Final answers script in beta

Final Answers
MATH221 April 2009

April 4, 2015

How to use this resource

- When you feel reasonably confident, simulate a full exam and grade your solutions. For your grading you can get the full solutions here.
- If you're not quite ready to simulate a full exam, we suggest you thoroughly and slowly work through each problem. Use this document with the final answers only to check if your answer is correct, without spoiling the full solution.
- Should you need more help, check out the hints and video lecture on the Math Education Resources.

Tips for Using Previous Exams to Study: Work through problems

Resist the temptation to read any of the final answers below before completing each question by yourself first! We recommend you follow the guide below.

1. **How to use the final answer:** *The final answer is not a substitution for the full solution!* The final answer alone will not give you full marks. The final answer is provided so that you can check the correctness of your work without spoiling the full solution.
   - To answer each question, only use what you could also use in the exam. Download the raw exam here.
   - If you found an answer, how could you verify that it is correct from your work only? E.g. check if the units make sense, etc. Only then compare with our result.
   - If your answer is correct: good job! Move on to the next question.
   - Otherwise, go back to your work and check it for improvements. Is there another approach you could try? If you still can't get to the right answer, you can check the full solution on the Math Education Resources.
2. **Reflect on your work:** Generally, reflect on how you solved the problem. Don't just focus on the final answer, but whether your mental process was correct. If you were stuck at any point, what helped you to go forward? What made you confident that your answer was correct? What can you take away from this so that, next time, you can complete a similar question without any help?
3. **Plan further studying:** Once you feel confident enough with a particular topic, move on to topics that need more work. Focus on questions that you find challenging, not on those that are easy for you. Once you are ready to tackle a full exam, follow the advice for the full exam (click here).

Please note that all final answers were extracted automatically from the full solution. It is possible that the final answer shown here is not complete, or it may be missing entirely. In such a case, please notify `mer-wiki@math.ubc.ca`. Your feedback helps us improve.

This pdf was created for your convenience when you study Math and prepare for your final exams. All the content here, and much more, is freely available on the Math Education Resources.

This is a free resource put together by the Math Education Resources, a group of volunteers that turn their desire to improve education into practice. You may use this material under the Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International licence.

Question 2

FINAL ANSWER. If we expand this out we get $(1-x)^3(x+3) = 3 - 8x + 6x^2 - x^4$ but this simplification is an optional step.

Question 4

FINAL ANSWER. and indeed we have it projects onto itself! The same thing would happen if we projected w_2.

Question 12 (j)

FINAL ANSWER. Hence v is also an eigenvector of $2A$ (corresponding to the eigenvalue 2λ).

Question 1

FINAL ANSWER. $\begin{bmatrix} x1 \\ x2 \\ x3 \\ x4 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -4 \\ 3 \\ 1 \\ 0 \end{bmatrix} * x_3$

Question 3

FINAL ANSWER. Note that this is a reasonable answer since the population at $t = 6$ was 9.

Question 5

FINAL ANSWER. $\left\{ \begin{pmatrix} -0.4 \\ 0.2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0.2 \\ -0.6 \\ 0 \\ 1 \end{pmatrix} \right\}$

Question 6

FINAL ANSWER. and thus our steady state solution is $\begin{pmatrix} x_\infty \\ y_\infty \end{pmatrix} = (1) \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and our limiting values would be $= \binom{2}{1}$

Question 7 (a)

FINAL ANSWER. Hence, we conclude $\left\{ \begin{pmatrix} 3 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \end{pmatrix} \right\}$ is a basis for $\mathbb{R}^2$ consisting of eigenvectors of T.

Question 7 (b)

FINAL ANSWER. $A \begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ and $A \begin{pmatrix} 1 \\ 3 \end{pmatrix} = - \begin{pmatrix} 1 \\ 3 \end{pmatrix}$.

Question 8

FINAL ANSWER. Hence, $A^k = P * D^k * P^{-1} = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 5^k & 0 \\ 0 & (-2)^k \end{bmatrix} \begin{bmatrix} 1/7 & 2/7 \\ 3/7 & -1/7 \end{bmatrix}$.

Question 9 (a)

FINAL ANSWER. thus the basis for $Nul(A) = \begin{pmatrix} 0 \\ -1 \\ -1 \\ 1 \end{pmatrix}$

Question 9 (b)

FINAL ANSWER. We first look back at our rref(A), $V = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ from this we can see that the pivot columns are the 2nd and 3rd columns of the matrix, corresponding to the columns in the original matrix A, we get $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ and these are the basis for $Col(A)$

Question 9 (c)

FINAL ANSWER. Thus, the coordinate vecctor of $\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ relative to basis of $Col(A)$ is $\binom{-1/3}{2/3}$

Question 9 (d)

FINAL ANSWER. the dimension of $Null(A^T)$ is therefore 1.

Question 10

FINAL ANSWER. Thus the inverse of AA^T is $\begin{bmatrix} 1 & -3 & 7 \\ -3 & 10 & -23 \\ 7 & -23 & 54 \end{bmatrix}$

Question 11 (a)

FINAL ANSWER. Thus, the nonzero vector v = $\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$

Question 11 (b)

FINAL ANSWER.

$$\begin{gathered} (1-\lambda)((-\lambda)(-1-\lambda)) + 2(0-(-1-\lambda)) = 0 \\ (1-\lambda)(\lambda+\lambda^2) + 2 + 2\lambda = 0 \\ \lambda^3 - 3\lambda - 2 = 0 \\ (\lambda+1)^2(\lambda-2) \\ \text{Hence,} \lambda_1 = \lambda_2 = -1, \lambda_3 = 2 \end{gathered} = \begin{gathered} (1-\lambda)((-\lambda)(-1-\lambda)) + 2(0-(-1-\lambda)) = 0 \\ (1-\lambda)(\lambda+\lambda^2) + 2 + 2\lambda = 0 \\ \lambda^3 - 3\lambda - 2 = 0 \\ (\lambda+1)^2(\lambda-2) \\ \text{Hence,} \lambda_1 = \lambda_2 = -1, \lambda_3 = 2 \end{gathered}$$

Question 11 (c)

FINAL ANSWER. Since V is formed by the eigenvectors of A, the matrix P = V.

Question 12 (a)

FINAL ANSWER. This means the constant term of $p(\lambda)$ (which is the product of the roots) must be strictly non-real, a contradiction since the matrix was assumed to be real.

Question 12 (b)

FINAL ANSWER. Therefore, we conclude $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ is a counterexample to the statement.

Question 12 (c)

FINAL ANSWER. We have $A = -A^T$ and thus, since determinants are transpose invariant, we have $\det(A) = -\det(A)$ or equivalently that $2\det(A) = 0$ and hence that $\det(A) = 0$.

Question 12 (d)

FINAL ANSWER. Then we can write $\mathbf{0} = 0 \cdot v + 0 \cdot w$; that is, $\mathbf{0}$ is a linear combination of v and w.

Question 12 (e)

FINAL ANSWER. So $v_1 + v_2$ is not an eigenvector of A.

Question 12 (f)

FINAL ANSWER. Therefore, v_1, v_2, v_3 are linearly independent.

Question 12 (g)

FINAL ANSWER. This precisely means that the columns of A are linearly dependent.

Question 12 (h)

FINAL ANSWER. However, clearly A has only one linearly independent column, so $\text{Rank}(A) = \#$ of distinct eigenvalues.

Question 12 (i)

FINAL ANSWER. $A = \begin{pmatrix} -9 & 2 & 8 \\ -3 & 1 & 3 \\ -13 & 2 & 12 \end{pmatrix}$.