen-Ye) elements and a cheapest finite element method for the Stokes equations
10:50-11:20	Coffee break
11:20-12:10	Daniele Boffi (University of Pavia, Italy) The finite element immersed boundary method for the approximation of fluid-structure interaction problems
12:00-14:00	Lunch break
	Session II
14:00-14:50	Martin Vohralik (INRIA Paris-Rocquencourt, France) Adaptive inexact Newton methods with a posteriori stopping criteria for nonlinear diffusion PDEs
14:50-15:30	Youngmok Jeon (Ajou University) The hybridized numerical schemes for flow and elasticity problems
15:30-16:10	Kwang-Yeon Kim (Kangwon National University) Improved a posteriori error estimates for the Stokes equation
16:10-16:40	Coffee break
	Session III
16:40-17:20	Thirupathi Gudi (Indian Institute of Science, India) A posteriori error control of discontinuous Galerkin methods for elliptic obstacle problem
17:20-17:50	Chunjae Park (Konkuk University) P1-nonconforming divergence-free finite element method on square grids for incompressible Stokes equations
18:00-	Banquet (Allen Hall)

* Registration and all lectures will take place at Chang Ki-Won seminar room (Yonsei Library 7F).

March 28 (Thursday)	
Time	Program
Session IV	
09:00-09:50	Dongbin Xiu (University of Utah, USA & CSE, Yonsei University) Multi-dimensional polynomial interpolation on arbitrary nodes
09:50-10:40	Chang-Ock Lee (KAIST) A dual iterative substructuring method with an optimized penalty parameter
10:40-11:10	Coffee break
11:10-12:00	Carsten Carstensen (Humboldt University of Berlin, Germany & CSE, Yonsei University) Guaranteed error control in CPDE
12:00-14:00	Lunch break
Session V	
14:00-14:50	Jun Hu (Peking University, China) Error estimates in the L^2 norm for finite elements of fourth order problems
14:50-15:30	Hyea Hyun Kim (Kyung Hee University) A staggered discontinuous Galerkin method for the Stokes system and its fast solvers
15:30-16:00	Dongwook Shin (Yonsei University) A hybrid discontinuous Galerkin method for convection-diffusion-reaction problems
16:00-16:30	Coffee break
Session VI	
16:30-17:10	Neela Natraj (IIT-Bombay, India) On a two-grid finite element scheme for the equations of motion arising in Kelvin-Voigt model
17:10-17:40	Eun-Hee Park (NIMS) A domain decomposition preconditioner for a discontinuous Galerkin method
17:40-18:10	Eunjung Lee (Yonsei University) FOSLL* for nonlinear partial differential equations

MARCH 27

COMPUTING THE SCHRÖDINGER EQUATION WITH NO FEAR OF COMMUTATORS

ARIEH ISERLES

Abstract. In this talk I report recent work on the solution of the linear Schrödinger equation (LSE) by exponential splitting in a manner that separates different frequency scales. The main problem in discretizing LSE originates in the presence of a very small parameter, which generates exceedingly rapid oscillation in the solution. However, it is possible to exploit the features of the graded free Lie algebra spanned by the Laplacian and by multiplication with the interaction potential to split the evolution operator in a symmetric Zassenhaus splitting so that the arguments of consecutive exponentials constitute an asymptotic expansion in the small parameter. Once we replace the Laplacian by an appropriate differentiation matrix, this results in a high-order algorithm whose computational cost scales like $O(N \log N)$, where N is the number of degrees of freedom and whose error is uniform in the small parameter.

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Key words and phrases. Schrödinger equation, exponential splitting, Lie-algebraic methods.

A class of nonparametric DSSY(Douglas-Santos-Sheen-Ye) elements and a cheapest finite element method for the Stokes equations [1]

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Abstract

In 1999 Douglas-Santos-Sheen-Ye (DSSY) [3] introduced (parametric) nonconforming quadrilateral element which are of four DOFs for rectangular (and parallelogram) meshes, but are of five DOFs for quadrilateral meshes [1]. Differently from the Rannacher-Turek quadrilateral element [6], one of the special characteristics of the DSSY element is the edgewise mean value property, which is fulfilled also by the Crouzeix-Raviart nonconforming simplicial element [2]. In this talk, we introduce a class of nonparametric DSSY elements of four DOFs for quadrilateral meshes [4].

