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EE407: Homework 2

Q1)
(a): $\frac{dw}{dt} = q_i - q_o$, $q_o = Kw \Rightarrow w = q_o/K$

$$\frac{dq_o}{dt} = Kq_i - Kq_o \Rightarrow \boxed{\frac{dq_o}{dt} + Kq_o = Kq_i}$$

(b): with zero initial conditions : $\xrightarrow{\mathcal{L}} sQ_o(s) + KQ_o(s) = K\underbrace{Q_i(s)}_{1/s}$

$$sQ_o(s) + KQ_o(s) = K\,1/s$$

$$Q_o(s)(s+K) = K \cdot 1/s \Rightarrow Q_o(s) = \frac{K}{s(s+K)} = \frac{A}{s} + \frac{B}{s+K}$$

$$\Rightarrow A = \frac{\cancel{K}}{\cancel{K}} = 1$$

$$B = \frac{K}{-K} = -1$$

$$Q_o(s) = 1/s - \frac{1}{s+K} \Rightarrow \boxed{q_o(t) = (1 - e^{-Kt})u(t)}$$

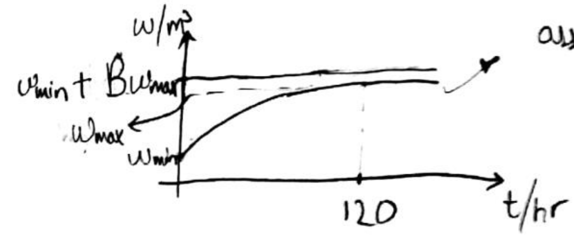
(c) $w(t) = \frac{q_o(t)}{K}$; $q_i(t) = u(t) \longrightarrow q_o(t) = (1-e^{-Kt})\,u(t)$

$q_i(t) = BKw_{max}u(t) \longrightarrow q_o(t) = (1-e^{-Kt})\,BKw_{max}\,u(t) + q_{o\,initial}$

$w(t) = (1-e^{-Kt})\,B\,w_{max}\,u(t) + \underbrace{w_{initial}}_{= w_{min}}$

not draining water before 5 days $\Rightarrow w_{max} \geq w(\overset{5\times 24\,hrs}{\overset{\shortparallel}{120}})$

$w(120) = (1-e^{-K120})\,B\,w_{max}\,\overbrace{u(120)}^{1} + w_{min}$



assuming $(1-e^{-K120}) \simeq 1 \Rightarrow$ steady state reached

$w(120) = w_{max} \Rightarrow$ water starts flowing over (reaches max) just after 5 days

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$$\omega_{max} = B\omega_{max} + \omega_{min} \Rightarrow \omega_{max}(1-B) = \omega_{min}$$

$$\boxed{\omega_{max} = \frac{\omega_{min}}{1-B}}$$

$$\omega_\delta = \omega_{max} - \omega_{min} = \omega_{min}\left(1 - \frac{1}{1-B}\right) = \boxed{\frac{B}{1-B}}, \quad \underline{B<0}$$

Q2:

(b)

