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# Lateral excitation of bridges by balancing pedestrians

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On its opening day, the London Millennium Bridge (LMB) experienced unexpected large amplitude lateral vibrations due to crowd loading. This form of pedestrian–structure interaction has since been identified on several other bridges of various structural forms. The mechanism has generally been attributed to 'pedestrian synchronous lateral excitation' or 'pedestrian lock-in'. However, some of the more recent site measurements have shown a lack of evidence of pedestrian synchronization, at least at the onset of the behaviour. This paper considers a simple model of human balance from the biomechanics field—the inverted pendulum model—for which the most effective means of lateral stabilization is by the control of the position, rather than the timing, of foot placement. The same balance strategy as for normal walking on a stationary surface is applied to walking on a laterally oscillating bridge. As a result, without altering their pacing frequency, averaged over a large number of cycles, the pedestrian effectively acts as a negative (or positive) damper to the bridge motion, which may be at a different frequency. This is in agreement with the empirical model developed by Arup from the measurements on the LMB, leading to divergent amplitude vibrations above a critical number of pedestrians.

**Keywords: bridges; dynamics; human–structure interaction; biomechanics; gait; inverted pendulum model**

## 1. Introduction

The problem of large amplitude lateral vibrations of bridges induced by crowds of pedestrians was brought to attention by the behaviour of the London Millennium Bridge (LMB) on its opening day in 2000. It was subsequently identified that several other bridges had experienced the same phenomenon (Dallard *et al.* 2001*a*,*b*; Fitzpatrick *et al.* 2001; Blekherman 2005), and indeed it had previously been reported on a tied arch footbridge in 1972 (see Bachmann & Ammann 1987) and a cable-stayed footbridge (the 'T-bridge') in 1993 (Fujino *et al.* 1993). More recently, measurements of the behaviour have been taken on at least five other bridges of different structural forms (Nakamura 2003; Brownjohn *et al.* 2004; SETRA 2006; Caetano *et al.* 2007; Macdonald 2008). The vibrations are characterized by sudden onset and the amplitudes achieved are greater than

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would be expected from random loading. It is clear that the motion of the bridge in some way affects the pedestrian loading, rather than the pedestrian simply applying an external force.

It has generally been assumed that the excitation mechanism involves the synchronization of the pedestrians to the bridge vibration frequency—so-called 'synchronous lateral excitation' or 'pedestrian lock-in'. Indeed, during large amplitude vibrations of the LMB and the T-bridge, video footage seemed to show the synchronization of a significant percentage of the pedestrians. However, analysis of some of the recent measurements on other bridges shows a lack of evidence of synchronization, so this is not necessarily the initiating mechanism. On the Clifton Suspension Bridge (CSB; Macdonald 2008), the following three observations cast doubt on this assumed mechanism.

- (i) The natural frequency of the dominant mode to be excited (Mode L2, 0.524 Hz) was approximately half the typical walking frequency. It seems unlikely that a large number of pedestrians would adjust their gait to lock-in to this low frequency, particularly when there are two other modes closer to normal walking frequencies (mode L3, 0.746 Hz and mode L4, 0.965 Hz).
- (ii) Once large vibrations had become established in mode L2, vibrations also built up in mode L3 simultaneously. Hence the mechanism was not lock-in to a single mode. It seems unlikely that if the mechanism were synchronization that a large number of people would synchronize to a weak stimulus from the second mode in the presence of a strong stimulus from the first mode, or indeed if they did, that a large number of pedestrians would continue to lock-in to the low-frequency first mode.
- (iii) Most significantly, if there were synchronization to the lateral natural frequencies, there would also be synchronization of vertical forcing at twice these frequencies, but this was hardly evident in the spectra of vertical responses. Rather there was relatively broadband vertical excitation in the 1.5–2.1 Hz frequency range, as would be expected from random walking.

On the Changi Mezzanine Bridge (CMB), Brownjohn *et al.* (2004) also noted the lack of a peak in the vertical spectrum at twice the lateral frequency and they noted that pedestrians were not observed to synchronize their motion while the bridge lateral amplitude increased significantly, although calculations suggested that if this were the mechanism, at least 26% of pedestrians should do so.

This paper presents a simple model of pedestrian gait, which, it is shown, does not require synchronization to cause lateral dynamic instability of bridges, which, it is believed, explains the above observations.

### (*a*) *Previous models of pedestrian lateral excitation*

From the measurements on the LMB, Dallard *et al.* (2001*b*; Fitzpatrick *et al.* 2001) suggest that pedestrians, while walking, act as negative dampers to lateral vibrations. Hence, above a certain critical number of pedestrians, this negative damping overcomes the positive structural damping, causing the onset of exponentially increasing vibrations. For uniform pedestrian density, the critical number of pedestrians, for a given mode of vibration, is (Fitzpatrick *et al.* 2001)

$$N_{\mathrm{crit}} = \frac{4\pi\zeta f_{\mathrm{n}} M}{k} \frac{L}{\int_0^L \phi^2 \,\mathrm{d}l}, \tag{1.1}$$

where $\zeta$ is the damping ratio to critical; $f_{\mathrm{n}}$ is the natural frequency; $M$ is the modal mass; $L$ is the length of the bridge; $\phi$ is the lateral mode shape; and $k$ is the 'lateral walking force coefficient', given as approximately $300\ \mathrm{N\,s\,m^{-1}}$ per person within the frequency range 0.5–1 Hz (Dallard *et al.* 2001*b*).

This relationship was found to estimate well the number of pedestrians for a sudden increase in lateral response of the Coimbra footbridge, being only approximately 70 (Caetano *et al.* 2007).

In this model, although synchronization was thought to be the cause, it was not a necessary assumption in the derivation. Rather it was based solely on the analysis of the LMB dynamic response. The same relationship was found from measurements on the CSB (Macdonald 2008), although in the absence of evidence of synchronization, as discussed earlier. The underlying mechanism therefore remained unclear.

Other tests on a 7 m long moving walkway and on the Solférino footbridge led SETRA (2006), the French Technical Department for Transport, Roads and Bridges, to suggest that synchronization occurs for lateral bridge acceleration above a threshold value of approximately $0.15\ \mathrm{m\,s^{-2}}$. However, the measurements on the LMB and the CSB show that the negative damping relationship is maintained even down to very low vibration amplitudes, and indeed the maximum acceleration amplitude on the CSB only just reached the above level. It therefore seems that even if such a threshold exists for synchronization, it does not explain the initiation of the vibration phenomenon.