In the second part of this talk, we introduce a cheapest finite element pair [5] for solving the Stokes equations, and also the Navier-Stokes equations. We precisely analyze the instability of the usual lowest order finite element pairs $([P_1]^2, P_0)$ for the Stokes problem. We conclude that adding a globally only one-dimensional bubble function suffices to make the system stable. Traditional stabilization method is an enrichment by adding bubble functions for all elements. Error estimates and numerical results will be presented.

References

[1] Z. Cai, J. Douglas, Jr., J. E. Santos, D. Sheen, and X. Ye. Nonconforming quadrilateral finite elements: A correction. *Calcolo*, 37(4):253–254, 2000.

[2] M. Crouzeix and P.-A. Raviart. Conforming and nonconforming finite element methods for solving the stationary Stokes equations. *R.A.I.R.O.– Math. Model. Anal. Numer.*, R-3:33–75, 1973.

[3] J. Douglas, Jr., J. E. Santos, D. Sheen, and X. Ye. Nonconforming Galerkin methods based on quadrilateral elements for second order elliptic problems. *ESAIM–Math. Model. Numer. Anal.*, 33(4):747–770, 1999.

[4] Y. Jeon, H. Nam, D. Sheen, and K. Shim. A class of nonparametric DSSY nonconforming quadrilateral elements. *ESAIM–Math. Model. Numer. Anal.*, 2013. submitted (arXiv:1301.1120).

[1]This talk is based on joint works with Youngmok Jeon (Ajou Univ.), Kwangshin Shim (SNU), Hyun Nam (SNU), Hyung Jun Choi (SNU), and Chunjae Park (Konkuk University)

[5] H. Nam, H. J. Choi, C. Park, and D. Sheen. A cheapest nonconforming rectangular finite element for the Stokes problem. *Comput. Methods Appl. Mech. Engrg.*, 257:77–86, 2013.

[6] R. Rannacher and S. Turek. Simple nonconforming quadrilateral Stokes element. *Numer. Methods Partial Differential Equations*, 8:97–111, 1992.

THE FINITE ELEMENT IMMERSED BOUNDARY METHOD FOR THE APPROXIMATION OF FLUID-STRUCTURE INTERACTION PROBLEMS

DANIELE BOFFI

Abstract. The Immersed Boundary Method (IBM) has been introduced by Peskin in the 70's in order to model and approximate fluid-structure interaction problems related to the blood flow in the heart. The original scheme makes use of finite differences for the discretization of the Navier–Stokes equations. We introduced a finite element formulation which has the advantage of handling the presence of the solid (modeled via a Dirac delta function) in a more natural way.

In this talk we review the finite element formulation of the IBM focusing, in particular, on the choice of the finite element spaces in order to guarantee a suitable mass conservation. Moreover, we present some links with the *fictitious domain* method.

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Key words and phrases. Fluid-Structure Interactions, Immersed Boundary Method, Finite Element Methods, Mass Conservation, CFL Condition.

ADAPTIVE INEXACT NEWTON METHODS WITH A POSTERIORI STOPPING CRITERIA FOR NONLINEAR DIFFUSION PDES

ALEXANDRE ERN AND MARTIN VOHRALÍK

ABSTRACT. We consider nonlinear algebraic systems resulting from numerical discretizations of nonlinear partial differential equations of diffusion type. To solve these systems, some iterative nonlinear solver, and, on each step of this solver, some iterative linear solver are used. We derive adaptive stopping criteria for both iterative solvers. Our criteria are based on an a posteriori error estimate which distinguishes the different error components, namely the discretization error, the linearization error, and the algebraic error. We stop the iterations whenever the corresponding error does no longer affect the overall error significantly. Our estimates also yield a guaranteed upper bound on the overall error at each step of the nonlinear and linear solvers. We prove the (local) efficiency and robustness of the estimates with respect to the size of the nonlinearity owing, in particular, to the error measure involving the dual norm of the residual. Our developments hinge on equilibrated flux reconstructions and yield a general framework. We show how to apply this framework to various discretization schemes like finite elements, nonconforming finite elements, discontinuous Galerkin, finite volumes, and mixed finite elements; to different linearizations like fixed point and Newton; and to arbitrary iterative linear solvers. Numerical experiments for the p-Laplacian illustrate the tight overall error control and important computational savings achieved in our approach. Details can be found in [1, 2].

REFERENCES

[1] ERN, A., AND VOHRALÍK, M. Adaptive inexact Newton methods with a posteriori stopping criteria for nonlinear diffusion PDEs. HAL Preprint 00681422 v2, submitted for publication, 2012.