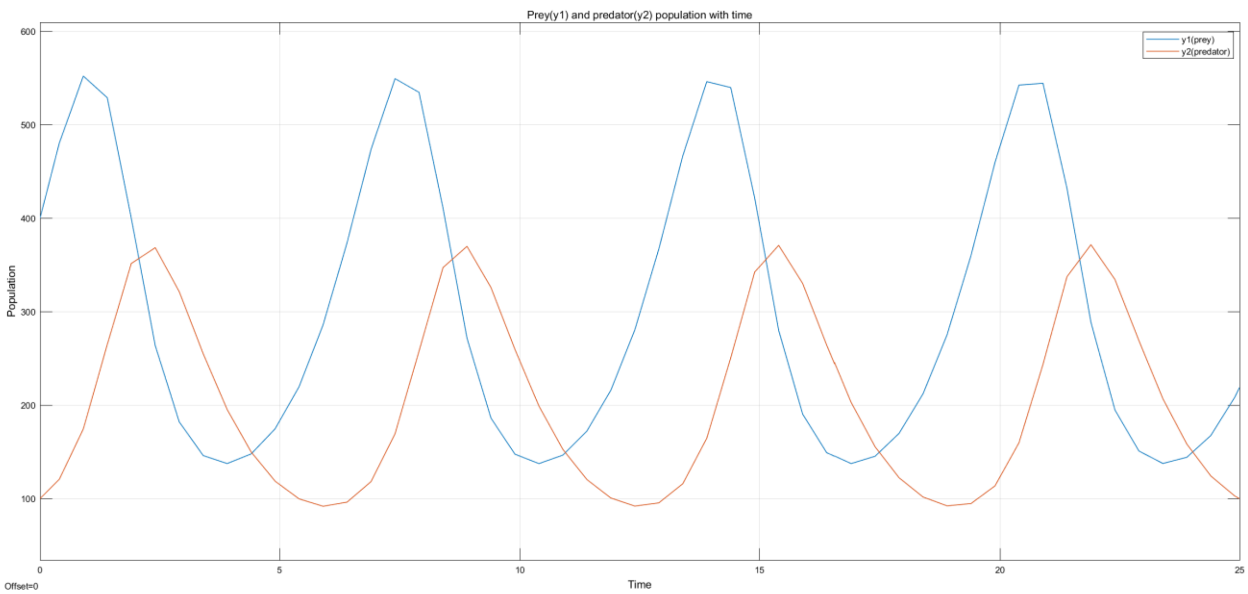

Figure 1: Predator Prey Population plot with given parameters

(c)

The population prey and predator show an oscillatory behavior with a certain phase. Upon close observation, it is noticeable that initially the predator and prey population are both on a rise. Once the predator population reaches close to the halfway point between its minimum and maximum value, the population of prey reaches its maximum and starts to decrease from there on. Once the prey population reaches close to the halfway between its maximum and minimum value, the predator population reaches its peak value and decreases from there on. After that point the population of both predator and prey decreases. The population of prey reaches its minimum first and starts to increase after. The prey then reaches its minimum and continues to increase after. This results in an oscillatory behavior of the population of the two species. The system is identical to

(e)

To find the equilibrium points, the derivative terms are equated to zero. Based on the assumption that the initial conditions are not zero, therefore, the values for y1 and y2 are not zero, the equilibrium point turns out to be (y1,y2) = (n1,n2). As the value of the population gets close to the equilibrium point, the period becomes very small and the difference between maxima and minima also becomes very small since the two populations are close to reaching a constant value, with no further change. Another simple nonlinear system that exhibits such behavior is the pendulum in a frictionless environment. Close to the equilibrium point $\Theta=0$, the system shows an oscillatory behavior with Θ and Θ', the two state variables having a phase difference, in the case there is no friction or damping.

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(g)

