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EE 407 HW-2

Q1) a)

$$\Delta p = p - p_0 = \rho g h$$

$$w_0 = k \sqrt{\rho g h}$$

$$\frac{dV(t)}{dt} = w_i(t) - k \sqrt{\rho g h(t)}$$

$$A \frac{dh(t)}{dt} = \frac{w_i - k \sqrt{\rho g h}}{A}$$

b) $\Delta p = \rho g h(t)$

$$\frac{d\Delta p(t)}{dt} = \rho g \frac{dh(t)}{dt}$$

$$\dot{\Delta p} = \frac{\rho g}{A} (w_i - k \sqrt{\Delta p})$$

c) Steady-state: $\frac{dh(t)}{dt} = 0$

$$w_i(t) = v_{ss}, \quad h(t) = r$$

$$0 = \frac{v_{ss}}{A} - \frac{k}{A} \sqrt{\rho g r}$$

$$v_{ss} = k \sqrt{\rho g r}$$

Assume A is constant despite the figure.

Q2)

a) Biomass accumulation = in flow by - out by flow + gen rate of substrate

Substrate accumulation = in flow by - out by flow - consumption

$$\dot{x}_1 = 0 = V_1 \left(\frac{F}{V} + \mu_1 \right) x_1$$

$$\dot{x}_2 = x_2 f \frac{F}{V} - x_2 \frac{F}{V} - \mu_2 x_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 + \mu_1 & 0 \\ -\mu_1/\gamma & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} x_{2f}$$

b)

$$\frac{dx_1(t)}{dt} = \frac{dx_2(t)}{dt} = 0$$

$$0 = -x_1 \mu_1 + x_1 \mu_1 \Rightarrow x_1 = 0$$

$$0 = x_{2f} \mu_1 - x_2 \mu_1 - x_1 \mu_1/\gamma \Rightarrow x_2 = x_{2f}$$

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EE407 Homework 2

Q3)

a)

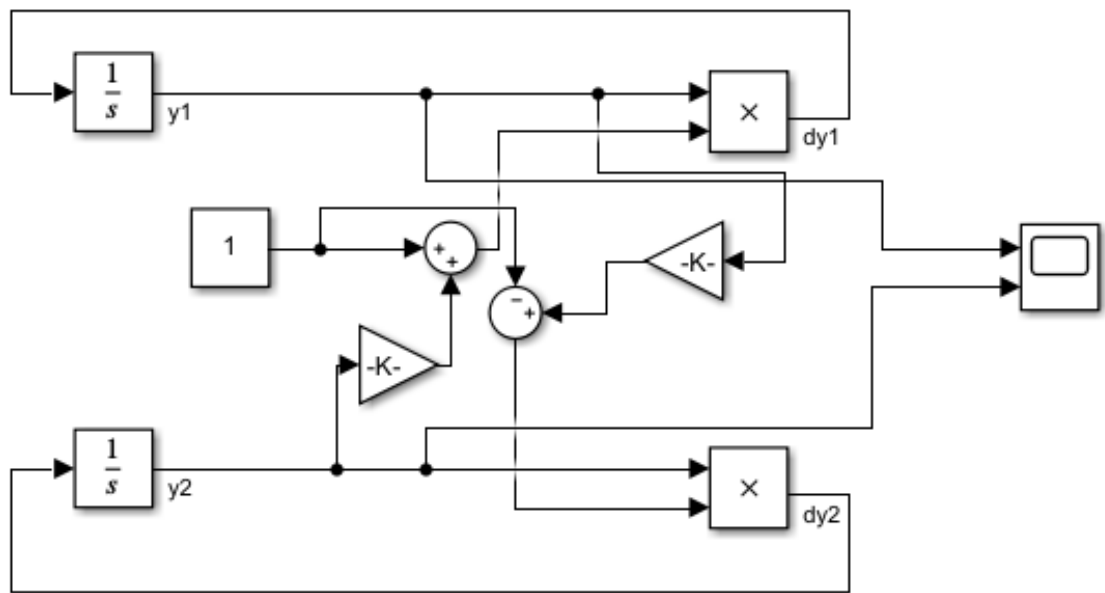


Figure 1: Prey-Predator System Implementation

b)

