NCERT Physics 12.7 Q19

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Question: Suppose the circuit in Exercise 7.18 (in Figure Fig. 1)has a resistance of 15 Ω. Obtain the average power transferred to each element of the circuit, and the total power absorbed.

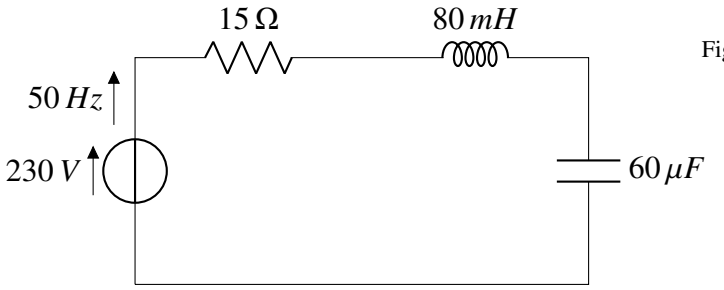

Fig. 1. LCR Circuit

Solution: In Figure Fig. 1 the following information is provided:

Symbol	Value	Description
L	80m H	Inductance
C	60 μF	Capacitance
R	15 Ω	Resistance
V	230 V	Voltage
f	50 Hz	Frequency

TABLE I
GIVEN PARAMETERS

Angular frequency of signal,

$\omega = 2\pi f = 2\pi \cdot (50) = 100\pi$

Applying Kirchoff's Voltage Law:

$$V(s) = RI(s) + sLI(s) + \frac{1}{sC}I(s) \tag{1}$$

$$\Rightarrow V(s) = I(s)\left(R + Ls + \frac{1}{sC}\right) \tag{2}$$

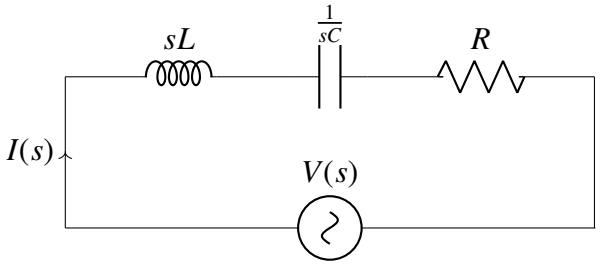

Fig. 2. LCR Circuit

$$\Rightarrow I(s) = \frac{V(s)}{\left(R + Ls + \frac{1}{sC}\right)} \tag{3}$$

Average Power transferred to the resistor is given by :

$$P_R = I^2(s) \cdot R \tag{4}$$

Average Power transferred to the inductor is given by :

$$P_L = I^2(s) \cdot \mathrm{Re}(Z_L(s)) \tag{5}$$

Average Power transferred to the capacitor is given by :

$$P_C = I^2(s) \cdot \mathrm{Re}(Z_C(s)) \tag{6}$$

Since the reactive components of inductor and capacitor have imaginary impedances,

$$\mathrm{Re}(Z_L) = 0 \tag{7}$$

$$\mathrm{Re}(Z_C) = 0 \tag{8}$$

Average power transferred to the capacitor, P_C = Average power transferred to the inductor, $P_L = 0$

$$H(s) = \frac{V(s)}{I(s)} \tag{9}$$

$$H(s) = R + sL + \frac{1}{sC} \tag{10}$$

Substituting s with $\jmath\omega$

$$H(j\omega) = R + j\omega L + \frac{1}{j\omega C} \tag{11}$$

$$\Rightarrow |H(j\omega)| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \tag{12}$$

Impedence is given by:

$$Z = \sqrt{R^2 + \left(\omega \cdot L - \frac{1}{\omega \cdot C}\right)^2} \tag{13}$$

$$= \sqrt{15^2 + \left(100\pi \cdot (80 \times 10^{-3}) - \frac{1}{100\pi \times 60 \times 10^{-6}}\right)^2} \tag{14}$$

$$= \sqrt{15^2 + (25.12 - 53.08)^2} \tag{15}$$

$$= 31.728 \ \Omega \tag{16}$$

Current flowing through the circuit I is :

$$I = \frac{V}{Z} = \frac{230}{31.728} \tag{17}$$

$$= 7.25 \ A \tag{18}$$

Average power transferred to resistance is given by :

$$P_R = I^2 \cdot R = (7.25)^2 \times 15 \tag{19}$$

$$= 788.44 \ W \tag{20}$$

Total power absorbed by circuit:

$$= P_R + P_C + P_L \tag{21}$$

$$= 788.44 + 0 + 0 \tag{22}$$

$$= 788.44 \ W \tag{23}$$

Total power absorbed by circuit is 788.44W

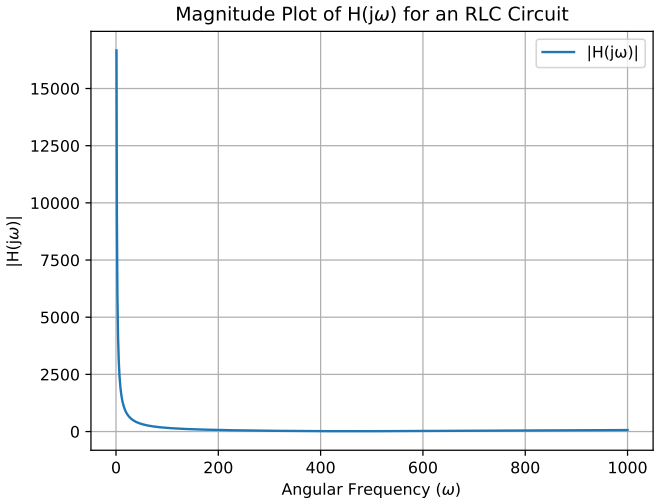

Fig. 3. Impedance vs ω