PEDMAS important for what order to manipulate equations

$$z = 4 * (y - 4) + (x + 1)^2$$

If we are given the values of $x = 3$ and $y = 2$, what would be the value of z?

✏️ Take a minute to solve this individually.

PEDMAS important for what order to manipulate equations

If we are given the values of and , what would be the value of ?

$x = -3$ $y = 2$ z

💡 Let's see a solution!

$$
\begin{aligned}
z &= 4 * (y - 4) + (x + 1)^2 \\
&= 4 * (2 - 4) + (-3 + 1)^2 \\
&= 4 * (-2) + (-2)^2 \\
&= -8 + 4 \\
&= -4.
\end{aligned}
$$

Often times we want flexible equations

Prices are important in economics, but not always available for environmental goods.

How do we get prices if we know quantity?

✏️ Take a minute to solve this individually.

$$Q = \frac{(400 - P)}{80}$$

Isolate P in terms of Q

Often times we want flexible equations

💡 Let's see a solution!

Prices are important in economics, but not always available for environmental goods.

How do we get prices if we know quantity?

$$
\begin{aligned}
Q &= \frac{(400 - P)}{80} && \text{Is} \\
80Q &= \frac{(400 - P)\cancel{80}}{\cancel{80}} && \text{Mult} \\
80Q - 400 &= \cancel{400} - \cancel{400} - P && \text{Subtr} \\
-1(80Q - 400) &= -P(-1) && \text{Mul} \\
400 - 80Q &= P && \text{Flip}
\end{aligned}
$$

It can be easy to make mistakes while doing algebra. Practice makes perfect!

💡 Let's see a solution!

Solve all in terms of x

a. $3x + 2 = 10x - 12$

b. $4 - 3(2x + 1) = 8 - \frac{3x}{2}$

It can be easy to make mistakes while doing algebra. Practice makes perfect!

💡 Let's see a solution!

Solve all in terms of

$$x$$

c.

$$3(x + 7a) - 5 = b + 2(c - 4x)$$

Practice Solutions

$$3x + 2 = 10x - 12$$
$$3x + 2 + 12 = 10x - 12(+12)$$
$$3x - 3x + 14 = 10x - 3x$$
$$14 = 7x$$
$$x = 2$$

$$4 - 3(2x + 1) = 8 - \frac{3x}{2}$$
$$4 - 3 - 6x = 8 - \frac{3x}{2}$$
$$1 - 6x = 8 - \frac{3x}{2}$$
$$2 - 12x = 16 - 3x$$
$$-9x = 14$$
$$x = \frac{-14}{9}$$

$$3(x + 7a) - 5 = b + 2(c -$$
$$3x + 21a - 5 = b + 2c -$$
$$11x + 21a - 5 = b + 2c$$
$$11x = 5 + b + 2$$
$$x = \frac{5 + b +}{1}$$

Exponents practice

Rule	Expr
Product Rule	$x^a \cdot x$
Quotient Rule	$\frac{x^a}{x^b} =$
Zero Exponent	$x^0 =$
Negative Exponent	$x^{-a} =$
Fractional Exponent	$x^{\frac{1}{n}} =$
Power of a Power	$(x^a)^b$
Power of a Product	$(xy)^a$
Power of a Quotient	$\left(\frac{x}{y}\right)^a$
n-th Root Definition	$a = \sqrt[n]{}$

✏️ Simplify the following.

a. $\frac{6^5}{6^3}$

b. $(x^2y)^4$

c. $\frac{x^5y^6}{xy^2}$

d. $\frac{24x^6}{12x^{-8}}$

Exponents practice

✏️ Let's see a solution.

a.
$$\frac{6^5}{6^3} = 6^{5-3} = 6^2$$

b.
$$(x^2y)^4 = (x^2)^4 * y^2 = x^8y^2$$

c.
$$\frac{x^5y^6}{xy^2} = \frac{x^5}{x} * \frac{y^6}{y^2} = x^{5-1} * y^{6-2} = x^4y^4 = (xy)^4$$

d.
$$\frac{24x^6}{12x^{-8}} = 2x^6x^{(-(-8))} = 2x^6x^8 = 2x^{6+8} = 2x^{14}$$

Quadratic Formula solves 2nd degree polynomials

For any second degree polynomial

$$ax^2 + bx + c = 0$$

The roots to can be found using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example:

