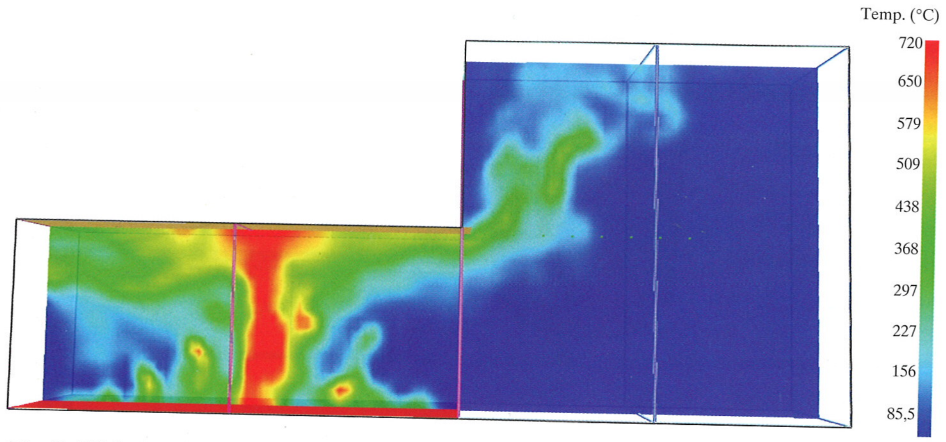

Fig. 8: FDS simulation for a typical simplified office compartment. Gas temperature visualisation with the windows open (glass broken)

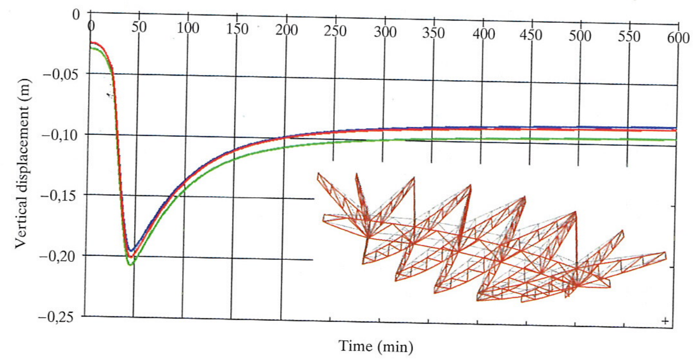

Fig 9: Maximal and residual deflections under a natural fire scenario

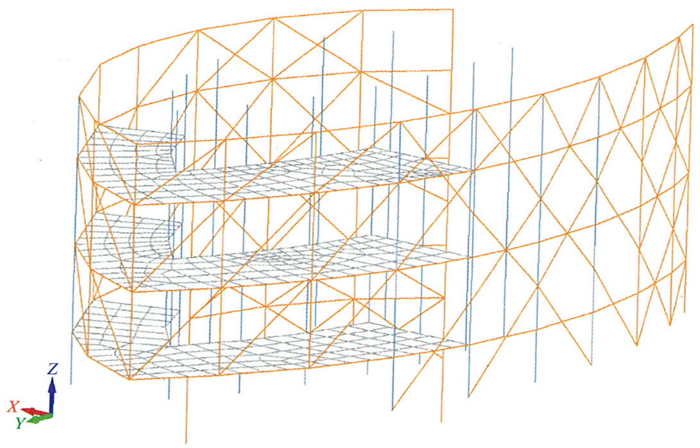
Fig. 10: SAFIR 3D structural model of the truss facade

However, using FEM together with the localised-fire-heated sections, it was possible to push the optimisation to its reasonable maximum (being not extreme as well as keeping safety in mind). An incremental procedure was used to find the critical load capacity under the natural and localised fire situations, knowing that this load is a function of the evolution of the section temperature that varies non-linearly with time. Many calculations were necessary to determine the critical load capacity, as every group of columns has a different load and buckling length. As a result of the procedure used, an optimisation of the section much closer to the effective critical temperature of each group of columns was achieved as compared to the values obtained with the much more conservative ISO fire approach.

Conclusions

The performance-based design approach used in this project has been a full success. The steel structure, as desired by the architect, remained unprotected and slender. The structural fire optimisation that was performed proves that the fire analysis and design process is really valuable and sustainable when achieved by a well-trained and experimented specialist. Thanks to the use of the new localised fire method proposed by Vassard and Zanon, the columns have been optimised within a conservative safety concept. The columns with lower load can stay unprotected, while the columns subjected to higher loads will be constructed as reinforced steel–concrete composite sections. The truss girder over the conference room and the façcade structure will stay completely unprotected, thanks to the active fire protection measures and at the price of sophisticated design verifications.

