Final Answers
MATH152 April 2011

April 4, 2015

How to use this resource

- When you feel reasonably confident, simulate a full exam and grade your solutions. For your grading you can get the full solutions here.
- If you're not quite ready to simulate a full exam, we suggest you thoroughly and slowly work through each problem. Use this document with the final answers only to check if your answer is correct, without spoiling the full solution.
- Should you need more help, check out the hints and video lecture on the Math Education Resources.

Tips for Using Previous Exams to Study: Work through problems

Resist the temptation to read any of the final answers below before completing each question by yourself first! We recommend you follow the guide below.

1. **How to use the final answer:** *The final answer is not a substitution for the full solution!* The final answer alone will not give you full marks. The final answer is provided so that you can check the correctness of your work without spoiling the full solution.
   - To answer each question, only use what you could also use in the exam. Download the raw exam here.
   - If you found an answer, how could you verify that it is correct from your work only? E.g. check if the units make sense, etc. Only then compare with our result.
   - If your answer is correct: good job! Move on to the next question.
   - Otherwise, go back to your work and check it for improvements. Is there another approach you could try? If you still can't get to the right answer, you can check the full solution on the Math Education Resources.
2. **Reflect on your work:** Generally, reflect on how you solved the problem. Don't just focus on the final answer, but whether your mental process was correct. If you were stuck at any point, what helped you to go forward? What made you confident that your answer was correct? What can you take away from this so that, next time, you can complete a similar question without any help?
3. **Plan further studying:** Once you feel confident enough with a particular topic, move on to topics that need more work. Focus on questions that you find challenging, not on those that are easy for you. Once you are ready to tackle a full exam, follow the advice for the full exam (click here).

Please note that all final answers were extracted automatically from the full solution. It is possible that the final answer shown here is not complete, or it may be missing entirely. In such a case, please notify `mer-wiki@math.ubc.ca`. Your feedback helps us improve.

This pdf was created for your convenience when you study Math and prepare for your final exams. All the content here, and much more, is freely available on the Math Education Resources.

Question A 01

FINAL ANSWER. $\mathbf{x}+\mathbf{y}=[1,-2,3]$ and so $\mathbf{x}+\mathbf{y}=[1,-2,3]$.

Question A 02

FINAL ANSWER. $\begin{vmatrix} \hat{\mathbf{i}}\hat{\mathbf{j}}\hat{\mathbf{k}} \\ 0-32 \\ 111 \end{vmatrix} = [-5,2,3]$ which is precisely the definition of the cross product. Therefore $\mathbf{x}\times\mathbf{y}=[-5,2,3]$.

Question A 03

FINAL ANSWER. $\text{proj}_{\mathbf{y}}\mathbf{x}=\left[-\frac{1}{3},-\frac{1}{3},-\frac{1}{3}\right]$.

Question A 04

FINAL ANSWER. $r \le \min(n,m)$. For this example we have m=2 and n=3 and so $r \le 2$. The dimension of the nullspace is n-r and in this case it is at least 1 which implies that there are non-trivial solutions to"$A\mathbf{x}$ "= $\mathbf{0}$ and in fact there are an infinite number of them. A solution to"$A\mathbf{x}$ "= $\boldsymbol{b}$ exists as long as **"b"** is spanned by the column space of A. If this is true then"$A\mathbf{x}$ "= $\boldsymbol{b}$ has a particular solution along with the infinite set of solutions belonging to the nullspace and therefore overall there are an infinite number of solutions. However, if $\boldsymbol{b}$ is **not** spanned by the column space of A then there are no solutions to"$A\mathbf{x}$ "= $\boldsymbol{b}$. Therefore, we conclude that a linear system with 2 equations in 3 unknowns has (d) either no solutions or an infinite number of solutions.

Question A 05

FINAL ANSWER. For our case, we have
$A=\begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}$
and Det$(A)=1$. Therefore,
$A^{-1}=\begin{bmatrix}2 & -3\\-1 & 2\end{bmatrix}$
is the inverse of the matrix.

Question A 06

FINAL ANSWER. $A=\begin{bmatrix}123\\456\\789\end{bmatrix}$. The semicolon at the end of the line indicates that output is suppressed and so the matrix will not be displayed.For the second line of code`A(2,:)`we recall that Matlab recognizes entries in a matrix first by row and column so a call A(i,j) would return an element in row i and column j. The colon instructs Matlab to take all the entries of a given field. Therefore A(:,j) instructs Matlab to output all entries in column j while A(i,:) instructs Matlab to output all of the entries in row i. Therefore A(2,:) instructs Matlab to output all of the entries from row 2.If we look at out matrix then the output would be`[4 5 6]`Notice the lack of semicolon on this output indicates that the output is **not** suppressed.

