# Full Solutions
# MATH110 December 2013

December 4, 2014

### How to use this resource

- When you feel reasonably confident, simulate a full exam and grade your solutions. This document provides full solutions that you can use to grade your work.
- If you're not quite ready to simulate a full exam, we suggest you thoroughly and slowly work through each problem. To check if your answer is correct, without spoiling the full solution, we provide a pdf with the final answers only. Download the document with the final answers here.
- Should you need more help, check out the hints and video lecture on the Math Educational Resources.

## Tips for Using Previous Exams to Study: Exam Simulation

*Resist the temptation to read any of the solutions below before completing each question by yourself first! We recommend you follow the guide below.*

1. **Exam Simulation:** When you've studied enough that you feel reasonably confident, print the raw exam (click here) without looking at any of the questions right away. Find a quiet space, such as the library, and set a timer for the real length of the exam (usually 2.5 hours). Write the exam as though it is the real deal.
2. **Reflect on your writing:** Generally, reflect on how you wrote the exam. For example, if you were to write it again, what would you do differently? What would you do the same? In what order did you write your solutions? What did you do when you got stuck?
3. **Grade your exam:** Use the solutions in this pdf to grade your exam. Use the point values as shown in the original exam.
4. **Reflect on your solutions:** Now that you have graded the exam, reflect again on your solutions. How did your solutions compare with our solutions? What can you learn from your mistakes?
5. **Plan further studying:** Use your mock exam grades to help determine which content areas to focus on and plan your study time accordingly. Brush up on the topics that need work:
   - Re-do related homework and webwork questions.
   - The Math Exam Resources offers mini video lectures on each topic.
   - Work through more previous exam questions thoroughly without using anything that you couldn't use in the real exam. Try to work on each problem until your answer agrees with our final result.
   - Do as many exam simulations as possible.

   Whenever you feel confident enough with a particular topic, move on to topics that need more work. Focus on questions that you find challenging, not on those that are easy for you. Always try to complete each question by yourself first.

This pdf was created for your convenience when you study Math and prepare for your final exams. All the content here, and much more, is freely available on the Math Educational Resources.



## Question 1 (a)

Solution 1. **Note: The first three parts of this question was identical to that of the October Midterm.**
**True:** There are multiple ways to see this.
Solve the quadratic equation.
The function expands to

$$f(x) = \frac{1}{2}x^2 + 3x + \frac{7}{2}.$$

Which then gives:

$$\begin{aligned} x &= \frac{-3 \pm \sqrt{9 - 4\left(\frac{1}{2}\right)\left(\frac{7}{2}\right)}}{2\left(\frac{1}{2}\right)} \\ &= \frac{-3 \pm \sqrt{9-7}}{1} \\ x &= -3 \pm \sqrt{2} \end{aligned}$$

Solution 2. We can also use the vertex formula by seeing that $f(x) = \frac{1}{2}(x+3)^2 - 1$, with vertex $(-3, -1)$ which is below the $x$-axis. Next, observe that a parabola is continuous (because its a polynomial). Combining this with the fact that the parabola has a positive coefficient of $\frac{1}{2}$, meaning that the parabola is curving upwards, we get that it must cross the $x$ axis.

Solution 3. We can view this as a graph transformation of $y = x^2$ which touches the $x$-axis. Then we shift it 3 to the left and then 2 down, so it still touches the $x$-axis. This is then followed by a vertical compression of 2 (or expansion by 0.5). This does not change direction of curvature of the function and so it must still touch the $x$-axis.

Solution 4. We can also use the IVT. The function is a quadratic and hence is continuous. We then look at $f(-3) = -1 < 0$ and then $f(0) = 3.5 > 0$. Thus there must be a point between $-3$ and $0$ that crosses the $x$-axis.

## Question 1 (b)

Solution. **False:** The easiest way to see this is to come up with a function where the limit exists but the function is not defined there. For example $f(x) = \frac{4(x-1)}{x-1}$. The limit exists and is equal to 4 but 1 is not in the domain, so $f(1)$ is not defined. Hence the statement is **false**.

## Question 1 (c)

Solution. **False:** To hunt for discontinuities, we have to check inside each piece and also at the boundary.