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SEI Data Block

Owner:
IHEID

Contractor:
Steiner - Total Services Contractor

Steel (t):	1700
Concrete (m^3):	20 000
Rebars (t):	2000
Pre-stressing steel (t):	5400
Estimated cost (CHF million):	170
Service date:	June 2013

The Lateral Dynamic Stability of Stockton Infinity Footbridge Using Complex Modes

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Abstract

The Arup criterion[5] for the lateral dynamic stability of footbridges under crowd loading is generalised using complex modes to the case of multiple modes and multiple tuned mass dampers. The procedure is applied to the design of the Infinity Footbridge, Stockton-on-Tees, where three tuned mass dampers were incorporated to deal with three horizontal modes, whose frequencies were predicted by finite element analysis to lie below 1,5 Hz.

Keywords: footbridges; dynamics; human-induced vibration; tuned mass dampers; complex modes.

Introduction

The Infinity Footbridge at Stockton-on-Tees, County Durham, was opened on 16 May 2009, following its structural design. It consists of two steel arches of 120 and 60 m span (*Figs. 1 and 2*). Although of bow-string configuration, arch thrusts are taken to the supports and the deck of precast concrete units is kept in compression by post-tensioned external cables. These cables, mounted externally with outriggers on both sides of the deck, also serve to increase the lateral stiffness of the bridge deck by ensuring that joints between units remain closed.

Given its lightweight nature, it was anticipated that pedestrian-induced vibration might prove problematic, and a full dynamic analysis formed part of the design stage. From the start, the precast deck unit geometry was designed to facilitate integration of any tuned mass dampers (TMDs) required in a visually unobtrusive manner beneath the deck.

Finite element analysis predicted three horizontal modes with frequencies below 1,5 Hz, three vertical modes in the 2 to 3 Hz region and a number of modes which had torsional components in the deck motion. The dynamic response of the bridge to pedestrian loading was considered from serviceability and ultimate limit state perspectives. Serviceability concerned vibration levels expected under a wide range of typical scenarios involving small groups of walkers or joggers. Many such scenarios were modelled in the time domain to assess acceptability of both vertical and horizontal displacements and accelerations for walker comfort, in a manner similar to that used on other bridges.[1–3] Such design studies led to the requirement for both vertical and horizontal tuned mass dampers to keep responses within comfortable limits as recommended by various codes of practice and design guides. The possibility of large amplitude lateral instability under a large crowd was considered as an ultimate limit state condition, and is the focus of this paper.

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Pedestrian Load Models for Lateral Stability

Lateral instability of a footbridge under crowd loading was first reported in Ref. [4]. From full-scale experiments conducted during remedial work on the London Millennium Bridge in 2000 to 2001, a model[5] was arrived at wherein the horizontal load effects of each walker could be represented as a force $f = c_p \dot{x}$ where $\dot{x}$ is the lateral velocity of the bridge deck at the walker's location and the constant c_p = 300 N s/m for bridge responses in the range 0,5 to 1 Hz. Each walker exerts other forces, but this component is in phase with the bridge velocity and acts like a negative damper to feed energy into any bridge motion. Should the negative damping effects of the walkers exceed the positive damping of the structure, the system becomes unstable. Consideration of a single mode with half-sinusoid mode shape and a uniformly distributed crowd leads to the Arup prediction[5] of the critical number of people that would destabilise such a bridge:

$$N_{cr} = \frac{8\pi\xi f M}{c_p}$$

where M is the modal mass, ξ is the fraction of critical damping and f is the frequency of the mode (in Hz). Considerable further research has since sought to provide models of human locomotion that explain the origin of the forces underlying this formula.[6–9] Although these provide insight, none as yet has sufficient theoretical strength to extend or adjust the experimental findings. In a more recent set of experiments on individuals walking on a laboratory treadmill subjected to forced lateral displacements at a variety of amplitudes and frequencies, average c_p values ranging from −100 to 214 N s/m for frequencies below 1,15 Hz have been measured,[10] with typical values *circa* 100 N s/m. These are substantially lower than the values, and were not available at the time of the Stockton design. These recent experimental findings have been incorporated into a stochastic load model.[11] However, this paper nevertheless recommends use of the higher values reported in Ref. [5], because laboratory experiments on individuals cannot be guaranteed to replicate the psychology of walking on a crowded bridge. Not only are person-to-person interactions absent, the visual parallax clues that guide walkers' perceptions about their state of balance are also markedly different in the two cases. Moreover, a walker on a treadmill has reduced freedom to adjust walking speed.