[2] ERN, A., AND VOHRALÍK, M. Adaptive inexact Newton methods: a posteriori error control and speed-up of calculations. *SIAM News 46*, 1 (2013), 1,4.

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Key words and phrases. nonlinear diffusion PDE, nonlinear algebraic system, a posteriori error estimate, adaptive linearization, adaptive algebraic solution, adaptive mesh refinement, stopping criterion.

THE HYBRIDIZED NUMERICAL SCHEMES FOR FLOW AND ELASTICITY PROBLEMS

YOUNGMOK JEON, EUN-JAE PARK & DONGWOO SHEEN

ABSTRACT. In this presentation we introduce the hybridized finite element methods for partial differential equations. The method was introduced by the speaker and his colleagues. The method is base on 1) constructing local solutions on each triangle of a triangulation 2) patching local solutions by flux continuity on intercell boundaries. Main advantage of our approach is that 1) it is a locally conservative numerical scheme 2) there is a lot of reduction in degree of freedoms, compared to the conventional finite element methods.

In this talk we present the hybridized numerical schemes for

- Stokes equations
- Elasticity problems

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Key words and phrases. elasticity problem, elliptic equation, finite element, hybridization, Stokes equations.

IMPROVED A POSTERIORI ERROR ESTIMATES FOR THE STOKES EQUATION

KWANG-YEON KIM

ABSTRACT. Nowadays adaptive mesh refinement based on a posteriori error estimates is an indispensable tool for increasing efficiency of numerical methods in scientific computing. There are several types of a posteriori error estimates developed and extensively studied in past decades. Recently, much attention has been paid to the type of error estimates which guarantee upper bounds on the actual errors measured in appropriate norms. For the Stokes equation such error estimates involve the global inf-sup constant due to the difficulty of handling the divergence-free constraint, but estimation of the global inf-sup constant can cause another difficulty in practical computations.

In this work we present improved a posteriori error estimates for the Stokes equation which guarantee upper bounds on the energy norm of the velocity error but do not involve the global inf-sup constant. The basic idea is to divide the global domain into a set of simple subdomains and consider the inf-sup constant local to each subdomain. This means that our new error estimate is fully computable when all the local inf-sup constants are available (which is true, e.g., if every subdomain is star-shaped with respect to a point). It turns out that the local mass conservation plays a crucial role in this localization argument. Thus some implementational details are given for the Crouzeix–Raviart and Fortin–Soulie nonconforming finite elements.

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Key words and phrases. a posteriori error estimation, computable upper bound, Stokes equation, nonconforming finite element method, inf-sup constant.

A POSTERIORI ERROR CONTROL OF DISCONTINUOUS GALERKIN METHODS FOR ELLIPTIC OBSTACLE PROBLEM

THIRUPATHI GUDI AND KAMANA PORWAL

Abstract. We present an a posteriori error analysis of discontinuous Galerkin methods for elliptic obstacle problem. By using a key property of discontinuous Galerkin methods, we derive error estimates for many methods in a general framework. Another ingredient we propose in the analysis is a nonlinear enriching function that controls nonconformity in the discontinuous Galerkin methods. Numerical results will be shown to illustrate the performance of the error estimates.

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1991 *Mathematics Subject Classification.* 65N12, 65N15, 65N30.

Key words and phrases. Computational Mathematics, Partial Differential Equations, Finite Element Methods.

*Thirupathi Gudi.

P_1-NONCONFORMING DIVERGENCE-FREE FINITE ELEMENT METHOD ON SQUARE GRIDS FOR INCOMPRESSIBLE STOKES EQUATIONS

CHUNJAE PARK

ABSTRACT. In this talk, we will introduce a locally divergence-free finite element space on square meshes whose elements are linear on each square and continuous on each midpoint of edge. The proposed finite element space approximates a continuous divergence-free function as much as does a usual space whose degrees of freedom are values at vertices, although the former is as small as about a half of the latter in their dimensions. Furthermore, when we solve elliptic problems reduced from incompressible Stokes problems for the velocity only in the proposed space, the concerning system of linear equations is split into two smaller ones for the red and black squares. After solving the velocity first, the pressure in the Stokes problem is obtained by an explicit method.

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Key words and phrases. Divergence-free, Nonconforming Finite Element Methods, Stokes.

