- You have **120 minutes** to answer the questions.
- Please write your **name** and **roll number** at appropriate places.
- Write the **key steps** of your solution of the problems in the space provided.
- You can solve the problems in the supplementary sheets provided. Please attach the same to this paper.

Name: ______________________ **Roll number:** ____________

1. Mark the typical(*approximate*) power rating of the following devices: **2 marks**

   (i) Ceiling Fan: (a) 10 W (b) 70 W (c) 500 W (d) 5 kW. 1

   (ii) Air Conditioner: (a) 15 W (b) 150 W (c) 1.5 kW (d) 5 kW. 1

2. A three phase device has its nameplate rating as 50 Hz, 3 kW, 415 V. The voltage rating refers to: **1 mark**

   (a) Line to line peak value (b) Line to line rms value 1
   (c) Line to neutral peak value (d) Line to neutral rms value.

3. The currents in the three phases of a star-connected load are $I_a = 4\angle 30°$ A, $I_b = 4\angle 170°$ A and $I_c = 4\angle -10°$ A. The current through the neutral is **1 mark**
   (a) 0 A (b) $4\angle 30°$ A (c) $2\sqrt{3}\angle 60°$ A (d) $2\angle 120°$ A. 1

   $4\angle 30° + 4\angle 170° + 4\angle -10°$
   $= 4\angle 30° + \cancel{4\angle 170°} - \cancel{4\angle 170°}$

4. Typical range of 50 Hz current that induces some perception (tingling sensation) in a human being is approximately **1 mark**
   (a) 1–5 μA (b) 1–5 mA (c) 1–5 A (d) 10–50 A 1

5. For the circuit shown in Figure 1, find the current I_R. **3 marks**

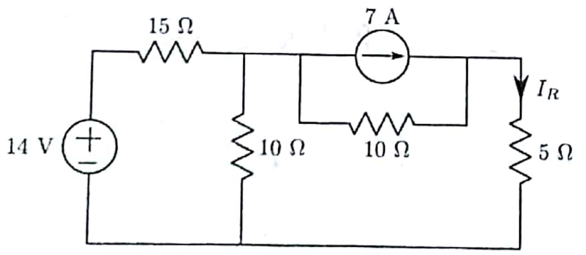
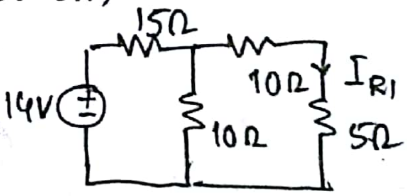


Figure 1: Circuit for Problem 5

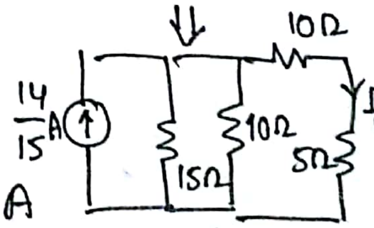
Applying superposition,

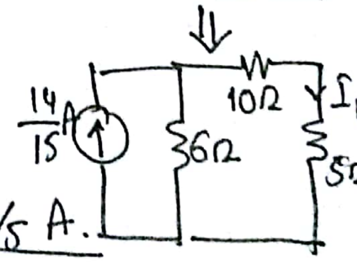


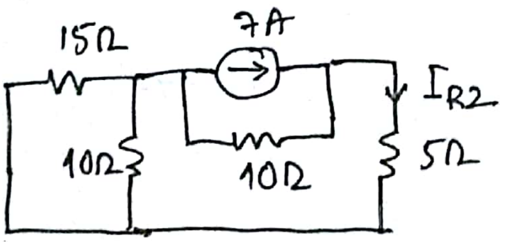
$I_{R1} = \frac{14}{15} \times \frac{6}{6+15}$ A

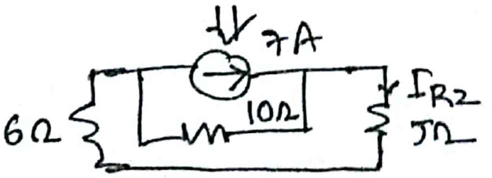
$= \frac{\cancel{14}^{2}}{15} \times \frac{\cancel{6}^{2}}{\cancel{21}_{3}}$ A

$= \frac{4}{15}$ A.