1. For the boundary to be continuous, we must have:

$$\begin{aligned}
\lim_{x\to 1^-} f(x) &= \lim_{x\to 1^+} f(x)\\
\lim_{x\to 1^-} \frac{3}{x+2} &= \lim_{x\to 1^+} \sqrt{x}\\
\frac{3}{1+2} &= \sqrt{1}\\
1 &= 1.
\end{aligned}$$

So the limits exists and is equal to 1 which happens to be $f(1) = 1$. So the function in continuous at the boundary.

2. On the right of $x = 1$, the function is continuous since $\sqrt{x}$ is only undefined for $x < 0$ which is not covered by this case.
3. On the left of $x = 1$, the function is discontinuous at $x = -2$ since the denominator is 0. This fall inside the region considered. So $f(x)$ is discontinuous at $x = -2$.

That means $f(x)$ is NOT continuous over all real numbers.

## Question 1 (d)

SOLUTION. **False:** If $f(x)$ is differentiable at $x = 2$, then by the definition of derivative at a point, we have $f'(2) = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h}$. So to paraphrase the statement, we are asking whether $f'(2) = 2$. Clearly, this does not have to be the case. Consider the example $f(x) = 1$. We have $f(x)$ is differentiable at $x = 2$ (and everywhere else too). But $f'(2) = 0$.

## Question 2

SOLUTION. We first recognise that parallel means that the slopes must be equal. Rearranging the equation, we get:

$$\begin{aligned}
x - 2y &= 2\\
x - 2 &= 2y\\
y &= \frac{x-2}{2}\\
y &= \frac{1}{2}x - 1.
\end{aligned}$$

So that means we are looking for points along the curve with slope equal to $\frac{1}{2}$. To get those points, we need the derivative of the curve:

$$\frac{\mathrm{d}}{\mathrm{d}x}(y) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{x-4}{x+4}\right)$$

Using the quotient rule, we get:

$$\begin{aligned}
u(x) &= x - 4 & u'(x) &= 1\\
v(x) &= x + 4 & v'(x) &= 1
\end{aligned}$$

So that means:

$$
\begin{aligned}
\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{v(x)u'(x) - u(x)v'(x)}{(v(x))^2} \\
&= \frac{(x+4)\cdot(1) - (x-4)\cdot(1)}{(x+4)^2} \\
&= \frac{8}{(x+4)^2}
\end{aligned}
$$

We are interested in points where the derivative is $\frac{1}{2}$

$$
\begin{aligned}
\frac{1}{2} &= \frac{8}{(x+4)^2} \\
(x+4)^2 &= 16 \\
x+4 &= \pm 4 \\
x &= 0, -8
\end{aligned}
$$

This means we have to tackle two potential $x$ values. At $x = 0$, we have:

$$
y = \frac{0-4}{0+4} = -1
$$

At $x = -8$, we have:

$$
y = \frac{-8-4}{-8+4} = \frac{-12}{-4} = 3
$$

So that means the tangent lines are are looking for are: for $x = 0$.

$$
\begin{aligned}
y &= \frac{1}{2}(x-0) + (-1) \\
&= \frac{1}{2}x - 1
\end{aligned}
$$

and for $x = -8$

$$
\begin{aligned}
y &= \frac{1}{2}(x-(-8)) + 3 \\
&= \frac{1}{2}(x+8) + 3 \\
&= \frac{1}{2}x + 7
\end{aligned}
$$

## Question 3 (a)

Solution. Using the power rule with $n = 6$.

$$
f'(x) = 6x^5.
$$

## Question 3 (b)

SOLUTION. **Parts b and c combined:** (These are all the rules I can think of).

