Final answers script in beta

Final Answers
MATH103 April 2014

April 16, 2015

How to use this resource

- When you feel reasonably confident, simulate a full exam and grade your solutions. For your grading you can get the full solutions here.
- If you're not quite ready to simulate a full exam, we suggest you thoroughly and slowly work through each problem. Use this document with the final answers only to check if your answer is correct, without spoiling the full solution.
- Should you need more help, check out the hints and video lecture on the Math Education Resources.

Tips for Using Previous Exams to Study: Work through problems

Resist the temptation to read any of the final answers below before completing each question by yourself first! We recommend you follow the guide below.

1. **How to use the final answer:** *The final answer is not a substitution for the full solution!* The final answer alone will not give you full marks. The final answer is provided so that you can check the correctness of your work without spoiling the full solution.
   - To answer each question, only use what you could also use in the exam. Download the raw exam here.
   - If you found an answer, how could you verify that it is correct from your work only? E.g. check if the units make sense, etc. Only then compare with our result.
   - If your answer is correct: good job! Move on to the next question.
   - Otherwise, go back to your work and check it for improvements. Is there another approach you could try? If you still can't get to the right answer, you can check the full solution on the Math Education Resources.
2. **Reflect on your work:** Generally, reflect on how you solved the problem. Don't just focus on the final answer, but whether your mental process was correct. If you were stuck at any point, what helped you to go forward? What made you confident that your answer was correct? What can you take away from this so that, next time, you can complete a similar question without any help?
3. **Plan further studying:** Once you feel confident enough with a particular topic, move on to topics that need more work. Focus on questions that you find challenging, not on those that are easy for you. Once you are ready to tackle a full exam, follow the advice for the full exam (click here).

Please note that all final answers were extracted automatically from the full solution. It is possible that the final answer shown here is not complete, or it may be missing entirely. In such a case, please notify `mer-wiki@math.ubc.ca`. Your feedback helps us improve.

This pdf was created for your convenience when you study Math and prepare for your final exams. All the content here, and much more, is freely available on the Math Education Resources.

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Question 1 (a)

FINAL ANSWER. **THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!**

The correct answer is A.

Question 1 (b) ii

FINAL ANSWER. Thus, the mean is also a value in $[0, 10]$.

Question 1 (b) i

FINAL ANSWER. In particular, $f'(5) = p(5) \geq 0$.

Question 1 (c) iii

FINAL ANSWER. *Note*: A common mistake that students tend to make in this type of question is interpreting the negative slope of the graph at (1.25,1.5) as the car returning to its original position.Keep in mind, the graph is illustrating the velocity of the car, not its displacement.

Question 1 (c) ii

FINAL ANSWER. Thus, the total area accumulated over $[0, 1]$ is 0.

Question 1 (c) i

FINAL ANSWER. When $t = 0$, we see from the graph that $v(t) = 0$.

Question 1 (d) iii

FINAL ANSWER. But since 0 and 1 are steady states, if this occurs then $x(t)$ will be 0 (respectively, 1) for all $t \geq s$.

Question 1 (d) ii

FINAL ANSWER. If the initial population is 0, then it will be 0 forever, as 0 is a steady state.

Question 1 (d) i

FINAL ANSWER. The answer is all of D, E and F.

Question 1 (e) ii

FINAL ANSWER. $1 + 0 + \frac{1}{2} + 0 + 0 + \frac{1}{3} + \cdots = 1 + \frac{1}{2} + \frac{1}{3} + \cdots$,which is just the harmonic series.But the harmonic series diverges (say by using the p-series test if need be)! Note the difference between a sequence and a series.