$I_{R2} = 7 \times \frac{10}{10+11}$ A

$= 7 \times \frac{10}{21}$ A

$= \frac{10}{3}$ A.

$I_R = \frac{4}{15} + \frac{10}{3} = \underline{\frac{18}{5}\text{ A.}}$

6. For the circuit shown in Figure 2, find the steady state value of I. 3 marks

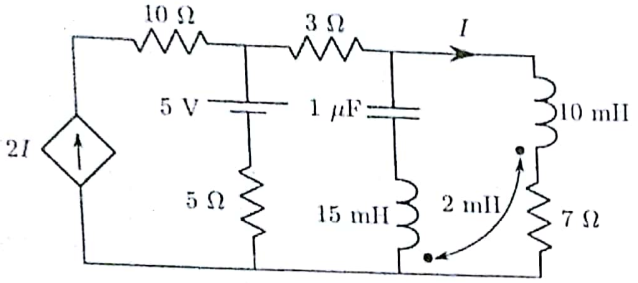
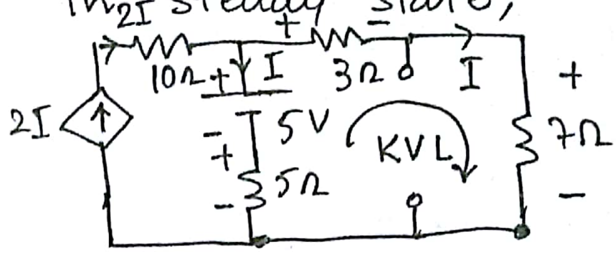


Figure 2: Circuit for Problem 6

In steady state,



1 mark

Applying KVL,

$5I + 5 - 3I - 7I = 0$. 1 mark.

or, $5 = 5I \Rightarrow \boxed{I = 1\ A}$.

1 mark.

3

7. A resistance R is connected to a linear network and the current flowing through it is I as shown in Figure 3. It is observed that for $R = 2\ \Omega$, I is 5 A and for $R = 3\ \Omega$, I is 4 A (all quantities are steady state values). Find the steady state value of I for $R = 8\ \Omega$. 3 marks

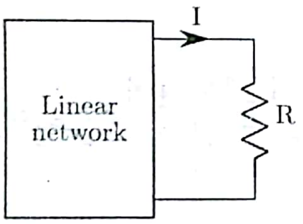
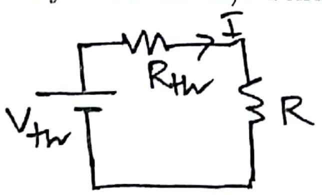


Figure 3: Circuit for Problem 7



$V_{th} = R_{th} \cdot I + R \cdot I$

$V_{th} - 5R_{th} = 10$ — (1)

$V_{th} - 4R_{th} = 12$ — (2)

By (2) − (1),

$R_{th} = 2$. — 1 mark

Putting in (1),

$V_{th} = 10 + 5 \times 2$

$= 20V$. — 1 mark

$$I = \frac{V_{th}}{R_{th} + R} = \frac{20}{2+8} = \boxed{2A}. \longrightarrow \text{1 mark.}$$

8. For a balanced three-phase load supplied by a balanced three-phase source, which of the following is/are true? (Tick the correcct option(s)) 1 mark

~~(a)~~ Three-phase instantaneous power is constant 0.5 mark

(b) Per phase instantaneous power is constant

(c) Instantaneous power in all phases are equal to each other

~~(d)~~ Reactive power in all phases are equal to each other. 0.5 mark.

Cont.

