NCERT Physics 12.7 Q19

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Question: Suppose the circuit in Exercise 7.18 (in Figure Fig. 1)has a resistance of 15 Ω. Obtain the average power transferred to each element of the circuit, and the total power absorbed.

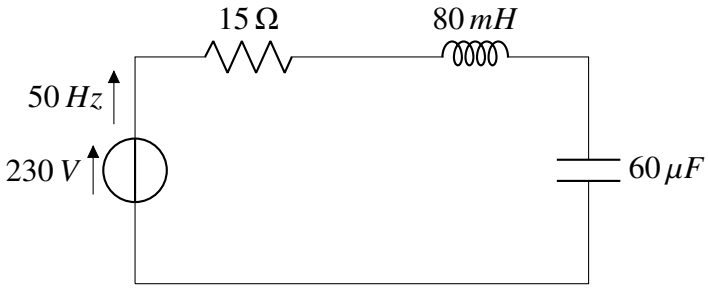

Fig. 1. LCR Circuit

Solution: In Fig. 1 the following information is provided:

Symbol	Value	Description
L	$80mH$	Inductance
C	$60\mu F$	Capacitance
R	$15\,\Omega$	Resistance
V_{rms}	$230\,V$	Voltage
f	$50\,Hz$	Frequency
ω	$2\pi f = 100\pi$	Angular Frequency
ϕ	?	Phase difference between current and voltage
I_{rms}	?	rms value of current

TABLE I
GIVEN PARAMETERS

Applying Kirchoff's Voltage Law in the Fig. 2

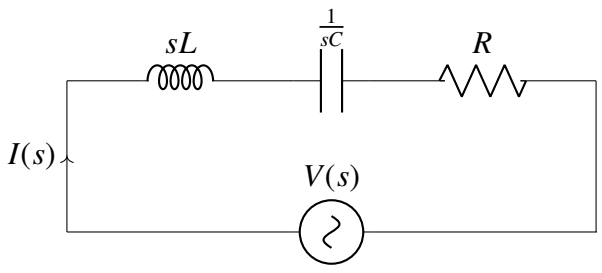

Fig. 2. s domain circuit

$$V(s) = RI(s) + sLI(s) + \frac{1}{sC}I(s) \tag{1}$$

$$= I(s)\left(R + Ls + \frac{1}{sC}\right) \tag{2}$$

$$I(s) = \frac{V(s)}{\left(R + Ls + \frac{1}{sC}\right)} \tag{3}$$

$$H(s) = \frac{V(s)}{I(s)} \tag{4}$$

$$H(s) = R + sL + \frac{1}{sC} \tag{5}$$

Substituting s with $\jmath\omega$

$$H(j\omega) = R + j\omega L + \frac{1}{j\omega C} \tag{6}$$

$$\Rightarrow |H(j\omega)| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \tag{7}$$

Average power transferred to an element in the circuit is given by:

$$P = VIcos(\phi) \tag{8}$$

a) Average power transferred to the capacitor, P_C: For a capacitor the phase angle is:

$$\phi = \frac{\pi}{2} \tag{9}$$

$$cos(\phi) = 0 \tag{10}$$

$$P_C = 0 \tag{11}$$

b) Average power transferred to the inductor, P_L: For an inductor the phase angle is:

$$\phi = -\frac{\pi}{2} \tag{12}$$

$$cos(\phi) = 0 \tag{13}$$

$$P_L = 0 \tag{14}$$

c) Average Power transferred to the resistor, P_R: $|H(j\omega)|$ is obtained by sustituting the numerical values from the Table I in equation (7):

$$|H(j\omega)| = 31.728\ \Omega \tag{15}$$

$$I_{rms} = \frac{V_{rms}}{H(j\omega)} = \frac{230}{31.728} = 7.25A \tag{16}$$

$$P_R = (I_{rms})^2 R = 788.44W \tag{17}$$

d)Total power absorbed by circuit:

$$= P_R + P_C + P_L \tag{18}$$
$$= 788.44 + 0 + 0 \tag{19}$$
$$= 788.44 \ \ W \tag{20}$$

Total power absorbed by circuit is 788.44W

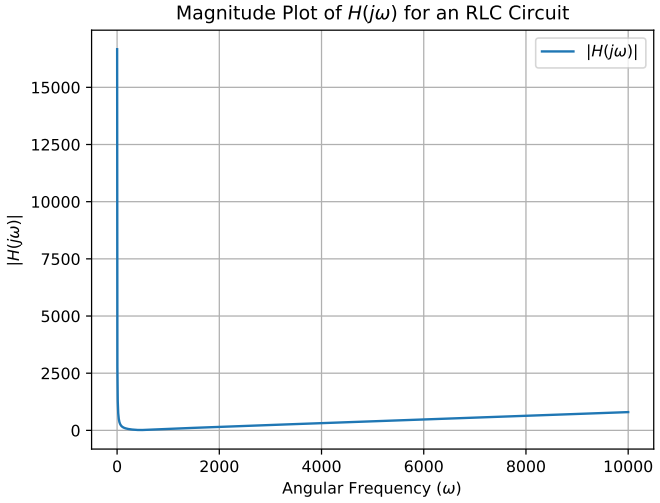

Fig. 3. $|H(j/omega)|$ vs ω