Various authors have proposed models of the pedestrian loading mechanism, based on the assumption that synchronization is the cause (Nakamura 2004; Newland 2004; Roberts 2005*a*,*b*; Strogatz *et al.* 2005; Eckhardt *et al.* 2007). Often these models have free parameters that are chosen to fit the data. Blekherman (2005) alternatively suggests that the mechanism is autoparametric resonance between vertical and lateral modes of vibration with a 2 : 1 frequency ratio. However, full-scale measurements show a lack of correlation between vertical and lateral vibrations (Dallard *et al.* 2001*b*; Brownjohn *et al.* 2004; Macdonald 2008). Piccardo & Tubino (2008) critically review other models of the excitation, classified as direct resonance, dynamic interaction and internal resonance. They then propose a parametric excitation model that could potentially excite a structure at half the lateral walking frequency, but this could not explain observed lateral vibrations at approximately 1 Hz and they admit that the model is based on an uncertain forcing assumption.

In contrast to the other models of the dynamic instability, based on synchronization or certain frequency ratios, an entirely different approach has been proposed by Barker (2002). The pedestrian is modelled as a point mass that moves in a straight line forwards, supported by an inclined leg, alternatively left and right, with the step timing and initial position of the foot, relative to the point mass, assumed unaffected by the bridge motion.

The weight is resolved along the supporting leg which gives a horizontal component on the bridge dependent on the bridge displacement. It was shown that, averaged over all possible phase angles, the pedestrian actually does work on the bridge, even when the pedestrian walks at a different frequency to the bridge motion. This is due to the asymmetry of the behaviour, with the lateral force increasing if the bridge moves away from the mass, but decreasing if the bridge moves towards it.

Hence, this model predicts lateral dynamic instability in the absence of synchronization, so it could go some way to explaining the observed behaviour. However, two shortcomings of the model are that movement of the pedestrian mass owing to the horizontal force has been neglected and there is no justification for the assumed position of foot placement for each step.

Inspired by this approach, a more refined model is developed to describe the pedestrian–structure interaction, based on a simple model of human balance from the biomechanics field.

## 2. Inverted pendulum pedestrian model

The pedestrian is modelled as an inverted pendulum, simply comprising a lumped mass at the centre of mass (CoM) supported by a rigid massless leg (figure 1). This model is commonly used to analyse the dynamics of human balance (Winter 2005). In particular, it has been found to be a reasonable model of whole body balance in the frontal (i.e. lateral) plane during walking, particularly during the single-stance phase (MacKinnon & Winter 1993; Lyon & Day 1997; Hof *et al.* 2007). MacKinnon & Winter (1993) found the moment exerted by the ankle to be negligible relative to other moments, although balance can be fine-tuned by the rocking of the foot or the swaying of the upper body about the hips. Similarly, Hof *et al.* (2007) concluded that the lateral position of the centre of pressure (CoP) of the foot is mainly determined by the position of foot placement and it is only modified slightly thereafter by ankle control.

In common with the studies by MacKinnon & Winter (1993) and Hof *et al.* (2007), the current model considers only the motion in the frontal plane, ignoring any coupling with motion in the sagittal plane (i.e. the vertical plane including the direction of progression). Bauby & Kuo (2000) have shown that such coupling is weak. From the above findings, during each step, the CoP is taken to be at a fixed position on the supporting surface and the motion of the CoM is given by rigid body motion considering the external forces. If motion of the supporting surface (bridge deck) is included, this gives an equivalent mass-proportional force on the CoM. For simplicity, it is assumed, as by Hof *et al.* (2007), that the transfer from one foot to the other is instantaneous.

Considering figure 1 (with balance on the right foot), taking moments about the CoP of the foot,

$$-mL^2\ddot{\theta} = mgL\cos\theta + m\ddot{x}L\sin\theta, \tag{2.1}$$

where $m$ is the mass of the pedestrian; $L$ is the distance from the CoP to the CoM; $g$ is the acceleration due to gravity; $x$ is the absolute horizontal displacement of the bridge deck; and $\theta$ is the angle of the support leg from the horizontal.

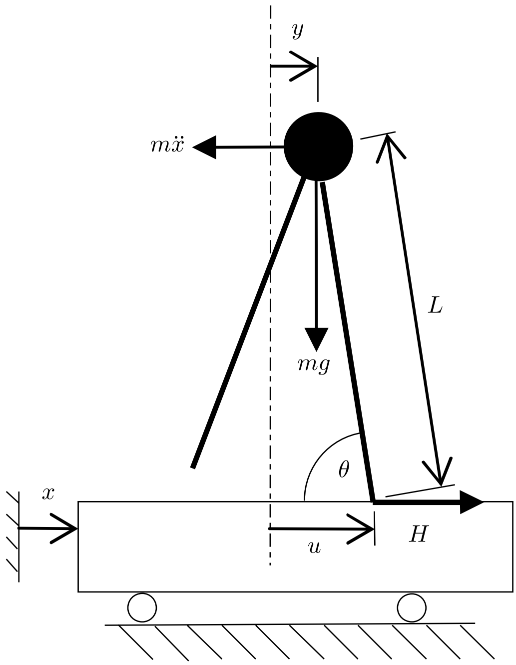


Figure 1. Inverted pendulum pedestrian model.

Defining $y$ and $u$ as the horizontal positions of the CoM and CoP, respectively, relative to an arbitrary reference point on the bridge deck (figure 1), and considering that $\theta$ is close to 90°, equation (2.1) can be approximated by

$$\ddot{y} + \Omega_p^2(u - y) = -\ddot{x}, \tag{2.2}$$

where $\Omega_p = \sqrt{g/L}$.

Equation (2.2) is equally valid for the support on the left foot, although $u$ is then (normally) less than $y$.

