Final answers script in beta

Final Answers
MATH152 April 2013

April 4, 2015

How to use this resource

- When you feel reasonably confident, simulate a full exam and grade your solutions. For your grading you can get the full solutions here.
- If you're not quite ready to simulate a full exam, we suggest you thoroughly and slowly work through each problem. Use this document with the final answers only to check if your answer is correct, without spoiling the full solution.
- Should you need more help, check out the hints and video lecture on the Math Education Resources.

Tips for Using Previous Exams to Study: Work through problems

Resist the temptation to read any of the final answers below before completing each question by yourself first! We recommend you follow the guide below.

1. **How to use the final answer:** *The final answer is not a substitution for the full solution!* The final answer alone will not give you full marks. The final answer is provided so that you can check the correctness of your work without spoiling the full solution.
   - To answer each question, only use what you could also use in the exam. Download the raw exam here.
   - If you found an answer, how could you verify that it is correct from your work only? E.g. check if the units make sense, etc. Only then compare with our result.
   - If your answer is correct: good job! Move on to the next question.
   - Otherwise, go back to your work and check it for improvements. Is there another approach you could try? If you still can't get to the right answer, you can check the full solution on the Math Education Resources.
2. **Reflect on your work:** Generally, reflect on how you solved the problem. Don't just focus on the final answer, but whether your mental process was correct. If you were stuck at any point, what helped you to go forward? What made you confident that your answer was correct? What can you take away from this so that, next time, you can complete a similar question without any help?
3. **Plan further studying:** Once you feel confident enough with a particular topic, move on to topics that need more work. Focus on questions that you find challenging, not on those that are easy for you. Once you are ready to tackle a full exam, follow the advice for the full exam (click here).

Please note that all final answers were extracted automatically from the full solution. It is possible that the final answer shown here is not complete, or it may be missing entirely. In such a case, please notify `mer-wiki@math.ubc.ca`. Your feedback helps us improve.

This pdf was created for your convenience when you study Math and prepare for your final exams. All the content here, and much more, is freely available on the Math Education Resources.

Question A 01

FINAL ANSWER. $a \times b = \begin{bmatrix} 15 \\ 0 \\ 15 \end{bmatrix}$

Question A 02

FINAL ANSWER. $\text{Proj}_c a = \frac{3}{13} \begin{bmatrix} 3 \\ -4 \\ 12 \end{bmatrix}$.

Question A 03

FINAL ANSWER. Therefore, $k = -2$ is the value for which a and d are orthogonal.

Question A 04

FINAL ANSWER. In other words, M has rank 2 exactly when *p=0* or *q=1*.

Question A 05

FINAL ANSWER. $b = \begin{pmatrix} -5 \\ -5 \\ 5 \end{pmatrix}$. Note that there are several choices that could be made and this method is just a quick and effective way for finding a suitable vector.

Question A 06

FINAL ANSWER. The answer then is that 3 vectors are always linearly dependent in $\mathbb{R}^2$

Question A 07

FINAL ANSWER. $\begin{bmatrix} n_1 n_2 n_3 \\ m_1 m_2 m_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$. Here we have two equations with three unknowns and this tells us that there is at least one free variable.Since the normal vectors are linearly independent, then there is only one free variable. Geometrically, one free variable corresponds to a line in space and so there is only one line which could contain the points of intersection of the two planes.

Question A 08

FINAL ANSWER. $x = 0$.

Question A 09

FINAL ANSWER. $A^{-1} = \begin{bmatrix} -8 & 3 \\ 3 & -1 \end{bmatrix}$

Question A 10

FINAL ANSWER. $AB = \begin{bmatrix} -83 - 1 \\ -218 - 3 \end{bmatrix}$

Question A 11

FINAL ANSWER. $C = \begin{bmatrix} 1 & 0 \\ 3 & 1 \\ 0 & 0 \end{bmatrix}$

Question A 12

FINAL ANSWER. and this is inconsistent since the last two imply that both a and b are zero and this contradicts the first line.Thus no such matrix D exists.

Question A 13

FINAL ANSWER. $M = \frac{1}{-1} \begin{bmatrix} 1 - 2 \\ -11 \end{bmatrix} = \begin{bmatrix} -12 \\ 1 - 1 \end{bmatrix}$.

Question A 14

FINAL ANSWER. $z^{2010} = i$ Since $\cos(\pi/2) = 0$ and $\sin(\pi/2) = 1$.

Question A 15

FINAL ANSWER. Then if $Av = 0$, we have $Bv = 0$, so 0 is an eigenvalue for B as well (in this case, v is also an eigenvector for this special eigenvalue).

Question A 16

FINAL ANSWER. Thus, the claim is false for all $n \geq 2$ but is true when $n = 1$.

Question A 17

FINAL ANSWER. $\neq T(x + y)$ and once again, we see that the map is not linear.The moral of this question is that the one in the middle row is causing the map to not be linear.

