Final Answers
MATH100 December 2014

Final answers script in beta

April 4, 2015

How to use this resource

- When you feel reasonably confident, simulate a full exam and grade your solutions. For your grading you can get the full solutions here.
- If you're not quite ready to simulate a full exam, we suggest you thoroughly and slowly work through each problem. Use this document with the final answers only to check if your answer is correct, without spoiling the full solution.
- Should you need more help, check out the hints and video lecture on the Math Education Resources.

Tips for Using Previous Exams to Study: Work through problems

Resist the temptation to read any of the final answers below before completing each question by yourself first! We recommend you follow the guide below.

1. **How to use the final answer:** *The final answer is not a substitution for the full solution!* The final answer alone will not give you full marks. The final answer is provided so that you can check the correctness of your work without spoiling the full solution.
   - To answer each question, only use what you could also use in the exam. Download the raw exam here.
   - If you found an answer, how could you verify that it is correct from your work only? E.g. check if the units make sense, etc. Only then compare with our result.
   - If your answer is correct: good job! Move on to the next question.
   - Otherwise, go back to your work and check it for improvements. Is there another approach you could try? If you still can't get to the right answer, you can check the full solution on the Math Education Resources.
2. **Reflect on your work:** Generally, reflect on how you solved the problem. Don't just focus on the final answer, but whether your mental process was correct. If you were stuck at any point, what helped you to go forward? What made you confident that your answer was correct? What can you take away from this so that, next time, you can complete a similar question without any help?
3. **Plan further studying:** Once you feel confident enough with a particular topic, move on to topics that need more work. Focus on questions that you find challenging, not on those that are easy for you. Once you are ready to tackle a full exam, follow the advice for the full exam (click here).

Please note that all final answers were extracted automatically from the full solution. It is possible that the final answer shown here is not complete, or it may be missing entirely. In such a case, please notify `mer-wiki@math.ubc.ca`. Your feedback helps us improve.

This pdf was created for your convenience when you study Math and prepare for your final exams. All the content here, and much more, is freely available on the Math Education Resources.

Question 1 (a)

FINAL ANSWER. log is contiuous on $(0, \infty)$ and $\log(1) = 0$.

Question 1 (b)

FINAL ANSWER. $\frac{1}{x} < 0$ for $x < 0$ and approaches $-\infty$ as $x \to 0$. Answer is C.

Question 1 (c)

FINAL ANSWER. $f'(x) = 2x + \frac{2}{\sqrt{x}}$, so the slope of the tangent line at $x = 4$ is $f'(4) = 9$.Answer is D.

Question 1 (d)

FINAL ANSWER. $\frac{d}{dx}(\cos x) = -\sin x$.Answer is B.

Question 1 (e)

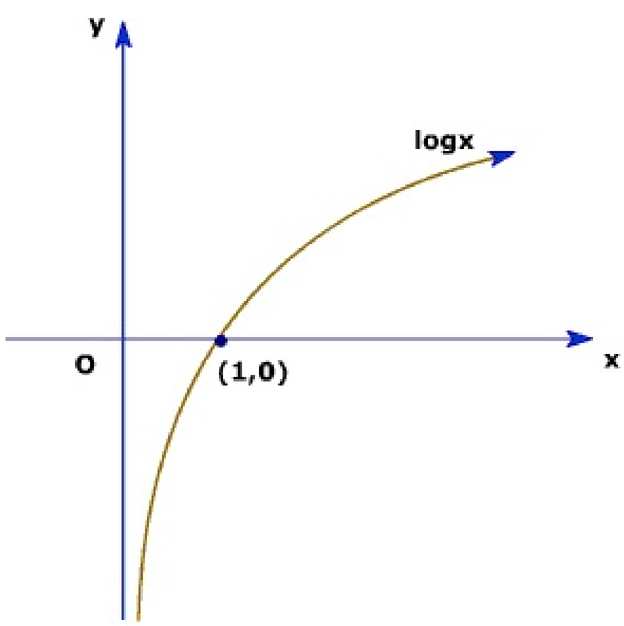
FINAL ANSWER. If $x = \sin y$ then $1 = \cos y \frac{dy}{dx}$ and so $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$
Answer is C.

Question 1 (f)

FINAL ANSWER. $\sin(-\frac{\pi}{6}) = -\frac{1}{2}$ and $-\frac{\pi}{2} \leq -\frac{\pi}{6} \leq \frac{\pi}{2}$ and so $\arcsin(-\frac{1}{2}) = -\frac{\pi}{6}$.Answer is B.

Question 1 (g)

FINAL ANSWER.



Question 1 (h)

FINAL ANSWER. Thus the concavity of f changes at $x = 0$ and $f(x)$ has a point of inflection at $x = 0$.Answer is D.

Question 1 (i)

FINAL ANSWER. $\tan(0) = 0$, so $g(x)$ is undefined at $x = 0$ and thus $g(x)$ has a discontinuity at $x = 0$.Answer is A.

Question 1 (j)

FINAL ANSWER. $h'(x) > 0$ for $0 < x < \frac{\pi}{2}$ and $h'(x) < 0$ for $-\frac{\pi}{2} < x < 0$, so $h(x)$ has a local minimum at $x = 0$.Answer is C.

Question 2 (a)

FINAL ANSWER. $\lim_{t\to 0} \arcsin(\cos t) = \pi/2$

Question 2 (b)

FINAL ANSWER. Hence the ratio goes to $-\infty$.

Question 2 (c)

FINAL ANSWER. Putting these two constraints together gives $0 < x < e^2$, so the domain of $f(x)$ is $(0, e^2)$.