MARCH 28

MULTI-DIMENSIONAL POLYNOMIAL INTERPOLATION ON ARBITRARY NODES

DONGBIN XIU

Abstract. Polynomial interpolation is well understood on the real line. In multi-dimensional spaces, one often adopts a well established one-dimensional method and fills up the space using certain tensor product rule. Examples like this include full tensor construction and sparse grids construction. This approach typically results in fast growth of the total number of interpolation nodes and certain fixed geometrical structure of the nodal sets. This imposes difficulties for practical applications, where obtaining function values at a large number of nodes is infeasible. Also, one often has function data from nodal locations that are not by "mathematical design" and are "unstructured".

In this talk, we present a mathematical framework for conducting polynomial interpolation in multiple dimensions using arbitrary set of unstructured nodes. The resulting method, least orthogonal interpolation, is rigorous and has a straightforward numerical implementation. It can faithfully interpolate any function data on any nodal sets, even on those that are considered degenerate by the traditional methods. We also present a strategy to choose "optimal" nodes that result in robust interpolation. The strategy is based on optimization of Lebesgue function and has certain highly desirable mathematical properties.

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A DUAL ITERATIVE SUBSTRUCTURING METHOD WITH AN OPTIMIZED PENALTY PARAMETER

CHANG-OCK LEE AND EUN-HEE PARK

Abstract. The FETI-DP [1] method is a domain decomposition method, which enforces the continuity across the subdomain interface by Lagrange multipliers. A dual iterative substructuring method with a penalty term was introduced in the previous works by the authors [2, 3], which is a variant of the FETI-DP method in terms of the way to deal with the continuity on the interface. The proposed method imposes the continuity not only by using Lagrange multipliers but also by adding a penalty term which consists of a positive penalty parameter η and a measure of the jump across the interface. Due to the penalty term, the proposed iterative method has a better convergence property than the standard FETI-DP method in the sense that the condition number of the resultant dual problem is bounded by a constant independent of the subdomain size and the mesh size.

In this talk, we will discuss a further study for a dual iterative substructuring method with a penalty term in terms of its convergence analysis and practical efficiency. On one hand, the convergence studies in [2, 3] rule out the case when a relatively small η is taken. On the other hand, the condition number estimate without any size limitation on η will be presented. As a result, a choice of a small η is not restricted so that we do not suffer from the ill-conditioned property of subdomain problems due to a large η. Moreover, based on the close relationship between the FETI-DP method and the proposed method, which results from the condition number estimate, it is shown that a penalty parameter chosen in a certain range, called a nearly optimal range of η, is sufficient to accelerate the convergence of the dual iteration. In addition, inner preconditioners for subdomain problems will be discussed for an improvement of practical efficiency.

References

[1] C. Farhat and M. Lesoinne and K. Pierson. A scalable dual-primal domain decomposition method. *Numer. Linear Algebra Appl.*, 7 (2000), pp. 687–714.
[2] C.-O. Lee and E.-H. Park. A dual iterative substructuring method with a penalty term. *Numer. Math.*, 112 (2009), pp. 89–113.
[3] C.-O. Lee and E.-H. Park. A dual iterative substructuring method with a penalty term in three dimensions. *Comput. Math. Appl.*, 64 (2012), pp. 2787–2805.

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Key words and phrases. Augmented Lagrangian, Dual Substructuring Method, FETI-DP, Penalty Parameter.

GUARANTEED ERROR CONTROL IN CPDE*

C. CARSTENSEN

Abstract. Five classes of up to 13 a posteriori error estimators compete in three second-order model cases, namely the conforming and non-conforming first-order approximation of the Poisson-Problem plus some conforming obstacle problem and non-conforming Crozeix- Raviart FEM for the Stokes problem. Since it is the natural first step, the error is estimated in the energy norm exclusively – hence the competition has limited relevance. The competition allows merely guaranteed error control and excludes the question of the best error guess. Even non-smooth problems can be included. For a variational inequality, Braess considers Lagrange multipliers and some resulting auxiliary equation to view the a posteriori error control of the error in the obstacle problem as computable terms plus errors and residuals in the auxiliary equation. Hence all the former a posteriori error estimators apply to this nonlinear benchmark example as well and lead to surprisingly accurate guaranteed upper error bounds. This approach allows an extension to more general boundary conditions and a discussion of efficiency for the affine benchmark examples. The Luce-Wohlmuth and the least-square error estimators win the competition in several computational benchmark problems. Novel equilibration of nonconsistency residuals and novel conforming averaging error estimators win the competition for Crouzeix-Raviart nonconforming finite element methods. Eventually, a novel postprocessing for the compared equilibration techniques leads to even higher accuracy at low costs. Our numerical results provide sufficient evidence that guaranteed error control in the energy norm is indeed possible with efficiency indices between one and two. Furthermore, accurate error control is slightly more expensive but pays off in all applications under consideration while adaptive mesh-refinement is sufficiently pleasant as accurate when based on explicit residual-based error estimates. Details of our theoretical and empirical ongoing investigations will be found in the papers quoted below.

