Case Study: Multi-Spring Mass Oscillation Model

1. Introduction

This case study analyzes the oscillatory behavior of a mass attached to three springs arranged in parallel. The objective is to determine the effective spring constant of the system and the time period of vibration. The study demonstrates the application of vibration theory and linear algebra in mechanical system modeling.

2. Problem Statement

Three springs are connected to a mass m as shown in the figure. The spring constants are:

Left spring = k

Middle spring = 2k

Right spring = k

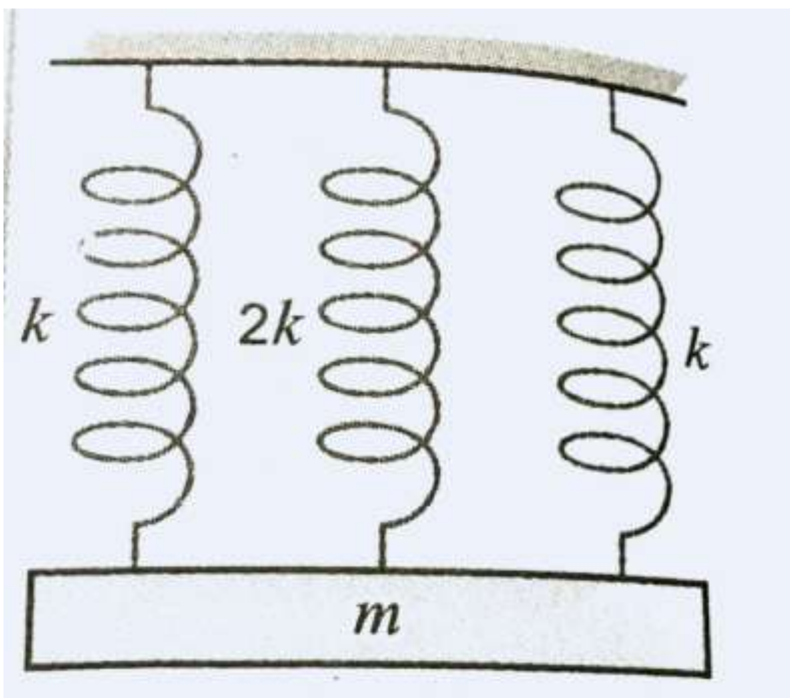
Given:

k = 2 N/m

m = 80 g = 0.08 kg

Determine the effective spring constant and time period of vibration.

System Diagram



3. Mathematical Formulation

$F_1 = kx,\ F_2 = 2kx,\ F_3 = kx$

$W = mg$

$\Sigma F = 0$

$kx + 2kx + kx - mg = 0$

$4kx = mg$

$x = \frac{mg}{4k}$

F on each spring

left → $F_1 = kx = k\left(\frac{mg}{4k}\right) = \frac{mg}{4}$

middle → $F_2 = 2kx = 2k\left(\frac{mg}{4k}\right) = \frac{mg}{2}$

right → $F_3 = kx = \frac{mg}{4}$

$\therefore\ F_1 + F_2 + F_3 = \frac{mg}{4} + \frac{mg}{2} + \frac{mg}{4} = \underline{\underline{mg}}$

For springs connected in parallel:

$k_{eff} = k + 2k + k = 4k$

Substituting $k = 2$ N/m:

$k_{eff} = 4(2) = 8$ N/m

Equation of motion:

$m x'' + k_{eff}\, x = 0$

Natural frequency:

$\omega_n = \sqrt{\frac{k_{eff}}{m}} / m$

$\omega_n = \sqrt{\frac{8}{0.08}} = 10$ rad/s

Time period:

$T = \frac{2\pi}{\omega_n}$

$T = \frac{2\pi}{10} \approx \underline{\underline{0.628 \text{ seconds}}}$

5. Results

Effective spring constant: 8 N/m

Natural frequency: 10 rad/s

Time period: approximately 0.63 seconds