9. For the circuit shown in Figure 4, the switch S was at position 1 for a very long time (1 hour or so). At $t = 0$, the switch S is now connected to position 2.
(a) Find the expression of $i(t)$ for $t \geq 0$.
(b) If the capacitor voltage is $v_c(t)$, calculate $\frac{dv_c(t)}{dt}$ at $t = 0^+$. 4 marks

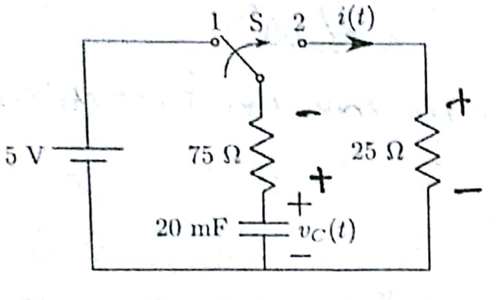


Figure 4: Circuit for Problem 9

Since it is at steady state,
$v_c(0^-) = 5V$. — 1 mark
At $t=0$, switching is done.
$i(t) = -C\frac{dv_c(t)}{dt}$. 2 marks

$v_c(t) - 100\, i(t) = 0 \Rightarrow i(t) = \frac{v_c(t)}{100}$.

$\therefore \frac{dv_c(t)}{dt} = -\frac{v_c(t)}{100.C} \Rightarrow \left.\frac{dv_c}{dt}\right|_{0^+} = -\frac{5}{100 \times 20 \times 10^{-3}}$

$\boxed{= -2.5 \text{ V/s.}}$ 1 mark

~~10.~~ A series R-L-C circuit is excited by a DC voltage source of 10 V. Assuming zero initial conditions, tick the following which is/are true. 1 mark
~~(a)~~ For $R = 15\ \Omega$, $L = 1$ mH, $C = 40\ \mu$F, response is non-oscillatory.
(b) For $R = 15\ \Omega$, $L = 1$ mH, $C = 40\ \mu$F, response is oscillatory.
(c) For $R = 5\ \Omega$, $L = 1$ mH, $C = 40\ \mu$F, response is non-oscillatory.
~~(d)~~ For $R = 5\ \Omega$, $L = 1$ mH, $C = 40\ \mu$F, response is oscillatory.

eigenvalues $= -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

11. Two single phase loads of power factors 0.8 (lag) and 0.9 (lag) respectively are connected in parallel. The overall power factor of the combined load is 1 mark
(a) < 0.8 (lag) ~~(b)~~ between 0.8 (lag) and 0.9 (lag) (c) > 0.9 (lag) (d) indeterminate.

$S_1 = P_1 + jQ_1 = S_1\angle\theta_1,\ S_2 = P_2 + jQ_2 = S_2\angle\theta_2$

$S = S_1 + S_2$
$\angle S \in [\theta_2, \theta_1]$

12. Calculate the transfer function $\frac{V_{out}(j\omega)}{V_{in}(j\omega)}$ for the circuit shown in Figure 5, where ω is in rad/s. Find the gain and the phase shift of the transfer function at (i) $f = 0.2$ Hz, (ii) $f = 2$ Hz, (iii) $f = 20$ Hz. Indicate the phase lead or lag appropriately. 4 marks

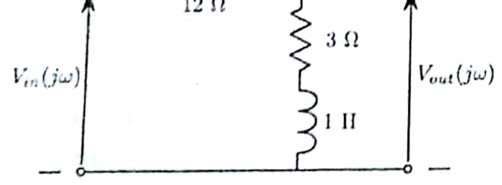


Figure 5: Circuit for Problem 12

$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{3 + j\omega.1}{3+12+j\omega.1}$

$= \frac{3+j\omega}{15+j\omega}$, ω is in rad/s — 1 mark.

$f = 0.2$ Hz $\Rightarrow \omega = 1.2566$ rad/s.

~~$|H(j\omega)|_{\omega=1.2566}$~~

Frequency | Gain | Phase (lead)

Scanned by CamScanner

13. For the circuit shown in Figure 6, the frequency of the voltage source $v_s(t)$ is 50 Hz. $\bar{I}_1$ and $\bar{I}_2$ are the phasor representations of $i_1(t)$ and $i_2(t)$ respectively and they are related as $\frac{\bar{I}_2}{\bar{I}_1} = 2\angle 30°$. **5 marks**

(a) Find the value of C.

(b) If $i_1(t) = 5\sin(2\pi \times 50t)$, find the expression of $v_s(t)$.