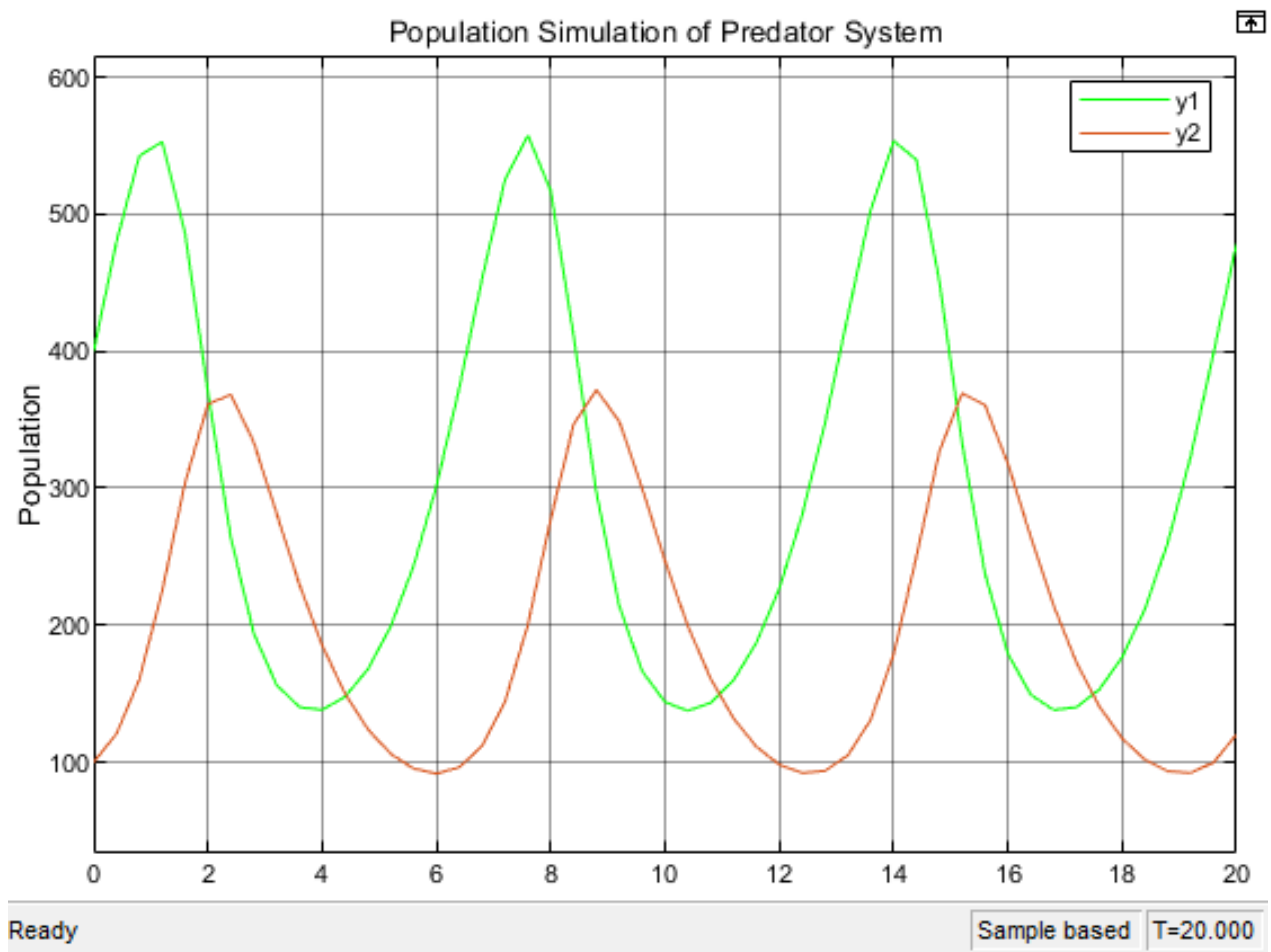


Figure 2: Prey-Predator System Simulation

c)

Population results are periodic with different phases. Peak values of prey are higher (minimum peak values are smaller) as we expect. Economical model has similar properties.

e)

Characteristics does not change

f)

