

NCERT Maths 11.9.2 Q6

EE23BTECH11212 - MANUGUNTA MEGHANA SAI*

Question: If the sum of certain number of terms in a AP 25,22,19,... is 116. Find the last term.

The sum of the terms of the sequence is computed using the residue theorem, expressed as R_i , which represents the residue of the Z-transform at $z = 1$ for the expression $Y(z)$.

Solution:

Symbol	Value	Description
$x(0)$	25	first term of AP
$x(1)$	22	second term of AP
$x(2)$	19	third term of AP
d	-3	common difference
$y(n)$	116	sum of terms

TABLE I
GIVEN PARAMETERS

$$x(n) = (25 - 3n)u(n)$$

Applying Z transform:

$$\begin{aligned} x(z) &= \frac{25}{1 - z^{-1}} - \frac{3z^{-1}}{(1 - z^{-1})^2} \\ &= \frac{25 - 28z^{-1}}{(1 - z^{-1})^2} \end{aligned}$$

Region of Convergence or R.O.C :

$$|z| > 1$$

For AP, the sum of first $n+1$ terms can be written as :

$$y(n) = x(n) * u(n) \quad (5)$$

Applying Z transform on both sides

$$Y(z) = x(z)u(z) \quad (6)$$

$$= \frac{25}{(1 - z^{-1})^2} - \frac{3z^{-1}}{(1 - z^{-1})^3} \quad (7)$$

Using contour integration to find inverse Z transform:

$$Y(n) = \frac{1}{2\pi j} \oint_C Y(z)z^{n-1}dz \quad (8)$$

$$= \frac{1}{2\pi j} \oint_C \left[\frac{25}{(1 - z^{-1})^2} - \frac{3z^{-1}}{(1 - z^{-1})^3} \right] z^{n-1} dz \quad (9)$$

$$R_i = R_1 + R_2 \quad (10)$$

R_1 and R_2 are residues calculated at the poles of the Z-transform.

$$R_1 = \frac{1}{(2-1)!} \left. \frac{d(25z^{n+1})}{dz} \right|_{z=1} \quad (11)$$

$$= 25(n+1) \quad (12)$$

$$R_2 = \frac{1}{(3-1)!} \left. \frac{d^2(-3z^{n+1})}{dz^2} \right|_{z=1} \quad (13)$$

$$= \frac{-3}{2}(n+1)(n) \quad (14)$$

(1) The sum of terms is given by R_i :

$$25(n+1) + \frac{-3}{2}n(n+1) = 116 \quad (15)$$

Solving the equation gives :

$$n = 7 \quad (16)$$

Upon substituting the value of n in 1 :

$$x(7) = 4 \quad (17)$$

Hence the last term of the given AP is 4.

