

NCERT Physics 12.7 Q19

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Question: Suppose the circuit in Exercise 7.18 (in Figure Fig. 1) has a resistance of $15\ \Omega$. Obtain the average power transferred to each element of the circuit, and the total power absorbed.

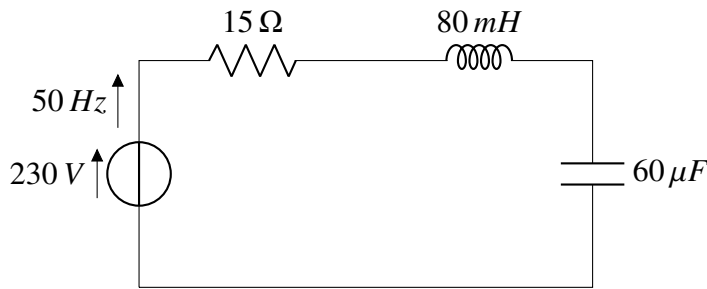


Fig. 1. LCR Circuit

Solution: In Fig. 1 the following information is provided:

Symbol	Value	Description
L	80 mH	Inductance
C	$60\ \mu\text{F}$	Capacitance
R	$15\ \Omega$	Resistance
V	$230\ \text{V}$	Voltage
f	$50\ \text{Hz}$	Frequency
ω	$2\pi f = 100\pi$	Angular Frequency
P	?	Instantaneous power
ϕ	?	Phase difference between current and voltage
I_{rms}	?	rms value of current

TABLE I
GIVEN PARAMETERS

Applying Kirchoff's Voltage Law in the Fig. 2

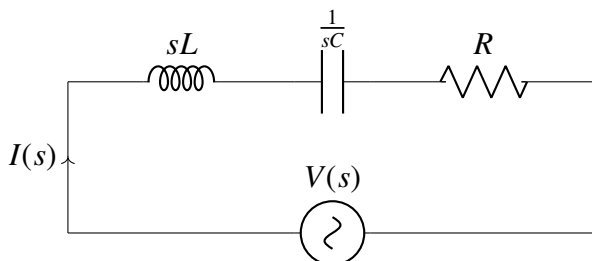


Fig. 2. s domain circuit

$$V(s) = RI(s) + sLI(s) + \frac{1}{sC}I(s) \quad (1)$$

$$= I(s) \left(R + Ls + \frac{1}{sC} \right) \quad (2)$$

$$I(s) = \frac{V(s)}{\left(R + Ls + \frac{1}{sC} \right)} \quad (3)$$

$$H(s) = \frac{V(s)}{I(s)} \quad (4)$$

$$H(s) = R + sL + \frac{1}{sC} \quad (5)$$

Substituting s with $j\omega$

$$H(j\omega) = R + j\omega L + \frac{1}{j\omega C} \quad (6)$$

$$\Rightarrow |H(j\omega)| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2} \quad (7)$$

Average power transferred to an element in the circuit is given by:

$$P = VI \cos(\phi) \quad (8)$$

a) Average power transferred to the capacitor, P_C :
For a capacitor the phase angle is:

$$\phi = \frac{\pi}{2} \quad (9)$$

$$\cos(\phi) = 0 \quad (10)$$

$$P_C = 0 \quad (11)$$

b) Average power transferred to the inductor, P_L :
For an inductor the phase angle is:

$$\phi = -\frac{\pi}{2} \quad (12)$$

$$\cos(\phi) = 0 \quad (13)$$

$$P_L = 0 \quad (14)$$

c) Average Power transferred to the resistor, P_R :
 $|H(j\omega)|$ is obtained by substituting the numerical values from the Table I in equation (7):

$$|H(j\omega)| = 31.728\ \Omega \quad (15)$$

$$I_{rms} = \frac{V_{rms}}{H(j\omega)} = \frac{230}{31.728} = 7.25A \quad (16)$$

$$P_R = (I_{rms})^2 R = 788.44W \quad (17)$$

d) Total power absorbed by circuit:

$$= P_R + P_C + P_L \quad (18)$$

$$= 788.44 + 0 + 0 \quad (19)$$

$$= 788.44 \text{ W} \quad (20)$$

Total power absorbed by circuit is 788.44W

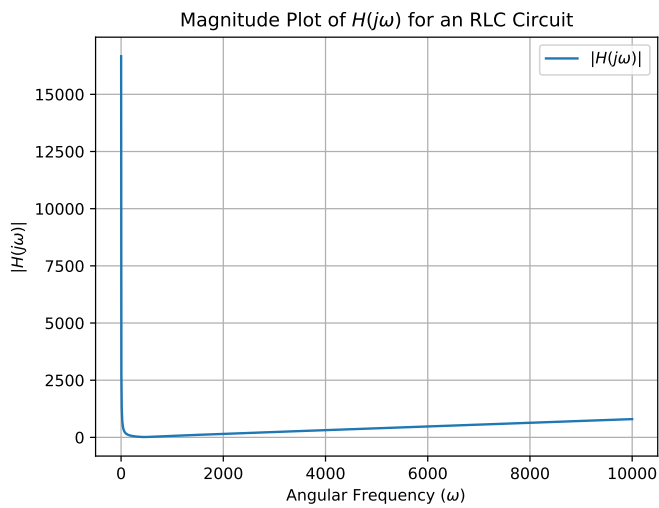


Fig. 3. $|H(j/\omega)|$ vs ω