Preliminary analysis of the Stockton bridge revealed that there were three lateral modes of concern with frequencies below 1,5 Hz, two of these

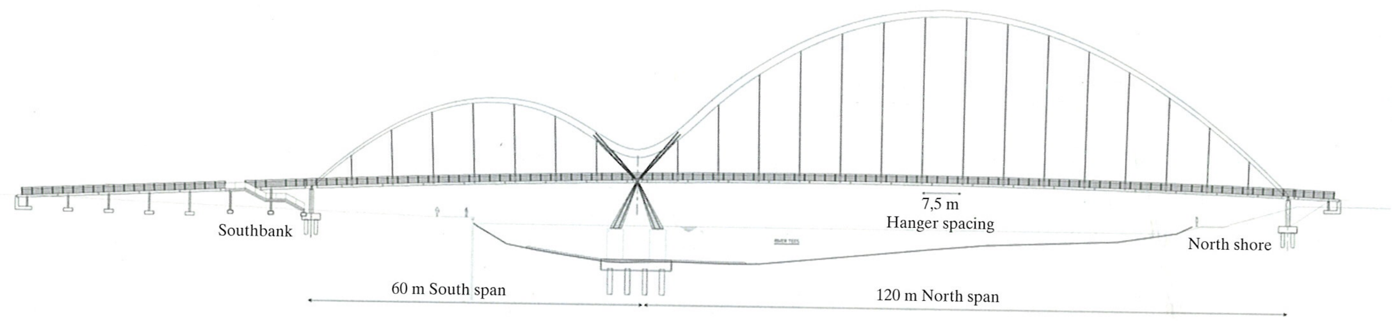

Fig. 1: Infinity Footbridge, Stockton-on-Tees. Elevation

(a) (b)

Fig. 2: Completed structure (2009); (a) elevation; (b) main span (Courtesy: Morley Von Sternberg)

having closely spaced frequencies near 1 Hz. Initial designs treated each resonance independently, adding a TMD to each and then estimating the increased effective damping due to the TMD using standard procedures.[12] The TMDs could then be sized independently such that each resonance satisfied the Arup criterion for a desired crowd size. However, the addition of TMDs to closely spaced modes couples the modes, and there is cross-talk such that nearby resonances cannot be treated in isolation. In the following section, therefore, the full theory is developed for the dynamic stability of N_m modes with N_T TMDs, taking full account of the interaction between nearby modes. The procedure involves the use of complex modes, the fundamentals of which are described in Ref. [13].

Lateral Stability for Multiple Modes and Multiple TMDs

Standard Preliminaries

The discretised equation of linear vibrations created by a finite element model of the bridge without dampers is:

$$M\ddot{X}+C\dot{X}+KX=F(t)$$

where, M, C and K are the mass, damping and stiffness matrices, respectively, X is a vector of nodal displacements and rotations and F is the vector of applied nodal forces. Specifically, this is the linearisation for small vibrations around the nonlinear static equilibrium under dead loads and prestress. This includes the "P-delta" effects of tension stiffening of cables and compression de-stiffening of the arches. The standard eigenvalue problem for free undamped oscillations $(K-\omega^2 M)X=0$ is then solved as usual for the eigenvalues $\omega_j^{\,2}$ and the corresponding mode shapes Φ_j.

Of the mode shape information, all that is required for the subsequent analysis are the horizontal deflected shapes of the deck. Each mode is thus rescaled and partitioned as $b\ \Phi_j = \Psi_j = [\Psi_{H,j};\ \Psi_{V,j}]$ where $\Psi_{H,j}$ are the horizontal displacements of the deck centreline in mode j and $\Psi_{V,j}$ are all other nodal displacements for that mode (such as vertical deflections of the deck or any displacement of the arch). The factor b is a renormalisation, which is convenient but not strictly necessary, and rescales each mode shape such that the maximum absolute lateral displacement of the deck is unity. Previously computed modal masses that correspond to a normalisation having a maximum displacement component of unity at any point on the bridge thus need to be rescaled by b^2 to match this renormalisation.

At any time t a general deflected shape X can be decomposed into the modal basis as $X=\Psi a$ where Ψ is the matrix of eigenvectors, each column of which is a mode shape Ψ_j and a is a vector of modal amplitudes. Substituting for X into the equation of motion and pre-multiplying by Ψ^T leads to the standard set of de-coupled equations $M_a\ddot{a}+C_a\dot{a}+K_a a=F_a$, where $M_a=\Psi^T M\Psi$ and $K_a=\omega^2 M_a$, with M_a being the diagonalised matrix of (renormalised) modal masses, and K_a and ω being the corresponding diagonal matrices of modal stiffnesses and natural frequencies.

As usual, it is assumed that the damping in each mode is small and can be represented via a fraction ξ_j of critical (and stored in the diagonal matrix ξ) to give a diagonal damping matrix $C_a=2\xi\omega M_a$. Finally, only the first N_m equations are retained, corresponding to the low frequency modes which have significant lateral deck displacements in their mode shapes.

The finite element model can now be dispensed with, as all further calculations require only the computed modal masses and frequencies and the lateral deck displacement components Ψ_H of the N_m mode shapes.

