Gate 2022 EC Q55

EE23BTECH11212 - Manugunta Meghana Sai*

Gate 2022 EE Q55

For a vector $\bar{x} = [x[0], x[1], \ldots, x[7]]$, the 8-point discrete Fourier transform (DFT) is denoted by $\bar{X} = \text{DFT}(\bar{x}) = [X[0], X[1], \ldots, X[7]]$, where

$$X[k] = \sum_{n=0}^{7} x[n] \exp\left(-j\frac{2\pi}{8}nk\right).$$

Here $j = \sqrt{-1}$. If $\bar{x} = [1, 0, 0, 0, 2, 0, 0, 0]$ and $\bar{y} = \text{DFT}(\text{DFT}(\bar{x}))$, then the value of $y[0]$ is

Solution:

Parameter	Description	Value
$\bar{X}$	$\text{DFT}(\bar{x})$	–
$\bar{x}$	vector	$[1, 0, 0, 0, 2, 0, 0, 0]$
$\bar{y}$	$\text{DFT}(\text{DFT}(\bar{x}))$	–

TABLE 0
GIVEN PARAMETERS

DFT of $\bar{x}$

$$X[k] = \sum_{n=0}^{7} x[n] \exp\left(-j\frac{2\pi}{8}nk\right) \tag{1}$$

As the only non-zero values in x are x[0] and x[4]:

$$X[k] = x[0] + x[4] \exp(-j\pi k) \tag{2}$$

After substituting the values of k ranging from 0 to 7,

$$\bar{X} = \text{DFT}(\bar{x}) = [X[0], X[1], \ldots, X[7]] \tag{3}$$
$$\bar{X} = [3, -1, 3, -1, 3, -1, 3, -1] \tag{4}$$

$$\bar{y} = \text{DFT}(\text{DFT}(\bar{x})) \tag{5}$$
$$\bar{y} = [3, -1, 3, -1, 3, -1, 3, -1] \tag{6}$$
$$y[0] = \sum_{n=0}^{7} x[n] \tag{7}$$
$$= x[0] + x[1] + \cdots + x[7] \tag{8}$$
$$= 3 - 1 + 3 - 1 + 3 - 1 + 3 - 1 = 8 \tag{9}$$