The horizontal force on the bridge is given by

$$H = -m(\ddot{x} + \ddot{y}) = m\Omega_p^2(u - y). \tag{2.3}$$

The first form of this expression is used by Newland (2004) and Roberts (2005*a*,*b*), although they assume that the pedestrian motion is sinusoidal (or periodic) and that a certain proportion of pedestrians synchronize with the bridge motion. The second form of the expression is used by Barker (2002), although he neglects the associated motion of the CoM described by equation (2.2).

## 3. Solution for no bridge motion

For no bridge motion (i.e. $\ddot{x} = 0$), the solution of equation (2.2) is (Hof *et al.* 2005)

$$y = u + (y_0 - u)\cosh[\Omega_p(t - t_0)] + \frac{v_0}{\Omega_p}\sinh[\Omega_p(t - t_0)], \tag{3.1}$$

where $t$ is time; and $t_0$, $y_0$ and $v_0$ are, respectively, the time, horizontal position and velocity of the CoM at the beginning of the current step.

Assuming a regular symmetrical gait with the frequency of a full cycle (right step+left step) equal to $f_{\mathrm{p}}$ and smooth transition from one foot to the other (Townsend 1985), at time $t = t_0 + 1/2f_{\mathrm{p}}$, the velocity is $-v_0$. Hence

$$\frac{v_0}{\Omega_p} = (u - y_0)\frac{\sinh(\Omega_p/2f_{\mathrm{p}})}{1+\cosh(\Omega_p/2f_{\mathrm{p}})} = (u - y_0)\tanh(\Omega_p/4f_{\mathrm{p}}), \tag{3.2}$$

where $y_0$ is the same for each step and can be defined as zero. Hence the displacement of the CoM and the force on the bridge are given by

$$y = u\{1 - \cosh[\Omega_p(t - t_0)] + \tanh(\Omega_p/4f_{\mathrm{p}})\sinh[\Omega_p(t - t_0)]\}, \tag{3.3}$$

$$H = mu\Omega_p^2\{\cosh[\Omega_p(t - t_0)] - \tanh(\Omega_p/4f_{\mathrm{p}})\sinh[\Omega_p(t - t_0)]\}, \tag{3.4}$$

where $u$ is equal to $\pm\delta/2$ (positive on the right foot, negative on the left), where $\delta$ is the step width. Taking $t=0$ at the start of the first step on the right foot, $t_0 = n/2f_{\mathrm{p}}$, where $n$ is an integer.

### (a) *Parameter values and numerical results*

The typical pedestrian mass is taken as 70 kg. The mean footfall frequency for normal walking is often taken to be 2.0 Hz, as measured by Matsumoto *et al.* (1978) from a sample of 505 people. However, Pachi & Ji (2005) found, from observations of 400 pedestrians walking naturally on two footbridges, that the mean footfall frequency was 1.83 Hz. In addition, given that pedestrians are likely to walk more slowly when in a dense crowd, a value of 1.8 Hz is used here. This gives a lateral walking frequency of 0.9 Hz.

Hof *et al.* (2005, 2007) stated that the effective inverted pendulum length for motion in the frontal plane is equal to 1.34 times the leg length. Based on a typical leg length of 0.9 m, this gives $L = 1.2$ m.

There is more uncertainty over the typical values of the step width, $\delta$. Donelan *et al.* (2001) gave an average measurement of 0.13 times leg length (117 mm) from 10 subjects, which was close to the step width they estimated would minimize metabolic cost, i.e. 0.12 times leg length (108 mm). However, Hof *et al.* (2007) found an average of 88 mm from the measurements on six normal subjects, and Townsend (1985) quoted values from previous comprehensive measurements by Murray and co-workers of 77 mm for men and 69 mm for women, at normal walking speeds. Given this variation of direct measurements, the value used here was chosen to match the measurements of the first harmonic of the lateral force, for which there is more consistency (see below), giving a value of 92 mm, in the middle of the range of values quoted above.

The typical pedestrian parameters used in this paper are summarized in table 1. Based on these values, the lateral displacement, velocity and acceleration of the inverted pendulum pedestrian model, for no bridge motion, are shown in figure 2. The patterns are similar to typical measurements presented by MacKinnon & Winter (1993). It is interesting to note that although the displacement appears to be approximately sinusoidal, the velocity and

Table 1. Estimated parameters for typical pedestrian.

| | |
|---|---|
| mass, $m$ | 70 kg |
| inverted pendulum length, $L$ | 1.2 m |
| lateral walking frequency, $f_{\mathrm{p}}$ | 0.9 Hz |
| step width for regular gait, $\delta$ | 92 mm |
| stability margin, $b_{\min}$ | 15.7 mm |

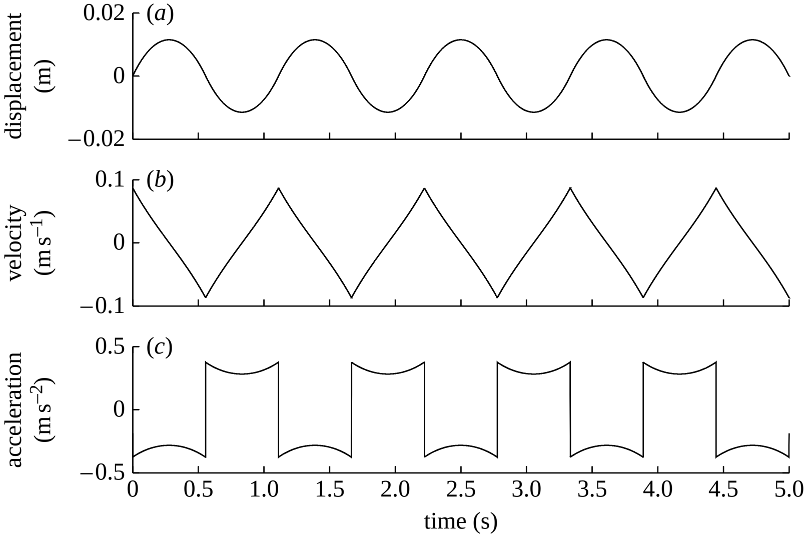


Figure 2. Motion of inverted pendulum pedestrian model for no bridge motion. Pedestrian parameters from table 1. (*a*) Displacement versus time, (*b*) velocity versus time, (*c*) acceleration versus time.

acceleration indicate that it departs from this significantly. The maximum displacement of the CoM is 11.4 mm. This compares with measured values of 16.2 mm for men and 10 mm for women, quoted by Townsend (1985).