Question 1 (e) i

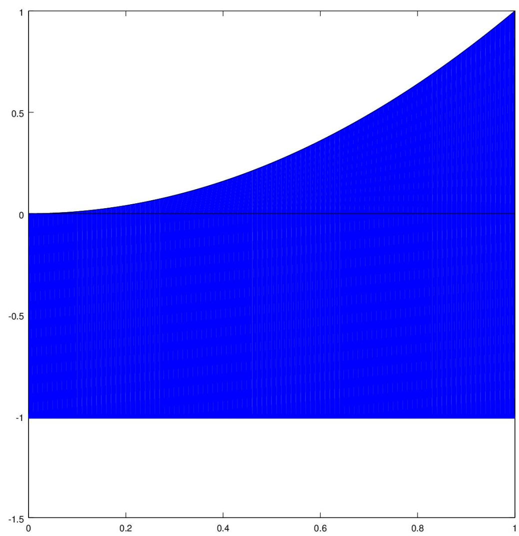
FINAL ANSWER. **THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!**

Thus, $a_k \to 0$.

Question 2 (a)

FINAL ANSWER. $\int_1^4 x^{3/2}\mathrm{d}x = \frac{62}{5}$.

Question 2 (b)



FINAL ANSWER.

Question 3 (a)

FINAL ANSWER. This probability is given by $\int_5^\infty e^{3-t}\mathrm{d}t = e^{-2}$.

Question 3 (b)

FINAL ANSWER. $-\lim_{b\to\infty} e^{3-t}(t+1)\big|_3^b = 4$.

Question 3 (c)

FINAL ANSWER. **THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!**

$q(s) = e^{s-e^s+4}$.

Question 4 (a)

FINAL ANSWER. **THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!**
$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \left(\frac{k}{n}\right)^2 = \frac{1}{3}$.

Question 4 (b)

FINAL ANSWER. We conclude that $\lim_{n\to\infty} a_n = 1$.

Question 4 (c)

FINAL ANSWER. Note: l′Hôpital′s rule is only applicable when its conditions are met by the numerator, the denominator, and even the ratio itself.

Question 5 (a)

FINAL ANSWER. Thus, $y = 1$.

Question 5 (b)

FINAL ANSWER. Note that we have taken the positive square root as we require the solution y to lie in the interval $[0, 1]$.

Question 5 (c)

FINAL ANSWER. $\Rightarrow t = \frac{21}{2}$.

Question 6 (a)

FINAL ANSWER. converges by the p-test with $p = 2$.

Question 6 (c)

FINAL ANSWER. $\sum_{k=1}^{\infty} \frac{1}{k(\ln k)^{103}} = \sum_{k=1}^{\infty} \frac{1}{k(\ln k)^{103}}$converges.

Question 7 (a)

FINAL ANSWER. For $n = 5$, this yields $a_{10} = -\frac{1}{120}$.

Question 7 (b)

FINAL ANSWER. For $n = 5$, this yields $b_{11} = -\frac{1}{11 \cdot 5!}$.

Question 7 (c)

FINAL ANSWER. e. $c_3 = \frac{1}{3}$.

Question 8 (a)

FINAL ANSWER. $2^{-2}\sum_{n=0}^{\infty}(2^{-3})^n = \frac{2}{7}$.

Question 8 (b)

FINAL ANSWER. We conclude that the only values for which the series converges are $-6 < x < 4$.

Question 9 (a)

FINAL ANSWER. $\left|F'\left(\frac{1}{3}\right)\right| = \frac{1}{2} < 1$. Thus 0 is an unstable fixed point and 1/3 is a stable fixed point.

Question 9 (b)

FINAL ANSWER. We can now repeat this procedure (replacing x_0 by $F(x_0)$, $F(x_0)$ by $F(F(x_0))$, etc.).

Question 9 (c)

FINAL ANSWER. Thus, the series $\sum_{n=0}^{\infty} a_n = \sum_{n=0}^{\infty} a_n$ diverges by the divergence test.

Question 10 (a)

FINAL ANSWER. $\int_1^e \ln y \mathrm{d}y = 2$.

Question 10 (b)

FINAL ANSWER. **THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!**

Note that the bounds of integration above did not change since $0^2 = 0$ and $1^2 = 1$.

Question 10 (c)

FINAL ANSWER. $\int_0^{\infty}\left(\frac{1}{\sqrt{x^2+1}}\right)^3 \mathrm{d}x = 1$.

Question 11

FINAL ANSWER. Thus, the series converges for $0 < \beta < 1$.

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