

Final Answers

MATH307 April 2012

April 16, 2015

Final answers script in beta

How to use this resource

- When you feel reasonably confident, simulate a full exam and grade your solutions. [For your grading you can get the full solutions here.](#)
- If you're not quite ready to simulate a full exam, we suggest you thoroughly and slowly work through each problem. Use this document with the final answers only to check if your answer is correct, without spoiling the full solution.
- Should you need more help, check out the hints and video lecture on the [Math Education Resources](#).

Tips for Using Previous Exams to Study: Work through problems

Resist the temptation to read any of the final answers below before completing each question by yourself first! We recommend you follow the guide below.

1. **How to use the final answer:** *The final answer is not a substitution for the full solution!* The final answer alone will not give you full marks. The final answer is provided so that you can check the correctness of your work without spoiling the full solution.
 - To answer each question, only use what you could also use in the exam. [Download the raw exam here.](#)
 - If you found an answer, how could you verify that it is correct from your work only? E.g. check if the units make sense, etc. Only then compare with our result.
 - If your answer is correct: good job! Move on to the next question.
 - Otherwise, go back to your work and check it for improvements. Is there another approach you could try? If you still can't get to the right answer, you can check the full solution on the [Math Education Resources](#).
2. **Reflect on your work:** Generally, reflect on how you solved the problem. Don't just focus on the final answer, but whether your mental process was correct. If you were stuck at any point, what helped you to go forward? What made you confident that your answer was correct? What can you take away from this so that, next time, you can complete a similar question without any help?
3. **Plan further studying:** Once you feel confident enough with a particular topic, move on to topics that need more work. Focus on questions that you find challenging, not on those that are easy for you. Once you are ready to tackle a full exam, follow the advice for the [full exam \(click here\)](#).

Please note that all final answers were extracted automatically from the full solution. It is possible that the final answer shown here is not complete, or it may be missing entirely. In such a case, please notify mer-wiki@math.ubc.ca. Your feedback helps us improve.

This pdf was created for your convenience when you study Math and prepare for your final exams. All the content here, and much more, is freely available on the [Math Education Resources](#).

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Question 1 (a)

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

So in total, $(N-1) + (N-1) = 2N - 2$ equations.

Question 1 (b)

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

So in total, $(N - 2) + (N - 2) = 2N - 4$ equations

Question 1 (c)

FINAL ANSWER. No content found.

Question 1 (d)

FINAL ANSWER. No content found.

Question 2 (a)

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

Thus, $f''(x_i) \approx \frac{f_{i+1} - 2f_i + f_{i-1}}{\Delta x^2}$ $i = 1, 2, 3, \dots, N$

Question 2 (b)

FINAL ANSWER. No content found.

Question 2 (c)

FINAL ANSWER. No content found.

Question 3

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

Therefore a basis for $R(A^T)$ is $\begin{bmatrix} 1 \\ 2 \\ 0 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix}$.

Question 4 (a)

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

$$N(D) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Question 4 (b)

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

$$\text{FINAL ANSWER } \vec{v} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

Question 4 (c)

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

$$N(L) = \text{span}\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right)$$

Question 4 (d)

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

$$L = \begin{bmatrix} 4 & -1 & -1 & -1 & -1 & 0 \\ -1 & 4 & -1 & 0 & -1 & -1 \\ -1 & -1 & 4 & -1 & 0 & -1 \\ -1 & 0 & -1 & 4 & -1 & -1 \\ -1 & -1 & 0 & -1 & 4 & -1 \\ 0 & -1 & -1 & -1 & -1 & 4 \end{bmatrix}$$

Question 4 (e)

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

```
%nodes n and m position
n = 1;
m = 6;

%find the size of the matrix L
lenL = length(L);

%swap nodes n and m to the 1st and 2nd position in the Laplacian matrix
swap = [n, m, 1:(n-1), (n+1):(m-1), (m+1):lenL];
L = L(swap, swap);

%compute matrix S which is the voltage to current map (Schur's complement).
A = L((1:2), (1:2));
B = L((3:lenL), (1:2));
C = L((3:lenL), (3:lenL));
S = A - (B = %nodes n and m position
n = 1;
m = 6;
```

```

%find the size of the matrix L
lenL = length(L);

%swap nodes n and m to the 1st and 2nd position in the Laplacian matrix
swap = [n, m, 1:(n-1), (n+1):(m-1), (m+1):lenL];
L = L(swap, swap);

%compute matrix S which is the voltage to current map (Schur's complement).
A = L((1:2), (1:2));
B = L((3:lenL), (1:2));
C = L((3:lenL), (3:lenL));
S = A - (B' * C\(-1) * B);

%finds the effective resistance which is the first entry of S.
r = 1/S(1,1); % = * C\(-1) * B);

%finds the effective resistance which is the first entry of S.
r = 1/S(1,1); You could run this for the 2 different Laplacian matrices to find the effective resistance for
each and compare them to confirm the conjecture is correct.

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Question 5 (a)

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

$$p(x) = a_1x^{99} + a_2x^{98} + \dots + a_{99}x + a_{100}$$

Question 5 (b)

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

$$\begin{bmatrix} x_1^{99} & x_1^{98} & \dots & x_1^2 & x_1 & 1 \\ x_2^{99} & x_2^{98} & \dots & x_2^2 & x_2 & 1 \\ \vdots & \vdots & & \ddots & & \vdots \\ x_{100}^{99} & x_{100}^{98} & \dots & x_{100}^2 & x_{100} & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \dots \\ \dots \\ a_{99} \\ a_{100} \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \dots \\ \dots \\ y_{99} \\ y_{100} \end{bmatrix}$$

However, since the Vandermonde matrix not very well conditioned (because Vandermonde matrices have large condition numbers, an error in the data points can cause a relatively large error in the solution), it is unlikely that the solution is accurate

Question 5 (c)

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

$$\text{plot}(\text{XX}, a(1)*\text{XX}.^2 + a(2)*\text{XX} = C3 * \text{ones}(1, \text{length}(\text{XX})))$$

Question 6 (a)

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

$$c_n = \frac{1}{T} \int_0^T e^{-2\pi i \frac{n}{T} t} y(t) dt$$

Question 6 (b)

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

This would probably be seasonal fluctuations, so it would be c_n for about 12 months.

Question 6 (c)

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

$\text{amp} = \text{sqrt}(2 \cdot 2^{\cos(2 \cdot \pi \cdot T \cdot (1 - \omega))}) ./ (2 \cdot \pi \cdot T \cdot \text{abs}(1 - \omega)); \text{plot}(\Omega, \text{amp});$

Question 6 (d)

FINAL ANSWER. No content found.

Question 7 (a)

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

and we would need the first three values of the sequence to calculate all the following values in the sequence $(x_0, x_1, \text{ and } x_3)$.

Question 7 (b)

FINAL ANSWER. No content found.

Question 7 (c)

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

$$\begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix}, \begin{bmatrix} -\pi \\ \pi \\ -\pi \end{bmatrix}, \text{ etc.}$$

Question 8 (a)

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

Therefore the eigenvalue is $\lambda = 1$.

Question 8 (b)

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

$\lambda_i \in (-1, 1)$.

Question 8 (c)

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

Lastly, we know that any constant multiple of the eigenvector is still the same eigenvector, therefore we can scale $[3, 8, 3]^T$ to what the answer that we found above.