Final Answers
MATH100 December 2012

How to use this resource

- When you feel reasonably confident, simulate a full exam and grade your solutions. For your grading you can get the full solutions here.

- If you're not quite ready to simulate a full exam, we suggest you thoroughly and slowly work through each problem. Use this document with the final answers only to check if your answer is correct, without spoiling the full solution.

- Should you need more help, check out the hints and video lecture on the Math Educational Resources.

Tips for Using Previous Exams to Study: Work through problems

Resist the temptation to read any of the final answers below before completing each question by yourself first! We recommend you follow the guide below.

1. **How to use the final answer:** *The final answer is not a substitution for the full solution!* The final answer alone will not give you full marks. The final answer is provided so that you can check the correctness of your work without spoiling the full solution.

   - To answer each question, only use what you could also use in the exam. Download the raw exam here.
   - If you found an answer, how could you verify that it is correct from your work only? E.g. check if the units make sense, etc. Only then compare with our result.
   - If your answer is correct: good job! Move on to the next question.
   - Otherwise, go back to your work and check it for improvements. Is there another approach you could try? If you still can't get to the right answer, you can check the full solution on the Math Educational Resources.

2. **Reflect on your work:** Generally, reflect on how you solved the problem. Don't just focus on the final answer, but whether your mental process was correct. If you were stuck at any point, what helped you to go forward? What made you confident that your answer was correct? What can you take away from this so that, next time, you can complete a similar question without any help?

3. **Plan further studying:** Once you feel confident enough with a particular topic, move on to topics that need more work. Focus on questions that you find challenging, not on those that are easy for you. Once you are ready to tackle a full exam, follow the advice for the full exam (click here).

Please note that all final answers were extracted automatically from the full solution. It is possible that the final answer shown here is not complete, or it may be missing entirely. In such a case, please notify mer-wiki@math.ubc.ca. Your feedback helps us improve.

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Question 1 (a)   Easiness: 82/100

FINAL ANSWER. $f(x) = 2\sin(x) - e^x + 1$

Question 1 (b)   Easiness: 89/100

FINAL ANSWER. Thus, $(2.001)^4 \approx L(2.001) = \frac{16032}{1000}$

Question 1 (c)   Easiness: 94/100

FINAL ANSWER. $y' = (\sin x)^{\sin x} \cos(x)(\ln(\sin x) + 1)$

Question 2 (a)   Easiness: 94/100

FINAL ANSWER. $f'(x) = e^{(\sin x)^2}(2\sin x)(\cos x)$

Question 2 (b)   Easiness: 89/100

FINAL ANSWER. $y' = -\cos(1)$ completing the question.

Question 2 (c)   Easiness: 83/100

FINAL ANSWER. $\frac{d}{dx}\sin^{-1}(\ln(x)) = \frac{1}{\sqrt{1-(\ln(x))^2}} \cdot \frac{1}{x}$

Question 2 (d)   Easiness: 89/100

FINAL ANSWER. $f'(\pi/4) = 2$

Question 3 (a)   Easiness: 88/100

FINAL ANSWER. As this line is tangent to the function at 1, we see that $m = f'(1)$ and $f(1) = 6$ completing the question.

Question 3 (b)   Easiness: 97/100

FINAL ANSWER. As these two limits are the same and our function $f(x)$ is bounded below and above by the two functions above, we have that $\lim_{x \to 4} f(x) = 7$ by the squeeze theorem.

Question 3 (c)   Easiness: 97/100

FINAL ANSWER. Thus the largest value $f(4)$ can be is -3.

Question 4 (a)   Easiness: 98/100

FINAL ANSWER. Thus we need $1 + a = $ -1 and so $\boldsymbol{a = }$ **-2.**

Question 4 (b)

FINAL ANSWER. $\lim_{x\to 0} \frac{e^{3x^2}-1}{\sin(x^2)} = 3$

Question 4 (c)

FINAL ANSWER. $\lim_{x\to\infty} \left(1+\frac{5}{x}\right)^x = e^5$

Question 5 (a)

FINAL ANSWER. Now, we are looking for the percentage remaining after 15 years (5 years after the initial 10 years), or $y(15)$, so plugging in the data gives $y(15) = 100(1/8)^{3/2}$ completing the question.

Question 5 (b)

FINAL ANSWER. Thus, the value of t that gives $y(t) = 50$ is $t = 10/3$

Question 6 (a)

FINAL ANSWER. Overall, the function in increasing on $(-\infty, 0) \cup (1, \infty)$ and decreasing on $(0, 1)$.

Question 6 (b)

FINAL ANSWER. Here since we have a decreasing function left of *1* and an increasing function right of *1*, we have that $x = 1$ is a local minimum.

Question 6 (c)

FINAL ANSWER. We test that the derivative at a point in the interval, say $x = 10$, which gives a value of $(1/25) * 10^{-12/5} * (-78) < 0$. Thus the function is concave down on this interval. This completes the question.
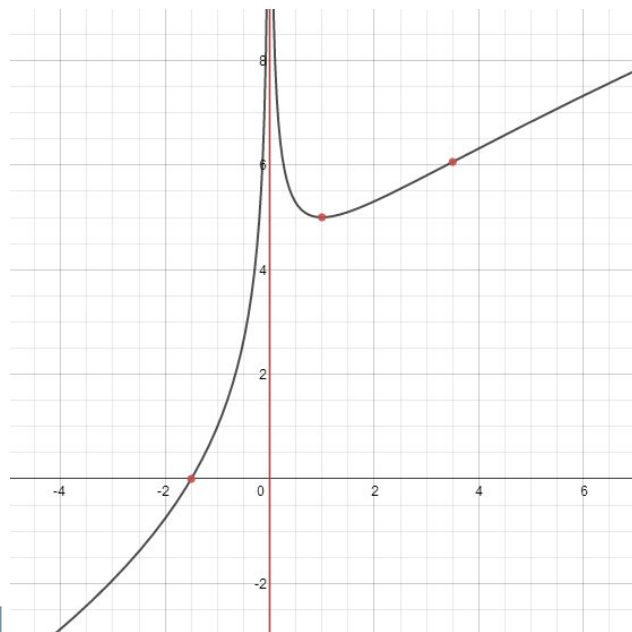
Question 6 (d)

FINAL ANSWER. By part c, we see immediately that concavity changes from concave up to concave down at the point $x = 7/2$. This is the only inflection point.

Question 6 (e)

FINAL ANSWER. Notice that the left hand limit is also positive infinity since the second term can be written as $3x^{-2/5} = \dfrac{3}{\sqrt[5]{x^2}}$ which is always positive.

Question 6 (f)

Question 7
Easiness: 33/100

FINAL ANSWER. The associated point P is given by $\left(\frac{1}{2}, \sqrt{1 - \left(\frac{1}{2} \right)^2} \right)$ or $\left(\frac{1}{2}, \frac{\sqrt{3}}{2} \right)$

Question 8
Easiness: 62/100

FINAL ANSWER. which is the final answer.

Question 9
Easiness: 56/100

FINAL ANSWER. This occurs when $-1/\sqrt{2} < x < 1/\sqrt{2}$

Question 10
Easiness: 92/100

FINAL ANSWER. Thus, our desired function is $y = \dfrac{x^4}{4} + \dfrac{3}{4}$.

Question 11
Easiness: 15/100

FINAL ANSWER. $0 \leq \cos(x) - \left(1 - \dfrac{x^2}{2} \right) = \dfrac{\cos(c)}{24}x^4 \leq \dfrac{1}{24}$ as required.