# Time Series HomeWork (11)

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1. 给定平稳序列 $\{X_t\}$ 的自协方差函数 $(\gamma_0, \gamma_1, ..., \gamma_4) = (5.61, -1.1, 0.23, 0.43, -0.1)$. 试为 $\{X_t\}$ 建立 ARMA(2,2) 模型.

**解.** *R 代码如下:*

1. 求 AR 部分的系数:

```
> ga34<-matrix(c(0.43,-0.1),byrow = TRUE,nrow = 2)
> ga34
      [,1]
[1,]  0.43
[2,] -0.10
> ga2132<-matrix(c(0.23,-1.1,0.43,0.23),byrow = TRUE,ncol = 2)
> ga2132
     [,1]  [,2]
[1,] 0.23 -1.10
[2,] 0.43  0.23
> ga2132ni<-solve(ga2132)
> a12<-ga2132%*%ga34
> a12
       [,1]
[1,] 0.2089
[2,] 0.1619
```

则 $a_1 = 0.2089$,$a_2 = 0.1619$, 转换为 MA 模型:

$$Y_t = X_t - 0.2089X_{t-1} - 0.1619X_{t-2} = \epsilon_t + b_1\epsilon_{t-1} + b_2\epsilon_{t-2}$$

2. 求 MA 序列 $\{Y_t\}$ 的自协方差函数 $\{\gamma_y(k)\}$:

```
> gak0<-matrix(c(5.61,-1.1,0.23,
+                 -1.1,5.61,-1.1,
+                 0.23,-1.1,5.61),byrow=TRUE,ncol=3)
> gak0
      [,1]  [,2]  [,3]
[1,]  5.61 -1.10  0.23
```

```
[2,] -1.10  5.61 -1.10
[3,]  0.23 -1.10  5.61
> gak1<-matrix(c(-1.1,0.23,0.43,
+                5.61,-1.1,0.23,
+                -1.1,5.61,-1.1),byrow=TRUE,ncol=3)
> gak1
      [,1]  [,2]  [,3]
[1,] -1.10  0.23  0.43
[2,]  5.61 -1.10  0.23
[3,] -1.10  5.61 -1.10
> gak2<-matrix(c(0.23,0.43,-0.1,
+                -1.1,0.23,0.43,
+                5.61,-1.1,0.23),byrow=TRUE,ncol=3)
> gak2
      [,1]  [,2]  [,3]
[1,]  0.23  0.43 -0.10
[2,] -1.10  0.23  0.43
[3,]  5.61 -1.10  0.23

> gay0<-a012%*%gak0%*%a012t
> gay1<-a012%*%gak1%*%a012t
> gay2<-a012%*%gak2%*%a012t
> gay0;gay1;gay2
         [,1]
[1,] 6.312563
          [,1]
[1,] -2.090825
           [,1]
[1,] -0.5287003
```

3. 求 MA 序列 $\{Y_t\}$ 的系数和 $\sigma^2$:

```
## Given \gamma_0, \gamma_1, \dots, \gamma_q,
## Solve MA(q) coefficients b_1, \dots, b_q, \sigma^2
## Using Li Lei's algorithm.
## Input: gms -- \gamma_0, \gamma_1, \dots, \gamma_q
> ma.solve <- function(gms, k=100){
        +    q <- length(gms)-1
        +    if(q==1){
                +      rho1 <- gms[2] / gms[1]
                +      b <- (1 - sqrt(1 - 4*rho1^2))/(2*rho1)
                +      s2 <- gms[1] / (1 + b^2)
```

```
            +      return(list(b=b, s2=s2))
            +    }
        +    A <- matrix(0, nrow=q, ncol=q)
        +    for(j in seq(2,q)){
            +      A[j-1,j] <- 1
            +    }
        +    cc <- numeric(q); cc[1] <- 1
        +    gamma0 <- gms[1]
        +    gammas <- numeric(q+k)
        +    gammas[1:(q+1)] <- gms
        +    gamq <- gms[-1]
        +    Gammak <- matrix(0, nrow=k, ncol=k)
        +    for(ii in seq(k)){
            +      for(jj in seq(k)){
                +        Gammak[ii,jj] <- gammas[abs(ii-jj)+1]
                +      }
            +    }
        +    Omk <- matrix(0, nrow=q, ncol=k)
        +    for(ii in seq(q)){
            +      for(jj in seq(k)){
                +        Omk[ii,jj] <- gammas[ii+jj-1+1]
                +      }
            +    }
        +    PI <- Omk %*% solve(Gammak, t(Omk))
        +    s2 <- gamma0 - c(t(cc) %*% PI %*% cc)
        +    b <- 1/s2 * c(gamq - A %*% PI %*% cc)
        +    return(list(b=b, s2=s2))
        + }

> gms<-c(gay0,gay1,gay2)
> ma.solve(gms,k=100)
$b
[1] -0.4454071 -0.1012276

$s2
[1] 5.222888
```

故我们得到 ARMA(2,2) 模型

$$X_t = 0.2089X_{t-1} + 0.1619X_{t-2} + \epsilon_t - 0.4454071\epsilon_{t-1} - 0.1012276\epsilon_{t-2}, \epsilon \sim WN(0, 5.222888)$$