Final answers script in beta

Final Answers
MATH220 April 2011

December 4, 2014

How to use this resource

- When you feel reasonably confident, simulate a full exam and grade your solutions. For your grading you can get the full solutions here.
- If you're not quite ready to simulate a full exam, we suggest you thoroughly and slowly work through each problem. Use this document with the final answers only to check if your answer is correct, without spoiling the full solution.
- Should you need more help, check out the hints and video lecture on the Math Educational Resources.

Tips for Using Previous Exams to Study: Work through problems

Resist the temptation to read any of the final answers below before completing each question by yourself first! We recommend you follow the guide below.

1. **How to use the final answer:** *The final answer is not a substitution for the full solution!* The final answer alone will not give you full marks. The final answer is provided so that you can check the correctness of your work without spoiling the full solution.
   - To answer each question, only use what you could also use in the exam. Download the raw exam here.
   - If you found an answer, how could you verify that it is correct from your work only? E.g. check if the units make sense, etc. Only then compare with our result.
   - If your answer is correct: good job! Move on to the next question.
   - Otherwise, go back to your work and check it for improvements. Is there another approach you could try? If you still can't get to the right answer, you can check the full solution on the Math Educational Resources.
2. **Reflect on your work:** Generally, reflect on how you solved the problem. Don't just focus on the final answer, but whether your mental process was correct. If you were stuck at any point, what helped you to go forward? What made you confident that your answer was correct? What can you take away from this so that, next time, you can complete a similar question without any help?
3. **Plan further studying:** Once you feel confident enough with a particular topic, move on to topics that need more work. Focus on questions that you find challenging, not on those that are easy for you. Once you are ready to tackle a full exam, follow the advice for the full exam (click here).

Please note that all final answers were extracted automatically from the full solution. It is possible that the final answer shown here is not complete, or it may be missing entirely. In such a case, please notify `mer-wiki@math.ubc.ca`. Your feedback helps us improve.

This pdf was created for your convenience when you study Math and prepare for your final exams. All the content here, and much more, is freely available on the Math Educational Resources.

Question 1 (a)

Easiness: 90/100

FINAL ANSWER. $f(C) = \{f(c) \mid c \in C\}$.

Question 1 (b)

Easiness: 100/100

FINAL ANSWER. The supremum of a set S of real numbers is defined to be the smallest real number M such that every number in S is less than or equal to M.

Question 1 (c)

Easiness: 100/100

FINAL ANSWER. The converse is:If I buy a new car, then I will win the lottery.

Question 1 (d)

FINAL ANSWER. $\exists x \in \mathbb{R}$ s.t. $\forall y \in \mathbb{N}\ x^2 + y \geq x + y^2$.

Question 1 (e)

Easiness: 100/100

FINAL ANSWER. We say that f is injective if whenever we have $f(x) = f(y)$ for $x, y \in A$ then $x = y$.

Question 1 (f)

Easiness: 100/100

FINAL ANSWER. We say that a sequence $\{a_n\}$ converges to a number L if for all $\epsilon > 0$ there exists an $n \in \mathbb{N}$ such that for all $n \geq N$ we have $|a_n - L| < \epsilon$

Question 1 (g)

Easiness: 80/100

FINAL ANSWER. We say that the series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} a_n$ converges provided the sequence $\{S_N\}$ converges to a number L.

Question 1 (h)

Easiness: 100/100

FINAL ANSWER. A set S is countable if there exists a bijection (or a one to one correspondence) between S and $\mathbb{N}$.

Question 1 (i)

FINAL ANSWER. Upon doing this, mathematical induction states that the statement $P(n)$ is true for all natural numbers n.

Question 1 (j)

Easiness: 100/100

FINAL ANSWER. $\mathcal{P}(A) = \{\emptyset, \{1\}, \{x\}, \{y\}, \{1, x\}, \{1, y\}, \{x, y\}, \{1, x, y\}\}$

Question 2

Easiness: 95/100

Final answer. $\bigcup_{n=1}^{\infty} [\frac{1}{n}, n] = (0, \infty)$ And so the infimum of this set is 0, while there isn't a supremum since the set does not have an upper bound.

Question 3 (a)

Easiness: 60/100

Final answer. To see that, check out ***Solution 2***.

Question 3 (b)

Easiness: 10/100

Final answer. for any value of n that is larger than $N_\epsilon = 3/\epsilon$; this concludes our proof.

Question 4

Easiness: 70/100

Final answer. Since the last column is the same for both tables (and since we order the truth of P, Q and R in the same order) we can conclude that the two statements are logically equivalent.

Question 5

Easiness: 100/100

Final answer. $f(x_r) = r$ as desired.(Note that this final justification is included only as a sanity check and is not needed in a solution to obtain full marks).

Question 6

Easiness: 100/100

Final answer. Since $2\ell - n - 1$ is an integer, the result is proven.

Question 7 (a)

Easiness: 80/100

Final answer. $a_{m+1} = 2^{m+1} - 1$ which is the claimed statement to be proved and so concludes our proof.

Question 7 (b)

Final answer. $1 \leq b_{m+1} \leq 2$ and hence conclude our proof.

Question 7 (c)

Final answer. $\leq \frac{b_m + b_m}{2} = b_m$ which concludes our proof.

Question 8

Final answer. **Note:** actually in *(d)*, you should easily convince yourself that one of the prime has to be 2 and so the other has to be what is called a twin prime, that is, a prime with the property that if you add 2 to it it is prime as well (those are, except for 2 and 3, the closest primes you might get and that's why we call them twin primes).For example, 5 and 7 are twin primes and so are 11 and 13; 17 and 19; 29 and 31.We do not know if there are infinitely many twin primes but we suppose it is the case (this is what we call the The twin prime conjecture).

Question 9 (a)

Easiness: 40/100

FINAL ANSWER. $(g \circ f)(a') = g(f(a'))$ we can say that the element *f(a')*, which is an element of the set B is the element b that we are looking for since it is mapped by g to the element a as requested. This explains why g is surjective.

Question 9 (b)

FINAL ANSWER. Notice that as shown in part (a) the function g is, and has to be, surjective.

Question 9 (c)

FINAL ANSWER. For example, in that case, no negative number is in the image of the function f and so no negative number can be in the image of $f \circ g$ either.

Question 10 (a)

FINAL ANSWER. $\inf(A) \leq \inf(B) \leq \sup(B) \leq \sup(A)$.

Question 10 (b)

FINAL ANSWER. ***Advanced note:*** *We call the limit of this convergent sequence the limit superior of the original sequence* $\{a_n\}$.