Final Answers
MATH307 April 2013

April 4, 2015

How to use this resource

- When you feel reasonably confident, simulate a full exam and grade your solutions. For your grading you can get the full solutions here.
- If you're not quite ready to simulate a full exam, we suggest you thoroughly and slowly work through each problem. Use this document with the final answers only to check if your answer is correct, without spoiling the full solution.
- Should you need more help, check out the hints and video lecture on the Math Education Resources.

Tips for Using Previous Exams to Study: Work through problems

Resist the temptation to read any of the final answers below before completing each question by yourself first! We recommend you follow the guide below.

1. **How to use the final answer:** *The final answer is not a substitution for the full solution!* The final answer alone will not give you full marks. The final answer is provided so that you can check the correctness of your work without spoiling the full solution.
   - To answer each question, only use what you could also use in the exam. Download the raw exam here.
   - If you found an answer, how could you verify that it is correct from your work only? E.g. check if the units make sense, etc. Only then compare with our result.
   - If your answer is correct: good job! Move on to the next question.
   - Otherwise, go back to your work and check it for improvements. Is there another approach you could try? If you still can't get to the right answer, you can check the full solution on the Math Education Resources.
2. **Reflect on your work:** Generally, reflect on how you solved the problem. Don't just focus on the final answer, but whether your mental process was correct. If you were stuck at any point, what helped you to go forward? What made you confident that your answer was correct? What can you take away from this so that, next time, you can complete a similar question without any help?
3. **Plan further studying:** Once you feel confident enough with a particular topic, move on to topics that need more work. Focus on questions that you find challenging, not on those that are easy for you. Once you are ready to tackle a full exam, follow the advice for the full exam (click here).

Please note that all final answers were extracted automatically from the full solution. It is possible that the final answer shown here is not complete, or it may be missing entirely. In such a case, please notify `mer-wiki@math.ubc.ca`. Your feedback helps us improve.

This pdf was created for your convenience when you study Math and prepare for your final exams. All the content here, and much more, is freely available on the Math Education Resources.

Question 1 (a)

FINAL ANSWER. $\|A\| = \max_{x:\, \|x\| \neq 0} \frac{\|Ax\|}{\|x\|}$

Question 1 (c)

FINAL ANSWER. $\left\|A\vec{x}\right\| = 33$ is not smaller than 2, so this is impossible.

Question 1 (d)

FINAL ANSWER. therefore, cond(A) = 2

Question Section 201 06 (a)

FINAL ANSWER. This is why all the entries in column 6 of matrix P contain the value 1/6.

Question 1 (b)

FINAL ANSWER. $\frac{\left\|\vec{\Delta x}\right\|}{\left\|\vec{x}\right\|} \leqslant cond(A) \frac{\left\|\vec{\Delta b}\right\|}{\left\|\vec{b}\right\|}$

Question 2 (a)

FINAL ANSWER. A hermitian matrix is a matrix A such that $A = A^*$.A matrix that is its own conjugate transpose.All symmetric matrices with all real entries are hermitian.

Question 2 (b)

FINAL ANSWER. Given that $\lambda_1 \neq \lambda_2$, this can only be true if $\langle x_1, x_2 \rangle = 0$

Question 2 (c)

FINAL ANSWER. False, because $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ has a repeated eigenvalue of 1 and is diagonalized by I

Question 2 (d)

FINAL ANSWER. A stochastic matrix is a square matrix with no negative entries, and the columns sum to 1.

Question 2 (e)

FINAL ANSWER. -> The largest eigenvalue is 1-> The eigenvector of the eigenvalue of 1 can be scaled to have no negative entries

Question 2 (f)

FINAL ANSWER. -> The largest eigenvalue is 1-> All other eigenvalues are less than 1-> The eigenvector of the eigenvalue of 1 can be scaled to have all positive entries

Question 3 (a)

FINAL ANSWER. 5) $\lambda = \langle x, Ax \rangle$ (x was normalized earlier).

Question 3 (b)

FINAL ANSWER. 5) $\lambda = \langle x, Ax \rangle$ (x was normalized earlier).

Question 3 (c)

FINAL ANSWER. lambda=dot(x,A*x)

Question 4 (a)

FINAL ANSWER. $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

Question 4 (b)

FINAL ANSWER. $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

Question Section 201 05 (a)

FINAL ANSWER. $\therefore R(D^T) = R(L)$

Question Section 201 05 (b)

FINAL ANSWER. Also by Rank-Nullity theorem: *dim(N(D^T)) = n - r(D) = 7 - 5 = 2* and hence these two loop current vectors form a basis in *N(D^T)* and any other loop current can be expressed using a linear combination of these two basis vectors.