1. Addition rule:

$$f(x) = \frac{1}{2}x^6 + \frac{1}{2}x^6 \qquad\qquad f'(x) = 3x^5 + 3x^5$$

2. Subtraction rule:

$$f(x) = 2x^6 - x^6 \qquad\qquad f'(x) = 12x^5 - 6x^5$$

3. Product rule:

$$f(x) = x^2 \cdot x^4 \qquad\qquad f'(x) = 2x \cdot x^4 + 4x^3 \cdot x^2$$

4. Quotient rule:

$$f(x) = \frac{x^7}{x} \qquad\qquad f'(x) = \frac{7x^6 \cdot x - x^7 \cdot 1}{x^2}$$

5. Chain rule:

$$f(x) = (x^3)^2 \qquad\qquad f'(x) = 3x^2 \cdot 2(x^3)$$

6. Log-diff:

$$\ln(f(x)) = 6\ln(x) \qquad\qquad f'(x) = \frac{6}{x} \cdot x^6$$

7. Exp-chain rule:

$$f(x) = \exp[6\ln(x)] \qquad\qquad f'(x) = \frac{6}{x} \cdot \exp[6\ln(x)]$$

8. Limit definition of derivative:

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^6 - x^6}{h} \qquad\qquad f'(x) = \lim_{h\to 0} \frac{6x^5h + 15x^4h^2 + \ldots + h^6}{h}$$

$$f'(x) = \lim_{a\to x} \frac{a^6 - x^6}{a - x} \qquad\qquad f'(x) = \lim_{a\to x} \frac{(a-x)(a^5 + a^4x + \ldots x^5)}{(a-x)}$$

## Question 4 (a)

SOLUTION. We simply substitute:

$$
\begin{aligned}
g(x) &= (f \circ f)(x) \\
&= f(f(x)) \\
&= f(\sqrt{x} - 1) \\
g(x) &= \sqrt{\sqrt{x} - 1} - 1.
\end{aligned}
$$

## Question 4 (b)

SOLUTION. Looking at the outside function, we want to find $\sqrt{x} - 1 \geq 0$ for $\sqrt{\sqrt{x} - 1}$ to be defined. So we want $\sqrt{x} \geq 1$. Looking at the inside function, we want $\sqrt{x} \geq 0$ so $x \geq 0$.
Combining the two, we get

$$x \geq 0$$

and $x \geq 1$.
Hence the domain of $g(x)$ is $\{x\mathbb{R} \mid x \geq 1\}$ or $x \in [1, \infty)$.

## Question 4 (c)

SOLUTION. We want to apply the chain rule. Set $u = \sqrt{x} - 1$.

$$
\begin{aligned}
g(u) &= \sqrt{u} - 1 & \qquad g'(u) &= \frac{1}{2\sqrt{u}} \\
u(x) &= \sqrt{x} - 1 & \qquad u'(x) &= \frac{1}{2\sqrt{x}}
\end{aligned}
$$

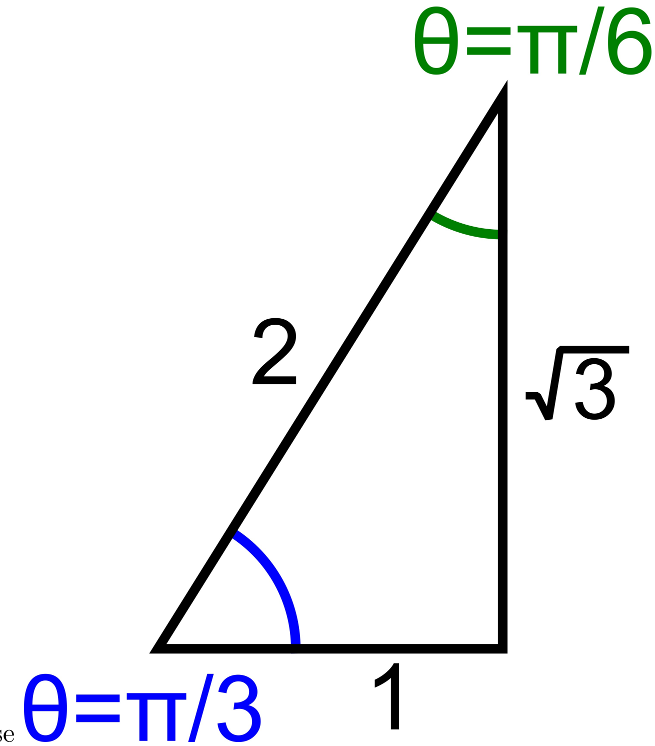
Combining, we get:

$$
\begin{aligned}
g'(x) &= \frac{1}{2\sqrt{x}} \cdot \frac{1}{2\sqrt{\sqrt{x} - 1}} \\
&= \frac{1}{4\sqrt{x} \cdot \sqrt{\sqrt{x} - 1}} \\
&= \frac{1}{4\sqrt{x\left(\sqrt{x} - 1\right)}}
\end{aligned}
$$