Inclusion of TMDs

The equations of motion, which are presently limited to the N_m modal equations of interest, can now be extended by appending the equations for the N_T TMDs. The generalised displacements are thus $X_{aT}=[a;\ X_T]$, this being a column vector containing the N_m modal amplitudes a of the lateral modes of concern, together with the absolute displacements X_T of the N_T TMDs.

This approach—using the original bridge modes to describe the bridge and appending the TMD displacements separately as additional unknowns—is superior to the alternative possibility of including TMDs in the original finite element analysis and performing a new modal analysis of the bridge/TMD system. The problem is that modes are extracted for the undamped case, and damping is usually added later with the assumption that the modes remain uncoupled. However, the high damping levels which are localised in the TMD degrees of freedom mean that it will no longer be possible to include a simple diagonalised damping matrix to the equations of motion. By the procedure adopted here, the effects of the highly localised damping can be properly modelled, and one consequence of this is that the new modes (of the damped bridge/TMD system) will now be complex. There is no additional level of approximation introduced here beyond the basic truncation of the original bridge modes: indeed it will be possible to keep as many of the original bridge modes as desired to properly represent the bridge in the coupled system. If all the original bridge modes are included, there is no approximation at this stage, and the modes are simply a change of coordinate.

Let the masses of the N_T TMDs be listed along the diagonal of the $N_T\times N_T$ matrix M_T. Trivially, the inertial term in the equation of motion becomes:

$$M_{aT}\ddot{X}_{aT}=\begin{bmatrix} M_a & 0 \\ 0 & M_T \end{bmatrix}\begin{bmatrix} \ddot{a} \\ \ddot{X}_T \end{bmatrix}$$

where 0 is a submatrix of zeroes of the dimension appropriate in each case. The resulting inertia matrix is thus diagonal.

Addition of the stiffness and damping effects of the TMDs is more subtle and the resulting matrices will no longer be diagonal. Assume the deck has N_d nodes along it, and the kth TMD is attached to deck node i_k. For each TMD a $1\times N_d$ vector is created, which has all zero entries, except that the i_kth entry is unity. Stacking N_T such vectors leads to a $N_T\times N_d$ matrix Δ with entries $\Delta_{k,ik}$ (this being the Kronecker delta). This matrix is now augmented with the $N_T\times N_T$ identity matrix I_T to obtain $B=[\Delta,-I_T]$.

If a spring of stiffness k connects a TMD to the bridge deck, the stiffness matrices need to be augmented with a stiffness submatrix:

$$\begin{bmatrix} k & -k \\ -k & k \end{bmatrix}=\begin{bmatrix} 1 \\ -1 \end{bmatrix}k\,[1\ -1]$$

meaning that k is added to the appropriate diagonal elements of the matrices K and K_T and $-k$ is added to the corresponding off-diagonal cross-terms. However, by this stage, the basic bridge stiffness matrix K has already been mode-generalised and has had its dimension reduced to $N_m\times N_m$ corresponding to the modes of interest. The TMD springs are thus included via:

$$K_{aT}=\begin{bmatrix} K_a & 0 \\ 0 & 0 \end{bmatrix}+B_{aT}^T\,K_T\,B_{aT}\ \text{with}$$

$$B_{aT}=[\Delta\Psi_H,-I_T]$$

with the B_{aT} matrix acting like the [1, −1] matrix above, the Kronecker Δ ensuring that the TMD stiffnesses are added into the correct matrix locations and Ψ_H taking account of the reduced effect of a TMD located away from the point of maximum deck displacement.

It follows similarly that the damping matrix becomes:

$$C_{aT}=\begin{bmatrix} C_a & 0 \\ 0 & 0 \end{bmatrix}+B_{aT}^T\,C_T\,B_{aT}$$

where C_T is the $N_T\times N_T$ diagonal matrix containing the damping coefficients for the dashpots connecting the TMDs to the deck. Again, the stiffness and damping submatrices for the TMDs can be defined relative to the TMD mass submatrix via diagonal matrices ω_T and ξ_T containing the appropriate (and possibly different) TMD frequencies and damping ratios, using $K_T=\omega_T^{\,2}\,M_T$ and $C_T=2\xi_T\omega_T M_T$.

The equation for free vibrations of the bridge plus TMDs is thus given by:

$$M_{aT}\ddot{X}_{aT}+C_{aT}\dot{X}_{aT}+K_{aT}X_{aT}=0$$

All that remains is to add the de-stabilising forces created by the pedestrians.

The Pedestrian-Induced Forces

Let there be N_p people on the bridge, and assume that N_p is sufficiently large so that average behaviour may be assumed over each short length δL of bridge around each deck node. Assuming that each pedestrian creates a net force on the bridge that is in phase with the bridge velocity, this force is given by $f_i=c_p\dot{x}(z_i)$ where z_i is the position of the walker on the deck. The walker will, of course, create forces at other frequencies (such as their walking frequency) but these will typically be at frequencies other than the bridge natural frequencies, and thus in the long term and when averaged over adjacent walkers, these may lead to fluctuations but do not affect the basic stability calculation.

