

# Final Answers

## MATH312 December 2012

December 4, 2014

### How to use this resource

- When you feel reasonably confident, simulate a full exam and grade your solutions. [For your grading you can get the full solutions here.](#)
- If you're not quite ready to simulate a full exam, we suggest you thoroughly and slowly work through each problem. Use this document with the final answers only to check if your answer is correct, without spoiling the full solution.
- Should you need more help, check out the hints and video lecture on the [Math Educational Resources](#).

### Tips for Using Previous Exams to Study: Work through problems

*Resist the temptation to read any of the final answers below before completing each question by yourself first! We recommend you follow the guide below.*

1. **How to use the final answer:** *The final answer is not a substitution for the full solution!* The final answer alone will not give you full marks. The final answer is provided so that you can check the correctness of your work without spoiling the full solution.
  - To answer each question, only use what you could also use in the exam. [Download the raw exam here.](#)
  - If you found an answer, how could you verify that it is correct from your work only? E.g. check if the units make sense, etc. Only then compare with our result.
  - If your answer is correct: good job! Move on to the next question.
  - Otherwise, go back to your work and check it for improvements. Is there another approach you could try? If you still can't get to the right answer, you can check the full solution on the [Math Educational Resources](#).
2. **Reflect on your work:** Generally, reflect on how you solved the problem. Don't just focus on the final answer, but whether your mental process was correct. If you were stuck at any point, what helped you to go forward? What made you confident that your answer was correct? What can you take away from this so that, next time, you can complete a similar question without any help?
3. **Plan further studying:** Once you feel confident enough with a particular topic, move on to topics that need more work. Focus on questions that you find challenging, not on those that are easy for you. Once you are ready to tackle a full exam, follow the advice for the [full exam \(click here\)](#).

Please note that all final answers were extracted automatically from the full solution. It is possible that the final answer shown here is not complete, or it may be missing entirely. In such a case, please notify [mer-wiki@math.ubc.ca](mailto:mer-wiki@math.ubc.ca). Your feedback helps us improve.

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### Question 1 (a)

FINAL ANSWER. So two possible answers are given by 18 and 53.

### Question 1 (b)

FINAL ANSWER. Thus, the divisors are 1, 3, 9, 27, 37, 111, 333, 999 and these are the potential  $m$  values.

### Question 1 (c)

FINAL ANSWER.  $\equiv 5 \pmod{10}$

### Question 1 (d)

FINAL ANSWER. Lastly, to be divisible by 4, we need the last two digits to be divisible by 4 so if  $X$  is one of 0, 4, or 8, then this condition is satisfied and the number will be divisible by 4.

### Question 1 (e)

FINAL ANSWER. Hence  $x \equiv 2(7)x \equiv 2(4) \equiv 8 \pmod{13}$  which completes the question.

### Question 1 (f)

FINAL ANSWER. valid since  $560 = 2^4 \cdot 5 \cdot 7$ . Thus 561 is a Carmichael number.

### Question 1 (g)

FINAL ANSWER.  $5^{16} \equiv (5^2)^8 \equiv 25^8 \equiv 2^8 \equiv 256 \equiv 26 \equiv 3 \pmod{23}$

### Question 2 (a)

FINAL ANSWER. Thus if  $\alpha + 1 = 3$  we have that  $\alpha = 2$  which is as claimed in the question.

### Question 2 (b)

FINAL ANSWER.  $5^7 \equiv 5 \cdot 5^6 \equiv 5 \cdot (25)^3 \equiv 5 \cdot 4^3 \equiv 5 \cdot 64 \equiv 5 \cdot (-12) \equiv -60 \equiv 16 \not\equiv 1 \pmod{19}$  and so 7 is not the order of 5.

### Question 2 (c)

FINAL ANSWER.  $7^{2 \cdot 3} \equiv 49^3 \equiv (-1)^3 \equiv -1 \pmod{25}$  Thus, 7 passes Miller's test. (Note that 25 is not prime but still 7 passes Miller's test).

### Question 2 (d)

FINAL ANSWER. Thus 7 divides  $2^{39} - 1$ .

### Question 3

FINAL ANSWER.  $n \in \{300, 301, 302, 303, 304\}$  completing the question.

### Question 4

FINAL ANSWER. Thus, as  $p - 1$  is even, we have that  $\alpha = 1$  (or  $n$  is odd) and further that  $p = 15$  which is not a prime. This is a contradiction and hence this equation has no solution.

### Question 5

FINAL ANSWER. (or easier, notice that  $\phi(16) = 8$  so Euler's theorem gives the same result) Thus as the number is 1, its inverse is itself.