Final answers script in beta

Final Answers
MATH220 December 2011

April 4, 2015

How to use this resource

- When you feel reasonably confident, simulate a full exam and grade your solutions. For your grading you can get the full solutions here.
- If you're not quite ready to simulate a full exam, we suggest you thoroughly and slowly work through each problem. Use this document with the final answers only to check if your answer is correct, without spoiling the full solution.
- Should you need more help, check out the hints and video lecture on the Math Education Resources.

Tips for Using Previous Exams to Study: Work through problems

Resist the temptation to read any of the final answers below before completing each question by yourself first! We recommend you follow the guide below.

1. **How to use the final answer:** *The final answer is not a substitution for the full solution!* The final answer alone will not give you full marks. The final answer is provided so that you can check the correctness of your work without spoiling the full solution.
   - To answer each question, only use what you could also use in the exam. Download the raw exam here.
   - If you found an answer, how could you verify that it is correct from your work only? E.g. check if the units make sense, etc. Only then compare with our result.
   - If your answer is correct: good job! Move on to the next question.
   - Otherwise, go back to your work and check it for improvements. Is there another approach you could try? If you still can't get to the right answer, you can check the full solution on the Math Education Resources.
2. **Reflect on your work:** Generally, reflect on how you solved the problem. Don't just focus on the final answer, but whether your mental process was correct. If you were stuck at any point, what helped you to go forward? What made you confident that your answer was correct? What can you take away from this so that, next time, you can complete a similar question without any help?
3. **Plan further studying:** Once you feel confident enough with a particular topic, move on to topics that need more work. Focus on questions that you find challenging, not on those that are easy for you. Once you are ready to tackle a full exam, follow the advice for the full exam (click here).

Please note that all final answers were extracted automatically from the full solution. It is possible that the final answer shown here is not complete, or it may be missing entirely. In such a case, please notify `mer-wiki@math.ubc.ca`. Your feedback helps us improve.

This pdf was created for your convenience when you study Math and prepare for your final exams. All the content here, and much more, is freely available on the Math Education Resources.

Question 1 (a)

Final answer. $(X \cap Y)^c = X^c \cup Y^c$ Here the c indicates the complement of the set.

Question 1 (b)

Final answer. The set Y is the *union* of all of the sets S_α; that is, Y is the set of all y such that there exists an $\alpha \in I$ such that $y \in S_\alpha$.

Question 1 (c)

Final answer. More precisely, that means there must exist at least one function $f\colon A \to B$ such that the function is injective and surjective; that is, for each element a of the set A there exists a unique element b in the set B such that $f(a) = b$ (that condition is the injectivity) AND for each element b of the B there exists a unique element a in the set A such that $f(a) = b$ (and this is the condition for surjectivity).

Question 1 (d)

Final answer. $\alpha \circ \beta = \begin{pmatrix} 1234 \\ 3241 \end{pmatrix}$

Question 1 (e)

Final answer. and so this really is the inverse of f.

Question 1 (f)

Final answer. Then for every natural number n, the statement $S(n)$ is true.

Question 2 (a)

Final answer. We also note that $Q(4)$ is the statement "17 is divisible by 3" whichis false.

Question 2 (b)

Final answer. $\neg Q(4) \Rightarrow \neg P(3)$ In words this is *"17 is not divisible by 3 implies that 3 is not divisible by 4"*.

Question 3

Final answer. $B = (\neg Q) \vee R \vee (\neg P) \vee Q$ which both are tautologies since they contain P OR *not* P and respectively *not* Q OR Q. So if both A and B are tautologies, then so is the conjunction of them into the statement A AND B.

Question 4

Final answer. It follows now, since $f(x_1) = y$ and $x_1 \in C \cap D$ that $y \in f(C \cap D)$ as claimed.

Question 5 (a)

FINAL ANSWER. Given a counting, that is such a bijection f the *first* element of the set A is the one that is mapped to 1 by the function, the *second* element is the one mapped to 2 and so on.Observe that we need to require the function to be bijective to guarantee that all the elements are counted and are counted in a unique way (we don't want to have two elements mapped to the same natural number).

Question 5 (b)

FINAL ANSWER. Since g is both surjective and injective it is a bijection and thus the set B is denumerable.

Question 6 (a)

FINAL ANSWER. This shows that our conjecture is true for $n = k+1$ and thus concludes our proof.

Question 6 (b)

FINAL ANSWER. Now, in the last line, the first summand is divisible by 133 and the second summand is also divisible by 133 by the induction hypothesis. Hence we must have that $12^{2(k+1)-1} + 11^{(k+1)+1}$ is divisible by 133 as required.

Question 7 (a)

FINAL ANSWER. $2n = m$ and is not surjective since clearly no odd integer will be in the image of the function f.

Question 7 (b)

FINAL ANSWER. Now the function is not injective since $g(3) = g(2) = 1$ (or more generally since $g(2k+1) = g(2k) = k$ for any integer k).

Question 8

FINAL ANSWER. we can conclude that if p is a prime larger than 4, then its square is always 1 mod 6.