The resulting force on the bridge deck is given in figure 3. Also shown is the total lateral force based on measured forces for single footfalls, averaged over 26 subjects, by Schneider & Chao (1983), assuming the footfalls to be regular and symmetric, and allowing for the timing of the single- and double-stance phases of walking. The dynamic load factors (DLFs), i.e. force divided by body weight, of the first five Fourier components are given in table 2, from the inverted pendulum model and measured values from different authors. Bachmann & Ammann's (1987) values are often quoted in the structural engineering field, but they were based on the measurements of only one pedestrian. Pizzimenti & Ricciardelli (2005) gave values averaged over 66 subjects. The DLF of just the first harmonic is presented by Dallard *et al.* (2001*b*), based on the measurements at Imperial College, London to address the LMB vibration problems. Only Schneider & Chao (1983) provided phase information from which the time histories can be directly compared. It can be seen that there is good agreement of the DLF of the first harmonic, which is matched by the inverted pendulum model with the chosen parameters.

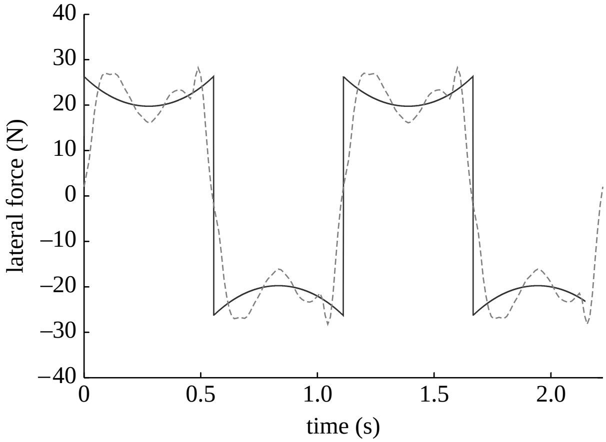


Figure 3. Lateral force on bridge deck for no bridge motion. Inverted pendulum (solid curve) with parameters from table 1, compared with the measured values by Schneider & Chao (1983; dotted curve) also scaled for pedestrian of mass 70 kg.

Table 2. Dynamic load factors of Fourier components of lateral forces, for no bridge motion.

| | dynamic load factors (% body weight) | | | | |
|---|---|---|---|---|---|
| | $f_{\mathrm{p}}$ | $2f_{\mathrm{p}}$ | $3f_{\mathrm{p}}$ | $4f_{\mathrm{p}}$ | $5f_{\mathrm{p}}$ |
| inverted pendulum model[a] | 3.9 | 0 | 1.6 | 0 | 1.0 |
| Schneider & Chao (1983) | 3.9 | 0 | 1.8 | 0 | 0.4 |
| Bachmann & Ammann (1987) | 3.9 | 1.0 | 4.3 | 1.2 | 1.5 |
| Pizzimenti & Ricciardelli (2005) | 4.0 | 0.8 | 2.3 | 0.4 | 1.1 |
| Dallard *et al.* (2001*b*) | 4.0 | — | — | — | — |

[a]Using parameter values from table 1.

The model gives a slight underestimate of the third harmonic and a reasonable value for the fifth harmonic. Bachmann & Ammann (1987) and Pizzimenti & Ricciardelli (2005) gave non-zero components of even harmonics due to asymmetry of the gait.

Overall it has been shown that, with the chosen parameters, the inverted pendulum model gives a reasonable approximation of the pedestrian motion and lateral forces on a stationary surface.

## 4. Solution with bridge motion

When bridge motion is introduced, equations ((2.2) and (2.3)) still hold, but the bridge motion effectively causes an inertia force on the pedestrian CoM, which modifies the pedestrian motion during each step.

Since the effect of an individual pedestrian on the global bridge behaviour is small and the damping of bridge vibrations is low, for the purposes of the pedestrian model in this study it is taken that the bridge is vibrating sinusoidally with small amplitude expressed as

$$x = X\sin[\omega_\mathrm{b}(t-\tau)], \tag{4.1}$$

where $X$ and $\omega_b$ are, respectively, the amplitude and circular natural frequency of the bridge motion, and $\tau$ is the time lag from the beginning of the first step on the right foot to the start of the following bridge vibration cycle.

The solution of equation (2.2), for any given step, is then

$$y = u + \{y_0 - u + A\sin[\omega_b(t_0-\tau)]\}\cosh[\Omega_p(t-t_0)] + \left\{\frac{v_0}{\Omega_p} + \frac{\omega_b}{\Omega_p}A\cos[\omega_b(t_0-\tau)]\right\}\sinh[\Omega_p(t-t_0)] - A\sin[\omega_b(t-\tau)], \tag{4.2}$$

where $A = X/(1+(\Omega_p/\omega_b)^2)$.

The remaining problem is to define the initial conditions for each step. The initial position of the CoM, $y_0$, must be equal to the final position from the previous step. Hence the remaining unknowns are the foot placement position, $u$, the velocity of the CoM at the beginning of the step, $v_0$, and the timing of the footstep, $t_0$. To determine these, it is necessary to consider the control strategy used to maintain lateral balance in human walking.

## (*a*) *Lateral balance control strategy*

Several authors have found that the lateral foot placement position is the most effective means of maintaining lateral balance during walking (Townsend 1985; MacKinnon & Winter 1993; Kuo 1999; Bauby & Kuo 2000; Hof *et al.* 2007). Of particular relevance to the current situation, Oddsson *et al.* (2004) applied perturbations to subjects during steady locomotion, in the form of sudden translations of a platform at an angle of 45°/−135° to the direction of progression. They found that the most common response of subjects was to alter the width of their following steps, in proportion to the amplitude of the disturbance. They concluded that the inverted pendulum model is valid, with the lateral foot position being the most important factor to control balance, including in response to external perturbations.

Vertical impulses are necessary at the transition from one foot to the other, considering the vertical motion of the CoM (particularly for sagittal plane dynamics). However, Townsend (1985) noted that it is desirable for there to be no net lateral impulse, which he stated is supported by most data. The measured lateral velocities presented by MacKinnon & Winter (1993) concur, showing no significant change at foot transition. Likewise, measured lateral forces do not display significant impulses but are comparable with the output from the inverted pendulum model with an assumed smooth transfer between feet (§3; figure 3). Therefore, it is assumed here that the initial lateral velocity of the CoM on each foot, $v_0$, is equal to the final velocity on the previous foot.

