Final answers script in beta

Final Answers
MATH103 April 2013

December 4, 2014

How to use this resource

- When you feel reasonably confident, simulate a full exam and grade your solutions. For your grading you can get the full solutions here.
- If you're not quite ready to simulate a full exam, we suggest you thoroughly and slowly work through each problem. Use this document with the final answers only to check if your answer is correct, without spoiling the full solution.
- Should you need more help, check out the hints and video lecture on the Math Educational Resources.

Tips for Using Previous Exams to Study: Work through problems

Resist the temptation to read any of the final answers below before completing each question by yourself first! We recommend you follow the guide below.

1. **How to use the final answer:** *The final answer is not a substitution for the full solution!* The final answer alone will not give you full marks. The final answer is provided so that you can check the correctness of your work without spoiling the full solution.
   - To answer each question, only use what you could also use in the exam. Download the raw exam here.
   - If you found an answer, how could you verify that it is correct from your work only? E.g. check if the units make sense, etc. Only then compare with our result.
   - If your answer is correct: good job! Move on to the next question.
   - Otherwise, go back to your work and check it for improvements. Is there another approach you could try? If you still can't get to the right answer, you can check the full solution on the Math Educational Resources.
2. **Reflect on your work:** Generally, reflect on how you solved the problem. Don't just focus on the final answer, but whether your mental process was correct. If you were stuck at any point, what helped you to go forward? What made you confident that your answer was correct? What can you take away from this so that, next time, you can complete a similar question without any help?
3. **Plan further studying:** Once you feel confident enough with a particular topic, move on to topics that need more work. Focus on questions that you find challenging, not on those that are easy for you. Once you are ready to tackle a full exam, follow the advice for the full exam (click here).

Please note that all final answers were extracted automatically from the full solution. It is possible that the final answer shown here is not complete, or it may be missing entirely. In such a case, please notify `mer-wiki@math.ubc.ca`. Your feedback helps us improve.

This pdf was created for your convenience when you study Math and prepare for your final exams. All the content here, and much more, is freely available on the Math Educational Resources.

Question 1 (a)

Easiness: 57/100

FINAL ANSWER. Since the numerator grows faster than the denominator (as $k^k \gg k!$), we get that this sequence diverges.

Question 1 (b)

Easiness: 51/100

FINAL ANSWER. $\lim_{n\to\infty} \frac{n^3+2}{\sqrt{n^6+3}+\sqrt[3]{27n^9+1}} = \frac{1}{4}$and so the sequence converges to one fourth.

Question 1 (c)

Easiness: 25/100

FINAL ANSWER. $\lim_{k\to\infty} \sum_{n=1}^{k} \frac{\pi}{k} \sin\left(\frac{\pi n}{k}\right) = 2$ and so the sum converges to 2.

Question 1 (d)

Easiness: 76/100

FINAL ANSWER. This limit is 0 since as n tends to infinity, the denominator tends to infinity.

Question 1 (e)

Easiness: 43/100

FINAL ANSWER. As this last limit diverges since $\ln|\ln(b)|$ tends to infinity as b tends to infinity, we have that the series diverges.**NOTE: Compare this to the previous question.Even though the sequence of terms converges to zero, the integral here still diverges!**

Question 1 (f)

Easiness: 21/100

FINAL ANSWER. Thus the set of x values where this series converges is defined by $-5 < x < 1$.

Question 2 (a)

Easiness: 80/100

FINAL ANSWER. $\displaystyle\int_1^{e^2} \frac{\ln x}{x}\, dx = 2$completing the problem.

Question 2 (b)

Easiness: 24/100

FINAL ANSWER. $I_b = x \arctan\left(\frac{1}{x}\right) + \frac{1}{2} \cdot \ln|x^2+1| + C$

Question 2 (c)

Easiness: 33/100

FINAL ANSWER. Since $\lim\limits_{b\to 1^-} \frac{1}{2} \ln|b-1|$ diverges, we have that the entire integral diverges and thus the integral does not exist.

Question 2 (d)

Easiness: 25/100

FINAL ANSWER. $I = \dfrac{1}{2} x(\sin(\ln x) - \cos(\ln x)) + C$

Question 3 (a)

Easiness: 20/100

FINAL ANSWER. If we missed the second last step, we could have instead simplified the other equation to see that $\overline{x} = a - \int_0^a F(x)$ and thus the two sides are equal.Either way this completes the proof.

Question 3 (b) i

Easiness: 5/100

FINAL ANSWER. and isolating for c gives us that $c = 3/4$.

Question 3 (b) ii

Easiness: 15/100

FINAL ANSWER. $\overline{x} = \frac{4}{5}$

Question 4 (a)

Easiness: 72/100

FINAL ANSWER. $V = \pi$

Question 4 (b)

