Time Series HomeWork (6)

钟瑜　222018314210044

2020 年 10 月 20 日

1. $X_t = \frac{1}{6}X_{t-1} + \frac{1}{6}X_{t-2} + \epsilon_t$, $\{\epsilon_t\} \sim WN(0, 0.25)$

   求它的平稳解以及自协方差函数 γ_k

   解. 平稳解的一般形式为:

$$X_t = \sum_{j=0}^{\infty} \psi_j \epsilon_{t-j} \tag{1}$$

先只需确定 ψ_j. 由题目可知 $p = 2$,$a_1 = a_2 = \frac{1}{6}$. 而

$$\psi_k = \begin{cases} 0 & k < 0 \\ 1 & k = 0 \\ \sum_{j=1}^{p} a_j \psi_{k-j} & k \geq 1 \end{cases}$$

当 $k \geq 1$ 时,$\psi_k = \frac{1}{6}(\psi_{k-1} + \psi_{k-2})$, 解该差分方程得

$$\psi_k = \frac{3}{5}(\frac{1}{2})^k + \frac{2}{5}(\frac{-1}{3})^k \tag{2}$$

故平稳解为:

$$X_t = \sum_{j=0}^{\infty} \left[\frac{3}{5}(\frac{1}{2})^j + \frac{2}{5}(\frac{-1}{3})^j\right] \epsilon_{t-j} \tag{3}$$

自协方差函数 γ_k 为:

$$\begin{aligned} \gamma_k &= \sigma^2 \sum_{j=1}^{\infty} \psi_j \psi_{j+k} \\ &= 0.25 \sum_{j=1}^{\infty} \left[\frac{3}{5}(\frac{1}{2})^j + \frac{2}{5}(\frac{-1}{3})^j\right] \left[\frac{3}{5}(\frac{1}{2})^{j+k} + \frac{2}{5}(\frac{-1}{3})^{j+k}\right] \\ &= \frac{1}{100}\left[3(\frac{1}{2})^{k-2} + \frac{4}{7}(\frac{-1}{3})^{k-2} + \frac{9}{7}(\frac{1}{2})^{k-2} + 2(\frac{-1}{3})^{k-2}\right] \end{aligned} \tag{4}$$

2. 设 $\{\epsilon_t\}$ 是 $WN(\mu, \sigma^2)$, $a_1, a_2, ..., a_p$ 满足最小相位条件. 求非中心化 AR(p) 模型:

$$X_t = a_0 + \sum_{j=1}^{p} a_j X_{t-j} + \epsilon_t \ , t \in \mathbb{Z} \tag{5}$$

   的平稳解和通解.

解.

$$
\begin{aligned}
X_t &= a_0 + \sum_{j=1}^{p} a_j X_{t-j} + \epsilon_t \\
&= \sum_{j=1}^{p} a_j X_{t-j} + a_0 + \epsilon_t
\end{aligned} \tag{6}
$$

而 $A(\mathcal{B})X_t = \epsilon_t + a_0$, 故

$$
\begin{aligned}
X_t &= A^{-1}(\mathcal{B})A(\mathcal{B})X_t \\
&= A^{-1}(\mathcal{B})(\epsilon_t + a_0) \\
&= \sum_{j=0}^{\infty} \psi_j \epsilon_{t-j} + a_0
\end{aligned} \tag{7}
$$

上式即为平稳解.

通解为:

$$
X_t = a_0 + \sum_{j=0}^{\infty} \psi_j \epsilon_{t-j} + \sum_{j=1}^{k} \sum_{l=0}^{r(j)-1} U_{l,j} t^l z_j^{-t}, t \in \mathbb{Z} \tag{8}
$$