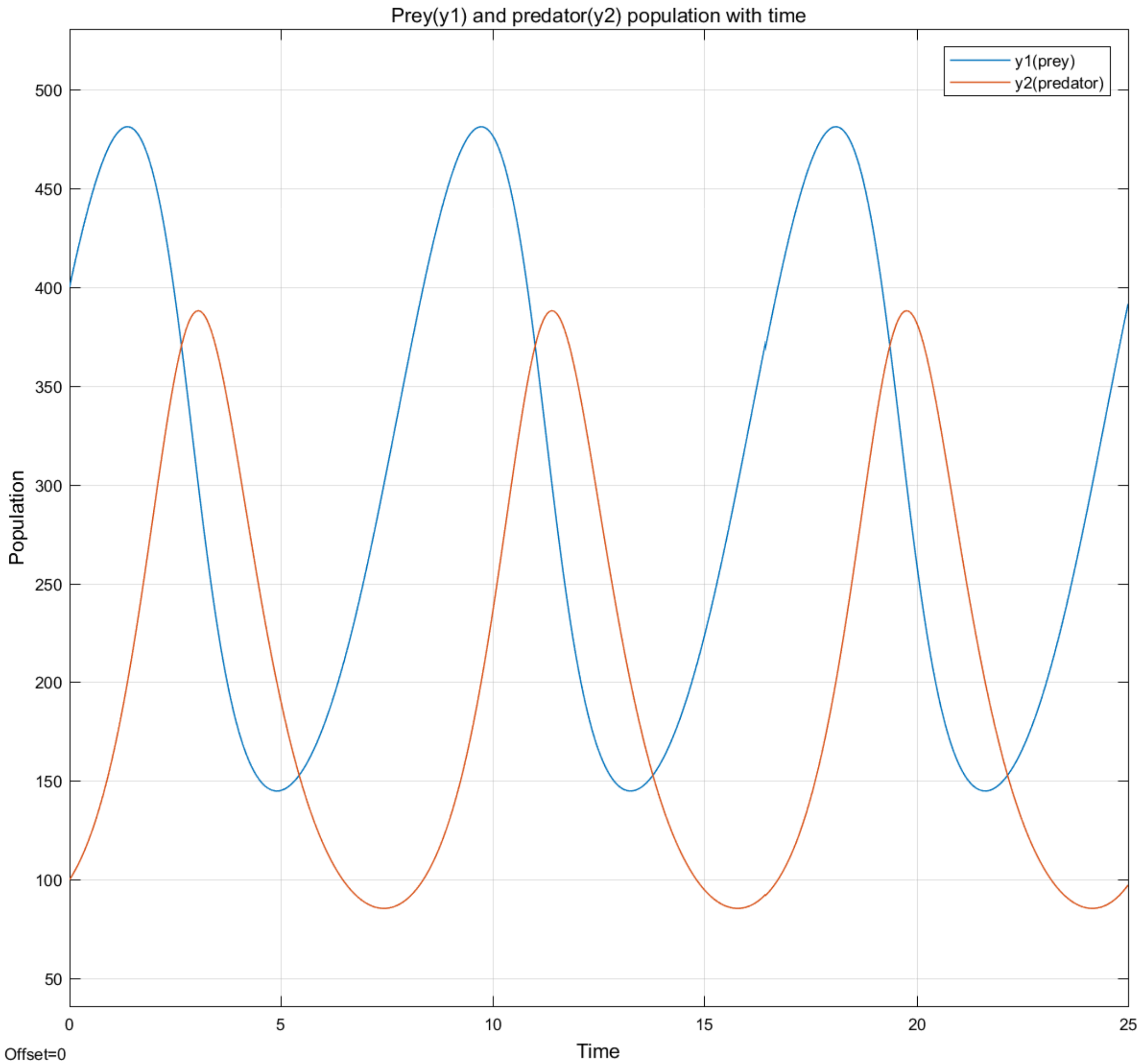

Figure 2: Predator Prey population plot with the prey population restricted

As per our expectations, the maximum of prey, which was earlier close to 550 has been limited to a value close to 480 now while the predator prey is the same as before. The shapes of the curves are identical to the ones from before, with the only difference being the extended period for this case. The elongated period is intuitive since the derivative term of the prey since to have decreased, taking longer for the change while the predator curve depends directly on the prey curve so the period of both has increased.

Q3:

(b)

$$K_p = \frac{\Delta y}{\Delta u} = \frac{1}{1} = 1$$

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$$\tau_p = \frac{1}{0.7}\left(t_{\frac{2}{3}} - t_{\frac{1}{3}}\right) = \frac{1}{0.7}(37 - 27) = 14.29$$

$$\theta_p = t_{\frac{1}{3}} - 0.4\tau_p = 27 - 0.4(14.29) = 21.29$$

(e)

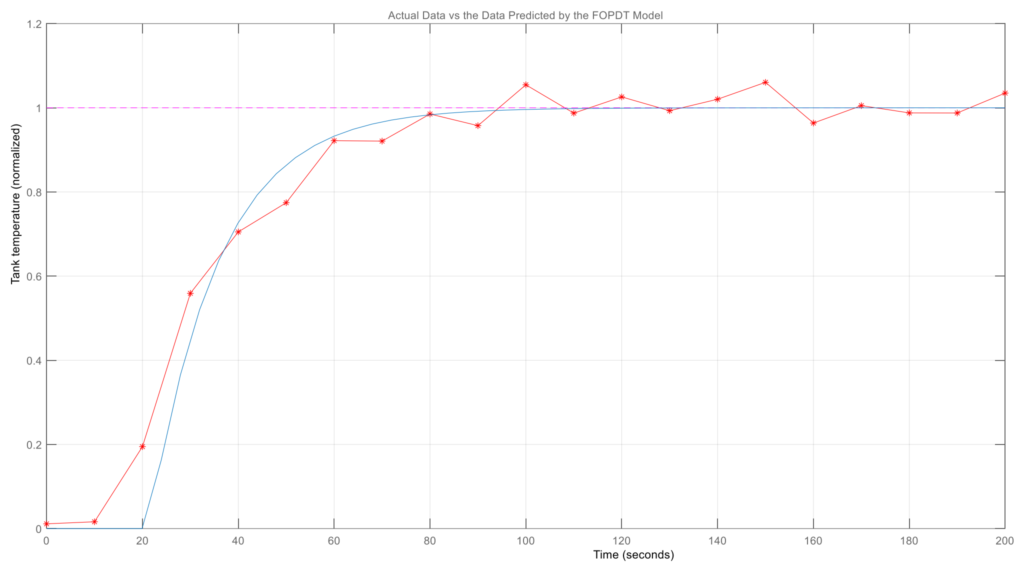

Figure 3: Unit Step Response of the FOPDT model of the heat exchanger system

(g)

