Final Answers
MATH221 December 2009

How to use this resource

- When you feel reasonably confident, simulate a full exam and grade your solutions. For your grading you can get the full solutions here.

- If you're not quite ready to simulate a full exam, we suggest you thoroughly and slowly work through each problem. Use this document with the final answers only to check if your answer is correct, without spoiling the full solution.

- Should you need more help, check out the hints and video lecture on the Math Education Resources.

Tips for Using Previous Exams to Study: Work through problems

Resist the temptation to read any of the final answers below before completing each question by yourself first! We recommend you follow the guide below.

1. **How to use the final answer:** *The final answer is not a substitution for the full solution!* The final answer alone will not give you full marks. The final answer is provided so that you can check the correctness of your work without spoiling the full solution.

   - To answer each question, only use what you could also use in the exam. Download the raw exam here.
   - If you found an answer, how could you verify that it is correct from your work only? E.g. check if the units make sense, etc. Only then compare with our result.
   - If your answer is correct: good job! Move on to the next question.
   - Otherwise, go back to your work and check it for improvements. Is there another approach you could try? If you still can't get to the right answer, you can check the full solution on the Math Education Resources.

2. **Reflect on your work:** Generally, reflect on how you solved the problem. Don't just focus on the final answer, but whether your mental process was correct. If you were stuck at any point, what helped you to go forward? What made you confident that your answer was correct? What can you take away from this so that, next time, you can complete a similar question without any help?

3. **Plan further studying:** Once you feel confident enough with a particular topic, move on to topics that need more work. Focus on questions that you find challenging, not on those that are easy for you. Once you are ready to tackle a full exam, follow the advice for the full exam (click here).

Please note that all final answers were extracted automatically from the full solution. It is possible that the final answer shown here is not complete, or it may be missing entirely. In such a case, please notify mer-wiki@math.ubc.ca. Your feedback helps us improve.

This pdf was created for your convenience when you study Math and prepare for your final exams. All the content here, and much more, is freely available on the Math Education Resources.

Question 1

FINAL ANSWER. $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \Rightarrow \vec{x} = \begin{bmatrix} 1 + 3x_3 \\ 1 - 2x_3 \\ x_3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$, $t \in \mathbb{R}$

Question 2 (a)

FINAL ANSWER. To generalize the above, just change every 5 above to an n.

Question 2 (b)

FINAL ANSWER. Thus $det(A) = -14$

Question 3 (a)

FINAL ANSWER. $\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -100 \\ 350 \\ -100 \end{bmatrix}$

Question 3 (b)

FINAL ANSWER. $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} x_3 + \begin{bmatrix} 150 \\ -150 \\ 0 \end{bmatrix}$

Question 4

FINAL ANSWER. Hence, our transformation matrix is $T = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

Question 5 (a)

FINAL ANSWER. $\therefore B = \left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 4 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \\ 8 \\ 2 \end{bmatrix} \right\}$ Where B forms a basis for the span of W

Question 5 (b)

FINAL ANSWER. $\{-3 - 410\}$

Question 5 (c)

FINAL ANSWER. No content found.

Question 6

FINAL ANSWER. To check that these matrices are correct, we can compute PDP^{-1}. When we do this calculation, we see that $PDP^{-1} = A$

Question 7 (a)

FINAL ANSWER. $\begin{bmatrix} 0.55 \\ 0.45 \end{bmatrix} = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$

Question 7 (b)

FINAL ANSWER. $ANS = \begin{bmatrix} .6 \\ .4 \end{bmatrix}$

Question 8

FINAL ANSWER. So $x_{30} = -5 \cdot 2^{30} + 4$

Question 9

FINAL ANSWER. So the parabola is $y = 0.1955 + 0.4884x + 1.3161x^2$

Question 10 (a)

FINAL ANSWER. we need to have $\frac{1}{3} = \frac{2}{a}$ Solve this to get $a = 6$.

Question 10 (b)

FINAL ANSWER. In order to get 2 free variables, we need to set $\frac{1}{1} = \frac{b}{-1}$ Thus $b = -1$

Question 11

FINAL ANSWER. $X = \begin{bmatrix} -\frac{58}{33} & \frac{10}{3} & \frac{43}{33} \\ \frac{71}{33} & -\frac{14}{3} & \frac{130}{33} \\ \frac{1}{33} & \frac{8}{3} & -\frac{73}{33} \end{bmatrix}$

Question 12 (a)

FINAL ANSWER. $x + y + z = 3$ $x = 1$ $y = 1$ $z = 1$ The system is consistent.

Question 12 (b)

FINAL ANSWER. $\begin{bmatrix} 1 & 0 & 0 & |a \\ 0 & 1 & 0 & |b \\ 0 & 0 & 1 & |c \\ 0 & 0 & 0 & |d \end{bmatrix}$

Question 12 (c)

FINAL ANSWER. The rank is the number of linearly independent columns of A, in this case is at the most 6, likewise $\dim(N(A)) \geq 3$.

Question 12 (d)

FINAL ANSWER. By the rank nullity theorem $dim(N(A)) = 6 - rank(A)$ $dim(N(A)) = 5 - 4$ $dim((N(A)) = 2$ Don't forget for the rank nullity theorem, its the number of columns minus the rank!

Question 12 (e)

FINAL ANSWER. $A = P^{-1}DP$ where D is a matrix with the eigenvalues of A as its diagonal. All projection matrices to the same dimension are diagonizable. Since all vectors in W project onto itself with that transformation, the eigenvalues of T are 0 and 1, hence $P^{-1}D^nP = A$.

Question 12 (f)

FINAL ANSWER. $rref(A) = \begin{bmatrix} 100 \\ 001 \\ 000 \end{bmatrix}$ number of nonzeros in diagonal is 1 but the rank is 2.

Question 12 (g)

FINAL ANSWER. If were linearly independent of each other then it can be said that n = 5 (i.e.is a basis), however, it is possible for n to be any number of vectors but still span R^5 given that some of the vectors are linearly dependent.

Question 12 (h)

FINAL ANSWER. Hence they satisfy $k_1v_1 + \cdots + k_mv_m \in W$ Hence the statement is **True**.

Question 12 (i)

FINAL ANSWER. Take $y = Ax$ then $Ay = b$

Question 12 (j)

FINAL ANSWER. $\begin{bmatrix} cos\theta - sin\theta \\ sin\theta cos\theta \end{bmatrix}$, Row 1, Column 1 and Row 2, Column 2 don't match.