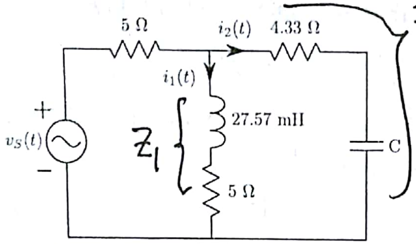


Figure 6: Circuit for Problem 13

Z_2

$\bar{I}_2/\bar{I}_1 = 2\angle 30°.$

Since, they are in parallel,

$\bar{I}_1 \bar{Z}_1 = \bar{I}_2 \bar{Z}_2 = \bar{V}$

or, $\bar{Z}_2/\bar{Z}_1 = \bar{I}_1/\bar{I}_2 = 0.5\angle -30°.$

$\bar{Z}_1 = 5 + j(2\pi 50 \times 27.57 \times 10^{-3}) = 10\angle 60° \Omega$

$\therefore \bar{Z}_2 = 0.5\angle -30° \times 10\angle 60° = 5\angle 30° \Omega$

Incorrect data.

14. Two lamps L_1 and L_2 are connected in series across a 200 V DC supply as shown in Figure 7. The ratings of L_1 and L_2 are (20 W, 200 V DC) and (80 W, 200 V DC) respectively. Which bulb glows brighter and what is the total power delivered by the source? **4 marks**

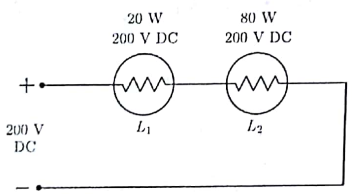
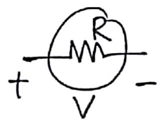


Figure 7: Circuit for Problem 14



$P = \frac{V^2}{R}.$

$L_1 : R_1 = \frac{V^2}{P}$

$= \frac{200^2}{20}$

$= 2000 \Omega.$

$L_2 : R_2 = \frac{200^2}{80}$ 1 mar

$= 500 \Omega.$

1 mark.

In series, I is constant.

$\therefore P_1 = I^2 R_1$, $P_2 = I^2 R_2$.

Since $R_1 > R_2$, so $P_1 > P_2$.

L_1 glows brighter. —— 1 mark.

Cont.

$I = \frac{200}{2000 + 500} = 0.08\,A.$

$P = I^2(R_1 + R_2)$

$= 0.08^2 \times 2500$

$= 16\,W.$ —— 1 mark

15. The voltage $v(t)$ across a particular component and the current $i(t)$ through it are given as

$$v(t) = 5 - 10\sin(2\pi \times 50t - 25^\circ)\ \text{V}.$$
$$i(t) = 2 + k\sin(2\pi \times 50t + 35^\circ)\ \text{A}.$$

Find k such that the average power dissipated in the component is zero. **2 marks**

$P_{dc} = 5 \times 2 = 10\ W.$ —— (0.5) marks

$P_{ac} = \frac{10 \times k}{2} \cos[35 - (180 - 25)] = 5k\cos(120) = -\frac{5k}{2}.$ (0.5) marks

$\therefore\ 10 - 5k/2 = 0 \Rightarrow \boxed{k = 4}$ (1) mark

16. A balanced star-connected three phase load of impedance $Z = 10\angle 30^\circ$ per phase is connected to a three phase voltage source given as

$$v_{an}(t) = \sqrt{2} \times 100 \sin(2\pi \times 50t)\ \text{V}$$
$$v_{bn}(t) = \sqrt{2} \times 100 \sin\left(2\pi \times 50t - \frac{2\pi}{3}\right)\ \text{V}$$
$$v_{cn}(t) = \sqrt{2} \times 100 \sin\left(2\pi \times 50t + \frac{2\pi}{3}\right)\ \text{V}.$$

(a) For $T = 0.02$ s, find the value of $\frac{\sqrt{3}}{T}\int_0^T (v_{bn}(t) - v_{cn}(t))\, i_a(t)dt$, where $i_a(t)$ is the phase a line current.
(b) Calculate the total reactive power absorbed by the load. **4 marks**

$T = 0.02\,s$ is one complete cycle of 50 Hz.