Kuo (1999) found that timing of footfalls is set primarily by dynamics in the sagittal plane. On this basis, here it is assumed that they occur at constant intervals of $t_0$, independent of lateral bridge motion.

Therefore, in the present model it is assumed that the lateral position of the CoP, $u$, is the only parameter used to control lateral balance. This is in contrast to the models of pedestrian synchronization, where it is the timing of footsteps that is assumed to be altered. The position of the CoP is taken as constant on each foot, representing the average position due to the combination of the major foot placement strategy and any minor adjustments from ankle movement.

A number of authors have considered control laws for the foot placement position (Townsend 1985; Redfern & Schumann 1994; Bauby & Kuo 2000), commonly concluding that the lateral displacement and velocity of the CoM prior to foot placement are the most important control parameters. Hof *et al.* (2007) also found this and they proposed a lateral balance model that can be applied directly in the present situation. Previously, Hof *et al.* (2005) noted from equation (3.1) that the condition for the CoM to not pass the CoP, which could lead to the pedestrian falling, is (on the right foot)

$$y_0 + \frac{v_0}{\Omega_p} \leq u. \tag{4.3}$$

This led to the definition of the 'position of the extrapolated CoM' (XcoM) as the quantity $y + \dot{y}/\Omega_p$. For dynamic stability, the CoP must be placed beyond the XcoM (to the right when stepping onto the right foot and to the left when onto the left foot). The additional distance beyond the XcoM is defined as the 'margin of stability', $b$. This is a minimum, equal to $b_{\min}$, when the foot is first placed.

Hof *et al.* (2007) showed that for normal walking there is variation of the displacements of the CoM and CoP from one step to another, but that there is a strong correlation between the position of the CoP averaged over each step and the position of the XcoM at the time of initial foot contact, which can be modelled as $b_{\min}$ being constant for each step. The mean measured values for normal subjects varied from 13.8 mm for walking at 0.75 m s$^{-1}$ to 18.4 mm at 1.25 m s$^{-1}$, although the differences were not statistically significant. In comparison, for the regular symmetrical gait of the inverted pendulum model, with the parameters given in §3*a* and table 1, the value of $b_{\min}$, at the transition from one foot to the other, is 15.7 mm.

It is assumed here that in the presence of bridge motion the same control law is still followed, so the position of foot placement for each step is given by

$$u = y_0 + \frac{v_0}{\Omega_p} \pm b_{\min} \quad (+\text{for right foot}, -\text{for left foot}), \tag{4.4}$$

where $y_0$ and $v_0$, the initial position and velocity of the CoM for the new step, are equal to the final values from the previous step, just before the foot placement, and the value of $b_{\min}$ is maintained as 15.7 mm. The effect of the bridge motion is to modify the motion of the pedestrian CoM during each step, in accordance with equation (4.2), thus modifying the final displacement and velocity and hence the foot placement position for the next step.

## 5. Simulation of pedestrian gait on laterally moving surface

Using the solution of the equation of motion for each step given by equation (4.2) together with the foot placement control law for the transition between steps given by equation (4.4), the motion of the pedestrian has been simulated step by step. For arbitrary initial conditions, simulations with no bridge motion

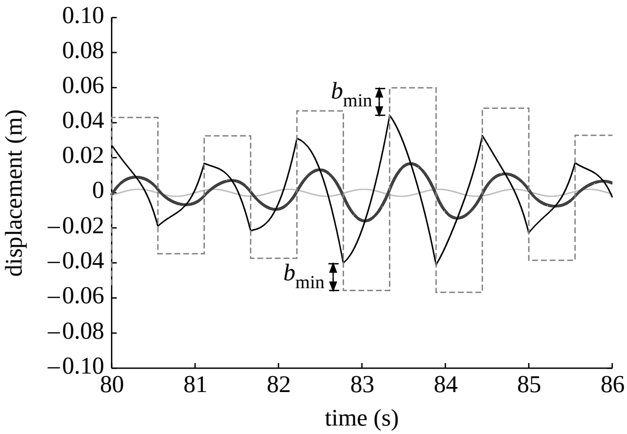


Figure 4. Lateral displacements of bridge and pedestrian, with bridge motion of 2 mm amplitude at 1.11 Hz and pedestrian parameters from table 1. Thin solid grey curve, bridge; thick solid black curve, CoM; thin solid black curve, XcoM; black dashed curve, CoP.

converged on the steady-state solution in §3 within a few steps, giving confidence in the results and showing that the balance control strategy gives a stable gait. Simulations with a laterally moving surface were also found to be stable and again the effect of the initial conditions was found to die out within a few steps.

Figure 4 shows typical displacements of the bridge and the pedestrian CoM, XcoM and CoP, using the pedestrian parameters in table 1 and with bridge motion of 2 mm amplitude at 1.11 Hz. It can be seen how the CoP is positioned a distance $b_{\rm min}$ beyond the XcoM at the time of foot placement. The amplitude of CoM oscillations varies as the relative phase between the pedestrian and bridge varies.

In comparison, Nakamura (2003) presented two records of bridge vibrations (at 0.88 Hz) together with pedestrian motions from belt-mounted accelerometers. One record, with maximum bridge amplitude 45 mm, shows the pedestrian to apparently repeatedly 'tune' and 'detune' with the bridge frequency. Rather than intermittent synchronization, as suggested, an alternative explanation would be that the pedestrian continued to walk with a different frequency from the bridge, so was periodically in phase and out of phase with it. A simulation using the method presented was able to produce similar results for the pedestrian motion as measured by Nakamura.