Question A 07

FINAL ANSWER. $A = \begin{bmatrix} 50 \\ 03 \\ 10 \end{bmatrix}$ is our created matrix.

Question A 08

FINAL ANSWER. $A = \begin{bmatrix} 1111 \\ 0-200 \\ 00-30 \\ 000-4 \end{bmatrix}$. Since there are no more lines of code, this is the final matrix.

Question A 09

FINAL ANSWER. $\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ v_1 \\ v_2 \end{bmatrix}$.

Question A 10

FINAL ANSWER. $0 = 9i_3 + 7(i_3 - i_1) + v_2$ as the linear equation for loop 3.

Question A 11

FINAL ANSWER. Do not worry about the presence of a negative sign, a negative loop current just means that the current (i_1 and i_3 in this case) actually flows counter-clockwise, not clockwise like we assumed.

Question A 12

FINAL ANSWER. $\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$will do the trick.

Question A 13

FINAL ANSWER. Therefore we have $[0,0]$ transforms to$[0,0]$, $[1,0]$ transforms to $[1,2]$, $[0,1]$ transforms to$[2,1]$ and $[1,1]$ transforms to $[3,3]$. The plot to the right400px|right|thumbnail shows the original points in blue while thetransformed points are in red. We complete the figure by connecting the corners.

Question A 14

FINAL ANSWER. Therefore we have $[0,0]$ transforms to$[0,0]$, $[1,0]$ transforms to $[1,1]$, $[0,1]$ transforms to$[1,1]$ and $[1,1]$ transforms to $[2,2]$. The plot to the right400px|right|thumbnail shows the original points in blue while thetransformed points are in red. We complete the figure by connecting the corners.Notice that in this case it may appear we have lost a vertex but really, as we computed above, two are just overlapping one another.We see that we have turned our square into a line, this is typical of a projection operator (recall that a projection operator maps vectors to a single vector).In this case, if we try to form a projection operator to the vector [1,1] we get (a multiple of) our linear transformation matrix A (try it yourself!)

Question A 15

Final answer. $T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \frac{\sqrt{3}+2}{10}\begin{bmatrix}1\\2\end{bmatrix}$.

Question A 16

Final answer. $\hat{\mathbf{v}} = \frac{1}{\sqrt{3}}[1, -1, -1]$.

Question A 17

Final answer. $(4-2i)(-1+3i) = 2+14i$

Question A 18

Final answer. $|-1+3i| = \sqrt{10}$

Question A 19

Final answer. $\frac{u}{z} = -1-i$

Question A 20

Final answer. $-1+i = \sqrt{2}e^{3\pi i/4}$

Question A 21

Final answer. There is better ways of solving $Ax = b$ than that, and $x= A\backslash b$ is one of them. $=$ than that, and $x= A\backslash b$ is one of them.It does not use the inverse matrix (which is costly to compute).

Question A 22

Final answer. $P\mathbf{x} = \begin{bmatrix}2/5\\2/5\\1/5\end{bmatrix}$. Therefore we get that **the probability of being in state 3 is 1/5**.Notice that the output probabilities sum to 1.This is a requirement since we only allow transitions from states 1 to 3, this summation to 1 simply says that no matter where we start we will end up somewhere in states 1 to 3.

Question A 23

Final answer. so that if we wanted the n^{th} transition probabilities we'd have $\mathbf{t_n} = P^n\,\mathbf{x}$.

Question A 24

Final answer. ***Note further:*** It may have crossed some minds that the equilibrium vector is always the eigenvector for the transition matrix corresponding to an eigenvalue of 1, whose entries sum to 1.For probability matrices where the columns sum to 1 there will always be such a vector.

Question A 25

FINAL ANSWER. Therefore we conclude that if $p > 0$ then the eigenvalues of A will be both **real** and **distinct**.

Question A 26

FINAL ANSWER. and so, for the matrix to be invertible, that is, for the matrix not to have a determinant of zero, we require that $a \neq 0$ and $a \neq b$.

Question A 27

FINAL ANSWER. Firstly this means that the row reduced form of A is the identity matrix (since it has n pivots) so **option b is correct** and secondly it means that there are no rows of zeros which means that $\det(A) \neq 0$, so **option e is correct**.Therefore, from starting with the definition of an inverse we were able to deduce that **all** of the options are correct.

