Final answers script in beta

# Final Answers
# MATH152 April 2012

April 4, 2015

## How to use this resource

- When you feel reasonably confident, simulate a full exam and grade your solutions. For your grading you can get the full solutions here.
- If you're not quite ready to simulate a full exam, we suggest you thoroughly and slowly work through each problem. Use this document with the final answers only to check if your answer is correct, without spoiling the full solution.
- Should you need more help, check out the hints and video lecture on the Math Education Resources.

## Tips for Using Previous Exams to Study: Work through problems

*Resist the temptation to read any of the final answers below before completing each question by yourself first! We recommend you follow the guide below.*

1. **How to use the final answer:** *The final answer is not a substitution for the full solution!* The final answer alone will not give you full marks. The final answer is provided so that you can check the correctness of your work without spoiling the full solution.
   - To answer each question, only use what you could also use in the exam. Download the raw exam here.
   - If you found an answer, how could you verify that it is correct from your work only? E.g. check if the units make sense, etc. Only then compare with our result.
   - If your answer is correct: good job! Move on to the next question.
   - Otherwise, go back to your work and check it for improvements. Is there another approach you could try? If you still can't get to the right answer, you can check the full solution on the Math Education Resources.
2. **Reflect on your work:** Generally, reflect on how you solved the problem. Don't just focus on the final answer, but whether your mental process was correct. If you were stuck at any point, what helped you to go forward? What made you confident that your answer was correct? What can you take away from this so that, next time, you can complete a similar question without any help?
3. **Plan further studying:** Once you feel confident enough with a particular topic, move on to topics that need more work. Focus on questions that you find challenging, not on those that are easy for you. Once you are ready to tackle a full exam, follow the advice for the full exam (click here).

**Please note that all final answers were extracted automatically from the full solution. It is possible that the final answer shown here is not complete, or it may be missing entirely. In such a case, please notify `mer-wiki@math.ubc.ca`. Your feedback helps us improve.**

This pdf was created for your convenience when you study Math and prepare for your final exams. All the content here, and much more, is freely available on the Math Education Resources.



## Question 1 (a)

FINAL ANSWER. Therefore, we get that the equation for $S_1$ is $2x_1 + x_2 - x_3 - 1 = 0$ while the equation for $S_2$ is $x_1 + x_2 - 1 = 0$.

## Question 1 (b)

FINAL ANSWER. This precisely the equation form of the line since any $x_1, x_2$ and $x_3$ that satisfy this will be on both planes and therefore be on the line of intersection.

## Question 1 (c)

FINAL ANSWER. $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$. This parametrically describes the line.

## Question 1 (d)

FINAL ANSWER. area $= \frac{1}{2}\sqrt{2}$.

## Question 1 (e)

FINAL ANSWER. volume $= 1$.

## Question 2 (a)

FINAL ANSWER. $\mathbf{x} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ As a sanity check, perform the matrix multiplication so that you can verify that this works.

## Question 2 (b)

FINAL ANSWER. for any number $t$.

## Question 2 (c)

FINAL ANSWER. $\mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$.

## Question 3 (a)

FINAL ANSWER. $\mathbf{e}_3 = -\mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3$.

## Question 3 (b)

FINAL ANSWER. $T(\mathbf{e}_1) = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$.

## Question 3 (c)

FINAL ANSWER. $A = \begin{bmatrix} 1-22 \\ -111 \\ -1-24 \end{bmatrix}$.

## Question 4 (a)

FINAL ANSWER. $\lambda = 1$ or 5 Thus, the eigenvalues of $A$are $\lambda_1 = 1,\ \lambda_2 = 5$.

## Question 4 (b)

FINAL ANSWER. and from this we see that the vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is an eigenvector.

## Question 4 (c)

FINAL ANSWER. $MDM^{-1} = \begin{bmatrix} 32 \\ 23 \end{bmatrix}$ as we were expecting.

