# FREQUENCY RESPONSE (FROM STATE-SPACE EQNS)

State-space equations:

$$\begin{bmatrix} \dot{\underline{y}} \\ \ddot{\underline{y}} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \begin{bmatrix} \underline{y} \\ \dot{\underline{y}} \end{bmatrix} + \begin{bmatrix} 0 \\ -M^{-1} \end{bmatrix} \begin{bmatrix} \underline{F}(t) \end{bmatrix}$$

ie $$\underbrace{\dot{\underline{x}}(t)}_{(2N\times1)} = \underbrace{[A_c]}_{(2N\times2N)} \underbrace{\underline{x}(t)}_{(2N\times1)} + \underbrace{[B]}_{(2N\times N)} \underbrace{\underline{F}(t)}_{(N\times1)}$$

Also: $$\underbrace{\underline{y}_0(t)}_{} = \underbrace{[C]}_{(N_0\times2N)} \underbrace{\underline{x}(t)}_{(2N\times1)} + \underbrace{[D]}_{(N_0\times N)} \underbrace{\underline{F}(t)}_{(N\times1)}$$

Note $\neq \underline{y}$. This is the output vector $(N_0\times1)$

These are "state-space equations" that define the problem

Frequency response is defined as follows:

$$\underline{y}_0(s) = [G(s)]\,\underline{F}(s)$$

Laplace Fourier
$s \Leftrightarrow j\omega$

$\underline{y}_0(s)$: Laplace (or Fourier) transform of $\underline{y}_0(t)$

$\underline{F}(s)$: Laplace (or Fourier) transform of $\underline{F}(t)$

Dimensions: $(N_0\times1) = (N_0\times N).(N\times1)$.

ie $[G(s)]$ is a $(N_0\times N)$ matrix, mapping inputs (force) to outputs.

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Properties of Laplace transform:

$$\mathcal{L}(\dot{\underline{y}}(t)) = s\,\underline{y}(s)$$

$$\mathcal{L}(\ddot{\underline{y}}(t)) = s\,\dot{\underline{y}}(s) \qquad (= s^2\underline{y}(s))$$

Laplace transform of state equations:

$$\left.\begin{aligned}\mathcal{L}(\dot{\underline{x}}(t)) &= [A_c]\underline{x}(s) + [B]\underline{F}(s)\\ \mathcal{L}(\underline{y}_o(t)) &= [C]\underline{x}(s) + [D]\underline{F}(s)\end{aligned}\right\}$$

$$\left.\begin{aligned}s\underline{x}(s) &= [A_c]\underline{x}(s) + [B]\underline{F}(s)\\ \underline{y}_o(s) &= [C]\underline{x}(s) + [D]\underline{F}(s)\end{aligned}\right\}$$

$$\Rightarrow \left(s\underbrace{[I]}_{\text{Identity matrix}} - [A_c]\right)\underline{x}(s) = [B]\underline{F}(s)$$

$$\Rightarrow \underline{x}(s) = \left(s[I]-[A_c]\right)^{-1}[B]\underline{F}(s)$$

$$\Rightarrow \underline{y}_o(s) = \left([C]\left(s[I]-[A_c]\right)^{-1}[B] + [D]\right)\underline{F}(s)$$

$$\Rightarrow \boxed{[G(s)] = [C]\left(s[I]-[A_c]\right)^{-1}[B] + [D]}$$

Defines $[G(s)]$ in terms of state-space matrices A, B, C, D.

Let $s = j\omega$ ($j = \sqrt{-1}$); this gives frequency response $[G(\omega)]$ (which is complex)

$$\boxed{[G(\omega)] = [C]\left(j\omega[I]-[A_c]\right)^{-1}[B] + [D]}$$