Assuming walkers are uniformly distributed across the bridge, the mode-generalised forces in phase with the bridge velocity are:

$$f_H=\Psi_H^T\left(N_p\frac{\delta L}{L}c_p\dot{X}\right)$$

$$=\Psi_H^T\left(N_p\frac{\delta L}{L}c_p\Psi_H\dot{a}\right)=N_pC_{pa}\dot{a}$$

with $C_{pa}=c_p\frac{\delta L}{L}\Psi_H^T\Psi_H$. This is then augmented to $C_p=[C_{pa},0;0,0]$ to apply to the full X_{aT} degrees of freedom which include the TMDs.

Finally the equations of motion for the combined bridge modes, TMDs and walkers are obtained:

$$M_{aT}\ddot{X}_{aT}+(C_{aT}-N_pC_p)\dot{X}_{aT}$$
$$+K_{aT}X_{aT}=0$$

The second order set of ODEs can be transformed by defining $Y=[X_{aT};\ V_{aT}]$ with $V_{aT}=\dot{X}_{aT}$ to obtain the first order set $\dot{Y}=AY$ with:

$$A=\begin{bmatrix} 0 & I \\ -M_{aT}^{-1}K_{aT} & -M_{aT}^{-1}(C_{aT}-N_pC_p) \end{bmatrix}$$

and the stability of the system can be checked by inspecting the eigenvalues of A. Some care is required with terminology as the original bridge modes are no longer eigenvectors of the full bridge–TMD–people system. The eigen-vectors of the full system matrix A will be complex, and each such full mode will in general involve motion in each of the original deck modes and each of the TMDs. For a particular complex mode, the motions of all the components will be at the same frequency but may be out of phase, and a phasor diagram of the elements of the complex eigenvector will reveal that phase relationship.

Frequency Dependence of C_p

The analysis has thus far assumed that the average parameter c_p is independent of the frequency of the bridge response, and the data in Ref. [5] suggests this is true for responses in the 0,5 to 1 Hz range. However, outside this range, the average c_p is expected to be lower, albeit that there is presently no clear guidance on how c_p falls away outside the critical frequency range. It is assumed that there is a function c_p (ω) that describes this frequency dependence. Determining the eigenstructure of the governing matrix A is thus complicated by the elements of A depending (via C_p) on the as-yet-undetermined eigenfrequencies of A. Fortunately, the means of handling such self-referential frequency dependence is familiar from the Theodorsen–Scanlan theory of flutter derivatives in wind engineering[14] wherein the eigenstructure is determined iteratively using the coefficients appropriate to the eigenfrequency of the mode under consideration. This will be illustrated in the following example.

Added Mass of the Pedestrians

Addition of the mass of the pedestrians to the model has two effects. First, their weight induces additional tensions and compressions to cables and arches which will affect the P-delta effects in the original equilibrium analysis. However, this makes a small change to an already second order effect, and moreover, it affects the stiffness rather than the damping. The effects of the vertical forces are thus not included here.

Second, some fraction of the pedestrians' mass should also be included in the horizontal dynamics. The appropriate fraction is denoted as ρ_p in Ref. [10]. It is similar to c_p, but relates to the component of the induced horizontal forces that are in phase with the bridge acceleration, rather than velocity. There is some uncertainty as to the appropriate values of ρ_p, although the experiments of Ref. [10] suggest that ρ_p is typically small and often negative. Thus whilst the mass matrix could be readily augmented with ρ_p terms, just as was done for the damping, this has not been done here because a small change to the inertia terms has little effect upon the system stability.

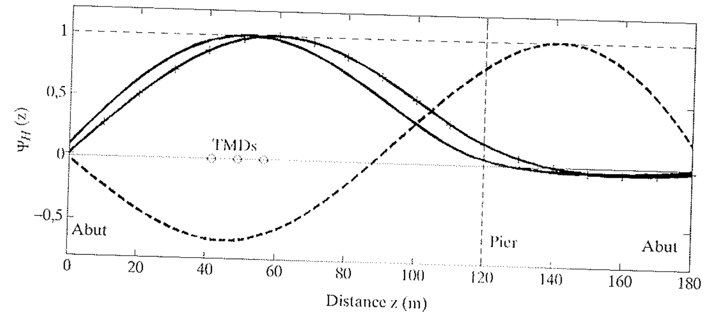

Fig. 3: Lateral deck components of the three modes of interest. The two modes close to 1 Hz (solid and +) involve the main span predominantly, and the mode near 1,33 Hz (dashed) involves both spans

Mode number	Frequency (Hz)	Renormalised modal mass (t)	Assumed damping (% of critical)
3	0,89	362	0,7
4	0,99	410	0,7
6	1,33	1791	0,7

Table 1: Horizontal mode properties

All the apparatus necessary to determine the critical number of people that will cause the bridge–TMDs–crowd system to lose lateral stability is now available.

