Final answers script in beta

Final Answers
MATH220 April 2005

December 4, 2014

How to use this resource

- When you feel reasonably confident, simulate a full exam and grade your solutions. For your grading you can get the full solutions here.
- If you're not quite ready to simulate a full exam, we suggest you thoroughly and slowly work through each problem. Use this document with the final answers only to check if your answer is correct, without spoiling the full solution.
- Should you need more help, check out the hints and video lecture on the Math Educational Resources.

Tips for Using Previous Exams to Study: Work through problems

Resist the temptation to read any of the final answers below before completing each question by yourself first! We recommend you follow the guide below.

1. **How to use the final answer:** *The final answer is not a substitution for the full solution!* The final answer alone will not give you full marks. The final answer is provided so that you can check the correctness of your work without spoiling the full solution.
   - To answer each question, only use what you could also use in the exam. Download the raw exam here.
   - If you found an answer, how could you verify that it is correct from your work only? E.g. check if the units make sense, etc. Only then compare with our result.
   - If your answer is correct: good job! Move on to the next question.
   - Otherwise, go back to your work and check it for improvements. Is there another approach you could try? If you still can't get to the right answer, you can check the full solution on the Math Educational Resources.
2. **Reflect on your work:** Generally, reflect on how you solved the problem. Don't just focus on the final answer, but whether your mental process was correct. If you were stuck at any point, what helped you to go forward? What made you confident that your answer was correct? What can you take away from this so that, next time, you can complete a similar question without any help?
3. **Plan further studying:** Once you feel confident enough with a particular topic, move on to topics that need more work. Focus on questions that you find challenging, not on those that are easy for you. Once you are ready to tackle a full exam, follow the advice for the full exam (click here).

Please note that all final answers were extracted automatically from the full solution. It is possible that the final answer shown here is not complete, or it may be missing entirely. In such a case, please notify `mer-wiki@math.ubc.ca`. Your feedback helps us improve.

This pdf was created for your convenience when you study Math and prepare for your final exams. All the content here, and much more, is freely available on the Math Educational Resources.

Question 1

Easiness: 100/100

FINAL ANSWER.

There exists a continuous function f on the interval $[a,b\,]$, *with* $a < b$ such that there exists a real number y between f(a) and f(b) such that for all real number c in the interval [a, b] we have f(c) $\neq$ y".

Question 2

Easiness: 100/100

FINAL ANSWER.

Well, sure enough, if one always choses $x = w + 1$, then clearly it is always going to be larger than the w that was picked in the first place.

Question 3 (a)

Easiness: 100/100

FINAL ANSWER. More precisely, that means there must exist at least one function $f\colon A \to B$ such that the function is injective and surjective; that is, for each element a of the set A there exists a unique element b in the set B such that $f(a) = b$ (that condition is the injectivity) AND for each element b of the B there exists a unique element a in the set A such that $f(a) = b$ (and this is the condition for surjectivity).

Question 3 (b)

Easiness: 100/100

FINAL ANSWER. In other words, there exists a bijective function $f : C \to \mathbb{N}$. (Equivalently, there exists a bijective function $g : \mathbb{N} \to C$.)

Question 3 (c)

Easiness: 100/100

FINAL ANSWER. Cantor's Diagonalization Argument is used to prove that the set of all real numbers is uncountable.More precisely, it says that if you claim that the set of all real numbers is actually countable, that is, you have a bijection between all real number and the natural numbers, then he would be able to exhibit you a real number (using the diagonalization argument) that is **not** listed by your bijection.This means it is a contradiction to assume that such a bijection could exist and hence the real numbers are uncountable.

Question 4

Easiness: 40/100

FINAL ANSWER. $S = \left(-\frac{3}{2}, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, \frac{3}{2}\right)$

Question 5

Easiness: 90/100

FINAL ANSWER. This finishes the induction step and so the result is true for all $n \in N$ by induction.

Question 6

Easiness: 20/100

FINAL ANSWER. It is easy to see that 1,2,4,7 are not in T.

Question 7

Easiness: 90/100

FINAL ANSWER. $\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \frac{1}{2}$ Since this is a number less than one, the ratio test guarantees that the series converges absolutely.

Question 8

Easiness: 10/100

FINAL ANSWER. $|b_m - b_n| < \dfrac{1}{m+n} \leq \dfrac{1}{2n} < \epsilon$ And since Cauchy sequences of real numbers are always convergent, we know that the limit of this sequence exists.

Question 9

Easiness: 66/100

FINAL ANSWER. $\left|\left(-\frac{2}{3}\right)^{17}\right| = 0.0006766$

Question 10

Easiness: 39/100

FINAL ANSWER. $|f(n)a_n| = \epsilon \quad$ for all $n > N_\epsilon$ This shows that $\{f(n)a_n\}$ converges to zero.

Question 11

FINAL ANSWER. $-\sqrt{5} < x_1 < q < x_2 < m < \sqrt{5}$ but in particular that means that q is in the interval (-,) and since it is rational, q is an element of the set M. But by construction, $q < m$ which contradicts the assumption that m could be a lower bound of the set M. This means that such a number m cannot exist and hence the infimum of the set M has to be -.