MET AD 616 Assignment 2

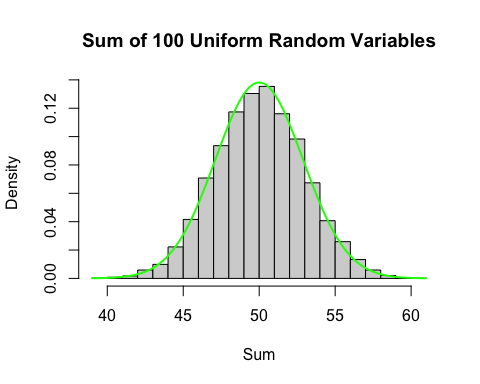
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# Question #1

## a)

num\_trials <- 10000  
sample\_size <- 100  
  
set.seed(100)   
sums <- replicate(num\_trials, sum(runif(sample\_size, 0, 1)))  
  
mean\_sums <- mean(sums)  
sd\_sums <- sd(sums)  
  
# i)  
hist(sums, breaks = 30, probability = TRUE, main = "Sum of 100 Uniform Random Variables",  
 xlab = "Sum", ylab = "Density")  
  
# ii)  
curve(dnorm(x, mean = sample\_size \* 0.5, sd = sqrt(sample\_size \* 1/12)),  
 col = "Green", lwd = 2, add = TRUE)



# iii)  
cat("The Mean of simulation is", mean\_sums, "\n")

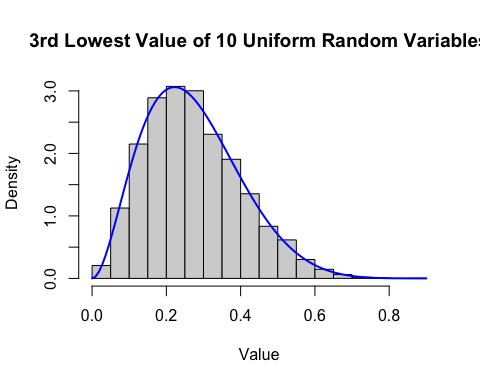
## The Mean of simulation is 50.03697

cat("The Standard deviation of simulation is", sd\_sums, "\n")

## The Standard deviation of simulation is 2.92671

## b)

num\_trials <- 10000  
sample\_size <- 10  
k <- 3  
  
set.seed(101)   
third\_lowest <- replicate(num\_trials, sort(runif(sample\_size, 0, 1))[k])  
  
mean\_third\_lowest <- mean(third\_lowest)  
sd\_third\_lowest <- sd(third\_lowest)  
  
# i)  
hist(third\_lowest, breaks = 30, probability = TRUE, main = "3rd Lowest Value of 10 Uniform Random Variables",  
 xlab = "Value", ylab = "Density")  
  
# ii)  
curve(dbeta(x, shape1 = k, shape2 = sample\_size + 1 - k), col = "blue", lwd = 2, add = TRUE)



# iii)  
cat("The Mean of simulation is", mean\_third\_lowest, "\n")

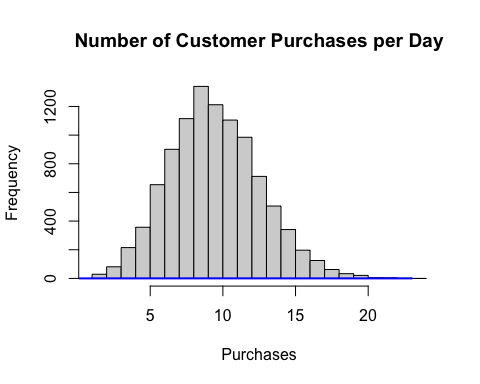
## The Mean of simulation is 0.2732965

cat("The Standard deviation of simulation is", sd\_third\_lowest, "\n")

## The Standard deviation of simulation is 0.1280419

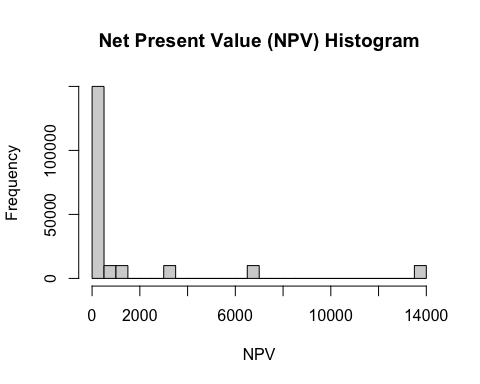
## Question #2

num\_trials <- 10000  
rate <- 10   
  
set.seed(101)  
purchases\_per\_day <- replicate(num\_trials, rpois(1, rate))  
  
hist(purchases\_per\_day, breaks = seq(min(purchases\_per\_day), max(purchases\_per\_day) + 1, by = 1),  
 main = "Number of Customer Purchases per Day", xlab = "Purchases", ylab = "Frequency")  
  
# Poisson Distribution with λ=10  
x <- 0:max(purchases\_per\_day)  
lines(x, dpois(x, rate), col = "blue", lwd = 2)



## Question #3

set.seed(103)  
num\_trials <- 10000  
shape <- 4.5  
scale <- 39  
term <- 20  
monthly\_payment <- 115  
death\_benefit <- 1000000  
lapsing\_prob <- 0.003  
discount\_rate <- 0.065  
initial\_cost <- monthly\_payment \* 12 \* term  
  
  
npv\_values <- replicate(num\_trials, {  
 cash\_flows <- initial\_cost  
 for (i in 1:term \* 12) {  
 prob\_death <- 1 - pweibull(i, shape, scale)  
 cash\_flows <- cash\_flows + monthly\_payment \* (1 - lapsing\_prob) \* (1 - prob\_death)  
 if (runif(1) < prob\_death && cash\_flows > death\_benefit) {  
 cash\_flows <- cash\_flows - death\_benefit  
 }  
 }  
 cash\_flows / (1 + discount\_rate)^(1:term\*12)  
})  
  