Question Section 201 05 (c)

FINAL ANSWER. $S = \frac{1}{R_{\text{eff}}} \begin{bmatrix} 1-1 \\ -11 \end{bmatrix}$

Question Section 201 06 (b)

FINAL ANSWER. Because Q projects onto a 1 dimension subspace, there is one eigenvalue equal to 1, and the rest are zero.

Question Section 201 06 (c)

FINAL ANSWER. $\underline{x_1} = (1/72)\begin{bmatrix}7\\10\\10\\10\\16\\19\end{bmatrix}$ From this solution, it is clear that the page with the highest rank after one step is page 6.

Question Section 201 06 (d)

FINAL ANSWER. It can be assumed that $\lambda_1 = 1$ is the first entry in D, the corresponding eigenvector is the first column of matrix V.To get the ranking we scale so that the sum is equal to 1.>> V(:,1) / sum(V(:,1))

Question Section 201 07 (a)

FINAL ANSWER. $\dim(A) = 2$

Question Section 201 07 (b)

FINAL ANSWER. $= \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}\sigma_1 & 0 & 0\\0 & \sigma_2 & 0\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt(2)} & 0 & -\frac{1}{\sqrt(2)}\\\frac{1}{\sqrt(2)} & 0 & \frac{1}{\sqrt(2)}\\0 & 1 & 0\end{bmatrix}$ It can be seen that the third column of V is an orthogonal basis for N(A), and both columns of U form the basis for R(A).

Question Section 201 07 (c)

FINAL ANSWER. $AA^* = U\Sigma^2U^T$From this equation, we can see that our eigenvalues are the same as the last.However, the corresponding eigenvectors are now the columns of U.

Question Section 202 05 (a)

FINAL ANSWER. $\begin{bmatrix}1\\2\\3\end{bmatrix}\begin{bmatrix}0\\1\\1\end{bmatrix}$

Question Section 202 05 (b)

FINAL ANSWER. If we let $x_2 = x_4 =$span($\begin{bmatrix}-1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}-1\\0\\-1\\1\end{bmatrix}$)

Question Section 202 05 (c)

FINAL ANSWER. Therefore the basis for R $(A^T) = \begin{bmatrix}1\\1\\0\\1\end{bmatrix}$ and $\begin{bmatrix}0\\0\\1\\1\end{bmatrix}$.

Question Section 202 05 (d)

FINAL ANSWER. hence $\dim(\mathrm{N}(A^T)) = 1$

Question Section 202 06 (a)

FINAL ANSWER. $c_n = \begin{cases} \frac{1}{2} & \text{if } n = 0 \\ 0 & \text{if } n \text{ is even, } n \neq 0 \\ \frac{-i}{\pi n} & \text{if } n \text{ is odd} \end{cases}$

Question Section 202 06 (b)

FINAL ANSWER. $\sum_{k=0}^{\infty} \frac{1}{(2k+1)^2} = \frac{\pi^2}{8}$

Question Section 202 07 (a)

FINAL ANSWER. $p_2(x) = a_2(x-1)^3 + b_2(x-1)^2 + c_2(x-1) + d_2$

Question Section 202 07 (b)

FINAL ANSWER. $2 = a_2 + b_2 + c_2 + d_2$

Question Section 202 07 (c)

FINAL ANSWER. $6a_1 + 2b_1 = 2b_2$

Question Section 202 07 (d)

FINAL ANSWER. $6a_2 + 2b_2 = 0$

Question Section 202 07 (e)

FINAL ANSWER.
```
A=[0,0,0,1,0,0,0,0; 1,1,1,1,0,0,0,0; 0,0,0,0,0,0,0,1; ...
0,0,0,0,1,1,1,1; 3,2,1,0,0,0,-1,0; 0,2,0,0,0,0,0,0; ...
0,0,0,0,6,2,0,0; 6,2,0,0,0,-2,0,0];
b=[1,0,0,2,0,0,0,0]=    A=[0,0,0,1,0,0,0,0; 1,1,1,1,0,0,0,0; 0,0,0,0,0,0,0,1; ...
0,0,0,0,1,1,1,1; 3,2,1,0,0,0,-1,0; 0,2,0,0,0,0,0,0; ...
0,0,0,0,6,2,0,0; 6,2,0,0,0,-2,0,0];
b=[1,0,0,2,0,0,0,0]';
a=A\b; =;
a=A\b;
```