Final Answers
MATH152 April 2010

April 4, 2015

How to use this resource

- When you feel reasonably confident, simulate a full exam and grade your solutions. For your grading you can get the full solutions here.
- If you're not quite ready to simulate a full exam, we suggest you thoroughly and slowly work through each problem. Use this document with the final answers only to check if your answer is correct, without spoiling the full solution.
- Should you need more help, check out the hints and video lecture on the Math Education Resources.

Tips for Using Previous Exams to Study: Work through problems

Resist the temptation to read any of the final answers below before completing each question by yourself first! We recommend you follow the guide below.

1. **How to use the final answer:** *The final answer is not a substitution for the full solution!* The final answer alone will not give you full marks. The final answer is provided so that you can check the correctness of your work without spoiling the full solution.
   - To answer each question, only use what you could also use in the exam. Download the raw exam here.
   - If you found an answer, how could you verify that it is correct from your work only? E.g. check if the units make sense, etc. Only then compare with our result.
   - If your answer is correct: good job! Move on to the next question.
   - Otherwise, go back to your work and check it for improvements. Is there another approach you could try? If you still can't get to the right answer, you can check the full solution on the Math Education Resources.
2. **Reflect on your work:** Generally, reflect on how you solved the problem. Don't just focus on the final answer, but whether your mental process was correct. If you were stuck at any point, what helped you to go forward? What made you confident that your answer was correct? What can you take away from this so that, next time, you can complete a similar question without any help?
3. **Plan further studying:** Once you feel confident enough with a particular topic, move on to topics that need more work. Focus on questions that you find challenging, not on those that are easy for you. Once you are ready to tackle a full exam, follow the advice for the full exam (click here).

Please note that all final answers were extracted automatically from the full solution. It is possible that the final answer shown here is not complete, or it may be missing entirely. In such a case, please notify `mer-wiki@math.ubc.ca`. Your feedback helps us improve.

This pdf was created for your convenience when you study Math and prepare for your final exams. All the content here, and much more, is freely available on the Math Education Resources.

Question A 01

FINAL ANSWER. $||\mathbf{x}|| = \sqrt{3}$.

Question A 02

FINAL ANSWER. $\begin{vmatrix} \hat{\mathbf{i}}\hat{\mathbf{j}}\hat{\mathbf{k}} \\ 1-1-1 \\ 112 \end{vmatrix} = [-1,-3,2]$ which is precisely the definition of the cross product. Therefore $\mathbf{x} \times \mathbf{y} = [-1,-3,2]$.

Question A 03

FINAL ANSWER. The first equation tells us k=1 but the other two equations then say that 1=-1 and 2=-1 which are both false and therefore there does not exist a unique k such that $\mathbf{x}$=$k\mathbf{y}$ and thus $\mathbf{x}$ and $\mathbf{y}$ are not multiples of one another so they are **not** linearly dependent.

Question A 04

FINAL ANSWER. Therefore we have constructed examples where we can have **unique solutions**, **no solutions**, or an **infinite number of solutions** and thus we see the answer is **(e)**

Question A 05

FINAL ANSWER. Therefore, we see that if A is not invertible then there is at least one zero eigenvalue with eigenvector $\mathbf{x}$. Conversely, if 0 is an eigenvalue then there exists a non-zero vector $\mathbf{x}$ such that A$\mathbf{x}$=0 which means A is not invertible.This concludes that A being non-invertible implies a zero eigenvalue and also that a zero eigenvalue implies that A is non-invertible.We just wrote that the only eigenvalues are 2 and 3 so therefore, 0 is not an eigenvalues which means A must be invertible.

Question A 06

FINAL ANSWER. Therefore, if a=-1/3 then the matrix will have a zero determinant and thus *not* be invertible.

Question A 07

FINAL ANSWER. $A^{-1} = \begin{bmatrix} -12 \\ 1-1 \end{bmatrix}$.

Question A 08

FINAL ANSWER. $\cos(\theta) = \frac{3}{\sqrt{420}}$.

Question A 09

FINAL ANSWER. $\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ v_1 \end{bmatrix}$.

Question A 10

Final answer. $0 = 2i_2 + 4(i_2 - i_3) + 7 + 3i_2 - v_1$ as the linear equation for loop 2.

Question A 11

Final answer. Do not worry about the presence of a negative sign, a negative loop current just means that the current (i_1 and i_3 in this case) actually flows counter-clockwise, not clockwise like we assumed.

Question A 12

Final answer. $A = \begin{bmatrix} 200 \\ 003 \end{bmatrix}$

Question A 13

Final answer. $A = \begin{bmatrix} -510 \\ 0-51 \\ 00-2 \end{bmatrix}$

Question A 14

Final answer. $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$

Question A 15

Final answer. $\lambda = \dfrac{6 \pm 2}{2}$ Therefore the eigenvalues are $\lambda = 2$ and $\lambda = 4$.

Question A 16

Final answer. $A = \begin{bmatrix} 13 \\ 2-1 \end{bmatrix}$

Question A 17

Final answer. To check our answer we could use the second basis vector [0,1] which would produce the second column of the matrix and once again we'd conclude the slope is -1/2.