Application to Stockton Infinity Footbridge

Finite element analysis of the bridge revealed three modes below 1,5 Hz, which involved significant lateral motion of the deck. The horizontal components Ψ_H of the deck are shown in *Fig. 3* and modal parameters are shown in *Table 1*. The two closely spaced modes near 1 Hz are approximately half-sine waves over the main span alone, with the lateral motions of the arch (not shown) being respectively in phase and out of phase with the deck, as would be expected for a simple two degree of freedom (arch, deck) system. The higher frequency mode at 1,33 Hz involves both spans.

Analysis of the serviceability scenarios suggested that three TMDs of the order of 5 t each would be required to limit horizontal vibrations near 1 Hz, and subsequent analysis indicated that tuning each to 0,88 Hz (just below the lowest mode) would be more effective than tuning one to each mode. The TMD parameters are given in *Table 2*.

The c_p value was taken to be 300 N s/m for 0,5 to 1 Hz. Above 1 Hz, c_p was assumed to fall linearly to zero at 1,5 Hz, although knowledge in this region is imprecise, and if critical, should be subject to sensitivity analysis and appropriate caution. With that caveat, the above information is all that is needed to construct the matrix A, for a given number N_p of people.

In the first instance, the model is constructed using only the two modes with frequencies below 1 Hz. The reason for this is clarity: for these two modes, c_p is a constant 300 N s/m, and thus no eigenfrequency iteration is required,

TMD Ref	Location along bridge (m)	Location across deck	TMD mass (kg)	Initial target frequency (Hz)	TMD damper (% of critical)	Maximum horizontal amplitude of TMD movement relative to deck (mm)
H1	41,2	Centroid on bridge centreline	5000	0,88	20	±20
H2	48,8		5000	0,88	20	±20
H3	56,3		5000	0,88	20	±20

Table 2: Horizontal TMD design properties

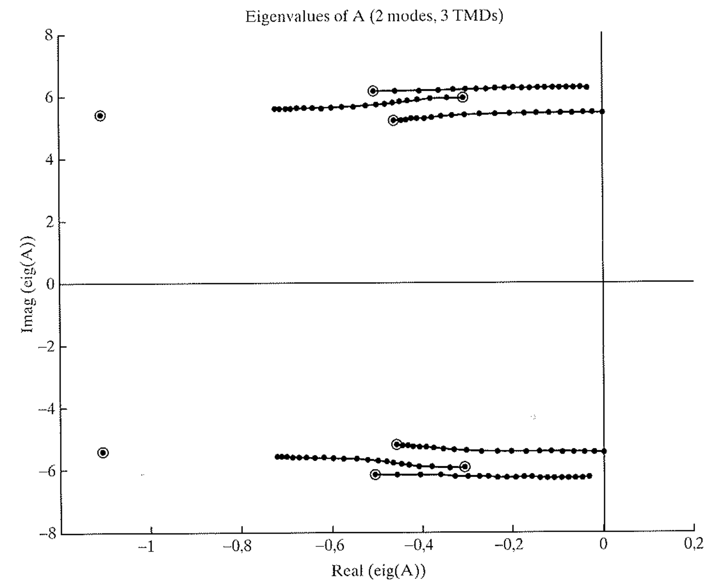

Fig. 4: Evolution of the ten complex eigenvalues of the system matrix A for the case of two modes (0,89, 0,99 Hz) and three TMDs (all tuned to 0,88 Hz). The circled points are the eigenvalues of the bridge–TMD system with no pedestrians. As pedestrian numbers increase (from 0 to 2000 in increments of 100) the effective damping of two of the modes decreases, with one mode losing stability when around 2000 people are on the bridge

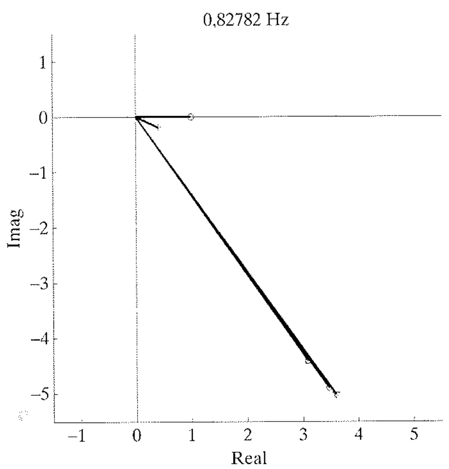

Fig. 6: The complex eigenvector for the two mode, three TMD models with no pedestrians, normalised such that the lowest frequency bridge mode has unit real amplitude

