Time Series HomeWork (12)

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1. 假设时间序列服从 AR(2) 过程

$$X_t = a_1 X_{t-1} + a_2 X_{t-2} + \epsilon_t, \{\epsilon_t\} \sim WN(0, \sigma^2)$$

其样本自协方差系数

$$\hat{\gamma}_0 = 1382.2, \hat{\gamma}_1 = 1114.4, \hat{\gamma}_2 = 591.73, \hat{\gamma}_3 = 96.216$$

试用这些估计值给出 a_1, a_2, σ^2 的 yule-walker 估计, 给出 a_1, a_2 的 95% 置信区间.

解. R 代码如下:

```
> g12<-matrix(c(1114.4,591.73),byrow = TRUE,ncol=1)
> g0110<-matrix(c(1382.2,591.73,591.73,1382.2),byrow = TRUE,ncol=2)
> g0110n<-solve(g0110)
> a12<-g0110n%*%g12
> a12
          [,1]
[1,] 0.7627729
[2,] 0.1015587
> sigma2<-1382.2-t(a12)%*%g12
> sigma2
          [,1]
[1,] 472.0706

#============== 求 sigmajj ================#
> sigma2<-as.numeric(sigma2)
> sigmajj<-sigma2*g0110n
> sigmajj
            [,1]        [,2]
[1,]  0.4181775 -0.1790249
[2,] -0.1790249  0.4181775
```

故

$$\hat{a}_1 = 0.7627729, \hat{a}_2 = 0.1015587, \sigma^2 = 472.0706$$

设 $x_1, x_2, ..., x_N$ 为观测数据, 于是 a_j 的近似 95% 置信区间为

$$[\hat{a}_j - 1.96\sqrt{\sigma_{jj/N}}, \hat{a}_j + 1.96\sqrt{\sigma_{jj/N}}]$$

i.e.

$$[0.7627729 - 1.96\sqrt{0.4181775/N}, 0.7627729 - 1.96\sqrt{0.4181775/N}]$$

$$[0.1015587 - 1.96\sqrt{0.4181775/N}, 0.1015587 - 1.96\sqrt{0.4181775/N}]$$