

Time Series HomeWork (9)

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1. 设 $\{X_t\}$ 为 ARMA(1,1) 序列, $X_t = a_1X_{t-1} + \epsilon_t + b_1\epsilon_{t-1}$, $\epsilon_t \sim WN(0, \sigma^2)$, 求 $\gamma_0, \gamma_1, \gamma_2$.

解. 由 γ_k 的递推公式可得

$$\begin{aligned}\gamma_0 &= a_1\gamma_1 + \sigma^2(b_0\psi_0 + b_1\psi_1) \\ \gamma_1 &= a_1\gamma_0 + \sigma^2b_1\psi_0 \\ \gamma_2 &= a_1\gamma_1\end{aligned}\tag{1}$$

解得

$$\begin{aligned}\gamma_0 &= \frac{\sigma^2(1 + 2a_1b_1 + b_1^2)}{1 - a_1^2} \\ \gamma_1 &= \frac{\sigma^2(a_1 + b_1)(1 + a_1b_1)}{1 - a_1^2} \\ \gamma_2 &= \frac{a_1\sigma^2(a_1 + b_1)(1 + a_1b_1)}{1 - a_1^2}\end{aligned}\tag{2}$$