Cosmological weak lensing of galaxy sources

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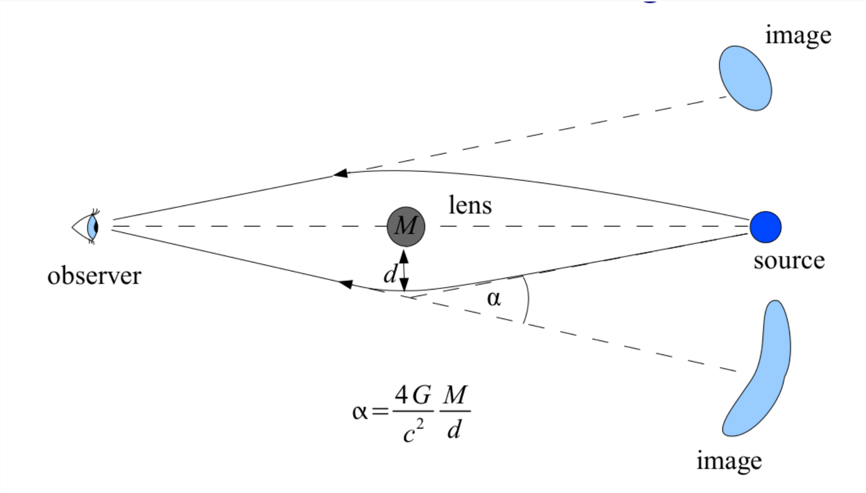
Curso de Primavera 2022 en “Weak Lensing: theory & estimators”

Curso IAC

1. Introduction and projected fields



Credit: Stefan Hilbert (MPA)

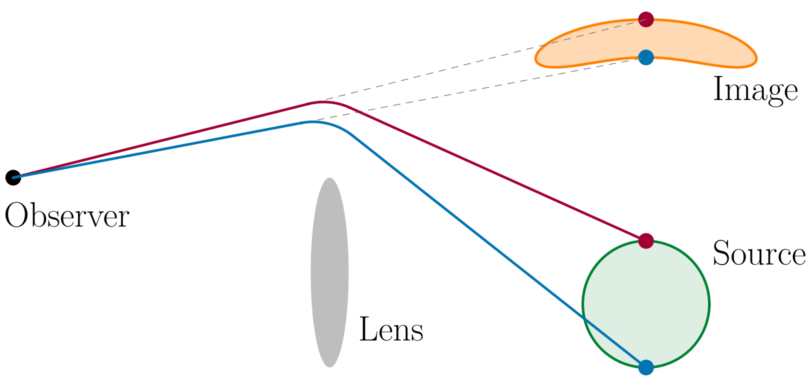


Image
Observer
Source
Lens

What this course is NOT about:

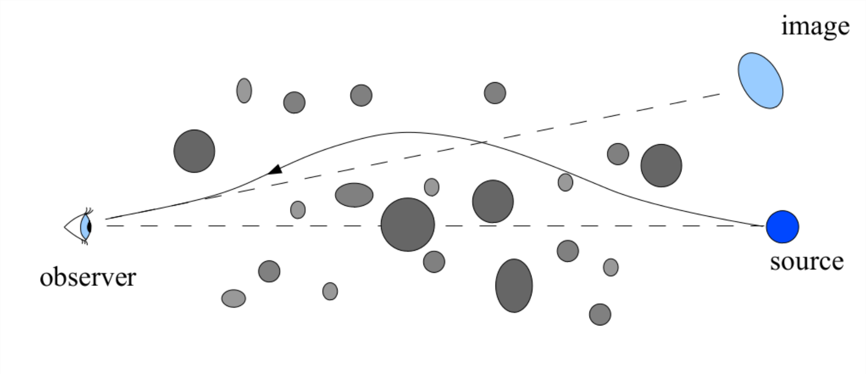
- Strong Lensing
- Time delays: e.g. H0LiCOW arxiv:1907.04869.
- Microlensing
- CMB lensing

This is not an introductory course to Cosmology. I will assume you know basic stuff ranging from background cosmology to transfer functions. I will assume you know what is a power spectrum and a correlation function (matter, 3-dim case) .

This is an introductory course to Weak Lensing

Gravitational lensing is an important tool because light paths respond directly to the gravitational potentials. That is, lensing probes the whole energy-momentum tensor, including, of course, its dark components.