References

[1] S. Bartels, C. Carstensen, R. Klose, *An experimental survey of a posteriori Courant finite element error control for the Poisson equation*, Adv. Comput. Math., 15 (2001), pp. 79-106.

[2] C. Carstensen, C. Merdon, *Estimator competition for Poisson problems*, J. Comp. Math., 28 (2010), pp. 309-330.

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[3] C. Carstensen, C. Merdon, *Computational survey on a posteriori error estimators for nonconforming finite element methods for Poisson problems*

[4] C. Carstensen, C. Merdon, *A posteriori error estimator competition for conforming obstacle problems*, Numer. Methods Partial Differential Eq.. doi: 10.1002/num.21728 (HU Preprint 12-01)

[5] C. Carstensen, C. Merdon, *Effective postprocessing for equilibration a posteriori error estimators* Numer. Math (2012)

[6] C. Carstensen, C. Merdon, *Computational survey On a posteriori Error estimators for nonconforming finite element methods for the Stokes problem*

ERROR ESTIMATES IN THE L^2 NORM FOR FINITE ELEMENTS OF FOURTH ORDER PROBLEMS

JUN HU

Abstract. This talk presents the L^2 norm error estimate of the lower order finite element methods for the fourth order problem. It is proved that the best error estimate in the L^2 norm of the finite element solution is of second order, which can not be improved generally. The main ingredients are the saturation condition established for these elements and an identity for the error in the energy norm of the finite element solution. The result holds for most of the popular lower order finite element methods in the literature including: the Powell-Sabin $C^1 - P_2$ macro element, the nonconforming Morley element, the $C^1 - Q_2$ macro element, the nonconforming rectangle Morley element, and the nonconforming incomplete biquadratic element, and the Adini element.

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A STAGGERED DISCONTINUOUS GALERKIN METHOD FOR THE STOKES SYSTEM AND ITS FAST SOLVERS

HYEA HYUN KIM, ERIC T. CHUNG, AND CHAK SHING LEE

ABSTRACT. In this talk, we will present a new staggered discontinuous Galerkin method for the Stokes system. The key feature of our method is that the discrete system preserves the structures of the continuous problem, which results from the use of our new staggered DG spaces. This also provides local and global conservation properties, which are desirable for fluid flow applications. The method is based on the first order mixed formulation involving pressure, velocity and velocity gradient. The velocity and velocity gradient are approximated by polynomials of the same degree while the choice of degree for pressure is flexible, namely the approximation degree for the pressure can be chosen as either that of velocity or one degree lower than that of velocity. In any case, stability and optimal convergence of the method are proved. Moreover, a superconvergence result with respect to a discrete H1-norm for the velocity is proved. In addition, we present a domain decomposition algorithm for fast solution of our DG formulation.

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Key words and phrases. staggered mesh, discontinuous Galerkin, Stokes system, preconditioner.

A HYBRID DISCONTINUOUS GALERKIN METHOD FOR CONVECTION-DIFFUSION-REACTION PROBLEMS

DONG-WOOK SHIN, YOUNGMOK JEON, AND EUN-JAE PARK

Abstract. A hybrid discontinuous Galerkin method introduced by Y. Jeon and E.-J. Park [Y. Jeon and E.-J. Park, *SIAM J. Numer. Anal.*, 48 (2010), pp.1968-1983] is applied to convection-diffusion-reaction problems. The hybrid discontinuous Galerkin method can be viewed a hybridizable discontinuous Galerkin method [B. Cockburn, J. Gopalakrishnan, and R. Lazarov, *SIAM J. Numer. Anal.*, 47 (2009), pp.1319-1365] using a Baumann-Oden type local solver. The method conserves the mass in each element and the average flux is continuous across the interelement boundary for even degree polynomial approximations. In this work, we propose upwind version of hybrid discontinuous Galerkin method to resolve layers arising in convection dominated diffusion equations. The stability of the formulation is proved, as is order $k + 1/2$ convergence for the convection-dominated case and order $k + 1$ convergence for the diffusive limit in the L^2 norm. Several numerical examples are presented to show the performance of the method.