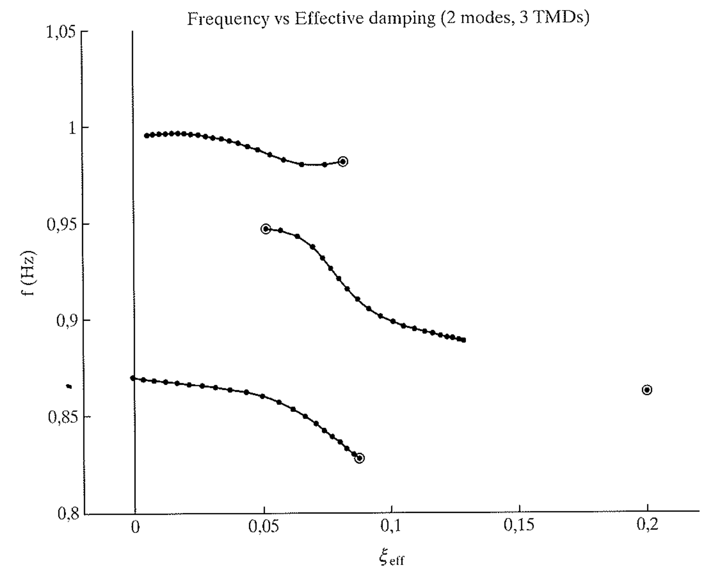

Fig. 5: The same information as Fig. 4, but rescaled such that the y-axis is the damped frequency in Hz and the x-axis is the effective damping as a fraction of critical

making it easier to see how the system loses stability as the number of people increases. Given that a two-mode description of the bridge may appear to be too severe a truncation, the model will later be expanded to include higher frequency bridge modes in order to see if these make any discernible difference.

Figure 4 shows the complex eigenvalue plot for the 0,89 and 0,99 Hz bridge modes plus three TMDs, as the number of people is increased from 0 to 2000 in increments of 100. The results with no people are circled. The same information is plotted in *Fig. 5*, showing the (damped) frequency (in Hz) against the effective damping in each mode. These are defined as follows: A second order damped system $\ddot{x}+2\xi\omega\dot{x}+\omega^2x=0$ with undamped natural frequency ω has solutions of the form $x = x_0\exp(st)$ requiring $s = -\xi\omega \pm i\sqrt{1-\xi^2}\,\omega$. For an eigenvalue of a general system one can thus define an effective damping as $\xi = -\mathrm{Re}(s)/|s|$ and equivalent damped and undamped natural frequencies as $\omega = |s|$ and $\omega_d = \sqrt{1-\xi^2}\,\omega = \mathrm{Im}(s)$.

The system is second order with five degrees of freedom, such that there are ten eigenvalues in five complex conjugate pairs. There are ten corresponding complex modes of the coupled bridge–TMD–people system. For the case of no pedestrians, it can be seen that there are complex resonances near 0,83 Hz, 0,95 Hz and 0,98 Hz (to the left of *Fig. 5*), with effective damping ratios between 5 and 9%, showing that the addition of the TMDs has succeeded in increasing the effective damping of the bridge modes from their low original values of 0,7%. The two remaining complex modes (practically coexistent at the left of *Fig. 4* and at the right of *Fig. 5*) involve TMD motions relative to an almost stationary deck and thus have damping fractions close to the TMDs' original 20%.

As the number of people is increased, the effective damping of two of the complex modes decreases. These are the two modes that involve most bridge motion. The eigenvalue trace of the lowest frequency mode crosses the zero damping stability axis when there are 1980 people on the bridge, corresponding to around 1320 people on the main span. This occurs just before the higher frequency mode loses stability with just over 2200 people.

The complex eigenvector of the lowest frequency is illustrated in *Fig. 6*, for the case with no pedestrians present. It can be seen that this complex mode involves responses in both the original bridge modes (o, +) and that these are out of phase with each other by approximately 30°. The mode also involves significant motion of the three TMDs (◊, , ∇), and it can be seen that these are essentially in phase with each other and thus acting largely as a single 15 t unit. As people are added to the bridge, the eigenvector evolves, but there is little of immediate interest other than that the three TMDs continue to act in unison.

Inclusion of the third bridge mode at 1,33 Hz leads to a 12×12 matrix A. The inclusion of this mode makes almost no difference to the eigenvalue plots for the behaviour below 1 Hz, due to its high modal mass and the large frequency mismatch with the other two modes. A new complex conjugate pair of eigenvalues is created with frequency near ±1,33 Hz. As the number of people increases, this complex mode-pair begins to lose stability. Resolution of the behaviour requires iteration on the frequency dependence of c_p used to construct A, and leads to a prediction of over 6000 people to cause loss of stability in this mode. Even if the c_p value remains at 300 N s/m in this region, this mode would lose stability if around 2100 people were on the bridge. It follows that this mode, although included, is not critical.

