# Gate 2022 EC Q55

EE23BTECH11212 - Manugunta Meghana Sai*

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For a vector $\bar{x} = [x[0], x[1], \ldots, x[7]]$, the 8-point discrete Fourier transform (DFT) is denoted by $\bar{X} = \text{DFT}(\bar{x}) = [X[0], X[1], \ldots, X[7]]$, where

$$X[k] = \sum_{n=0}^{7} x[n] \exp\left(-j\frac{2\pi}{8}nk\right).$$

Here $j = \sqrt{-1}$. If $\bar{x} = [1, 0, 0, 0, 2, 0, 0, 0]$ and $\bar{y} = \text{DFT}(\text{DFT}(\bar{x}))$, then the value of $y[0]$ is

**Solution:** DFT of $\bar{x}$

For $k = 0$ :

$$X[0] = \sum_{n=0}^{7} x[n] \tag{1}$$
$$= x[0] + x[1] + \cdots + x[7] \tag{2}$$
$$= 1 + 0 + 0 + 0 + 2 + 0 + 0 + 0 \tag{3}$$
$$= 3 \tag{4}$$

For $k = 1$ :

$$X[1] = \sum_{n=0}^{7} x[n] \exp\left(-j\frac{2\pi}{8}n\right) \tag{5}$$
$$= x[0] + x[1] \exp\left(-j\frac{2\pi}{8}\right) + x[2] \exp\left(-j\frac{4\pi}{8}\right) + \cdots + x[7] \exp\left(-j\frac{14\pi}{8}\right) \tag{6}$$
$$= 1 - 2 \tag{7}$$
$$= -1 \tag{8}$$

For $k = 2$ :

$$X[2] = \sum_{n=0}^{7} x[n] \exp\left(-j\frac{2\pi}{8}2n\right) \tag{9}$$
$$= x[0] + x[1] \exp\left(-j\frac{4\pi}{8}\right) + x[2] \exp\left(-j\frac{8\pi}{8}\right) + \cdots + x[7] \exp\left(-j\frac{28\pi}{8}\right) \tag{10}$$
$$= 1 + 2 \tag{11}$$
$$= 3 \tag{12}$$

For $k = 3$ :

$$X[3] = \sum_{n=0}^{7} x[n] \exp\left(-j\frac{2\pi}{8}3n\right) \tag{13}$$

$$= x[0] + x[1]\exp\left(-j\frac{6\pi}{8}\right) + x[2]\exp\left(-j\frac{12\pi}{8}\right) + \cdots + x[7]\exp\left(-j\frac{42\pi}{8}\right) \tag{14}$$

$$= 1 - 2 \tag{15}$$

$$= -1 \tag{16}$$

For $k = 4$ :

$$X[4] = \sum_{n=0}^{7} x[n] \exp\left(-j\frac{2\pi}{8}4n\right) \tag{17}$$

$$= x[0] + x[1]\exp\left(-j\frac{8\pi}{8}\right) + x[2]\exp\left(-j\frac{16\pi}{8}\right) + \cdots + x[7]\exp\left(-j\frac{56\pi}{8}\right) \tag{18}$$

$$= 1 + 2 \tag{19}$$

$$= 3 \tag{20}$$

For $k = 5$ :

$$X[5] = \sum_{n=0}^{7} x[n] \exp\left(-j\frac{2\pi}{8}5n\right) \tag{21}$$

$$= x[0] + x[1]\exp\left(-j\frac{10\pi}{8}\right) + x[2]\exp\left(-j\frac{20\pi}{8}\right) + \cdots + x[7]\exp\left(-j\frac{70\pi}{8}\right) \tag{22}$$

$$= 1 - 2 \tag{23}$$

$$= -1 \tag{24}$$

For $k = 6$ :

$$X[6] = \sum_{n=0}^{7} x[n] \exp\left(-j\frac{2\pi}{8}6n\right) \tag{25}$$

$$= x[0] + x[1]\exp\left(-j\frac{12\pi}{8}\right) + x[2]\exp\left(-j\frac{24\pi}{8}\right) + \cdots + x[7]\exp\left(-j\frac{84\pi}{8}\right) \tag{26}$$

$$= 1 + 2 \tag{27}$$

$$= 3 \tag{28}$$

For $k = 7$ :

$$X[7] = \sum_{n=0}^{7} x[n] \exp\left(-j\frac{2\pi}{8}7n\right) \tag{29}$$

$$= x[0] + x[1]\exp\left(-j\frac{14\pi}{8}\right) + x[2]\exp\left(-j\frac{28\pi}{8}\right) + \cdots + x[7]\exp\left(-j\frac{98\pi}{8}\right) \tag{30}$$

$$= 1 - 2 \tag{31}$$

$$= -1 \tag{32}$$

$$\bar{X} = \mathrm{DFT}(\bar{x}) = [X[0], X[1], \ldots, X[7]] \tag{33}$$

$$\bar{X} = [3, -1, 3, -1, 3, -1, 3, -1] \tag{34}$$

$$\bar{y} = \mathrm{DFT}(\mathrm{DFT}(\bar{x})) \tag{35}$$

$$\bar{y} = [3, -1, 3, -1, 3, -1, 3, -1] \tag{36}$$

$$\begin{aligned} y[0] &= \sum_{n=0}^{7} x[n] && (37) \\ &= x[0] + x[1] + \cdots + x[7] && (38) \\ &= 3 - 1 + 3 - 1 + 3 - 1 + 3 - 1 = 8 && (39) \end{aligned}$$