



input: w_i
output: h

$$\Delta p = p - p_a \quad w_o = k \sqrt{\Delta p} \quad p = \rho g h + p_a$$

a) Accumulation = Inflow - Outflow

$$\frac{dh}{dt} = \alpha (w_i - w_o)$$

α is a constant relating Δw with the rate of height change.

Assuming $\alpha = 1$,

$$\dot{h} = w_i - w_o = w_i - k \sqrt{\Delta p} = w_i - k \sqrt{p - p_a} = w_i - k \sqrt{\rho g h + p_a - p_a}$$

$$\rightarrow \boxed{\dot{h} = w_i - k \sqrt{\rho g h}}$$

b) $\Delta p = \rho g h$

$$\Delta \dot{p} = \rho g \dot{h}$$

Then, $\Delta \dot{p} = \rho g (w_i - \underbrace{k \sqrt{\rho g h}}_{\Delta p})$

$$\boxed{\Delta \dot{p} = \rho g (w_i - k \sqrt{p})}$$

c) For constant r , we need $\dot{h} = 0$.

This means $w_i = w_o$, or $w_i - k \sqrt{\rho g h} = 0$

The input u_{ss} keeping output (h) at constant r ,

$$\boxed{u_{ss} = k \sqrt{\rho g r}}$$