1.

Definition 0.1. *The* **average value** *of a continuous function $f(x,y)$ over a rectangle $R = [a,b] \times [c,d]$ is defined as*

$$f_{ave} = \frac{1}{A(R)} \iint\limits_R f(x,y)\, dA$$

where $A(R) = (b-a)(d-c)$ is the area of the rectangle R.

Suppose that the temperature in degrees Celsius at a point (x,y) on a flat metal plate is $T(x,y) = 10 - 8x^2 - 2y^2$, where x and y are in meters. Find the average temperature of the rectangular portion of the plate for which $0 \le x \le 1$ and $0 \le y \le 2$.

2. Give a geometric argument to show that

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \sqrt{1 - x^2 - y^2}\, dx\, dy = \frac{\pi}{6}.$$

3.

Definition 0.2. *The* **average value** *of a continuous function $f(x,y)$ over a region R in the xy-plane is defined as*

$$f_{ave} = \frac{1}{A(R)} \iint\limits_R f(x,y)\, dA$$

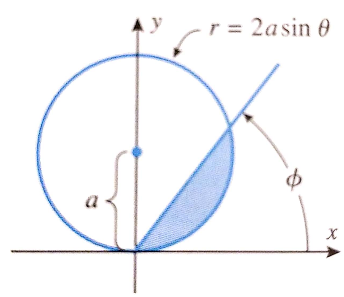
where $A(R)$ is the area of the region R.

Find the average value of $f(x,y) = x^2 - xy$ over the region enclosed by $y = x$ and $y = 3x - x^2$.

4. Use a double integral in polar coordinates to find the volume of the oblate spheroid

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{c^2} = 1, \quad (0 < c < a).$$

5. Show that the shaded area in the accompanying figure is $a^2\phi - \frac{1}{2}a^2 \sin 2\phi$.



6. Evaluate $\iint\limits_R x^2 \, dA$ over the region R in the accompanying figure.