Final answers script in beta

Final Answers
MATH102 December 2014

April 16, 2015

How to use this resource

- When you feel reasonably confident, simulate a full exam and grade your solutions. For your grading you can get the full solutions here.
- If you're not quite ready to simulate a full exam, we suggest you thoroughly and slowly work through each problem. Use this document with the final answers only to check if your answer is correct, without spoiling the full solution.
- Should you need more help, check out the hints and video lecture on the Math Education Resources.

Tips for Using Previous Exams to Study: Work through problems

Resist the temptation to read any of the final answers below before completing each question by yourself first! We recommend you follow the guide below.

1. **How to use the final answer:** *The final answer is not a substitution for the full solution!* The final answer alone will not give you full marks. The final answer is provided so that you can check the correctness of your work without spoiling the full solution.
   - To answer each question, only use what you could also use in the exam. Download the raw exam here.
   - If you found an answer, how could you verify that it is correct from your work only? E.g. check if the units make sense, etc. Only then compare with our result.
   - If your answer is correct: good job! Move on to the next question.
   - Otherwise, go back to your work and check it for improvements. Is there another approach you could try? If you still can't get to the right answer, you can check the full solution on the Math Education Resources.
2. **Reflect on your work:** Generally, reflect on how you solved the problem. Don't just focus on the final answer, but whether your mental process was correct. If you were stuck at any point, what helped you to go forward? What made you confident that your answer was correct? What can you take away from this so that, next time, you can complete a similar question without any help?
3. **Plan further studying:** Once you feel confident enough with a particular topic, move on to topics that need more work. Focus on questions that you find challenging, not on those that are easy for you. Once you are ready to tackle a full exam, follow the advice for the full exam (click here).

Please note that all final answers were extracted automatically from the full solution. It is possible that the final answer shown here is not complete, or it may be missing entirely. In such a case, please notify mer-wiki@math.ubc.ca. Your feedback helps us improve.

This pdf was created for your convenience when you study Math and prepare for your final exams. All the content here, and much more, is freely available on the Math Education Resources.

Question A 01

FINAL ANSWER. **THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!**

If $g(x)$ is the inverse function of $f(x)$, then $g(a) = c$ and $g'(a) = \frac{1}{b}$. Knowing a point (a, c) on the tangent line and the slope $\frac{1}{b}$, we can write down the equation of the tangent line, which is answer (A) $y = c + \dfrac{1}{b}(x - a)$.

Question A 02

FINAL ANSWER. **THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!**

Using the L'Hopital's rule we find $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$. Then to make the original function $f(x)$ continuous, we just need $a = 1$. It gives $a = 1$ which is answer (B).Note: This question can also be solved using the Squeeze Theorem.

Question A 03

FINAL ANSWER. **THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!**

So the correct answer is (b).

Question A 04

FINAL ANSWER. **THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!**

Thus the only possible answer is (c).

Question A 05

FINAL ANSWER. **THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!**

1. Healthy cells have a constant increasing rate P.
2. Healthy cells can become infected when they encounter infected virus(from the $\pm aCV$ term in equations of C and I).
3. Virus is produced at a rate proportional to the current infected cell density(from the βI term in the equation for V).
4. Healthy cells, infected cells and virus die at a rate proportional to its own density(from the $-\gamma_1 C$, $-\gamma_2 I$ and $-\gamma_3 V$ terms).

Thus the correct statement is (b).

Question A 06

FINAL ANSWER. **THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!**

where the only possible answer is (d).

Question A 07

FINAL ANSWER. **THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!**

So comparing plots of (1) and (3) we find the k for (1) was larger than the value of k for (3) as (1) approaches to its steady state faster than (3)(so (c),(d) are wrong).This leaves the only choice (e).

Question A 08

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which is answer (C).

Question B 01

FINAL ANSWER. **THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!**

So $x_{1,2}$ are inflection points.

Question B 02

FINAL ANSWER. **THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!**

When $P = 800$, solving for t we get $100(\sqrt{2})^t = 800$ and hence $t = 6$.

Question B 03

FINAL ANSWER. **THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!**

So the equation of the tangent line is $L(x) = 1 + 2(x - \frac{\pi}{4})$.

Question B 04

FINAL ANSWER. **THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!**

So $\varphi = -19\frac{2\pi}{24}$ and the final function is $T(t) = 6 + 4\cos(\frac{2\pi}{24}t - \frac{2\pi}{24}19)$. Note the expression of the solution is not unique, it can be written as $T(t) = 6 - 4\cos(\frac{2\pi}{24}t - \frac{2\pi}{24}7)$, or $T(t) = 6 + 4\cos(\frac{2\pi}{24}t + \frac{2\pi}{24}5)$,

Question B 05

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$y(1/4) = \frac{3}{2}$.

Question B 06

FINAL ANSWER. **THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!**

$L(0.95) = -0.05$.

Question B 07

FINAL ANSWER. **THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!**

Note: The third line of the equation above is where the observation that $x' = 0$ is applied into the question as indicated by the substitution of y' with v, the variable of x with 30, and the rate of x' with 0.

Question C 01

FINAL ANSWER. **THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!**

Note: the triangle drawn on the right diagram can be solely constructed from the equation of $x = \cot(\theta)$

Question C 02 (a)

FINAL ANSWER. **THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!**

Substitute values of P_m, k and r and simplify the expression, we obtain $\frac{1}{5}c(c-5)(c-20) = 0$, which gives $c = 0,\ 5,\ 20$ are the steady states.

Question C 02 (b)

FINAL ANSWER. **THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!**

Similarly, we find $c = 5$ is unstable and $c = 20$ is stable.

Question C 02 (c)

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So c would increase with time and finally approach the steady state $c = 20$.

Question C 02 (d)

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Same as in C2(a), to solve the steady states, we just need to solve the following equation $c(rc^2 - P_m c + k^2 rc) = 0$, Obviously $c = 0$ is one of the steady states and other steady states are roots of the quadratic equation $rc^2 - P_m c + k^2 rc = 0$. To get only two steady state, from the quadratic formula we need to have $b^2 - 4ac = 0$ and in this case $(P_m)^2 - 4 \cdot r \cdot k^2 r = 0$. Thus with the given values of P_m and k we have $r^2 = \frac{1}{16}$, To obtain positive steady state c, we get $r = \frac{1}{4}$.

Question C 03

FINAL ANSWER. **THIS QUESTION HAS NOT YET BEEN REVIEWED! THE ANSWER BELOW MAY CONTAIN MISTAKES!**

So the angle is maximized at $x = 4$.