$\frac{1}{T}\int_0^T v_{bc}(t)\, i_a(t)\, dt = (V_{bc})_{rms} \cdot (I_a)_{rms} \cos\angle_{I_a}^{V_{bc}}$

$(V_{an})_{rms} = 100\,V,\ (V_{bc})_{rms} = \sqrt{3} \times 100\,V.$ —— (1) mark

$(i_a)_{rms} = \frac{(V_{an})_{rms}}{|Z|} = 10\,A.$ —— (1) mark.

$v_{bc}(t) = v_{bn}(t) - v_{cn}(t) = \sqrt{2} \times 100\sqrt{3} \sin(\omega t - 90^\circ).$

$i_a(t) = v_{an}(t)/|Z| = \sqrt{2} \times \cancel{100}\ 10 \sin(\omega t - 30^\circ).$

$\therefore\ \frac{\sqrt{3}}{T}\int_0^T v_{bc}(t)\, i_a(t)\, dt = \sqrt{3} \times \sqrt{3} \times 100 \times 10 \times \cos(60^\circ)$

$= \boxed{1500}.$ —— (1) mark.

Cont.

$Q = 3 \times (V_{an})_{rms} \times (i_a)_{rms} \cdot \cancel{\cos}\ \sin(30^\circ) = 3 \times 100 \times 10 \times \sin(30^\circ)$
$= 1500\ VAR$ — (1) mark.

17. A single-phase load consumes 5 kW active power and 2 kVAr of reactive power from a 415 V (rms, 50 Hz) terminal. A capacitor C is to be connected in parallel to the load to improve the power factor. Find the value of C required to improve the power factor to 0.95 (lead). **3 marks**

$S = P + jQ$, $\tan(\theta) = Q/P$.

After correction, $S_1 = P + jQ_1$, $\tan(\theta_1) = \frac{Q_1}{P}$.

$\theta = \tan^{-1}(2/5) = 21.801°$.

$\theta_1 = -\cos^{-1}(0.95) = -18.19°$. — 2 marks

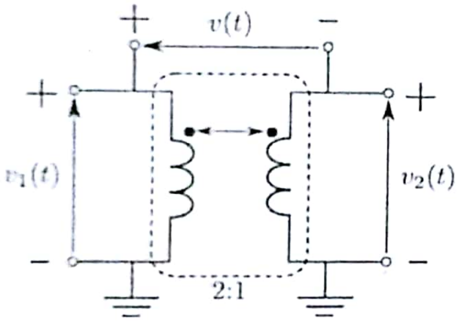
$Q_1 = Q \times \frac{\tan(\theta_1)}{\tan\theta}$

$= -1.643$ kVAr.

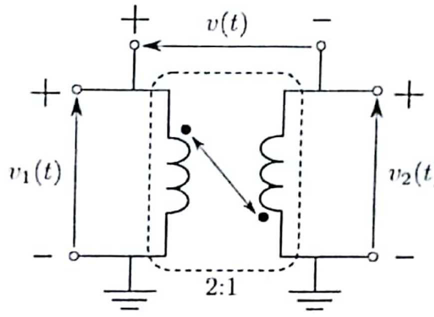
$Q_C = Q - Q_1 = 3.643$ kVAr.

$Q_C = \omega C V^2 \Rightarrow C = Q_C / \omega V^2 = \frac{3.643 \times 10^3}{2\pi \times 50 \times 415^2} = 67.33 \mu F$ — 1 mark

18. An ideal transformer has a turns ratio of 2:1 as shown in Figure 8. If the primary winding is excited with $v_1(t) = \sqrt{2} \times 50 \sin(2\pi \times 50t)$ V, find the expressions of the secondary voltage $v_2(t)$ and the voltage $v(t)$ for both the connections as shown in Figure 8a and Figure 8b. **4 marks**



(a) Configuration 1



(b) Configuration 2

Figure 8: Circuits for Problem 18

$v_1(t)$ N_1

19. For the circuit shown in Figure 9, the transfer function of the individual blocks are $G_A(j\omega)$ and $G_B(j\omega)$. Find the transfer function $\frac{V_{out}(j\omega)}{V_{in}(j\omega)}$. **2 marks**