Let's use the quadratic formula to find the solutions to .

$$x^2 - x - 12 = 0$$

Team Assessment

1. Identify which numbers you should plug into which variable of the quadratic formula (e.g. a,b,c)

$$4x^2 + x - 14 = 0$$

2. Identify which numbers you should plug into which variable of the quadratic formula (e.g. a,b,c)

$$256 - \sqrt{44}x^2 + .23x = 10$$

3. What happens if is negative in the quadratic formula?

$$b^2 - 4ac$$

4. Expand

$$(3x - 6)(2x + 1)$$

Solutions

$$4x^2 + x - 14 = 0$$

$$a = 4$$

$$b = 1$$

$$c = -14$$

Solutions

$$256 - \sqrt{44}x^2 + .23x = 10$$

$$a = \sqrt{44}$$

$$b = .23$$

$$c = 246$$

This will probably be a nasty calculation, but that is what computers are for. The order does not matter, only that the a corresponds to the square term, the b to the 1st degree term, and the c to the constant.

Solutions

3. What happens if $b^2 - 4ac$ is negative in the quadratic formula

There are no real solutions, but complex (imaginary) solutions. This means numbers of the form $a + ib$ where $i = \sqrt{-1}$. These can still be useful - but that's for another class!

Solutions

4. Expand

$$(3x - 6)(2x + 1)$$

$$(3x - 6)(2x + 1)$$

$$6x^2 + 3x - 12x - 6$$

$$6x^2 - 9x - 6$$

Graphs

Team Assessment

Your team measured the concentrations of pesticides in a lake exposed to agricultural runoff. You have the following equation describing the total amount of pesticides in the lake if runoff is stopped from the farm by a new policy incentive reducing pesticides use:

$$y = (8 - 2t)(t + 2)$$

Where y is pesticde concentration in ppb and t is time in years.

Work with your team to

- **first** discuss conceptually how you would solve *all* the following tasks, do not write anything!
- **second** work out with pen and paper the solutions to verify the ideas you discussed.

1. What kind of equation is this? Would it be useful to write it in another form?
2. How long will it take for the pesticide concentration in the lake to reach zero? Since the equation is a polynomial describe why one solution is more applicable than the other.
3. Present your findings (choose between a graph or table).
4. What is the average change in concentration from year 0 to year 4?
5. Explain to your client why concentrations might behave the way they were modeled.

Solution 1

Let's expand out the equation so it becomes easier to graph.

$$y = (-2t + 8)(t + 2)$$

$$y = \overbrace{-2t^2}^{\text{First}} - \overbrace{4t}^{\text{Outside}} + \overbrace{8t}^{\text{Inside}} + \overbrace{16}^{\text{Last}}$$

