NCERT Physics 12.7 Q19

EE23BTECH11212 - MANUGUNTA MEGHANA SAI*

Question: Suppose the circuit in Exercise 7.18 (in Figure Fig. 1)has a resistance of 15 Ω. Obtain the average power transferred to each element of the circuit, and the total power absorbed.

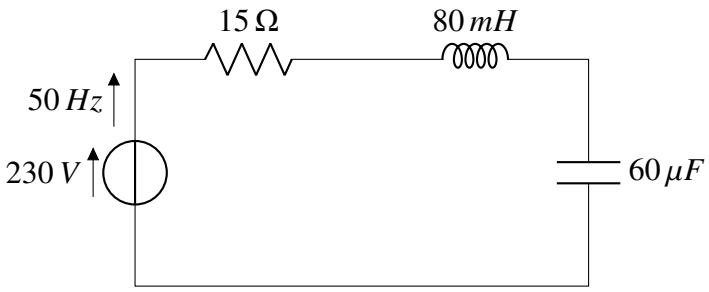

Fig. 1. LCR Circuit

Solution: In Fig. 1 the following information is provided:

Symbol	Value	Description
L	$80m$ H	Inductance
C	$60\,\mu F$	Capacitance
R	$15\,\Omega$	Resistance
V	$230\,V$	Voltage
f	$50\,Hz$	Frequency
ω	$2\pi f = 100\pi$	Angular Frequency

TABLE I
GIVEN PARAMETERS

Applying Kirchoff's Voltage Law in the Fig. 2

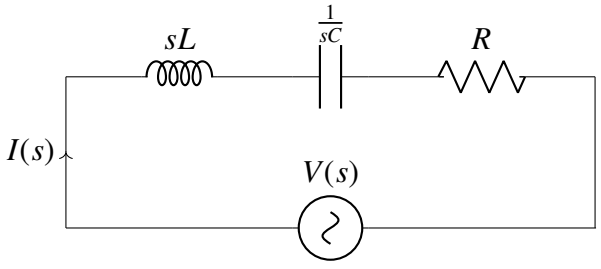

Fig. 2. s domain circuit

$$V(s) = RI(s) + sLI(s) + \frac{1}{sC}I(s) \tag{1}$$

$$= I(s)\left(R + Ls + \frac{1}{sC}\right) \tag{2}$$

$$I(s) = \frac{V(s)}{\left(R + Ls + \frac{1}{sC}\right)} \tag{3}$$

$$H(s) = \frac{V(s)}{I(s)} \tag{4}$$

$$H(s) = R + sL + \frac{1}{sC} \tag{5}$$

Substituting s with $\jmath\omega$

$$H(j\omega) = R + j\omega L + \frac{1}{j\omega C} \tag{6}$$

$$\Rightarrow |H(j\omega)| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \tag{7}$$

a) Average power transferred to the capacitor, P_C: In the first half cycle, the capacitor stores energy in the form of electric field. In the second half of the cycle it dissipates the same amount of energy. So net energy over a given time period is zero, hence

$$P_C = 0 \tag{8}$$

b) Average power transferred to the inductor, P_L: In the first half cycle, the inductor stores energy in the form of magnetic field. In the second half of the cycle it dissipates the same amount of energy. So net energy over a given time period is zero, hence the average power of an inductor is zero.

$$P_L = 0 \tag{9}$$

c) Average Power transferred to the resistor, P_R:

$$P_R = (I(j\omega))^2 R \tag{10}$$

$|H(j\omega)|$ is obtained by sustituting the numerical values from the Table I in equation (7):

$$|H(j\omega)| = 31.728\ \ \Omega \tag{11}$$

Current flowing through the circuit $I(j\omega)$ is :

$$I(j\omega) = \frac{V}{|H(j\omega)|} = \frac{230}{31.728} \tag{12}$$

$$= 7.25 \ A \tag{13}$$

$$P_R = (I(j\omega))^2 \cdot R = (7.25)^2 \times 15 \tag{14}$$

$$= 788.44 \ W \tag{15}$$

d)Total power absorbed by circuit:

$$= P_R + P_C + P_L \tag{16}$$

$$= 788.44 + 0 + 0 \tag{17}$$

$$= 788.44 \ W \tag{18}$$

Total power absorbed by circuit is 788.44W

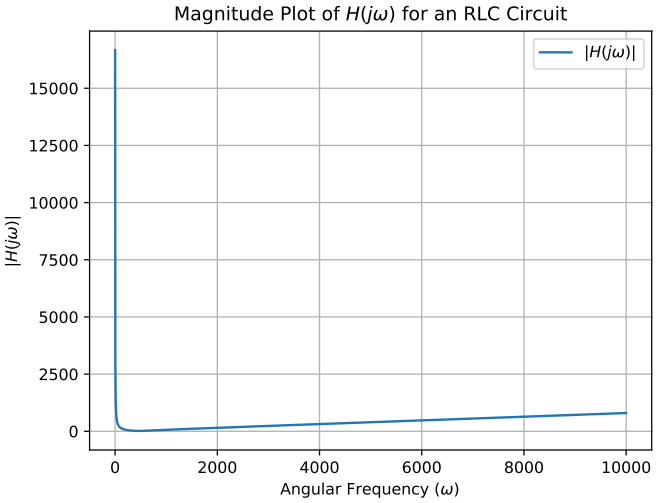

Fig. 3. $|H(j/omega)|$ vs ω