NCERT Maths 11.9.2 Q6

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Question: If the sum of certain number of terms in a AP 25,22,19,... is 116. Find the last term.

Solution:

Symbol	Value	Description
$x(0)$	25	first term of AP
d	-3	common difference
$x(n)$	$(25-3n)u(n)$	n-th term of AP
$y(n)$	116	sum of terms

TABLE I
GIVEN PARAMETERS

$$x(n) = (25-3n)u(n) \tag{1}$$

Applying Z transform:

$$x(z) = \frac{25}{1-z^{-1}} - \frac{3z^{-1}}{(1-z^{-1})^2} \tag{2}$$

$$= \frac{25-28z^{-1}}{(1-z^{-1})^2} \tag{3}$$

Region of Convergence or R.O.C :

$$|z| > 1 \tag{4}$$

For AP, the sum of first n+1 terms can be written as :

$$y(n) = x(n) * u(n) \tag{5}$$

Applying Z transform on both sides

$$Y(z) = X(z)U(z) \tag{6}$$

$$= \frac{25}{(1-z^{-1})^2} - \frac{3z^{-1}}{(1-z^{-1})^3} \tag{7}$$

Using contour integration to find inverse Z transform:

$$y(n) = \frac{1}{2\pi j}\oint_C Y(z)z^{n-1}dz \tag{8}$$

$$= \frac{1}{2\pi j}\oint_C \left[\frac{25}{(1-z^{-1})^2} - \frac{3z^{-1}}{(1-z^{-1})^3}\right] z^{n-1}\,dz \tag{9}$$

The sum of the terms of the sequence is computed using the residue theorem, expressed as R_i, which represents the residue of the Z-transform at $z = 1$ for the expression $Y(z)$.

$$R_i = R_1 + R_2 \tag{10}$$

R_1 and R_2 are residues calculated at the poles of the Z-transform.

$$R_1 = \frac{1}{(2-1)!}\left.\frac{d(25z^{n+1})}{dz}\right|_{z=1} \tag{11}$$

$$= 25(n+1) \tag{12}$$

$$R_2 = \frac{1}{(3-1)!}\left.\frac{d^2(-3z^{n+1})}{dz^2}\right|_{z=1} \tag{13}$$

$$= \frac{-3}{2}(n+1)(n) \tag{14}$$

The sum of terms is given by R_i:

$$25(n+1) + \frac{-3}{2}n(n+1) = 116 \tag{15}$$

Solving the equation gives :

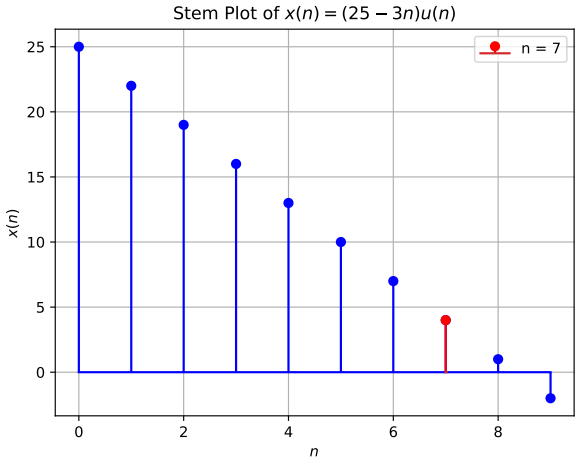
$$n = 7 \tag{16}$$

$$n = 8.667 \tag{17}$$

Since n can take only integer values, $n = 8.667$ is rejected. Upon substituting the value of n in equation (1) :

$$x(7) = 4 \tag{18}$$

Hence the last term of the given AP is 4.


Stem Plot of $x(n) = (25 - 3n)u(n)$
n = 7
$x(n)$
n
25
20
15
10
5
0
0
2
4
6
8