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Key words and phrases. hybridization, discontinuous Galerkin, convection-diffusion-reaction.

On a two-grid finite element scheme for the equations of motion arising in Kelvin-Voigt model

Saumya Bajpai, Neela Nataraj and Amiya K. Pani

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Abstract : In the talk, we discuss a two level method based on Newton's iteration for the nonlinear system arising from Galerkin finite element approximation to the equations of motion described by Kelvin-Voigt viscoelastic fluid flow model. The two-grid algorithm is based on three steps. In the first step, the nonlinear system is solved on a coarse mesh $\mathcal{T}_H$ to obtain an approximate solution u_H. In the second step, the nonlinear system is linearized around u_H based on Newton's iteration and the linear system is solved on a finer mesh $\mathcal{T}_h$. Finally, in the third step, a correction to the results obtained in the second step is achieved by solving a linear problem with a different right hand side on $\mathcal{T}_h$. Optimal error estimates are discussed.

A DOMAIN DECOMPOSITION PRECONDITIONER FOR A DISCONTINUOUS GALERKIN METHOD

SUSANNE C. BRENNER, EUN-HEE PARK, AND LI-YENG SUNG

Abstract. In this talk we will discuss a nonoverlapping domain decomposition preconditioner for a symmetric interior penalty Galerkin (SIPG) method for the heterogeneous elliptic problem. The proposed preconditioner is based on balancing domain decomposition by constraints (BDDC) methodology. Theoretical results on the condition number estimate of the preconditioned system will be presented along with numerical results.

The linear system resulting from the SIPG discretization is ill-conditioned with respect to the discontinuous coefficient ρ in the heterogeneous elliptic problem as well as mesh refinements. In the design of our preconditioner, the discontinuity of finite element functions causes the major difficulty in localizing the bilinear form and deriving a norm equivalence relationship. Firstly, a subspace decomposition is introduced for the localization which is crucial in constructing a BDDC preconditioner. Secondly, an enriching technique of the discontinuous finite element space plays a major role in deriving the norm equivalence which is crucial for condition number estimates of preconditioned Schur complement systems. As results, it is shown that the condition number of the preconditioned system grows at the rate of $O((1 + \ln(H/h))^2)$ independent of ρ, where H and h are the subdomain size and the mesh size.

References

[1] J. Douglas, Jr. and T. Dupont. Interior penalty procedures for elliptic and parabolic Galerkin methods. *Lecture Notes in Phys. 58*, Springer-Verlag, Berlin, 1976, pp. 207–216.

[2] M.F. Wheeler. An elliptic collocation-finite-element method with interior penalties. *SIAM J. Numer. Anal.*, 15 (1978), pp. 152–161.

[3] D.N. Arnold. An interior penalty finite element method with discontinuous elements. *SIAM J. Numer. Anal.*, 19 (1982), pp. 742–760.

[4] C.R. Dohrmann. A preconditioner for substructuring based on constrained energy minimization. *SIAM J. Sci. Comput.*, 25 (2003), pp. 246–258.

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Key words and phrases. Nonoverlapping Domain Decomposition, BDDC Preconditioner, Symmetric Interior Penalty Method.

FOSLL* FOR NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

EUNJUNG LEE, THOMAS A. MANTEUFFEL AND CHAD R. WESTPHAL

Abstract. The Newton's method with first-order system least squares (FOSLS) finite element method has been widely used to approximately solve a system of nonlinear partial differential equations [1, 2, 3]. In this paper, we propose to use first order system LL* method to find a correction in each Newton's iteration which provides L^2-approximation while the typical Newton-FOSLS method provides H^1-approximation.

[1] J. H. Adler, T. A. Manteuffel, S. F. McCormick, and J. W. Ruge. *First-order system least squares for incompressible resistive magnetohydrodynamics* SIAM J. Sci. Comput., 32(1):229.248, 2010.
[2] A. L. Codd, T. A. Manteuffel, and S. F. McCormick. *Multilevel first-order system least squares for nonlinear elliptic partial differential equations* SIAM J. Numer. Anal., 41(6):2197.2209, 2003.
[3] T. A. Manteuffel, S. F. McCormick, J. G. Schmidt, and C. R. Westphal. *First-order system least squares for geometrically nonlinear elasticity* SIAM J. Numer. Anal., 44(5):2057.2081 (electronic), 2006.

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