Since the objective is servo control, i.e. to match the change in the input, the following formula would be utilized to calculate the proportional gain:

$$K_c = \frac{0.202}{K_p}\left(\frac{\theta_p}{\tau_p}\right)^{-1.219} = 0.124$$

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(i)

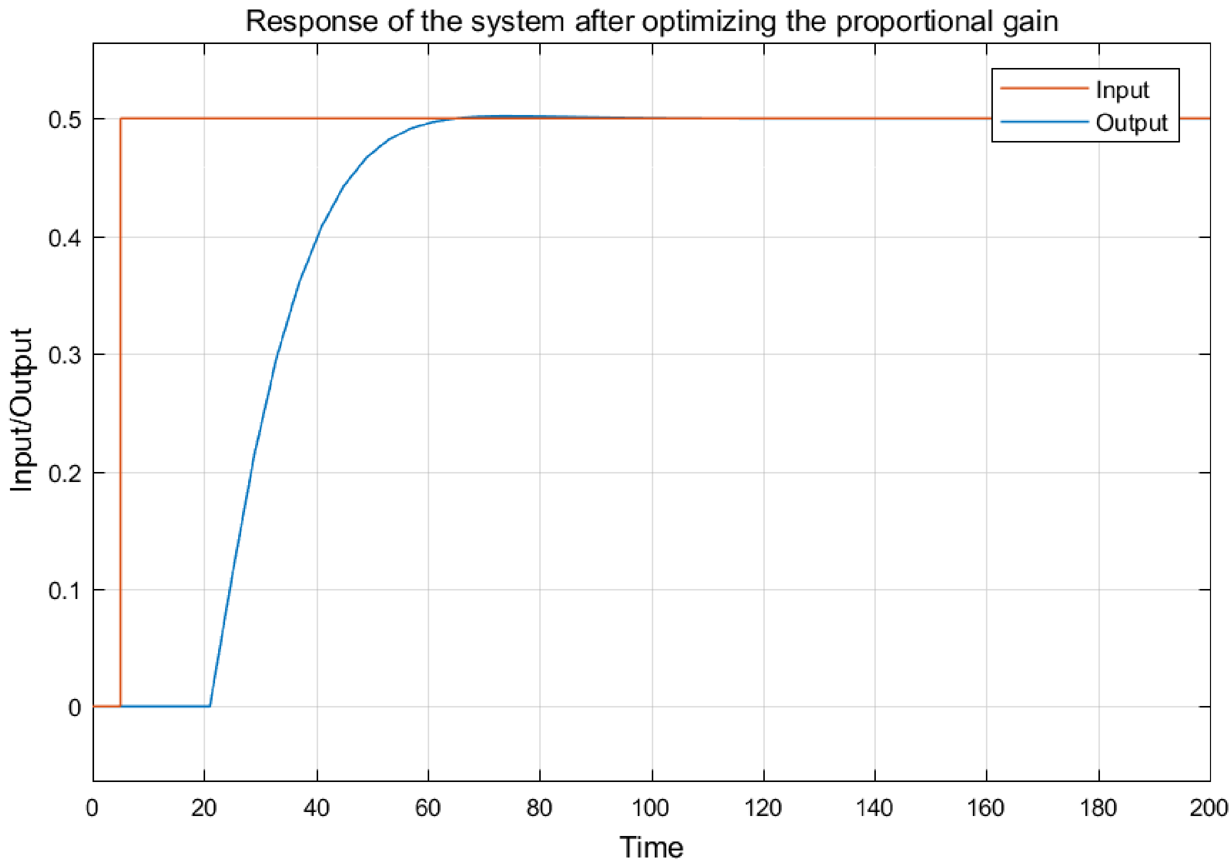

Figure 4: The response of the system after optimizing the proportional gain

The optimal parameter turned out to be extremely close to the predicted value as calculated above so the initial guess was accurate to a fairly high degree. The final value of Kc is 0.11, which is very close to the value calculated above. The tuning procedure was fairly simple. The value of gain was altered close to the guessed value of the Kc in order to obtain the best response. It did require a bit of trial and error but eventually a decent value was obtained.