$$y = -2t^2 + 4t + 16$$

Solution 2

Use the Quadratic Formula

$$0 = -2t^2 + 4t + 16$$

$$0 = \frac{-4 \pm \sqrt{4^2 - 4(-2)(16)}}{2(-2)}$$

$$0 = \frac{-4 \pm \sqrt{16 + 128}}{-4}$$

$$t = 4, t = -2$$

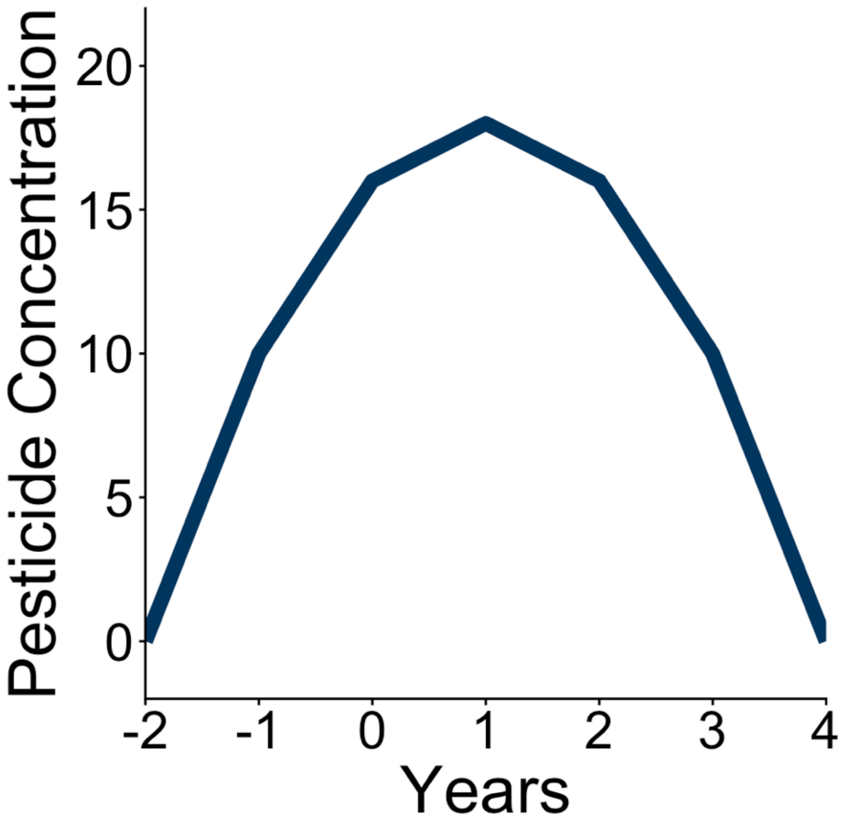
Use the factors

$$2t - 8 = 0$$

$$t = \frac{8}{2}$$

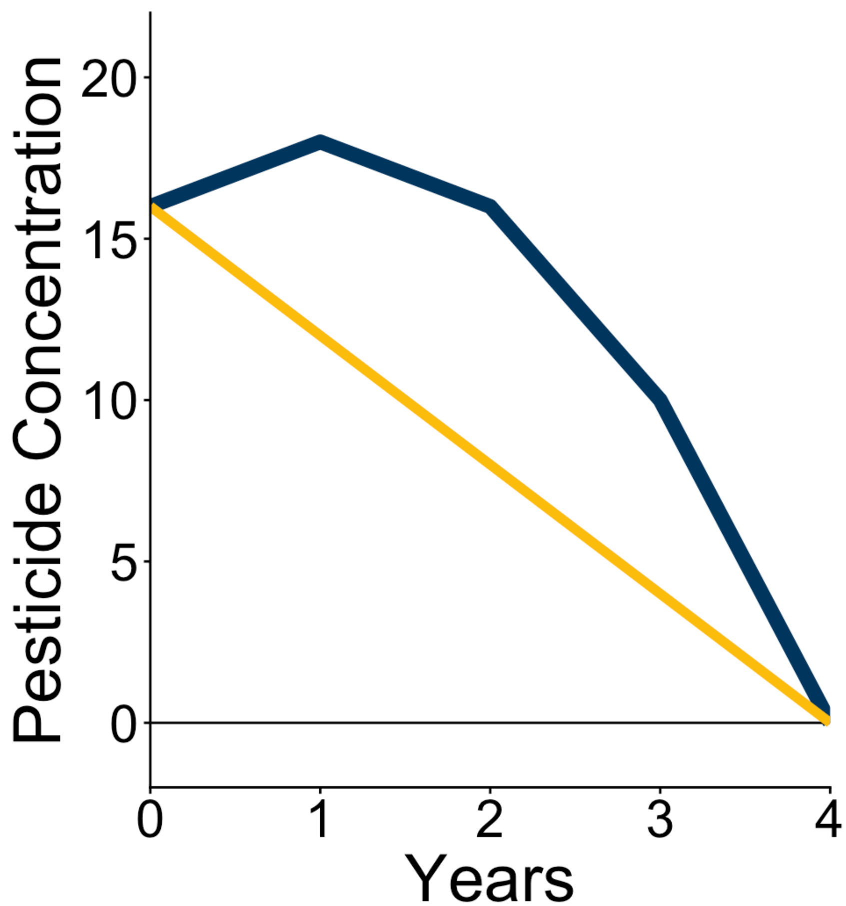
$$t = 4$$

Solution 3




```
t=seq(-2,4)
y=-2*t^2+4*t+16
df<-data.frame(t=t,y=y)
p<-df %>%
  ggplot(aes(x=t,y=y))+
  geom_line(color="#003660",linewidt
  labs(x="Years",y="Pesticide Concen
  scale_x_continuous(expand = c(0, 0
  scale_y_continuous(expand = c(0, 0
  theme_classic()+
  theme(text = element_text(size = 2

p
```