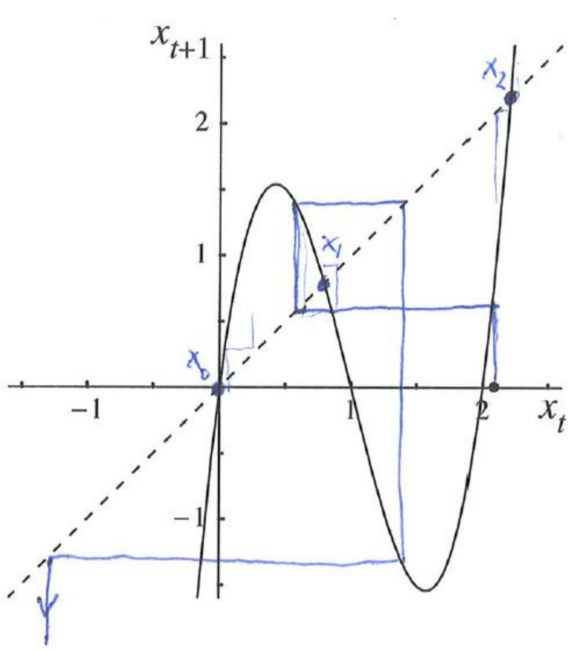
Easiness: 40/100

FINAL ANSWER. $\int_1^\infty \frac{1}{x}\, dx = \lim_{b\to\infty} \ln|b| - \ln(1)$ diverges, we have that our integral diverges by the integral comparison test.Thus the surface area diverges.NOTE: Even though you can fill the horn with a finite amount of paint, you cannot paint the inside with a finite amount of paint.

Question 5 (a)

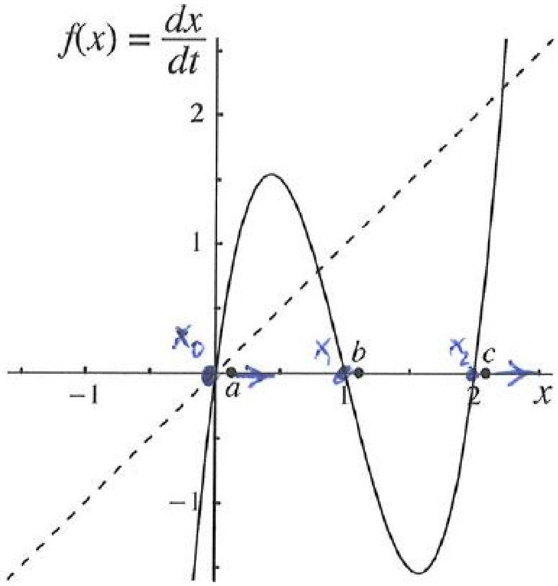
Easiness: 15/100

FINAL ANSWER.



Question 5 (b)

Easiness: 36/100



FINAL ANSWER.

Question 6 (a)

Easiness: 17/100

FINAL ANSWER. $F(x) = \frac{2}{\pi}\left(\arctan\left(\frac{x}{2}\right) + \frac{\pi}{4}\right)$ completing the problem.

Question 6 (b)

Easiness: 18/100

FINAL ANSWER. $p(x) = \frac{4}{\pi(4+x^2)}$ completing the question.

Question 6 (c)

Easiness: 18/100

FINAL ANSWER. $\overline{x} = 0$by symmetry of an odd function.

Question 7 (a)

Easiness: 78/100

FINAL ANSWER. and so either $M = 0$ or $1 - \frac{M}{K} = 0$, that is $M = K$.

Question 7 (b)

Easiness: 43/100

FINAL ANSWER. Setting the derivative in this differential equation to 0 gives either $M = 0$ or $1 - \frac{M}{K} - \frac{h}{2} = 0$ and isolating for M gives $M = K\left(1 - \frac{h}{2}\right)$

Question 7 (c)

Easiness: 4/100

FINAL ANSWER. As $\alpha = 2$, we have that $h^* = 2$ and in fact, $h < h^*$ must be true in order for the population to survive.

Question 7 (d)

Easiness: 26/100

FINAL ANSWER. (Note any of the last three lines would be an acceptable answer)

Question 7 (e)

Easiness: 53/100

FINAL ANSWER. Isolating for H gives $H = \frac{3}{8}K$

Question 8 (a)

Easiness: 87/100

FINAL ANSWER. $I_a = -\cos(\sin(x))$

Question 8 (b)

Easiness: 61/100

FINAL ANSWER. Since integrals give you areas, which are just constants, the derivative of a constant is 0.

Question 8 (c)

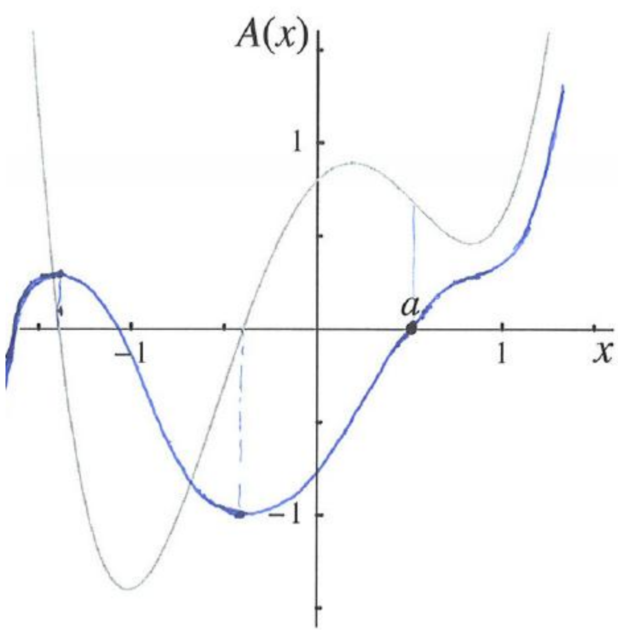
Easiness: 35/100

FINAL ANSWER. $\lim_{x\to 0} \frac{1}{x^3} \int_0^x \frac{t^2}{1+t^6}\, dt = \frac{1}{3}$Notice that in the first equality, we also used the fundamental theorem of calculus.

Question 9

Easiness: 47/100

FINAL ANSWER.



Question 10

Easiness: 19/100

Final answer. $T_4(x) = \dfrac{x^3}{3}$ as required.