## Question 5 (a)

SOLUTION. We evaluate these values by following three steps:

1. Reduce the parameter to be in the interval $[0, 2\pi)$.

2. Apply the relevant special triangle, in this case 

3. Check the sign of the result by remembering the $\cos(t)$ gives the $x$-coordinate and $\sin(t)$ gives the $y$-coordinate.

1. $\sin\left(\frac{5\pi}{3}\right)$.
   (a) The angle $\frac{5\pi}{3}$ is in the desired interval.
   (b) The angle $\frac{5\pi}{3}$ is $\frac{\pi}{3}$ below the $x$-axis. Applying the special triangle, we get $\sin\left(\frac{5\pi}{3}\right) = \pm\frac{\sqrt{3}}{2}$.
   (c) We are in the fourth quadrant so the $y$-coordinate is negative.

   We have
$$\sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}.$$

2. $\cos\left(-\frac{\pi}{3}\right)$
   (a) The angle $-\frac{\pi}{3}$ is not in the desired interval. So we apply periodicity:
$$\begin{aligned}\cos\left(-\frac{\pi}{3}\right) &= \cos\left(-\frac{\pi}{3} + 2\pi\right)\\ &= \cos\left(-\frac{\pi}{3} + \frac{6\pi}{3}\right)\\ &= \cos\left(\frac{5\pi}{3}\right)\end{aligned}$$
   (b) The angle $\frac{5\pi}{3}$ is $\frac{\pi}{3}$ below the $x$-axis. Applying the special triangle, we get $\cos\left(\frac{5\pi}{3}\right) = \pm\frac{1}{2}$.
   (c) We are in the fourth quadrant so the $x$-coordinate is positive.

   We have
$$\cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}.$$

3. $\tan\left(\frac{11\pi}{3}\right)$.

(a) The angle $\frac{11\pi}{3}$ is not in the desired interval. So we apply periodicity:

$$\begin{aligned}\tan\left(\frac{11\pi}{3}\right) &= \tan\left(\frac{11\pi}{3} - 2\pi\right)\\ &= \tan\left(\frac{11\pi}{3} - \frac{6\pi}{3}\right)\\ &= \tan\left(\frac{5\pi}{3}\right)\end{aligned}$$

(b) The angle $\frac{5\pi}{3}$ is $\frac{\pi}{3}$ below the $x$-axis. Applying the special triangle, we get $\tan\left(\frac{5\pi}{3}\right) = \pm\sqrt{3}$.

(c) We are in the fourth quadrant and since $\tan(t) = \frac{\sin(t)}{\cos(t)}$, we get that $\tan(t)$ is negative.

Alternative, we can make use of the previous parts to get:
$\tan\left(\frac{11\pi}{3}\right) = \tan\left(\frac{5\pi}{3}\right) = \frac{\sin\left(\frac{11\pi}{3}\right)}{\cos\left(\frac{11\pi}{3}\right)} = \frac{\frac{-\sqrt{3}}{2}}{\frac{1}{2}}$
Either way, we have

$$\tan\left(\frac{11\pi}{3}\right) = -\sqrt{3}$$

## Question 5 (b)

**Easiness: 50/100**

SOLUTION. To find the equation of the tangent line, we need to find the slope via the derivative. We can either apply the quotient rule or the power rule (by recognising $y = \tan(x)^{-1}$. Using the quotient rule:

$$u(x) = \cos(x) \qquad u'(x) = -\sin(x)$$
$$v(x) = \sin(x) \qquad v'(x) = \cos(x)$$

This gives:

$$\begin{aligned}\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{vu' - uv'}{v^2}\\ &= \frac{\sin(x)\cdot(-\sin(x) - \cos(x)\cdot\cos(x)}{[\sin(x)]^2}\\ &= \frac{-\sin^2(x) - \cos^2(x)}{\sin^2(x)}\\ &= \frac{-\left(\sin^2(x) + \cos^2(x)\right)}{\sin^2(x)}\\ &= \frac{-1}{\sin^2(x)}\end{aligned}$$