Finally, to check that the two or three bridge mode truncations have not unduly approximated any important features of the behaviour, a number of higher frequency bridge modes were incorporated into the formulation. These all had frequencies above 1,5 Hz, and thus the associated c_p values were zero. No iteration was thus required on the new eigenvalues, nor did these eigenvalues evolve as more people were added. More importantly though, their inclusion had almost negligible influence on the behaviour below 1,5 Hz, indicating that the simpler two and three bridge mode models had already done a good job at capturing the fully coupled behaviour.

Fig. 8: The fitted TMDs (photograph courtesy GERB)

In summary, the two modes below 1 Hz are critical and instability is predicted to occur when there are more than 1980 people on the bridge, with oscillations growing in the lowest frequency mode.

TMD Fitting and Full-Scale Testing

The three 5 t TMDs for lateral stability were fitted as designed, and located in the recess on the underside of the precast deck units (see *Figs. 7 and 8*). The TMDs have a thickness of only 260 mm.

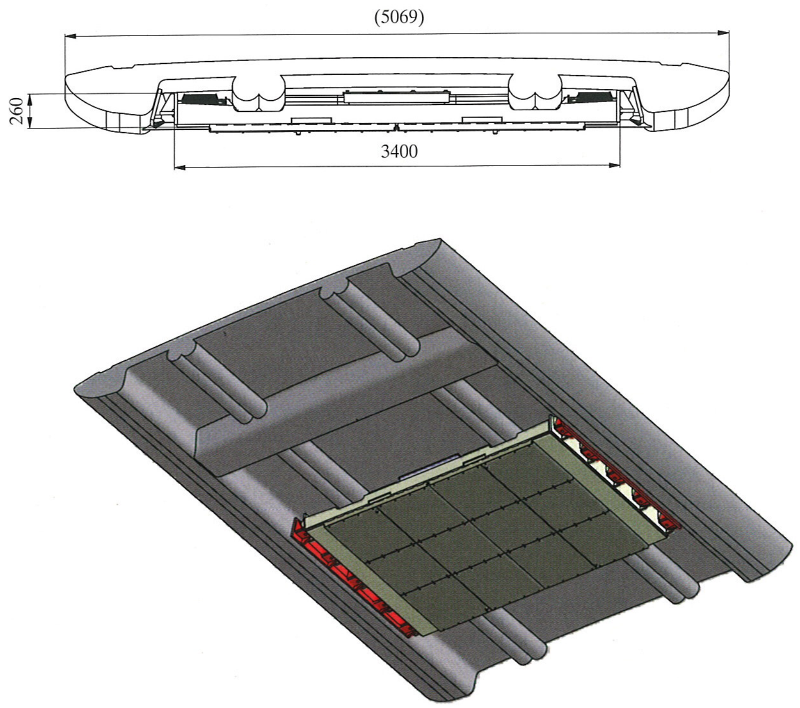

Fig. 7: The location of the TMDs below the deck (Units: mm)

Dynamic testing of the completed structure was undertaken[15] in January 2009. This involved a variety of ambient and human-induced vibrations tests with various groups of walkers, for the cases of TMDs either locked or unlocked. The primary purpose of the tests was to determine if the design predictions of modal frequencies and mode shapes were accurate, and to then estimate the improvements to damping when the TMDs were unlocked.

The test involving 10 people walking on the undamped main span showed two closely spaced lateral resonances near 1 Hz, the lowest of which had a frequency around 0,93 Hz. This is in reasonable agreement with the finite element modelling and design assumptions. Serviceability tests were undertaken with TMDs unlocked, and as a result it was decided that the horizontal TMDs did not require any further tuning from the as-designed 0,88 Hz. With TMDs unlocked, an effective damping ratio of 5,1% was indirectly calculated[15] for the lateral response near 1 Hz, although this calculation was complicated by the beat phenomena, presumably originating from the two closely spaced modes there. No crowd load tests were conducted which could validate or invalidate the prediction of the critical number of people to cause lateral instability.

Conclusion

The Arup criterion for the critical number of people was generalised to the multi-modal, multi-TMD case. Addition of TMDs can couple bridge modes with nearby frequencies and the standard assumptions of uncoupled structural modes each with its own TMD are no longer applicable. However, the coupled system equations can be described using complex modes, and this description can be readily extended to include the load model which treats human-induced lateral forces as negative damping. The eigenvalues of the resulting system matrix then determine the stability.

The procedure was applied to assess the lateral stability under crowd loading of the Stockton Infinity Footbridge. This involved three lateral modes below 1,5 Hz and three TMDs. Eigenvalue analysis of the resulting complex mode formulation predicts that the critical number of people for the Stockton structure is around 2000.

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