Question A 18

Final answer. We plug this value of t back into the equation of the line to get that the point on the line that intersects the plane is (1,1,0).

Question A 19

Final answer. $QR \neq RQ$.

Question A 20

FINAL ANSWER. $T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}-9\\-2\\6\end{bmatrix}$.

Question A 21

FINAL ANSWER. $T = \begin{bmatrix}14-9\\4-2\\-56\end{bmatrix}$ where we recall that the columns of the transformation matrix are just the transformation of the basis vectors.

Question A 22

FINAL ANSWER. $2u - z = 5 + 5i$

Question A 23

FINAL ANSWER. $\overline{u} = 3 - 2i$.

Question A 24

FINAL ANSWER. $\frac{u}{z} = \frac{1}{2} + \frac{5}{2}i$

Question A 25

FINAL ANSWER. $(1+i)^{10} = 32i$.

Question A 26

FINAL ANSWER. Notice here that I reinitialize `x` to be the new transition vector so that I can minimize the amount of code I have to write.Once this is done we have the transition vector after 10 time steps.However, we're interested in the probability of being in the 5th state.This is the 5th entry of the vector.Therefore we type,`x(5)`Notice the lack of semicolon tells us that the entry of x(5) will be output to the screen for us to see.

Question A 27

FINAL ANSWER. (That's because calculating the inverse of a matrix A is an overkill here, there is faster ways of finding x that don't require the matrix inverse.)

Question A 28

FINAL ANSWER. *Therefore, a,d, and e are true.*

Question A 29

FINAL ANSWER. *Therefore, only (a) and (c) are true.*

Question A 30

FINAL ANSWER. Now we notice that the resulting matrix is upper-triangular, and thus the determinant is just the product of the entries on the diagonal: $\Delta = 24$.

Question B 01 (a)

FINAL ANSWER. $\mathbf{b} = \begin{bmatrix} 0 \\ 6 \\ 14 \end{bmatrix}$.

Question B 01 (b)

FINAL ANSWER. $\begin{bmatrix} 1-200 \\ 8086 \\ 16161614 \end{bmatrix}$

Question B 01 (c)

FINAL ANSWER. $x_1 = \frac{1}{4}$ and so Hartosh paints 1/4 rooms per hour (or 1 room every 4 hours).We have found the number of rooms that Hartosh, Mark, and Keiko can paint in an hour to be 1/4, 1/8, and 1/2 respectively.

Question B 02 (a)

FINAL ANSWER. $\mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. Of course, any multiple of this is also an eigenvector (expect multiplying by 0).

Question B 02 (b)

FINAL ANSWER. $\mathbf{x}_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$. Any non-trivial linear combination of these two vectors is also an eigenvector, which you can easily verify.

Question B 03 (a)

FINAL ANSWER. (Equivalently, to answer this question you could look at the second entry in the first column of the matrix P).

Question B 03 (b)

FINAL ANSWER. $\rightarrow \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Question B 03 (c)

FINAL ANSWER. $\frac{1}{11}\left(3 + 8\left(\frac{1}{12}\right)^n\right)$

Question B 03 (d)

FINAL ANSWER. $\hat{\mathbf{v}} = \begin{bmatrix} 3/11 \\ 8/11 \end{bmatrix}$. Hence, it is more likely, that the walker will be be in state 2 after a large number of steps, since the 2nd entry in the vector is larger than the first entry.

Question B 04 (a)

FINAL ANSWER. $\mathbf{x} = s \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ and there are an infinite number of solutions.

Question B 04 (b)

FINAL ANSWER. $\mathbf{x} = t \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$. There are an infinite number of solutions.

Question B 04 (c)

FINAL ANSWER. $\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -4 \\ -1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -5 \\ -3 \\ 0 \\ 1 \end{bmatrix}$. There are an infinite number of solutions.

Question B 04 (d)

FINAL ANSWER. Therefore $x_1 = 5$, $x_2 =$ -1, and $x_3 = 2$.

Question B 04 (e)

FINAL ANSWER. $\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -2 \\ -2 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$. There are an infinite number of solutions.

Question B 05 (a)

FINAL ANSWER. this *system* is just a collection of single equations whose solutions do not affect each other.

Question B 05 (b)

FINAL ANSWER. And impose the initial condition $\mathbf{y}_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ direcly to find that $C_1 = -\frac{i}{6}$.Then compute the real part to get the same result as above.The amount of algebra involved will be slightly less.

Question B 06 (a)

FINAL ANSWER. $T = \frac{\sqrt{19}}{2}$

Question B 06 (b)

FINAL ANSWER. $\vec{N} = (-3, -3, 1)$ The above vector gives the normal direction.

Question B 06 (c)

FINAL ANSWER. Dividing both sides by -3 gives $b = \frac{1}{3}$

Question B 06 (d)

FINAL ANSWER. Thus *(0, 0,-1)* is on P.

Question B 06 (e)

FINAL ANSWER. the point *(0, 0, 1)*, it wouldn't be solvable and some inconsistencies in the linear equations will result.