Final answers script in beta

Final Answers
MATH257 December 2011

April 16, 2015

How to use this resource

- When you feel reasonably confident, simulate a full exam and grade your solutions. For your grading you can get the full solutions here.
- If you're not quite ready to simulate a full exam, we suggest you thoroughly and slowly work through each problem. Use this document with the final answers only to check if your answer is correct, without spoiling the full solution.
- Should you need more help, check out the hints and video lecture on the Math Education Resources.

Tips for Using Previous Exams to Study: Work through problems

Resist the temptation to read any of the final answers below before completing each question by yourself first! We recommend you follow the guide below.

1. **How to use the final answer:** *The final answer is not a substitution for the full solution!* The final answer alone will not give you full marks. The final answer is provided so that you can check the correctness of your work without spoiling the full solution.
   - To answer each question, only use what you could also use in the exam. Download the raw exam here.
   - If you found an answer, how could you verify that it is correct from your work only? E.g. check if the units make sense, etc. Only then compare with our result.
   - If your answer is correct: good job! Move on to the next question.
   - Otherwise, go back to your work and check it for improvements. Is there another approach you could try? If you still can't get to the right answer, you can check the full solution on the Math Education Resources.
2. **Reflect on your work:** Generally, reflect on how you solved the problem. Don't just focus on the final answer, but whether your mental process was correct. If you were stuck at any point, what helped you to go forward? What made you confident that your answer was correct? What can you take away from this so that, next time, you can complete a similar question without any help?
3. **Plan further studying:** Once you feel confident enough with a particular topic, move on to topics that need more work. Focus on questions that you find challenging, not on those that are easy for you. Once you are ready to tackle a full exam, follow the advice for the full exam (click here).

Please note that all final answers were extracted automatically from the full solution. It is possible that the final answer shown here is not complete, or it may be missing entirely. In such a case, please notify `mer-wiki@math.ubc.ca`. Your feedback helps us improve.

This pdf was created for your convenience when you study Math and prepare for your final exams. All the content here, and much more, is freely available on the Math Education Resources.

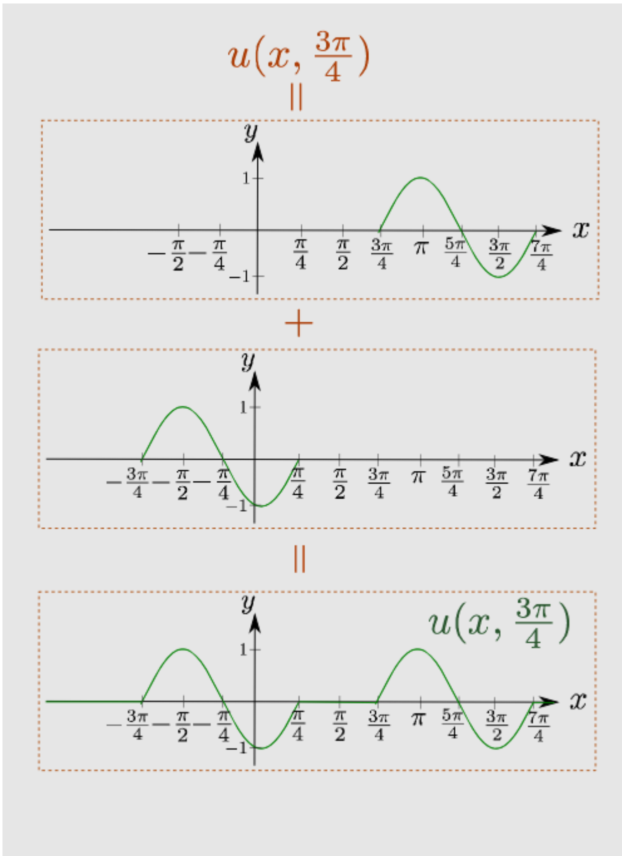
Question 1

Easiness: 5.0/5

FINAL ANSWER. $y(x) = a_0 x^{\frac{1}{2}} - \frac{1}{10} a_0 x^{\frac{3}{2}} + \frac{3}{280} a_0 x^{\frac{5}{2}} + \cdots$

Question 2 (a)

Easiness: 4.0/5



FINAL ANSWER.

Question 2 (b)

FINAL ANSWER. $\phi_{tt}(t) = 0$. We will start by solving the equation for $\psi(x)$ because it has the two boundary conditions required for a complete solution. We start by analyzing different possible values of λ.