### (*a*) *Frequency content of pedestrian forcing, in the presence of bridge motion*

For the same simulation as in figure 4, the resulting force on the bridge is shown in figure 5. Figure 6 shows the magnitude of the fast Fourier transform (FFT) of the force on the bridge, over a period of 100 bridge cycles or 81 pedestrian cycles. There are force components at odd harmonics of the pedestrian walking frequency ($f_{\rm p}$), which are no different from the case without bridge motion (table 2). However, there are additional components at frequencies $nf_{\rm p} \pm (f_{\rm b} - f_{\rm p})$, where $n$ is an odd integer. In particular, this includes a component at the bridge vibration frequency itself ($f_{\rm b}$). Hence the bridge motion causes

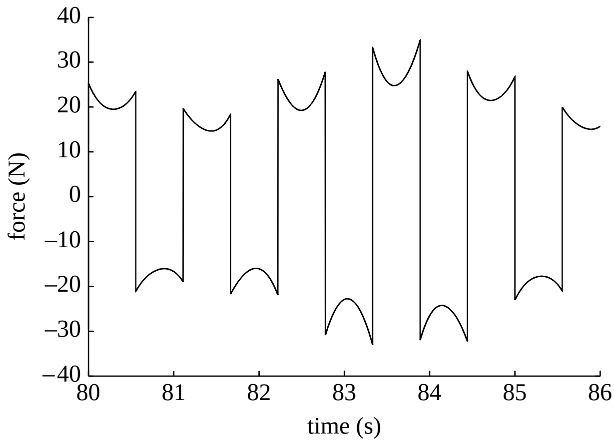


Figure 5. Lateral force on bridge, corresponding to figure 4.

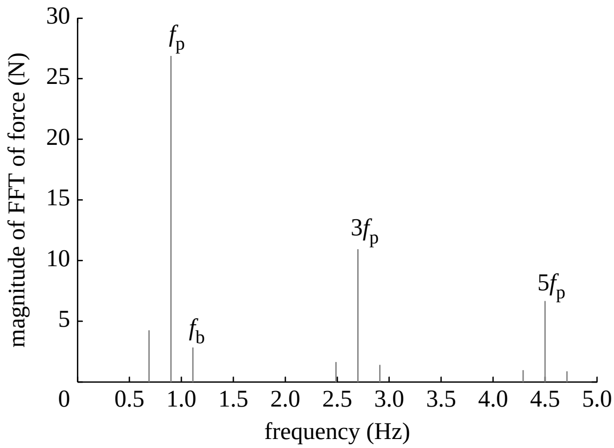


Figure 6. Fourier components of lateral force on bridge over a period of 100 bridge cycles or 81 pedestrian cycles with bridge motion of 2 mm amplitude at 1.11 Hz ($=f_b$) and pedestrian parameters from table 1 ($f_p$=0.9 Hz).

a self-excited force from its effect on the pedestrian, which is in addition to the normal pedestrian excitation force. The self-excited force is due to a combination of the interaction between the human and bridge motions in the single-stance phase, described by equation (4.2), and the balance corrections from the CoM displacement and velocity at the end of the previous step, which determine the CoP position, $u$. The additional components of the force at frequencies other than $f_b$ are unimportant since they are quite small in magnitude and are detuned from the bridge frequency.

The results are consistent with the measurements by Pizzimenti & Ricciardelli (2005) from pedestrians walking on a laterally oscillating instrumented treadmill. They found that for most of their tests the spectra of the lateral forces contained components at both the normal walking frequency and the oscillation frequency.

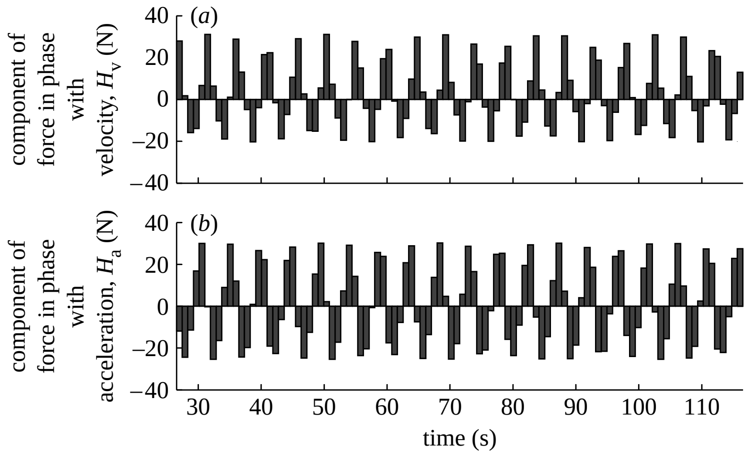


Figure 7. Magnitudes of components of force on bridge in phase with (*a*) bridge velocity and (*b*) acceleration for each bridge vibration cycle, from simulation over 100 bridge cycles or 81 pedestrian cycles with bridge motion of 2 mm amplitude at 1.11 Hz and pedestrian parameters from table 1.

### (*b*) *Effective damping and added mass per pedestrian*

The self-excited harmonic force can be expressed as two components, in phase with the bridge velocity and acceleration, respectively, with magnitudes given by

$$H_\mathrm{v} = \frac{2}{T}\int_{T_0}^{T_0+T} H\cos[\omega_b(t-\tau)]\mathrm{d}t, \tag{5.1}$$

$$H_\mathrm{a} = \frac{-2}{T}\int_{T_0}^{T_0+T} H\sin[\omega_b(t-\tau)]\mathrm{d}t. \tag{5.2}$$

For steady-state synchronized motion of the bridge and pedestrian, $T$ can be taken as the period of one bridge vibration cycle, but with the pedestrian not synchronized in frequency with the bridge, $H_\mathrm{v}$ and $H_\mathrm{a}$ calculated on this basis vary from cycle to cycle, as shown, for example, in figure 7 for the simulation above. It can be seen that they fluctuate at the beating frequency $|f_\mathrm{b}-f_\mathrm{p}|$ $(=0.21\ \mathrm{Hz}$ in this case), as the relative phase between the bridge and pedestrian changes. $H_\mathrm{v}$ does work on the bridge, which is periodically positive and negative.

Whereas it may have been anticipated that the average self-excited forces would be zero for unsynchronized motion, this is not necessarily the case since $H$ is a nonlinear function of the bridge motion and the step width can vary from step to step. The expected values of $H_\mathrm{v}$ and $H_\mathrm{a}$ can be calculated over a large number of vibration cycles, with the time lag between the beginning of the bridge cycle and the pedestrian cycle evenly covering the full range of possibilities. The results of such a simulation in figure 7 show that the mean values are non-zero, giving the net self-excited forces. The data in figure 7 are re-plotted in figure 8 against the time lag, showing how some lags give positive values and others negative, but the behaviour is slightly asymmetric.