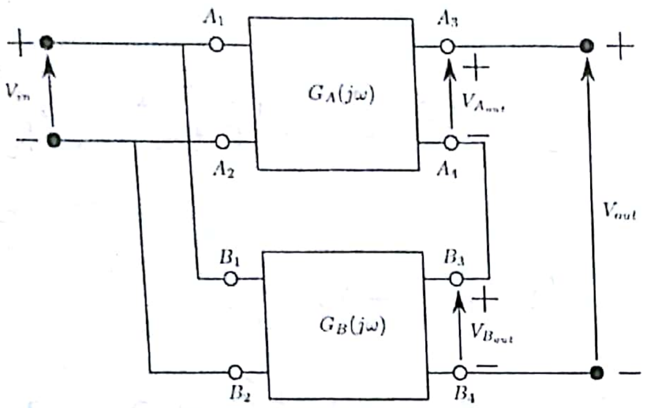


Figure 9: Circuit for Problem 19

$$\frac{V_{Aout}(j\omega)}{V_{in}(j\omega)} = G_A(j\omega)$$

$$\frac{V_{Bout}(j\omega)}{V_{in}(j\omega)} = G_B(j\omega).$$

$$V_{out}(j\omega) = V_{Aout}(j\omega) + V_{Bout}(j\omega)$$

$$= [G_A + G_B](j\omega)\, V_{in}(j\omega)$$

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = G_A(j\omega) + G_B(j\omega).$$

(2) marks

20. For the circuit shown in Figure 10, find the value of R such that maximum power is transferred to the load (shown as dotted block in Figure 10) when the source frequency is 50 Hz. **3 marks**

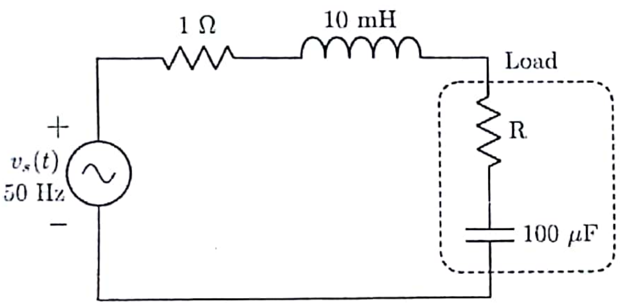


Figure 10: Circuit for Problem 20

$\cancel{For}$ I_{rms}

$$= \frac{V_s}{\sqrt{(R_s+R)^2+(X_L-X_c)^2}}$$

$$P_L = I_{rms}^2 R$$

$$\frac{\partial P_L}{\partial R} = 0 \Rightarrow R = \sqrt{R_s^2 + (X_L - X_c)^2} \text{ . for max } P_L.$$

For max Q_L, since C is fixed,

$Q_L = I_{rms}^2 X_c$ is max when I_{rms} is max.

So, I_{rms} will be max when $R = 0$.

For max Q_L ; $R = 0$.

$$S = I_{rms}^2 Z \qquad S = \frac{V_s^2}{(R_s+R)^2+(X_e)^2} \cdot \sqrt{R^2 + X_c^2} \qquad X_e = X_\ell - X_c$$

The End.

$\frac{\partial S}{\partial R} = 0 \Rightarrow$ Solving for R, we shall get R required for max S.

$$S = \frac{V_s^2}{(R_s+R)^2+(X_s-X_c)^2} \cdot \sqrt{R^2+X_c^2}$$

$$\frac{\partial S}{\partial R} = 0$$

$$\Rightarrow \left[(R_s+R)^2 + X_e^2\right] \cdot \frac{1}{\cancel{2}\sqrt{R^2+X_c^2}} \cdot \cancel{2}R - \sqrt{R^2+X_c^2} \cdot 2(R_s+R) = 0.$$

$$\left[(R_s+R)^2 + X_e^2\right] \cdot R = 2(R^2+X_c^2)(R_s+R).$$

$$\cancel{2R_sR^2} + R_s^2 R + R^3 + X_e^2 R = 2\cancel{R^2R_s} + 2R^3 + 2X_c^2 R_s + 2X_c^2 R.$$

$$R^3 + (2X_c^2 - X_e^2 - R_s^2)R + 2X_c^2 R_s = 0.$$

Solving $R = 0.8407 \pm j\,34.7\ \Omega, \ -1.6815\ \Omega.$