Question A 18

FINAL ANSWER. $S = \begin{bmatrix} 0 - 11 \\ 111 \\ -1 - 20 \end{bmatrix}$

Question A 19

FINAL ANSWER. Then $Ax = b$ corresponds to the given problem.

Question A 20

FINAL ANSWER. $(A(BC)^{-1}D)^T = D^T(B^{-1})^T(C^{-1})^T A^T$

Question A 21

Final answer. Thus, $\det B = -6$.

Question A 22

Final answer. The probability of moving from location 3 to location 1 in one time step is $P_{1,3} = 0.1626$.

Question A 23

Final answer. We know $(P^{10})_{2,1} = 0.3843$, $(P^{10})_{2,2} = 0.3753$, and $(P^{10})_{2,3} = 0.3786$, so to maximize the probability of getting to location 2, you should start from location 1.

Question A 24

Final answer. The probability of moving from location 3 to location 1 in 10 time steps is $(P^{10})_{1,3} = 0.2875$

Question A 25

Final answer. and hence every **nonzero** vector v is an eigenvector.

Question A 26

Final answer. and so the eigenvalues are given by $\lambda = \pm 1$

Question A 27

Final answer. The identity matrix has every non-zero vector as an eigenvector.So does the zero matrix.

Question A 28

Final answer. The answer is 2.

Question A 29

Final answer. Keep in mind that the code b=3:4:11; gives us the second row that begins at 3, increments entries by 4 and then goes up to 11.

Question A 30

Final answer.

```
C=zeros(20);
for j=1:20;
C(j,j)=j;
end;
```

Question B 01 (a)

FINAL ANSWER. $\hat{a} = \begin{bmatrix} 4/5 \\ 3/5 \end{bmatrix}$

Question B 01 (b)

FINAL ANSWER. $\mathbf{A} = \begin{bmatrix} \frac{7}{25} & \frac{24}{25} \\ \frac{24}{25} & \frac{-7}{25} \end{bmatrix}$

Question B 01 (c)

FINAL ANSWER. $\text{Ref}_L \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 7/25 \\ 24/25 \end{bmatrix}$

Question B 01 (d)

FINAL ANSWER. $(A + I)\mathbf{v} = \mathbf{0} \quad \rightarrow \quad \mathbf{v} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$. Any non-zero constant multiple of $\mathbf{v}$ is also an acceptable answer.

Question B 02 (a)

FINAL ANSWER. Area $= \frac{\sqrt{6}}{2}$

Question B 02 (b)

FINAL ANSWER. Thus, vectors $\vec{BA}$ and $\vec{CB}$ are orthogonal and so the triangle is right angled.

Question B 02 (c)

FINAL ANSWER. $4x + 2y - 2z = -2.$)

Question B 02 (d)

FINAL ANSWER. $D = (0, 2, 3)$

Question B 03 (a)

FINAL ANSWER. Hence the general solution to is $\begin{bmatrix} \mathbf{x_1} \\ \mathbf{x_2} \\ \mathbf{x_3} \end{bmatrix} = a \begin{bmatrix} 7 \\ -4 \\ 1 \end{bmatrix}$, where a is any real number.

Question B 03 (b)

FINAL ANSWER. As we see, regardless of a and b we always find $\mathbf{x}_1 \neq \mathbf{x}_2$.Thus using the given information, we cannot tell which solution set is correct.

Question B 03 (c)

FINAL ANSWER. $\mathbf{x} = a \begin{bmatrix} 7 \\ -4 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$

Question B 04 (a)

FINAL ANSWER. with $a \neq 0$ is an eigenvector of A with eigenvalue 3.

Question B 04 (b)

FINAL ANSWER. Solving for the roots gives the eigenvalues $\lambda = -6, -5, 3$

Question B 04 (c)

FINAL ANSWER. is an eigenvector associated to $\lambda = -6$ (to get all eigenvectors, multiply this vector by any non-zero number).

Question B 05 (a)

FINAL ANSWER. $P^2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$ which tells us that the probability of being in state 1 is 1/2.

Question B 05 (b)

FINAL ANSWER. $P^{20} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{-1/(2^{19})+1}{3} \\ \frac{1/(2^{19})+2}{3} \end{bmatrix}$ and this completes the problem.

Question B 05 (c)

FINAL ANSWER. $\lim_{n\to\infty} P^n \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \end{bmatrix}$ and this completes the problem.

Question B 06 (a)

FINAL ANSWER. This equation is equivalent to $\mathbf{y}'(\mathbf{t}) = \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix} \mathbf{y}(\mathbf{t})$

Question B 06 (b)

FINAL ANSWER. But for the real solution, it is enough to only consider $v_a e^{\lambda_a t}$."

Question B 06 (c)

FINAL ANSWER. $y(t) = e^t \begin{bmatrix} -2\sin(2t) - \cos(2t) \\ -3\sin(2t) + \cos(2t) \end{bmatrix}$