The average component of the force in phase with the bridge velocity can be equated to a damping term in the equation of motion of the bridge, with the equivalent damping coefficient per pedestrian given by

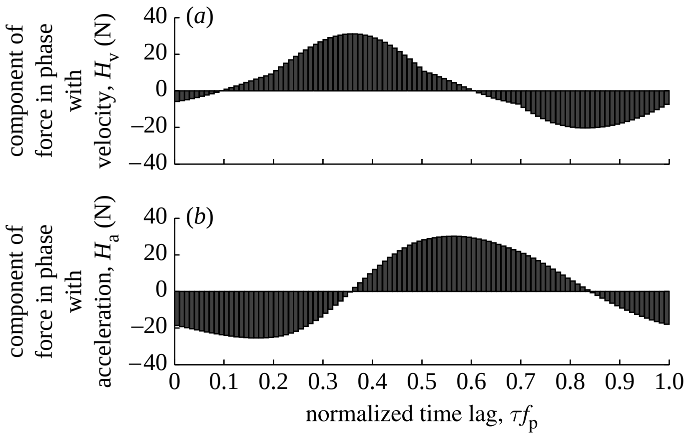


Figure 8. Magnitudes of components of force in phase with (*a*) bridge velocity and (*b*) acceleration for each bridge vibration cycle as in figure 7, re-plotted against normalized time lag of start of bridge cycle from previous right footfall.

$$c_{\mathrm{p}} = -k = \frac{-\bar{H}_{\mathrm{v}}}{X\omega_b}, \tag{5.3}$$

where the overbar represents the average and $k$ is the lateral walking force coefficient proposed by Dallard *et al.* (2001*b*; in equation (1.1)), although it could be amplitude (as well as frequency) dependent.

Similarly, the component in phase with the acceleration is equivalent to the bridge having an added mass per pedestrian of

$$\Delta M = \frac{-\bar{H}_{\mathrm{a}}}{X\omega_b^2}. \tag{5.4}$$

Simulations were conducted for wide ranges of bridge frequencies and amplitudes. The equivalent damping and added mass per pedestrian were found from averages over at least 100 bridge cycles, starting after 30 cycles to allow initial transients to decay. The results from equations ((5.3) and (5.4)) were found to be independent of bridge amplitude, meaning that the forces at the bridge frequency are indeed well represented as added damping and mass, rather than as external forces. It was also confirmed that the basic pedestrian excitation forces, described by the harmonics of the pedestrian walking frequency (table 2), are invariant with bridge motion. Furthermore, it was found that they are proportional to the initial margin of stability, $b_{\mathrm{min}}$ (taken as constant), but it has no effect on the self-excited forces.

The added damping and mass are, however, frequency dependent, as shown in figure 9 (solid curve). Also shown in figure 9*a* are values of $c_{\mathrm{p}}$ found from full-scale measurements on three different bridges.

Although the numerical values obtained from the analysis do not match the full-scale measurements accurately, the predicted behaviour is similar. For bridge frequencies in the range of 0.7–1.7 Hz (0.8–1.9 times lateral walking frequency), the pedestrian acts as a negative damper to bridge vibrations, as proposed by Dallard *et al.* (2001*b*; Fitzpatrick *et al.* 2001). In the worst case the lateral walking force coefficient, $k$, is just over 200 N s m$^{-1}$, at a frequency of 1.2 Hz (1.3 times walking frequency).

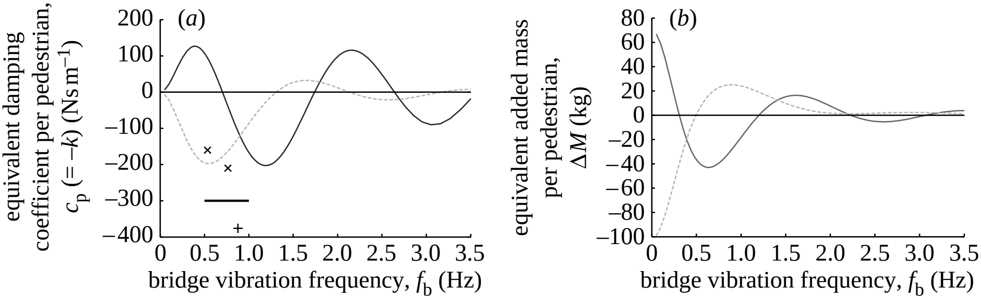


Figure 9. (*a*) Equivalent damping coefficient per pedestrian (thin black curves), along with estimated values from site measurements on three bridges: thick black line, LMB (Dallard *et al.* 2001*b*); +, CMB (Brownjohn *et al.* 2004); X, CSB (Macdonald 2008). (*b*) Equivalent added mass per pedestrian. Pedestrian parameters from table 1. Results from multiple simulations, each over 200 bridge cycles, for different bridge frequencies. Results shown for both the standard foot placement control law, based on the relative velocity of the CoM with respect to the bridge (solid curve; §4*a*), and the alternative based on the absolute velocity of the CoM (dashed curve; §5*c*).

Apart from the inaccuracy due to the simplicity of the model, the discrepancy with the full-scale measurements could be partly due to the parameter values chosen. In particular, in crowded conditions, the average walking frequency would be expected to decrease. Both Matsumoto *et al.* (1978) and Pachi & Ji (2005) observed the lowest lateral walking frequencies of approximately 0.7 Hz in normal (presumably uncrowded) conditions. Using this value in the analysis gives a maximum value of $k$ of 280 N s m$^{-1}$ at a bridge frequency of 0.9 Hz, which is much closer to the values from full-scale observations. The value of $k$ is also proportional to the pedestrian mass (here taken as 70 kg). The value of 300 N s m$^{-1}$ proposed by Dallard *et al.* (2001*b*; Fitzpatrick *et al.* 2001) for the frequency range of 0.5–1 Hz is conservative relative to the results obtained here, so seems reasonable for design, although the analysis suggests that it should be applied up to higher frequencies.