## Question 4 (d)

FINAL ANSWER. $A^k = \left(\frac{1}{2}\right) \begin{bmatrix} 1+5^k-1+5^k \\ -1+5^k 1+5^k \end{bmatrix}$

## Question 5 (a)

FINAL ANSWER. $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$

## Question 5 (b)

FINAL ANSWER. This is zero when $p = 0$ or when $p = 2$.

## Question 6 (a)

FINAL ANSWER. $M = \begin{bmatrix} 10a \\ 214 \\ -135 \end{bmatrix} \rightarrow \begin{bmatrix} 10a \\ 014-2a \\ 035+a \end{bmatrix} \rightarrow \begin{bmatrix} 10a \\ 014-2a \\ 00-7+7a \end{bmatrix}$, we can see that the last row will be a row of zeros if $a = 1$. Therefore, the vectors $\mathbf{v}_1,\ \mathbf{v}_2,\ \mathbf{v}_3$ arelinearly dependent if $a = 1$.

## Question 6 (b)

FINAL ANSWER. $\mathrm{Det}(A) = \mathrm{Det}(B) = 12$

## Question 6 (c)

FINAL ANSWER. (6): Since $C$ is a projection, it satisfies (6) by definition and so (6) is true.

## Question 6 (d)

FINAL ANSWER. $x + y + z + u + v = 0$ In this example, there is a four-parameter family of solutions since the associated coefficient matrix has rank 1.(4) is FALSE.See (1) and (3) for counterexamples.

## Question 6 (e)

FINAL ANSWER. ii) $S(c\mathbf{u}) = \begin{bmatrix} -cu_1 \\ c^2 u_2 u_3 \\ c(u_3 + u_1) \end{bmatrix} \neq cS(\mathbf{u})$ Therefore, S is NOT linear.

## Question 7 (a)

FINAL ANSWER. $\mathbf{p}^{(2)} = \begin{bmatrix} 1/4 \\ 1/2 \\ 1/4 \end{bmatrix}$ Therefore, the probability of being in the third state at the second step is 1/4.

## Question 7 (b)

FINAL ANSWER. $\lambda = 0, 1/2, 1$.

## Question 7 (c)

FINAL ANSWER. $\begin{bmatrix} -3/41/41/4 \\ 3/4 - 1/41/4 \\ 00 - 1/2 \end{bmatrix} \bar{\mathbf{p}} = \begin{bmatrix} 1/4 \\ 3/4 \\ 0 \end{bmatrix}$. Therefore, the stationary probabilities are probability 1/4 of being in state 1, probability 3/4 of being in state 2, and zero probability of being in state 3.

## Question 7 (d)

FINAL ANSWER. This means that only the component of the initial distribution that is parallel to the stationary distribution will maintain its length over repeated applications of P. Since the 3x3 matrix $P$ has three distinct eigenvalues, every vector of initial conditions can be composed to a sum of eigenvectors. Hence, all components of the initial distribution that are not parallel to the stationary distribution will decay with repeated applications of P.Hence, all initial distributions eventually converge to the stationary distribution computed in part (c).

## Question 8 (a)

FINAL ANSWER. Hence, the eigenvalues $\lambda_{1,2}$ are $-1 + 2i$ and $-1 - 2i$.

## Question 8 (b)

FINAL ANSWER. There is a free choice like before and we choose $y_1 = 1$, then $y_2 = -i$ and have the eigenvector $y = \begin{pmatrix} 1 \\ -i \end{pmatrix}$.

## Question 8 (c)

FINAL ANSWER. $\vec{\mathbf{x}}(t) = C_1 e^{-t}\begin{pmatrix}\cos 2t\\ \sin 2t\end{pmatrix} + C_2 e^{-t}\begin{pmatrix}\sin 2t\\ -\cos 2t\end{pmatrix}$

## Question 8 (d)

FINAL ANSWER. Since $e^{-t}$ goes to 0 as $t \to \infty$, the solution goes to 0 as well.