# a)   
hist(npv\_values, breaks = 30, main = "Net Present Value (NPV) Histogram",  
 xlab = "NPV", ylab = "Frequency")



# b)  
mean\_npv <- mean(npv\_values)  
sd\_npv <- sd(npv\_values)  
cat("The Mean NPV is", mean\_npv, "\n")

## The Mean NPV is 1311.242

cat("The Standard Deviation of NPV is", sd\_npv, "\n")

## The Standard Deviation of NPV is 3269.385

# c)   
ci\_95 <- quantile(npv\_values, c(0.025, 0.975))  
cat("The 95% Confidence Interval for Mean NPV is", ci\_95, "\n")

## The 95% Confidence Interval for Mean NPV is 0.008082359 13907.49

# d)   
desired\_half\_width <- 200  
  
iterations\_for\_99\_ci <- ceiling(qnorm(0.995) \* sd\_npv / desired\_half\_width)^2  
cat("The number of Iterations for 99% CI with $200 Half-Width is", iterations\_for\_99\_ci, "\n")

## The number of Iterations for 99% CI with $200 Half-Width is 1849

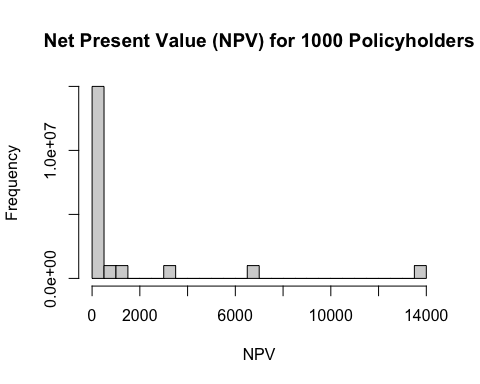
# e) NPV at 90% and 99% Confidence  
npv\_90\_percent <- quantile(npv\_values, 0.1)  
npv\_99\_percent <- quantile(npv\_values, 0.01)  
cat("The NPV at 90% Confidence is", npv\_90\_percent, "\n")

## The NPV at 90% Confidence is 0.03469478

cat("The NPV at 99% Confidence is", npv\_99\_percent, "\n")

## The NPV at 99% Confidence is 0.008082359

# Now with the assumption of having 1,000 policyholders  
set.seed(104)  
num\_trials <- 1000  
shape <- 4.5  
scale <- 39  
term <- 20  
monthly\_payment <- 115  
death\_benefit <- 1000000  
lapsing\_prob <- 0.003  
discount\_rate <- 0.065  
initial\_cost <- monthly\_payment \* 12 \* term  
  
npv\_values\_all\_policyholders <- list()  
  
for (j in 1:num\_trials) {  
 npv\_values <- replicate(num\_trials, {  
 cash\_flows <- initial\_cost  
 for (i in 1:term \* 12) {  
 prob\_death <- 1 - pweibull(i, shape, scale)  
 cash\_flows <- cash\_flows + monthly\_payment \* (1 - lapsing\_prob) \* (1 - prob\_death)  
 if (runif(1) < prob\_death && cash\_flows > death\_benefit) {  
 cash\_flows <- cash\_flows - death\_benefit  
 }  
 }  
 cash\_flows / (1 + discount\_rate)^(1:term\*12)  
 })  
 npv\_values\_all\_policyholders[[j]] <- npv\_values  
}  
  
all\_npv\_values <- unlist(npv\_values\_all\_policyholders)  
  
# a)  
hist(all\_npv\_values, breaks = 30, main = "Net Present Value (NPV) for 1000 Policyholders",  
 xlab = "NPV", ylab = "Frequency")



# b)   
mean\_npv\_all <- mean(all\_npv\_values)  
sd\_npv\_all <- sd(all\_npv\_values)  
cat("The Mean NPV for 1000 Policyholders is", mean\_npv\_all, "\n")

## The Mean NPV for 1000 Policyholders is 1311.242

cat("The Standard Deviation of NPV for 1000 Policyholders is", sd\_npv\_all, "\n")

## The Standard Deviation of NPV for 1000 Policyholders is 3269.377

# c)  
ci\_95\_all <- quantile(all\_npv\_values, c(0.025, 0.975))  
cat("The 95% Confidence Interval for Mean NPV of 1000 Policyholders is", ci\_95\_all, "\n")

## The 95% Confidence Interval for Mean NPV of 1000 Policyholders is 0.008082359 13907.49

# d)   
iterations\_for\_99\_ci\_all <- ceiling(qnorm(0.995) \* sd\_npv\_all / desired\_half\_width)^2  
cat("The number of Iterations for 99% CI with $200 Half-Width for 1000 Policyholders is", iterations\_for\_99\_ci\_all, "\n")

## The number of Iterations for 99% CI with $200 Half-Width for 1000 Policyholders is 1849

# e) NPV at 90% and 99% Confidence  
npv\_90\_percent\_all <- quantile(all\_npv\_values, 0.1)  
npv\_99\_percent\_all <- quantile(all\_npv\_values, 0.01)  
cat("The NPV at 90% Confidence for 1000 Policyholders is", npv\_90\_percent\_all, "\n")

## The NPV at 90% Confidence for 1000 Policyholders is 0.03469478

cat("The NPV at 99% Confidence for 1000 Policyholders is", npv\_99\_percent\_all, "\n")

## The NPV at 99% Confidence for 1000 Policyholders is 0.008082359