

Final Answers

MATH221 December 2007

April 16, 2015

How to use this resource

- When you feel reasonably confident, simulate a full exam and grade your solutions. [For your grading you can get the full solutions here.](#)
- If you're not quite ready to simulate a full exam, we suggest you thoroughly and slowly work through each problem. Use this document with the final answers only to check if your answer is correct, without spoiling the full solution.
- Should you need more help, check out the hints and video lecture on the [Math Education Resources](#).

Tips for Using Previous Exams to Study: Work through problems

Resist the temptation to read any of the final answers below before completing each question by yourself first! We recommend you follow the guide below.

1. **How to use the final answer:** *The final answer is not a substitution for the full solution!* The final answer alone will not give you full marks. The final answer is provided so that you can check the correctness of your work without spoiling the full solution.
 - To answer each question, only use what you could also use in the exam. [Download the raw exam here.](#)
 - If you found an answer, how could you verify that it is correct from your work only? E.g. check if the units make sense, etc. Only then compare with our result.
 - If your answer is correct: good job! Move on to the next question.
 - Otherwise, go back to your work and check it for improvements. Is there another approach you could try? If you still can't get to the right answer, you can check the full solution on the [Math Education Resources](#).
2. **Reflect on your work:** Generally, reflect on how you solved the problem. Don't just focus on the final answer, but whether your mental process was correct. If you were stuck at any point, what helped you to go forward? What made you confident that your answer was correct? What can you take away from this so that, next time, you can complete a similar question without any help?
3. **Plan further studying:** Once you feel confident enough with a particular topic, move on to topics that need more work. Focus on questions that you find challenging, not on those that are easy for you. Once you are ready to tackle a full exam, follow the advice for the [full exam \(click here\)](#).

Please note that all final answers were extracted automatically from the full solution. It is possible that the final answer shown here is not complete, or it may be missing entirely. In such a case, please notify mer-wiki@math.ubc.ca. Your feedback helps us improve.

This pdf was created for your convenience when you study Math and prepare for your final exams. All the content here, and much more, is freely available on the [Math Education Resources](#).

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Question 1

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

In this case, we conclude that $\left\{ \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$ is a basis for the null space of A .

Question 2

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

$$B^{-1} = \begin{pmatrix} 1/2 & -3 & 1 & -5 \\ 0 & 1 & 0 & 0 \\ 0 & -3 & 1 & -5 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Question 3

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

$$A = \begin{pmatrix} -1 & 2 \\ 1 & -2 \end{pmatrix}.$$

Question 4

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

Since this is impossible, we have shown that v does not belong to W .

Question 5 (a)

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

$$\det(B) = 2t - 29.$$

Question 5 (b)

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

Therefore, B is invertible if and only if $t \neq 29/2$.

Question 6 (a)

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

Therefore, by the two bulleted points above, \mathcal{B} is a basis for \mathbb{R}^2 .

Question 6 (b)

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 17/2 \\ -3/2 \end{pmatrix}.$$

Question 6 (c)

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

$$[T]_{\overline{B}} = \begin{pmatrix} 1/2 & 3/2 \\ -3/2 & 7/2 \end{pmatrix}.$$

Question 7 (a)

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

Therefore the determinant of the above matrix is zero, and hence $\det(A - I) = 0$.

Question 7 (b)

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

Therefore, the other eigenvalue of A is 2.

Question 7 (c)

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

$\dim \text{null}(A - 2I) = 2$. In conclusion,

- For the eigenvalue 1, the dimension of its eigenspace is 1.
- For the eigenvalue 2, the dimension of its eigenspace is 2.

Question 8 (a)

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

We therefore have two (linearly independent) eigenvectors for T , $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$, which form a basis for \mathbb{R}^2 .

Question 8 (b)

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

Therefore T has no eigenvectors.

Question 8 (c)

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

In particular, the standard basis $\left\{\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right\}$ is a basis of \mathbb{R}^2 consisting of eigenvectors of T .

Question 8 (d)

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

As a basis of \mathbb{R}^2 consists of two linearly independent vectors, there exists no basis consisting of eigenvectors of T .

Question 9 (a)

FINAL ANSWER. No content found.

Question 9 (b)

FINAL ANSWER. No content found.

Question 9 (c)

FINAL ANSWER. No content found.

Question 9 (d)

FINAL ANSWER. No content found.

Question Section 101 10 (a)

FINAL ANSWER. No content found.

Question Section 101 10 (b)

FINAL ANSWER. No content found.

Question Section 102 10 (a)

FINAL ANSWER. No content found.

Question Section 102 10 (b)

FINAL ANSWER. No content found.

Question Section 102 10 (c)

FINAL ANSWER. No content found.

Question Section 103 10 (a)

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

$$y_n = 2[4^{n+1} + (-1)^n] .$$

Question Section 103 10 (b)

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

$$\lim_{n \rightarrow \infty} \frac{x_{n+1}}{x_n} = 4.$$

Question Section 103 10 (c)

FINAL ANSWER. THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!

$$\text{As } \lim_{n \rightarrow \infty} \left(\frac{-1}{4}\right)^n = 0, \text{ we conclude } \lim_{n \rightarrow \infty} \frac{x_n}{y_n} = 1.$$