Exercises - Part 1

EXERCISES ON THE SUMMATION

Exercise 1. Re-write each of the following expressions using the summation symbol:

1. $x_1^2 + x_2^2 + x_3^2 + x_4^2 + \ldots + x_{15}^2$ $\left[\sum_{i=1}^{15} x_i^2\right]$
2. $ax_1 + ax_2 + ax_3 + \ldots + ax_4$ $\left[\sum_{i=1}^{4} ax_i = a\sum_{i=1}^{4} x_i\right]$
3. $(x_1 + y_1) + (x_2 + y_2) + \ldots + (x_8 + y_8)$ $\left[\sum_{i=1}^{8}(x_i + y_i)\right]$
4. $b_1x_1^3 + b_2x_2^3 + \ldots + b_{40}x_{40}^3$ $\left[\sum_{i=1}^{40} b_i x_i^3\right]$
5. $\frac{2a_1+2a_2+2a_3+2a_4+2a_5}{b_1+b_2+b_3+b_4+b_5}$ $\left[\frac{\sum_{i=1}^{5} 2a_i}{\sum_{i=1}^{5} b_i} = \frac{2\sum_{i=1}^{5} a_i}{\sum_{i=1}^{5} b_i}\right]$
6. $\frac{2a_1}{b_1} + \frac{2a_2}{b_2} + \frac{2a_3}{b_3} + \frac{2a_4}{b_4} + \frac{2a_5}{b_5}$ $\left[\sum_{i=1}^{5} \frac{2a_i}{b_i} = 2\sum_{i=1}^{5} \frac{a_i}{b_i}\right]$
7. $x_1 + x_2^2 + x_3^3 + x_4^4 + x_5^5$ $\left[\sum_{i=1}^{5} x_i^i\right]$

Exercise 2. Consider the following data:

a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
2	3	1	2	2	4	8	5	5	3	4	6	3	5	0

and solve the following expressions:

1. $\sum_{i=1}^{15} a_i$

   [53]

2. $\sum_{i=1}^{5} a_i^2$

   [22]

3. $\sum_{i=5}^{8} a_i$

   [19]

4. $\sum_{i=10}^{13} 3a_i$

   [48]

5. $\frac{\sum_{i=1}^{4} a_i}{\sum_{i=2}^{8} a_i}$

   [0.32]

6. $\sum_{i=1}^{3} a_i^i$

   [12]

7. $\sum_{i=1}^{5} a_i + \sum_{i=6}^{8} a_i$ [27]

   [2]

8. $\sum_{i=2}^{5} (a_i - 2)^2$

EXERCISES ON THE DISTRIBUTIONS

Exercise 1. A group of 120 students participates to a memory test. For each individual, the number of errors is recorded (X); the error distribution follows:

x_i	n_i
1	12
2	48
3	12
4	36
5	6
6	6
Total	120

1. Report the cumulative frequency distribution.

x_i	N_i
1	12
2	60
3	72
4	108
5	114
6	120

2. Compute the relative frequency distribution.

x_i	f_i
1	0.10
2	0.40
3	0.10
4	0.30
5	0.05
6	0.05

3. What is the mode?

   The mode is 2.

4. What is the median number of errors?

   The median number of errors is 2.5.

5. Compute the average number of errors.

   The average number of errors is 2.95.

6. Produce the new frequency distribution according to the following class organization:

x_i	n_i
$0 \dashv 3$	72
$3 \dashv 5$	42
$5 \dashv 6$	6
Total	120

Exercise 2. The following table includes data about the cholesterol concentration in a sample of patients:

Concentration (mg/dl)	f_i
$0 \dashv 20$	0.05
$20 \dashv 40$	?
$40 \dashv 60$	0.30
$60 \dashv 200$	0.45
Total	1.00

1. Fill the table.

Concentration (mg/dl)	f_i
$0 \dashv 20$	0.05
$20 \dashv 40$	0.20
$40 \dashv 60$	0.30
$60 \dashv 200$	0.45
Total	1.00

2. Given that the sample is constituted of 300 patients, recover the distribution of the absolute frequencies.

Concentration (mg/dl)	n_i
$0 \dashv 20$	15
$20 \dashv 40$	60
$40 \dashv 60$	90
$60 \dashv 200$	135
Total	300

3. How many patients have a cholesterol concentration that is not less than 40 mg/dl?
   Patients with a cholesterol concentration not less than 40 mg/dl are 225.

4. Compute the midpoint of each class.

Concentration (mg/dl)	$\bar{x}_i$
$0 \dashv 20$	10
$20 \dashv 40$	30
$40 \dashv 60$	50
$60 \dashv 200$	130

5. Compute the frequency density for each class.

Concentration (mg/dl)	h_i
$0 \dashv 20$	0.75
$20 \dashv 40$	3.00
$40 \dashv 60$	4.50
$60 \dashv 200$	0.96

6. Compute the average concentration of cholesterol.
   The average cholesterol concentration is equal to 80 mg/dl.

7. Determine the modal class.
   The modal class is $40 \dashv 60$ mg/dl.

8. Determine the median.
   The median class is $40 \dashv 60$ mg/dl. The median is:

$$m = 40 + \frac{0.5 - F_{20\dashv 40}}{f_{40\dashv 60}}(60-40) = 40 + \frac{0.50 - .25}{0.30}20 = 56.67$$