Question A 28

FINAL ANSWER. Consider if the original angle is clockwise of the reflector at $\alpha/2$.In this case if we reflect it so that it is now counter-clockwise to the reflector and rotate it even further counter-clockwise then there is no way of returning to the original vector.In order for it to be an identity operator, the operation has has to return the original vector for any arbitrary initial vector.We have shown with this class of examples that the operation will not produce the original vector ever and so it can't hold for any arbitrary initial vector.Therefore this is **not** an identity matrix operation.Therefore, we conclude that only **(a)** and **(b)** are equivalent to identity matrix operations.

Question A 29

FINAL ANSWER. *Note that only one counterexample would be enough, so your solution is just as good, if you choose numbers for a and b, e.g. $a = 1$.*

Question A 30

FINAL ANSWER. Therefore we see that the vector $\mathbf{v} \times (\mathbf{v} \times \mathbf{w})$ is orthogonal to the normal vector of the plane and thus lies in the plane itself.Therefore, the statement is **true**.

Question B 01 (a)

FINAL ANSWER. $$\begin{bmatrix} \frac{1}{3}\frac{1}{3}\frac{1}{3} \\ \\ 0\frac{1}{2}\frac{1}{2} \\ \\ \frac{1}{2}\frac{1}{2}0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 10 \\ 16 \end{bmatrix}$$

Question B 01 (b)

Final answer. $\begin{bmatrix} \frac{1}{3}\frac{1}{3}\frac{1}{3}12 \\ 0\frac{1}{2}\frac{1}{2}10 \\ \frac{1}{2}\frac{1}{2}016 \end{bmatrix}$

Question B 01 (c)

Final answer. $\begin{bmatrix} 11136 \\ 01120 \\ 0014 \end{bmatrix}$ which of course leads to the same conclusion but some arithmetic is involved.This type of situation is often the trade off in deciding which process to choose.Doing a row-echelon form is often quicker but because it only guarantees entries **below** pivots are zero, then arithmetic will almost always be required to get the solution. Conversely, the reduced row-echelon form takes longer to obtain but then the unknowns are clearly presented.

Question B 02 (a)

Final answer. $\mathbf{y}(t) = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} \exp(-t) + c_2 \begin{bmatrix} 2 \\ 3 \end{bmatrix} \exp(-2t).$

Question B 02 (b)

Final answer. $\mathbf{y}(t) = - \begin{bmatrix} 1 \\ 2 \end{bmatrix} \exp(-t) + \begin{bmatrix} 2 \\ 3 \end{bmatrix} \exp(-2t).$

Question B 03 (a)

Final answer. $\lambda_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Question B 03 (b)

Final answer. Therefore we conclude that det(A)=0 and thus that the matrix A is **not** invertible.

Question B 03 (c)

Final answer. $A = \begin{bmatrix} 11 \\ 11 \end{bmatrix}$. Notice that we can indeed confirm that this matrix has eigenvalues 2 and 0 and is not invertible.

Question B 04 (a)

Final answer. Therefore we have concluded that T is a linear transformation.

Question B 04 (b)

Final answer. *which is indeed the same result.*

Question B 04 (c)

FINAL ANSWER. $\mathbf{a} \times (k\mathbf{a}) = \mathbf{0}$.

Question B 04 (d)

FINAL ANSWER. Therefore a vector such that T($\mathbf{x}$)=$\mathbf{0}$ is $\mathbf{x}$=$\mathbf{a}$ (or any vector of the form $k\boldsymbol{a}$, due to the linearity of T).

Question B 05 (a)

FINAL ANSWER. $\mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

Question B 05 (b)

FINAL ANSWER. $\lambda_3 = 1 - i$.

Question B 05 (c)

FINAL ANSWER. *If you still have trouble computing eigenvectors, it is a worthwhile exercise to actually compute* $\mathbf{x}_3$ *using the method above and comparing with this result.*

Question B 06 (a)

FINAL ANSWER. $\begin{bmatrix} 102 \\ 013 \end{bmatrix}$

Question B 06 (b)

FINAL ANSWER. $\begin{bmatrix} 10-11 \\ 0102 \end{bmatrix}$

Question B 06 (c)

FINAL ANSWER. Notice in this example, if the solution vector $\mathbf{x}$ has components x_1, x_2, and x_3 then we are trying to claim that $0x_3 = 7$ which is never true.Therefore this system has no solution.

Question B 06 (d)

FINAL ANSWER. $\begin{bmatrix} 102 \\ 013 \\ 002 \end{bmatrix}$ we see the other type of indicator for no solution that occurred in part (c).

Question B 06 (e)

FINAL ANSWER. $e \neq ac + bd$.