NO.5 应用时间序列

姓名:魏程 学号:222018314210015

[*1* 练习]

若$\epsilon_t \sim WN(0,\sigma^2)$,求$X_t = \sum\limits_{i=0}^{\infty} \frac{1}{3^i}\epsilon_{t-i}$的频密度

解:

由于$f(\lambda) = \frac{\sigma^2}{2\pi}|\sum\limits_{j=\infty}^{\infty} a_j e^{j\lambda i}|^2$,对于$X_t$我们有:

$a_j = \frac{1}{3^j}$故:

$$
\begin{aligned}
f(\lambda) &= \frac{\sigma^2}{2\pi}|\sum_{j=0}^{\infty}\frac{1}{3^j}e^{j\lambda i}|^2 = \frac{\sigma^2}{2\pi}|\sum_{j=0}^{\infty}(\frac{e^{\lambda i}}{3})^j|^2 && (1)\\
&= \frac{\sigma^2}{2\pi}|\frac{1}{1-\frac{e^{\lambda i}}{3}}|^2 \qquad\qquad (|\frac{e^{\lambda i}}{3}| < 1) && (2)\\
&= \frac{\sigma^2}{2\pi}|\frac{3}{3-e^{\lambda i}}|^2 && (3)\\
&= \frac{\sigma^2}{2\pi}|\frac{3}{3-cos\lambda - isin\lambda}|^2 && (4)\\
&= \frac{\sigma^2}{2\pi}|\frac{3(3-cos\lambda + isin\lambda)}{(3-cos\lambda)^2+sin^2\lambda}|^2 && (5)\\
&= \frac{\sigma^2}{2\pi}(\frac{3}{(3-cos\lambda)^2+sin^2\lambda})^2((3-cos\lambda)^2+sin^2\lambda) && (6)\\
&= \frac{\sigma^2}{2\pi}\frac{9}{(3-cos\lambda)^2+sin^2\lambda} && (7)\\
&= \frac{\sigma^2}{2\pi}\frac{1}{1+\frac{1}{9}-\frac{2}{3}cos\lambda} && (8)
\end{aligned}
$$

[*2* 作业]

求齐次差分方程$X_t = \sqrt{2}X_{t-1} - X_{t-2}$的通解

解:

由题知该其次差分方程的特征多项式为:$A(z) = 1-\sqrt{2}z+z^2$,且

其$\Delta = b^2-4ac = -2 < 0$,故有两个重数为一的共轭复根:

$z_1 = \frac{\sqrt{2}}{2}(1+i) \qquad z_2 = \frac{\sqrt{2}}{2}(1-i)$

故齐次差分方程的通解为

$$X_t = U_{0.1}z_1^{-t} + U_{0.2}z_2^{-t} = U_{0.1}\frac{\sqrt{2}}{2}(1+i)^{-t} + U_{0.2}\frac{\sqrt{2}}{2}(1-i)^{-t}$$

其中$U_{i.j}$为随机变量,可由初值(X_0, X_1)唯一决定.

[*3 作业*]

设$\epsilon_t \sim WN(0, \sigma^2)$,现有一阶非齐次差分方程$X_t - 0.5X_{t-1} = \epsilon_t$

1).判断并证明$X_t^0 = \sum\limits_{j=0}^{\infty}(0.5)^j\epsilon_{t-j}$是否为上述非齐次差分方程的一个特解?

2).如上定义的X_t^0是(弱)平稳序列吗?为什么?

3).求出上述一阶非齐次差分方程的通解X_t,并且写出当$t \to \infty$时,该通解X_t的收敛性质.

解:

1).由于$X_t^0 = \sum\limits_{j=0}^{\infty}(0.5)^j\epsilon_{t-j}$,故$X_{t-1}^0 = \sum\limits_{j=0}^{\infty}(0.5)^j\epsilon_{t-1-j}$,将$X_t, X_{t-1}$代入,

$$X_t - 0.5X_{t-1} = \sum_{j=0}^{\infty}(0.5)^j\epsilon_{t-j} - 0.5(\sum_{j=0}^{\infty}(0.5)^j\epsilon_{t-1-j}) \tag{9}$$

$$= \sum_{j=0}^{\infty}(0.5)^j\epsilon_{t-j} - \sum_{j=0}^{\infty}(0.5)^{j+1}\epsilon_{t-(j+1)} \tag{10}$$

$$= \epsilon_t + \sum_{j=1}^{\infty}(0.5)^j\epsilon_{t-j} - \sum_{j=0}^{\infty}(0.5)^{j+1}\epsilon_{t-(j+1)} \tag{11}$$

$$= \epsilon_t \tag{12}$$

故$X_t^0 = \sum\limits_{j=0}^{\infty}(0.5)^j\epsilon_{t-j}$是上述非齐次差分方程的一个特解.

2).注意到X_t^0作为零均值白噪声$\{\epsilon_t\}$的单边无穷滑动,要验证X_t^0平稳,只需考察系数为绝对可和,即验证$\sum\limits_{j=0}^{+\infty}|(0.5)^j| < \infty$

实际上:

$$\sum_{j=0}^{+\infty}|(0.5)^j| = \sum_{j=0}^{+\infty}(0.5)^j = \frac{1}{1-0.5} = 2$$

故X_t^0是(弱)平稳序列.