Figure 9*b* shows that for bridge frequencies between 0.3 and 1.2 Hz (0.3–1.3 times pedestrian frequency), the equivalent added mass per pedestrian is negative, with a minimum value equal in magnitude to 61% of the body mass (i.e. −43 kg in this case). This slightly surprising result is actually consistent with the site measurements from the CSB, which showed that the vibration frequencies of both pedestrian-excited lateral modes increased with more pedestrians (Macdonald 2008), contrary to expectations were they to have acted as passive mass. The result is also in agreement with the measurements by Pizzimenti & Ricciardelli (2005) who found that, over the frequency range they tested (0.6–0.92 Hz), the component of the self-excited force in phase with the displacement (or acceleration) always acted as positive stiffness (or negative mass). They also found that the component in phase with the velocity acts as positive damping at lower frequencies but as negative damping at higher frequencies, which agrees with figure 9*a*.

It should be noted that the results presented here are expected values over a large number of footsteps and/or pedestrians. Over short time periods and for pedestrians (each with different parameters) walking randomly, the actual loading can vary considerably, so the vibrations may grow or decay. On the CMB, Brownjohn *et al.* (2004) found that in one test lateral vibrations built up with 100 pedestrians, but this was not repeatable.

### (c) Variations of the model

In simulations, various other algorithms for the lateral foot placement position have been trialled, as alternatives to equation (4.4). Using either a constant step width, from one foot to the other, or a constant half-step width, from the CoM to the foot being placed (similar to Barker's (2002) model), yields an unstable gait, even without bridge motion. Any perturbation from the exact solution (§3) leads to increasing pedestrian motion, soon giving unreasonable values. Furthermore, no control law based only on the CoM displacement and an offset has been found to be stable.

Redfern & Schumann (1994) suggested that the foot should be placed symmetrically with the other foot, with respect to the CoM. This gives the correct solution for no bridge motion (§3). However, with bridge motion, although it gives a stable gait, the pedestrian staggers off to one side ($y_0$ continues to increase or decrease), and step widths can become unreasonably large, even for bridge vibrations as small as 2 mm amplitude.

A variation of the model from Hof *et al.* (2007) was investigated, still based on equation (4.4) but using the displacement and velocity of the CoM before the end of the previous step, at the time of actual foot placement considering the double-stance phase. However, without any subsequent adjustment of the CoP or the velocity, as could occur in actual human gait, this was generally found to be unstable, even if additional factors were applied to $y_0$ and $v_0$ in equation (4.4).

The analysis thus far has considered the motion of the CoM relative to the bridge. However, since the original lateral balance model by Hof *et al.* (2007) did not consider motion of the surface, it could alternatively be interpreted to apply to the absolute CoM motion. It seems likely that it is more appropriate to use the relative motion, since the pedestrian's local frame of reference will be the bridge and, in particular, Bauby & Kuo (2000) have demonstrated that visual feedback is important for lateral balance. However, the vestibular system senses absolute motion, so it could have an influence on the behaviour. To investigate the potential effect, the model was modified to use the absolute motion. In practice, the only difference is the use of the absolute, rather than relative, velocity of the CoM in the foot placement control law (equation (4.4)). This yielded stable solutions with reasonable pedestrian motions and step widths. With bridge motion, the modified model yields the equivalent damping and added mass depicted by dashed curves in figure 9. The minimum damping per pedestrian is similar to before, but it occurs at a lower frequency, while the added mass is positive for all frequencies above 0.5 Hz. Hence, the equivalent damping is arguably more in keeping with the full-scale measurements, but the added mass does not agree with the observations on the CSB (Macdonald 2008) or the pedestrian tests by Pizzimenti & Ricciardelli (2005). It seems likely that the actual pedestrian response will be somewhere between the behaviour considering relative and absolute motions, probably more based on the relative motion for the reasons given above.

Many other lateral balance strategies can be imagined. Additional factors can be applied to $y_0$ and $v_0$ in equation (4.4), which, within certain ranges, give stable gaits and can significantly affect the values of the effective damping and added mass. In particular, a factor greater than unity applied to $y_0$ can correct any lateral drift of the CoM. However, there is currently no basis for the choice of such additional factors. Other variations could include using additional sensory information or applying corrective lateral impulses at the foot transition. Such modifications could easily be incorporated into the model, but at present, in the absence of any further data on actual pedestrian lateral balance control, the original model proposed (equations (4.2) and (4.4)), using the relative velocity, appears to be the most reasonable.

A final variation of the model was to introduce random (Gaussian) variations of the foot placement position, $u$, and the timing of foot placement, $t_0$. Although this gives some scatter of results and low-level forcing over a broad frequency range, it causes little difference to either the mean force components at harmonics of the walking frequency or the mean values of the pedestrian effective damping and added mass.

## 6. Conclusions

A simple model of pedestrian lateral balance has been adopted from a previous study of human gait (Hof *et al.* 2007). The model comprises an inverted pendulum with instantaneous transfer of support from one foot to the other. Balance is achieved by varying the lateral position of foot placement for each step, based only on the final displacement and velocity of the CoM from the previous step. The model gives a reasonable representation of the lateral pedestrian forces applied to the ground (or bridge) in the absence of bridge motion, compared against previous measurements.

It has been shown that in the presence of lateral bridge motion, without changing the pedestrian walking frequency (unsynchronized with the bridge) and applying the same foot placement strategy to maintain lateral balance, the model causes additional lateral force components at the bridge frequency. In certain ranges of bridge frequencies, on average, these additional forces are equivalent to the pedestrian providing negative damping (and negative added mass) to the bridge. Hence, without synchronization, this would explain the underlying mechanism of the empirical negative damping model derived from tests on the LMB (Dallard *et al.* 2001*b*; Fitzpatrick *et al.* 2001). It is also consistent with the measurements on the CMB (Brownjohn *et al.* 2004) and CSB (Macdonald 2008), where there was evidence of a lack of pedestrian synchronization.

Unfortunately, with the chosen parameters, the present model does not accurately match the numerical results from the full-scale measurements, although it is possible to modify the results considerably using different parameters or balance strategies. In addition, the model does not cover the behaviour for larger bridge amplitudes for which pedestrians may change their gait, possibly including synchronizing their motion with the bridge or stopping walking. Therefore, to refine the model, further data are required on pedestrian behaviour when perturbed by bridge motion.

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