Solution 4



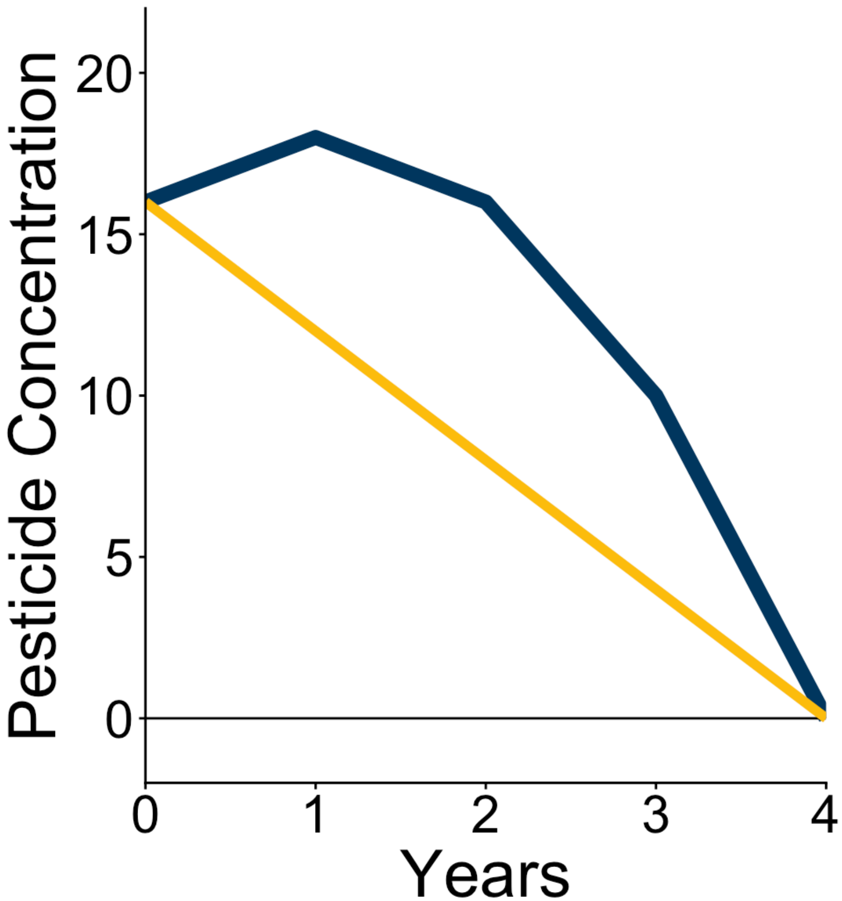

```r
p2<-df %>%
  filter(t>=0) %>%
  ggplot(aes(x=t,y=y))+
  geom_line(color="#003660",linewidtl
  labs(x="Years",y="Pesticide Concent
  scale_x_continuous(expand = c(0, 0)
  scale_y_continuous(expand = c(0, 0)
  geom_hline(yintercept = 0)+
  annotate("segment",x=0,xend=4,y=16,
  theme_classic()+
  theme(text = element_text(size = 28

p2
```

Use rise over run:

$$\frac{\Delta y}{\Delta x} = \frac{0-16}{4-0} = -4$$

Pesticides are removed from the lake at an average rate of 4 ppb per year.

Solution 5



Pesticide concentrations might initially increase in the lake from residual particles in the soil being washed into the lake. Then a mixture microbial activity and other chemical process reduce the pesticides to more inert components.

(You will learn the actual answer in ESM 202: Environmental Biogeochemistry)