At $x = \frac{5\pi}{3}$, we have $\sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$ (from part (a)), and so that means that the slope of the curve at $x = \frac{5\pi}{3}$ is

$$
\begin{aligned}
y' &= \frac{-1}{\sin^2(x)} \\
&= \frac{-1}{\sin^2\left(\frac{5\pi}{3}\right)} \\
&= \frac{-1}{\left[-\frac{\sqrt{3}}{2}\right]^2} \\
&= \frac{-1}{\frac{3}{4}} \\
&= \frac{-4}{3}
\end{aligned}
$$

We will also need the $y$-value at the point

$$
\begin{aligned}
y &= \frac{\cos(x)}{\sin(x)} \\
&= \frac{1}{\tan\left(\frac{5\pi}{3}\right)} \\
&= \frac{1}{\tan\left(\frac{2\pi}{3}\right)} \\
&= \frac{1}{-\sqrt{3}} \\
&= -\frac{\sqrt{3}}{3}
\end{aligned}
$$

(here we used the fact that the function $\tan(x)$ is $\pi$ periodic and then applied the computation from part (a)).
This means our tangent line is:
$y = -\frac{\sqrt{3}}{3} - \frac{4}{3}\left(x - \frac{5\pi}{3}\right)$

## Question 6 (a)

SOLUTION. We first simplify using the exponential rules to get $f(x) = e^{x^3+2x^2+x-3}$. Then we can solve for $f(x) = 0$.

$$
\begin{aligned}
e^{x^3+2x^2+x-3} &= 0 \\
x^3 + 2x^2 + x - 3 &= \ln(0).
\end{aligned}
$$

By taking the natural logarithm of both sides, we get a $\ln(0)$ on the right hand side which is bad since 0 is not in the domain of the natural logarithm. So we conclude that there cannot be a solution to the equation $f(x) = 0$. Hence, we have 0 solutions.

## Question 6 (b)

SOLUTION 1. Since we have simplified, we can simply apply the chain rule to it using $u = x^3 + 2x^2 + x - 3$.

$$
\begin{aligned}
&f(u) = e^u && f'(u) = e^u \\
&u(x)x^3 + 2x^2 + x - 3 && u'(x) = 3x^2 + 4x + 1.
\end{aligned}
$$

Combining, we get:
$f'(x) = \left(3x^2 + 4x + 1\right) \cdot e^{x^3+2x^2+x-3}$

SOLUTION 2. Alternatively, we can start with the initial function and use log-diff:

$$\begin{aligned}
f(x) &= \frac{e^{\left(x^3\right)}e^{\left(2x^2\right)}e^x}{e^3} \\
\ln(f(x)) &= \ln\left(\frac{e^{\left(x^3\right)}e^{\left(2x^2\right)}e^x}{e^3}\right) \\
&= \ln\left(e^{\left(x^3\right)}\right) + \ln\left(e^{\left(2x^2\right)}\right) + \ln\left(e^x\right) - \ln\left(e^3\right) \\
&= x^3\ln(e) + 2x^2\ln(e) + x\ln(e) - 3\ln(e) \\
\ln(f(x)) &= x^3 + 2x^2 + x - 3 \\
\frac{\mathrm{d}}{\mathrm{d}x}\ln(f(x)) &= \frac{\mathrm{d}}{\mathrm{d}x}\left(x^3 + 2x^2 + x - 3\right) \\
\frac{f'(x)}{f(x)} &= 3x^2 + 4x + 1 \\
f'(x) &= \left(3x^2 + 4x + 1\right) f(x) \\
f'(x) &= \left(3x^2 + 4x + 1\right) \cdot \frac{e^{\left(x^3\right)}e^{\left(2x^2\right)}e^x}{e^3}
\end{aligned}$$

## Question 6 (c)

SOLUTION. Recall that
$f'(x) = \left(3x^2 + 4x + 1\right) \cdot e^{x^3+2x^2+x-3}$.
From part (a), we know that $e^{x^3+2x^2+x-3} \neq 0$. That implies that $f'(x) = 0$ when $3x^2+4x+1 = 0$. Factoring, we get $(3x+1)(x+1) = 0$, which means $x = -\frac{1}{3}, -1$. That means there are two solutions for when $f'(x) = 0$.

