

Gate 2022 EC Q55

EE23BTECH11212 - Manugunta Meghana Sai*

Gate 2022 EC Q50

Two linear time-invariant systems with transfer functions

$$G_1(s) = \frac{10}{s^2 + s + 1}$$

and

$$G_2(s) = \frac{10}{s^2 + s\sqrt{10} + 10}$$

have unit step responses $y_1(t)$ and $y_2(t)$, respectively. Which of the following statements is/are true?

- 1) $y_1(t)$ and $y_2(t)$ have the same percentage peak overshoot.
- 2) $y_1(t)$ and $y_2(t)$ have the same steady state values.
- 3) $y_1(t)$ and $y_2(t)$ have the same damped frequency of oscillation.
- 4) $y_1(t)$ and $y_2(t)$ have the same 2% settling time.

Solution: The general second-order transfer function is given by:

Parameter	Description	value
$x_1(s)$	input	$\frac{1}{s}$
$x_2(s)$	input	$\frac{1}{s}$
$G_1(s)$	transfer function	$\frac{10}{s^2+s+1}$
$G_2(s)$	transfer function	$\frac{10}{s^2+s\sqrt{10}+10}$
$y_1(t)$	unit step response	—
$y_2(t)$	unit step response	—
ω_n	natural frequency	—
ζ	damping ratio	—

TABLE 4
GIVEN PARAMETERS

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (1)$$

After comparing the coefficients of $G_1(s)$ and $G_2(s)$, as $\zeta = \frac{1}{2}$ is less than 1, the system is underdamped.

Tranfer function	ω_n	ζ
$G_1(s)$	1	$\frac{1}{2}$
$G_1(s)$	$\sqrt{10}$	$\frac{1}{2}$

TABLE 4
GIVEN PARAMETERS

$$Y(s) = X(s) G(s) \quad (2)$$

$$= \frac{1}{s} \left(\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \right) \quad (3)$$

Applying inverse laplace transform,

$$y(t) = 1 - \frac{e^{-\zeta\omega_n t}}{1 - \zeta^2} \sin(\omega_d t + \phi) \quad (4)$$

where ω_d is the damped frequency of oscillation.

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} \quad (5)$$

The percentage peak overshoot (PO) is a measure of how much a system's response exceeds its steady-state value in terms of a percentage of the steady-state value:

$$PO = \left(\frac{y_{\max} - y_{ss}}{y_{ss}} \right) \times 100\% \quad (6)$$

y_{\max} is obtained by differentiating (4) with respect to time and equating it to zero, substituting the value in (4),

$$y_{\max} = 1 + \frac{1}{\sqrt{1 - \zeta^2}} \quad (7)$$

y_{ss} is obtained by final value theorem,

$$y_{ss} = \lim_{s \rightarrow 0} sY(s) \quad (8)$$

$$= \lim_{s \rightarrow 0} s \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \frac{1}{s} \quad (9)$$

$$= 1 \quad (10)$$

Substituting the values of y_{\max} and y_{ss} in (6),

$$PO = \frac{1}{\sqrt{1 - \zeta^2}} \times 100\% \quad (11)$$

$y_1(t)$ and $y_2(t)$ have same ζ , they have same percentage peak overshoot. So, option (1) is correct.

The steady state value of $y(t)$ is given by final value theorem:

$$y_{1ss} = \lim_{s \rightarrow 0} sY_1(s) \quad (12)$$

$$= \lim_{s \rightarrow 0} s \frac{10}{s^2 + s + 1} \frac{1}{s} \quad (13)$$

$$= 10 \quad (14)$$

$$y_{2ss} = \lim_{s \rightarrow 0} sY_2(s) \quad (15)$$

$$= \lim_{s \rightarrow 0} s \frac{10}{s^2 + s\sqrt{10} + 10} \frac{1}{s} \quad (16)$$

$$= 1 \quad (17)$$

as both the unit step responses have different steady state values, option (2) is incorrect.

From (6), as ω_n is different for $y_1(t)$ and $y_2(t)$, they have different damped frequency of oscillation. Hence option (3) is incorrect.

Settling time T_s is the time required for the transient damped oscillations to reach and stay within 2% of the steady-state value.

$$T_s = \frac{4}{\zeta\omega_n} \quad (18)$$

As, ω_n is different for $y_1(t)$ and $y_2(t)$, they have different 2% settling time, Hence option (4) is incorrect. So, only option (1) is correct.