Case 1:$\lambda < 0$.
If $\lambda < 0$ then
$\psi_{xx}(x) = -\lambda\psi(x)$
has solution $\psi(x) = a_1 \exp(\sqrt{-\lambda}x) + b_1 \exp(-\sqrt{-\lambda}x)$ which after applying the conditions $\psi(0) = \psi(\pi) = 0$ results in $a_1 = b_1 = 0$. Therefore $\lambda < 0$ is **not** permitted if we want to have non-trivial solutions.

Case 2:$\lambda = 0$
If $\lambda = 0$ then
$\psi_{xx}(x) = -\lambda\psi(x)$
has solution $\psi(x) = a_2 x + b_2$ which after applying the conditions $\psi(0) = \psi(\pi) = 0$ results in $a_2 = b_2 = 0$. Therefore $\lambda = 0$ is **not** permitted if we want to have non-trivial solutions.

Case 3:$\lambda > 0$
If $\lambda > 0$ then

$\psi_{xx}(x) = -\lambda\psi(x)$
has solution $\psi(x) = a_3\sin(\sqrt{\lambda}x) + b_3\cos(\sqrt{\lambda}x)$. Applying the condition at $x = 0$,
$\psi(0) = a_2\sin(0) + b_3\cos(0) = b_3 = 0$
and so we conclude that $b_3 = 0$. At the other endpoint, $x = \pi$,
$\psi(\pi) = a_2\sin(\sqrt{\lambda}\pi) = 0$.
In order for this condition to be satisfied we require that $\sqrt{\lambda}\pi = n\pi$ or that $\lambda = n^2$ with $n = 1, 2, 3\ldots$, an integer. We must start at $n = 1$ because we already concluded that $\lambda = 0$ is invalid. Therefore, we conclude (ignoring the constant out front) that,
$\psi(x) = \sin(nx)$.
Before, we continue, note that had I taken λ instead of $-\lambda$ for the original constant, I would have done the same analysis and recovered the same function, only with $\lambda < 0$ being the requirement. We choose the constant to be $-\lambda$ from the start so that we have positive values of λ but this is really just a matter of style.
Now for the other differential equation, knowing $\lambda = n^2$ we have
$\phi_{tt} = -n\phi(t)$
which, recalling $n > 0$, has solution, $\phi(t) = a_4\cos(nt) + b_4\sin(nt)$. Applying the condition that $\phi_t(0) = 0$ we conclude that $b_4 = 0$ so that,
$\phi(t) = \cos(nt)$
where we once again haven't included the constant out front of the solution. Finally, combining all this information we get that $u(x,t) = A_n\sin(nx)\cos(nt)$ but this must hold for all n and therefore, by the principle of superposition,
$u(x,t) = \sum_{n=1}^{\infty} A_n\sin(nx)\cos(nt)$.
Now we know that $u(x,0) = \sum_{n=1}^{\infty} A_n\sin(nx) = f(x)$ and so we could apply the usual Fourier series orthogonality to get that,
$A_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin(nx)\mathrm{d}x$
which will work just fine. However, in this case, $f(x) = 2\sin(2x)$ is special because it is one of the eigenfunctions (i.e. it is of the form, $\sin(nx)$ with $n = 2$). Therefore, $\sin(2x)$ is orthogonal to every eigenfunction of the form $\sin(nx)$ except for when $n = 2$. Therefore, only the coefficient A_2 will be non-zero and is given by,
$A_2 = \frac{2}{\pi}\int_0^{\pi} 2\sin(2x)^2\mathrm{d}x = 2$.
If you are unconvinced of how this works, try getting the other coefficients using the formula above and you will quickly see that they are zero! Therefore, we conclude that the solution $u(x,t)$ to the PDE, $u_{tt} = u_{xx}$ is
$u(x,t) = 2\sin(2x)\cos(2t)$

Question 3 (a)

Easiness: 4.0/5

FINAL ANSWER. $Cf(x) = \frac{1}{2} + \sum_{n=1}^{\infty} 2(\frac{1}{n\pi})^2[(-1)^n - 1]\cos(n\pi x)$.

Question 3 (b)

Easiness: 3.0/5

FINAL ANSWER. $Sf(1) = 0$ as expected.

Question 4 (a)

FINAL ANSWER. $u_n^1 = 1, 2, 3, \cdots, N$. Once we have u_n^1, we can iterate the process to find $u_n^2, u_n^3, \cdots$

Question 4 (b)

FINAL ANSWER. for $\lambda_n = \frac{(2n+1)\pi}{2}$.

Question 5 (a)

FINAL ANSWER. $2^{\lambda} = 0$. However, this contradicts with our original assumption that $\lambda^2 > 0$.

Question 5 (b)

FINAL ANSWER. $u(r,\theta) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{\ln(2)} \ln(r)\right) \sinh\left(\lambda_n \theta\right).$