## Question 7

SOLUTION 1. We can solve this problem by using logarithmic differentiation.

$$\begin{aligned}
\ln(f(x)) &= \ln\left((x^2+1)^7(x^4+2)^5(x^6+3)^3(x^8+4)\right) \\
&= 7\ln(x^2+1) + 5\ln(x^4+2) + 3\ln(x^6+3) + \ln(x^8+4)
\end{aligned}$$

Differentiating implicitly yields

$$\frac{1}{f(x)}f'(x) = \frac{7}{x^2+1}\cdot 2x + \frac{5}{x^4+2}\cdot 4x^3 + \frac{3}{x^6+3}\cdot 6x^5 + \frac{1}{x^8+4}\cdot 8x^7$$

So

$$f'(x) = (x^2+1)^7(x^4+2)^5(x^6+3)^3(x^8+4)\left(\frac{14x}{x^2+1} + \frac{20x^3}{x^4+2} + \frac{18x^5}{x^6+3} + \frac{8x^7}{x^8+4}\right)$$

**Solution 2.** Solution not found, please notify the MER wiki team.

## Question 8

**Easiness: 70/100**

**Solution.** The condition for $f$ to be continuous is

$$\begin{aligned}\lim_{x\to t^-} f(x) &= \lim_{x\to t^+} f(x)\\ e^{-t} &= 2(t+1)\end{aligned}$$

Since both piece of the function are continuous, we can use direct substitution and find $t$. We must show there exists a solution to the above equation. Equivalently we want to show that the following function $g(t)$ has a zero

$$g(t) = e^{-t} - 2(t+1)$$

Note that $g$ is continuous since it is constructed from continuous functions. We aim to apply IVT. Observe

$$\begin{aligned}g(0) &= e^0 - 2 < 0\\ g(-1) &= e^1 - 2(-1+1) = e > 0\end{aligned}$$

Hence by the IVT there exists $c \in (-1, 0)$ such that $g(c) = 0$. At $t = c$, we have $e^{-c} = 2(c+1)$. In turn, this means we have the two one sided limits equalling each other and hence $f(x)$ will be continuous by choosing $x = c$.

## Question 9

**Solution.** Before we start, let's confirm that the point $(0, \pi)$ is indeed on the line, as the statement suggests:

$$\begin{aligned}\sin(0+\pi) &= 0\cdot\pi\\ \sin(\pi) &= 0\\ 0 &= 0\end{aligned}$$

The point is on the line. Next we want to find the derivative of the function. We apply implicit differentiation and differentiate both sides:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\sin(x+y) &= \frac{\mathrm{d}}{\mathrm{d}x}xy\\ \cos(x+y)\left(1+\frac{\mathrm{d}y}{\mathrm{d}x}\right) &= x\frac{\mathrm{d}y}{\mathrm{d}x} + y\end{aligned}$$

The left hand side was obtained via the chain rule and the right hand side was obtained via a product rule.

$$\begin{aligned}\cos(x+y) + \frac{\mathrm{d}y}{\mathrm{d}x}\cos(x+y) &= x\frac{\mathrm{d}y}{\mathrm{d}x} + y\\ \frac{\mathrm{d}y}{\mathrm{d}x}\cos(x+y) - x\frac{\mathrm{d}y}{\mathrm{d}x} &= y - \cos(x+y)\\ \frac{\mathrm{d}y}{\mathrm{d}x}\left(\cos(x+y) - x\right) &= y - \cos(x+y)\\ \frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{y-\cos(x+y)}{\cos(x+y)-x}\end{aligned}$$

Substitute in the given point.

