

NCERT Physics 12.7 Q19

EE23BTECH11212 - MANUGUNTA MEGHANA SAI*

Question: Suppose the circuit in Exercise 7.18 (in Figure Fig. 1) has a resistance of 15Ω . Obtain the average power transferred to each element of the circuit, and the total power absorbed.

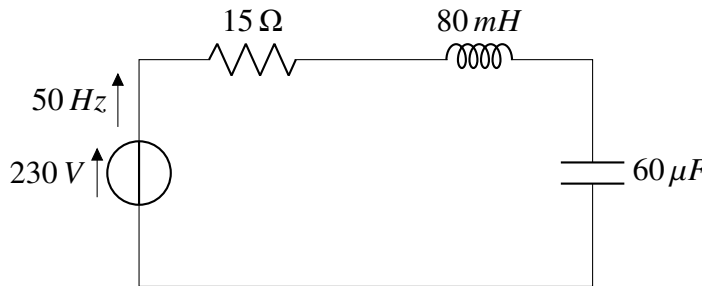


Fig. 1. LCR Circuit

Solution: In Figure Fig. 1 the following information is provided:

Symbol	Value	Description
L	80m H	Inductance
C	60 μF	Capacitance
R	15 Ω	Resistance
V	230 V	Voltage
f	50 Hz	Frequency

TABLE I
IMPEDENCES

Angular frequency of signal,

$$\omega = 2\pi f = 2\pi \cdot (50) = 100\pi$$

Applying Kirchoff's Voltage Law:

$$V(s) = RI(s) + sLI(s) + \frac{1}{sC}I(s) \quad (1)$$

$$\Rightarrow V(s) = I(s) \left(R + Ls + \frac{1}{sC} \right) \quad (2)$$

$$\Rightarrow I(s) = \frac{V(s)}{\left(R + Ls + \frac{1}{sC} \right)} \quad (3)$$

Average Power transferred to the resistor is given by :

$$P_R = I^2(s) \cdot R \quad (4)$$

Average Power transferred to the inductor is given by :

$$P_L = I^2(s) \cdot \text{Re}(Z_L(s)) \quad (5)$$

Average Power transferred to the capacitor is given by :

$$P_C = I^2(s) \cdot \text{Re}(Z_C(s)) \quad (6)$$

Since the reactive components of inductor and capacitor have imaginary impedances,

$$\text{Re}(Z_L) = 0 \quad (7)$$

$$\text{Re}(Z_C) = 0 \quad (8)$$

Average power transferred to the capacitor, $P_C = 0$
Average power transferred to the inductor, $P_L = 0$

$$H(s) = \frac{V(s)}{I(s)} \quad (9)$$

$$H(s) = R + sL + \frac{1}{sC} \quad (10)$$

Substituting s with $j\omega$

$$H(j\omega) = R + j\omega L + \frac{1}{j\omega C} \quad (11)$$

$$\Rightarrow |H(j\omega)| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2} \quad (12)$$

Impedance is given by:

$$Z = \sqrt{R^2 + \left(\omega \cdot L - \frac{1}{\omega \cdot C} \right)^2} \quad (13)$$

$$= \sqrt{15^2 + \left(100\pi \cdot (80 \times 10^{-3}) - \frac{1}{100\pi \times 60 \times 10^{-6}} \right)^2} \quad (14)$$

$$= \sqrt{15^2 + (25.12 - 53.08)^2} \quad (15)$$

$$= 31.728 \Omega \quad (16)$$

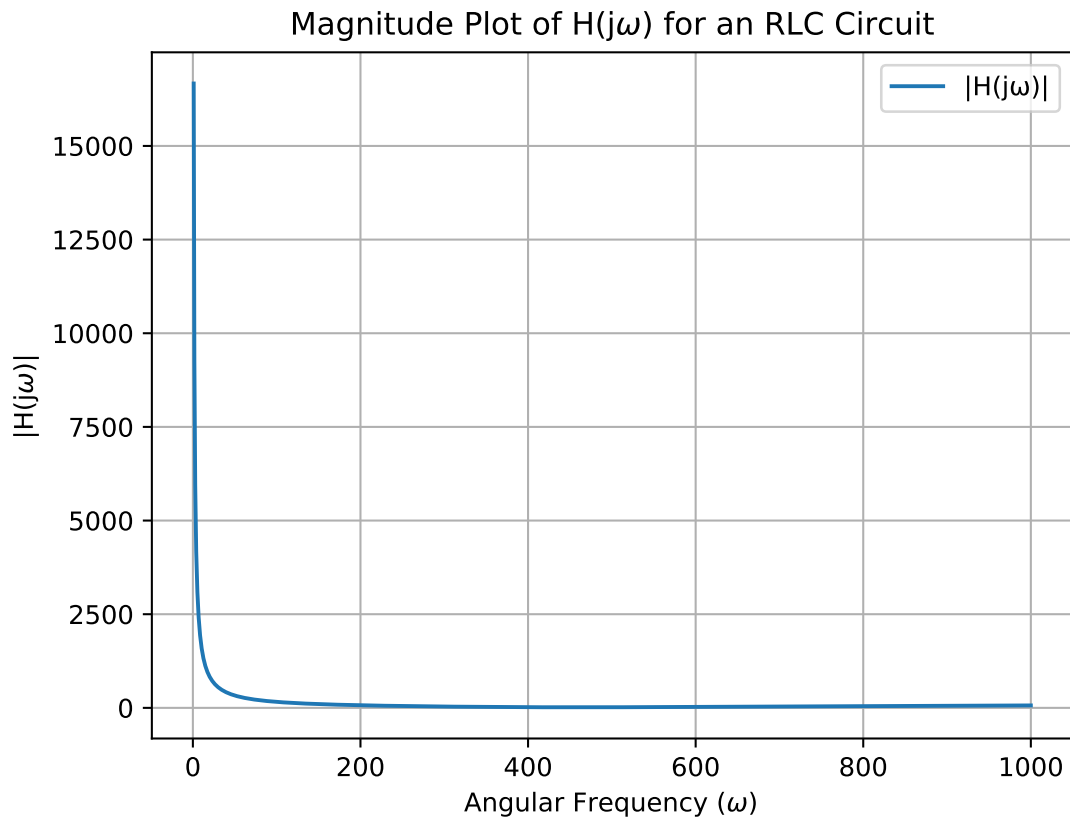


Fig. 2. Absolute value of $H(j\omega)$ with respect to ω for RLC Circuit

Current flowing through the circuit I is :

$$I = \frac{V}{Z} = \frac{230}{31.728} \quad (17)$$

$$= 7.25 \text{ A} \quad (18)$$

Average power transferred to resistance is given by :

$$P_R = I^2 \cdot R = (7.25)^2 \times 15 \quad (19)$$

$$= 788.44 \text{ W} \quad (20)$$

Total power absorbed by circuit:

$$= P_R + P_C + P_L \quad (21)$$

$$= 788.44 + 0 + 0 \quad (22)$$

$$= 788.44 \text{ W} \quad (23)$$

Total power absorbed by circuit is 788.44W