Question 2 (d)

FINAL ANSWER. $\lim_{x\to-\infty} \frac{x}{\sqrt{x^2+1}-x} = -\frac{1}{2}$

Question 2 (e)

FINAL ANSWER. This follows by the Squeeze Theorem since $-1 \le \cos(x) \le 1$, so $-\frac{1}{x} \le \frac{\cos(x^2)}{x} \le \frac{1}{x}$ and then taking the limit as $x \to \infty$, we get $\lim_{x\to+\infty} \frac{\cos(x^2)}{x} = 0$ and so the limit is $e^0 = 1$

Question 3 (a)

FINAL ANSWER. $f'(x) = e^x(\cos(\pi x) - \pi \sin(\pi x))$

Question 3 (b)

FINAL ANSWER. $\frac{dy}{dx} = \frac{\cos(\log x)}{x \sin(\log x)}$

Question 3 (c)

FINAL ANSWER. $\frac{dy}{dx} = 2\log(x) \cdot x^{\log(x)-1}$

Question 3 (d)

FINAL ANSWER. L'Hôpital's Rule states $\lim_{t\to a} \frac{f(t)}{g(t)} = \lim_{t\to a} \frac{f'(t)}{g'(t)}$ Using $f(t) = \sqrt{t+1} - e^t$ and $g(t) = t$, we obtain $\lim_{t\to 0} \frac{\frac{1}{2\sqrt{t+a}} - e^t}{1} = -\frac{1}{2}$

Question 3 (e)

FINAL ANSWER. We have $V = \frac{4\pi r^3}{3}$ and so $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$. Thus $\frac{dr}{dt} = \frac{1}{16\pi}$ cm/s

Question 3 (f)

FINAL ANSWER. $\arcsin\left(\sin(\frac{31\pi}{11})\right) = \frac{2\pi}{11}$

Question 4 (a)

FINAL ANSWER. $\lim_{x\to 0} \frac{-\cos x - (4x^2 e^{x^2} + 2e^{x^2})}{2} = -\frac{3}{2}$

Question 4 (b)

FINAL ANSWER. $f(1) = -\frac{1}{2}$

Question 4 (c)

FINAL ANSWER. Hence $L(x) = x$ and $L(1/10) = 1/10$.

Question 5 (a)

FINAL ANSWER. Hence $a = -2$.

Question 5 (b)

FINAL ANSWER. So the global minimum is $4 - 4\sqrt{2}$ with coordinates $(x, y) = (\sqrt{2}, 4 - 4\sqrt{2})$.

Question 5 (c)

FINAL ANSWER. which finally gives $f(x) = 2x^{\frac{3}{2}} + \frac{5}{x} - 11$.

Question 6 (a)

FINAL ANSWER. $g'(x) = \frac{2}{(2+3x)^2}$

Question 6 (b)

FINAL ANSWER. $\lim_{x\to+\infty}(\sqrt{x^2+2x}-\sqrt{x^2-2x})=2$

Question 7 (a)

FINAL ANSWER. $d=\sqrt{5t^2-24t+36}$

Question 7 (b)

FINAL ANSWER. By the closed interval method, the minimum distance is reached when $t=\frac{12}{5}$ seconds.

Question 8 (a)

FINAL ANSWER. So for their ratio to be defined, we need $x>0$ and the domain of f is $(0,\infty)$.

Question 8 (b)

FINAL ANSWER. Since the function is defined only for $x>0$ there is no y-intercept. Solving $f(x)=0$ gives $x=1$ so the only intercept is an x-intercept at $(1,0)$.

Question 8 (c)

FINAL ANSWER. So the only horizontal asymptote is $y=0$.

Question 8 (d)

FINAL ANSWER. Hence $\lim_{x\to 0^+}\frac{\log x}{\sqrt{x}}=-\infty$ This asymptote at $x=0$ is the only vertical asymptote.

Question 8 (e)

FINAL ANSWER. Hence $x=e^2$ gives the only local maximum at the coordinates $(e^2,f(e^2))=(e^2,\frac{2}{e})$ and there is no local minimum.

Question 8 (f)

FINAL ANSWER. So, the only inflection point is at $(e^{\frac{8}{3}},f(e^{\frac{8}{3}}))=(e^{\frac{8}{3}},\frac{8}{3e^{4/3}})$.

Question 9 (a)

FINAL ANSWER. Thus $q=0$ satisfies the equation having at least two solutions.

Question 9 (b)

FINAL ANSWER.

- ie $f(a)=0$.

- Now since f is a polynomial it is continuous and differentiable everywhere, so we can apply the Mean Value Theorem or Rolle′s theorem.
- By Rolle′s theorem there must be points p, q with p between a and b and q between b and c so that $f'(p) = 0$ with $p < q$.
- But $f'(x) = 4(x^3 + 1)$ which only has a single real zero at $x = -1$.
- Hence the derivative cannot have 2 distinct zeros, so we have reached a contradiction and so f cannot have 3 solutions.

Question 10

FINAL ANSWER. So by the Squeeze Theorem, $f(x) \to 0$ as $x \to 0^+$. Thus $f(x)$ is continuous at $x = 0$ when $a > 0$.

Question 11 (a)

FINAL ANSWER. Differentiating gives $\frac{d}{dx} f(x) = e^x + e^{-x}$ which is nowhere near zero since $e^x > 0$ for all x Thus the maximum value is $e - e^{-1}$ and the minimum value is $-(e - e^{-1})$.

Question 11 (b)

FINAL ANSWER. And since $2 < e < 3$ we have $\frac{1}{3} < \frac{1}{e} < \frac{1}{2}$ and so $R_2(1) \leq \frac{e-e^{-1}}{6} \leq \frac{3-\frac{1}{3}}{6} = \frac{4}{9}$