NCERT Maths 12.7 Q19

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Question: If the sum of certain number of terms in a AP 25,22,19,... is 116. Find the last term.

Solution:

Symbol	Value	Description
$x(0)$	25	first term of AP
$x(1)$	22	second term of AP
$x(2)$	19	third term of AP
d	−3	common difference
$y(n)$	116	sum of terms

TABLE I
GIVEN PARAMETERS

$$x(n) = (25 - 3n)u(n) \tag{1}$$

Applying Z transform:

$$x(z) = \frac{25}{1 - z^{-1}} - \frac{3z^{-1}}{(1 - z^{-1})^2} \tag{2}$$

$$= \frac{25 - 28z^{-1}}{(1 - z^{-1})^2} \tag{3}$$

Region of Convergence or R.O.C :

$$|z| > 1 \tag{4}$$

For AP, the sum of first n+1 terms can be written as :

$$y(n) = \sum_{k=0}^{n} x(k) \tag{5}$$

$$= \sum_{k=-\infty}^{\infty} x(k)u(n - k) \tag{6}$$

$$y(n) = x(n) * u(n) \tag{7}$$

Applying Z transform on both sides

$$Y(z) = x(z)u(z) \tag{8}$$

$$= \frac{25}{(1 - z^{-1})^2} - \frac{3z^{-1}}{(1 - z^{-1})^3} \tag{9}$$

Using contour integration to find inverse Z transform:

$$Y(n) = \frac{1}{2\pi j} \oint_C Y(z)z^{n-1}dz \tag{10}$$

$$= \frac{1}{2\pi j} \oint_C [\frac{25}{(1 - z^{-1})^2} - \frac{3z^{-1}}{(1 - z^{-1})^3}]z^{n-1}dz \tag{11}$$

$$R_i = R_1 + R_2 \tag{12}$$

$$R_1 = \frac{1}{(2 - 1)!} lim_{z\to 1} \frac{d(25z^{n+1})}{dx} \tag{13}$$

$$= 25(n + 1) \tag{14}$$

$$R_2 = \frac{1}{(3 - 1)!} lim_{z\to 1} \frac{d^2(-3z^{n+1})}{dx^2} \tag{15}$$

$$= \frac{-3}{2}(n + 1)(n) \tag{16}$$

The sum of terms is given by R_i:

$$25(n + 1) + \frac{-3}{2}n(n + 1) = 116 \tag{17}$$

Solving the equation gives :

$$n = 7 \tag{18}$$

Upon substituting the value of n in 1 :

$$x(7) = 4 \tag{19}$$

Hence the last term of the given AP is 4.

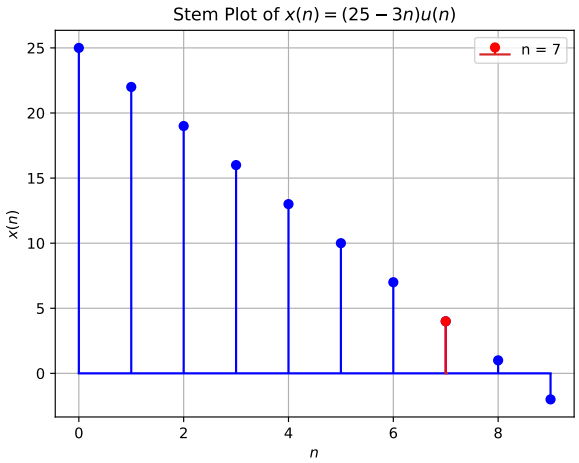

Fig. 1. $|H(j/omega)|$ vs ω