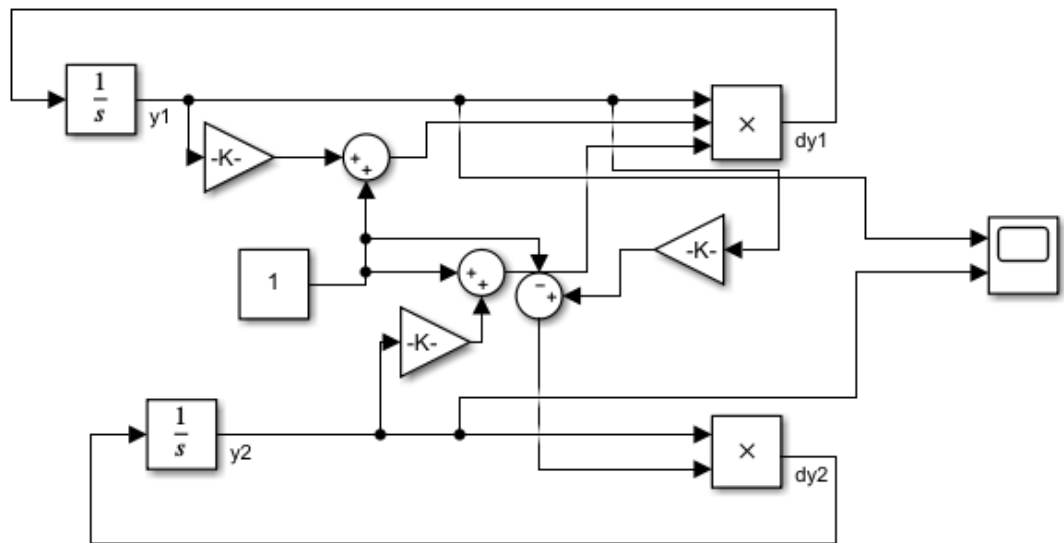


Figure 3: Prey-Predator System Implementation with Grow Limiting

g)

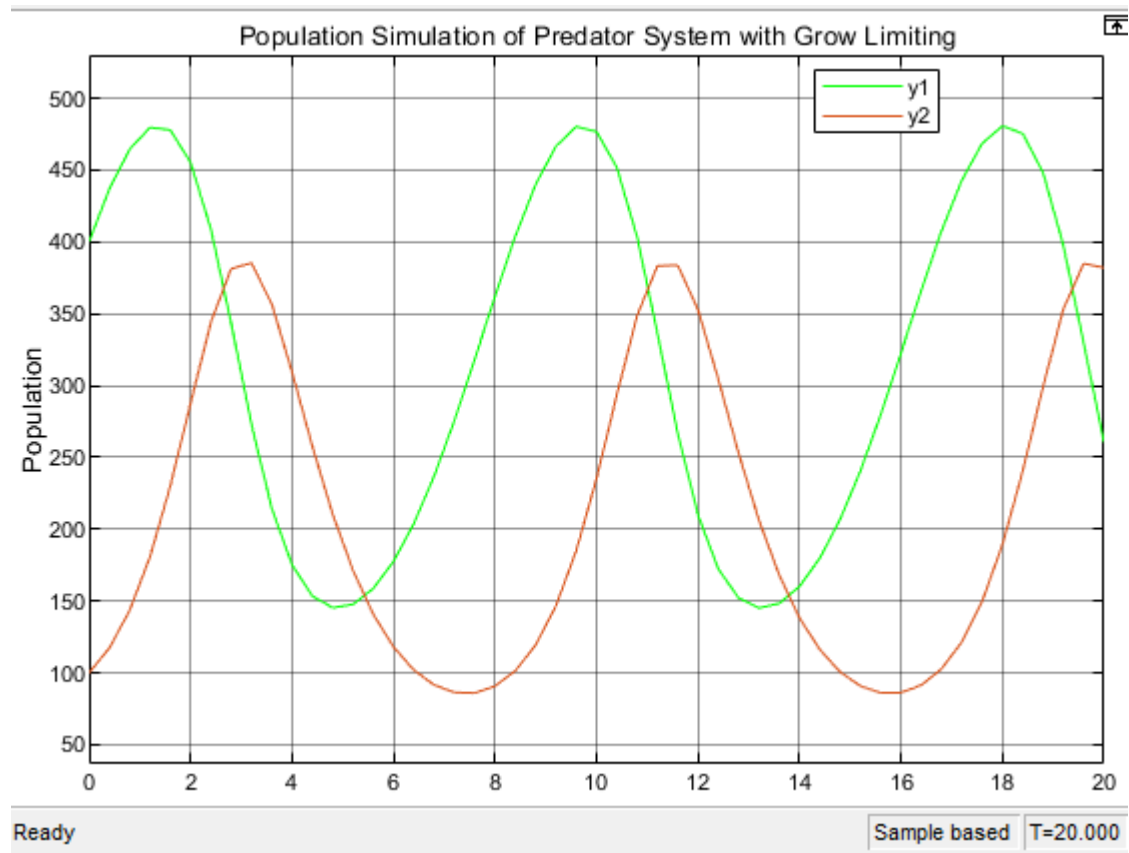


Figure 4: Prey-Predator System with Grow Limiting Simulation

System gets slower