$$\frac{\mathrm{d}y}{\mathrm{d}x}(0) = \frac{\pi - \cos(0+\pi)}{\cos(0+\pi) - 0}$$
$$\frac{\mathrm{d}y}{\mathrm{d}x}(0) = \frac{\pi - (-1)}{(-1) - 0}$$
$$\frac{\mathrm{d}y}{\mathrm{d}x}(0) = \frac{\pi + 1}{-1} = -(\pi + 1)$$

## Question 10 (a)

SOLUTION. A function $f(x)$ is differentiable at the point $x = 2$ if the following limit exists

$$\lim_{x\to 2} \frac{f(x) - f(2)}{x - 2}$$

Equivalently, $f(x)$ is differentiable at the point $x = 2$ if the following limit exists

$$\lim_{h\to 0} \frac{f(2+h) - f(2)}{h}$$

## Question 10 (b)

SOLUTION. As suggested in the hint, this function is continuous everywhere except for possibly at the point $x = 2$. There we need to check continuity and differentiability there. For continuity, we require that the following three quantities are all the same
$\lim_{x\to 2^+} f(x) = f(2) = \lim_{x\to 2^-} f(x)$
Since $f(2) = (2)^2 - 2(2) + 1 = 1$ we have to check that both limits above also equal 1. Calculating the limits we find that

$$\lim_{x\to 2^-} f(x) = (2)^2 - 2(2) + 1 = 1$$
$$\lim_{x\to 2^+} f(x) = a(2) + b = 2a + b$$

Thus we require $2a + b = 1$. Rearranging this, we see that $b - 1 = -2a$. The second piece of information come from checking that the function is differentiable. For differentiability, we require that the following two limits are the same

$$\lim_{x\to 2^+} \frac{f(x) - f(2)}{x - 2} = \lim_{x\to 2^-} \frac{f(x) - f(2)}{x - 2}$$

Calculating these limits we find

$$\begin{aligned}\lim_{x\to 2^+} \frac{f(x) - f(2)}{x - 2} &= \lim_{x\to 2^+} \frac{x^2 - 2x + 1 - 1}{x - 2} \\ &= \lim_{x\to 2^+} \frac{x(x-2)}{x - 2} \\ &= \lim_{x\to 2^+} x \\ &= 2\end{aligned}$$

and using $b-1=-2a$ from above we find that the left-handed limit is

$$\begin{aligned}\lim_{x\to 2^-}\frac{f(x)-f(2)}{x-2} &= \lim_{x\to 2^-}\frac{ax+b-1}{x-2}\\ &= \lim_{x\to 2^-}\frac{ax+(-2a)}{x-2}\\ &= a\lim_{x\to 2^-}\frac{x-2}{x-2}\\ &= a\lim_{x\to 2^-}1\\ &= a\end{aligned}$$

For the previous two limits to be equal we require that $a=2$. Substituting back into $2a+b=1$ shows us that $b=-3$.

*Note.* We could also have taken a bit of a short cut and seen that this function is differentiable if the derivatives of the two halves are equal at 2, that is $\frac{d}{dx}(x^2-2x+1)=2x-2$ at $x=2$ has to equal $\frac{d}{dx}(ax+b)=a$.

## Question Section 001 10 (a)

**Easiness: 24/100**

SOLUTION. We have 4 slots to fill and 7 people to fill them. So that means, we have 7 choices for the first slot, 6 for the second, 5 for the third and 4 for the fourth. Since the number of choices for each slot does not dependent on the choice of the previous spots we need to multiply these numbers. So the total number of ways to fill the slots is $7\cdot 6\cdot 5\cdot 4=\frac{7!}{3!}=840$. There are 840 ways to fill the slots.

## Question Section 001 10 (b)

SOLUTION. When selecting courses, the order does not matter. So I am really interested in a combination here. I have want to select 5 of the 9 available courses, so I am looking at $\binom{9}{5}$.

$$\begin{aligned}\binom{9}{5} &= \frac{9!}{5!(9-5)!}\\ &= \frac{9!}{5!}\cdot\frac{1}{4!}\\ &= \frac{9\cdot 8\cdot 7\cdot 6}{4\cdot 3\cdot 2\cdot 1}\\ &= 9\cdot 2\cdot 7\\ &= 126\end{aligned}$$

There are 126 ways to fill the 5 timeslots.

# Good Luck for your exams!