

Gate 2021 BM Q8

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For a linear stable second order system, if the unit step response is such that peak time is twice the rise time, then the system is .

- 1) underdamped
- 2) undamped
- 3) overdamped
- 4) critically damped

Solution:

Parameter	Description
ω_n	natural frequency
ζ	damping ratio
θ	is the angle in the complex plane corresponding to the pole location

TABLE 4
GIVEN PARAMETERS

The rise time is given by:

$$t_r = \frac{\pi - \theta}{\omega_n \sqrt{1 - \zeta^2}} \quad (1)$$

The peak time is given by:

$$t_p = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}} \quad (2)$$

as, peak time is twice the rise time:

$$t_p = 2t_r \quad (3)$$

$$\frac{\pi}{\omega_n \sqrt{1 - \zeta^2}} = 2 \frac{\pi - \theta}{\omega_n \sqrt{1 - \zeta^2}} \quad (4)$$

$$\theta = \frac{\pi}{2} \quad (5)$$

as, $\theta = \frac{\pi}{2}$, both roots of the system are imaginary, so

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (6)$$

So, for the denominator to have two imaginary roots $2\zeta\omega_n$ should be zero. so,

$$\zeta = 0 \quad (7)$$

ζ is zero, hence system is undamped.