- Dodelson & Schmidt, *Modern Cosmology*. Second Edition. Academic Press
- M. Kilbinger, *Cosmology with cosmic shear observations: a review*. Rep. Prog. Phys. 78 (2015) 086901. [arxiv:1411.0115].
- R. Mandelbaum, *Weak lensing for precision cosmology*. Ann. Rev. Astron. Astrophys. 56 (2018) 393. [arxiv:1710.03235]



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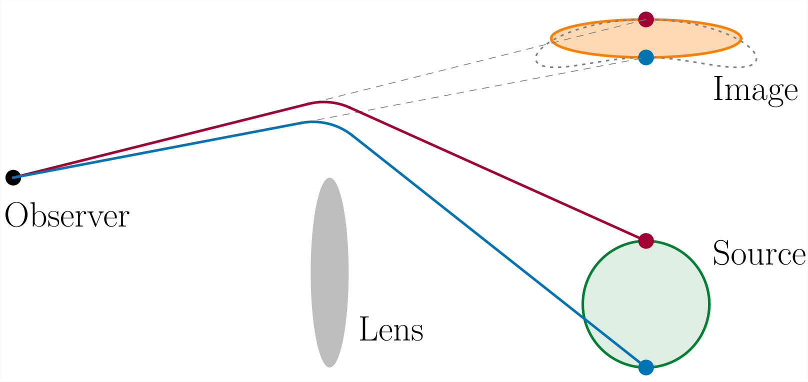
Image
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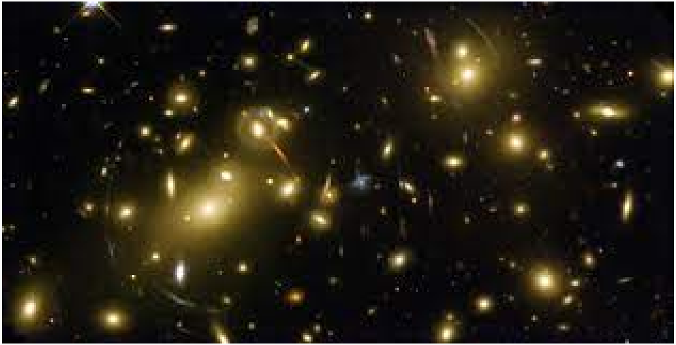
The statistical properties of the cosmic shear are directly linked to the statistical properties of density fluctuations (that is, to the total matter power spectrum). Hence, contrary to other Cosmological tests, WL is probing the dark matter itself



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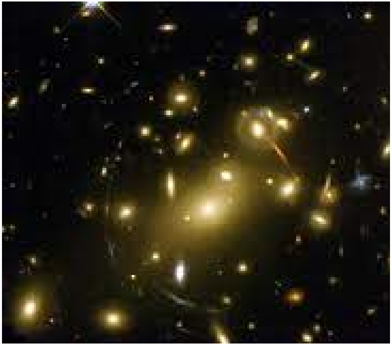
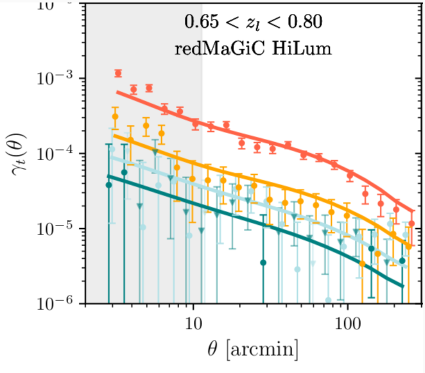
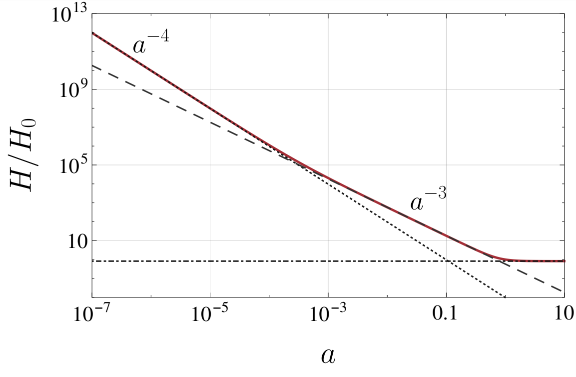




Background (homogeneous and isotropic) Cosmology

Cosmological Principle $\quad + \quad$ General Relativity

$$\Longrightarrow \qquad H^2 = \frac{8\pi G}{3}\sum \bar{\rho}_A + \frac{\Lambda}{3} - \frac{k}{a^2}, \qquad \dot{\bar{\rho}}_A = -3H(\bar{\rho}_A + \bar{P}_A)$$



Comoving radial distance χ

The starting point for the calculation of distances in cosmology is the comoving distance. Consider the comoving distance between a distant light source and us. In a small time interval dt, light travels a comoving *radial* distance

$$d\chi = \frac{dt}{a} \qquad \text{for} \quad d\chi = \frac{dr}{\sqrt{1-kr^2}}$$

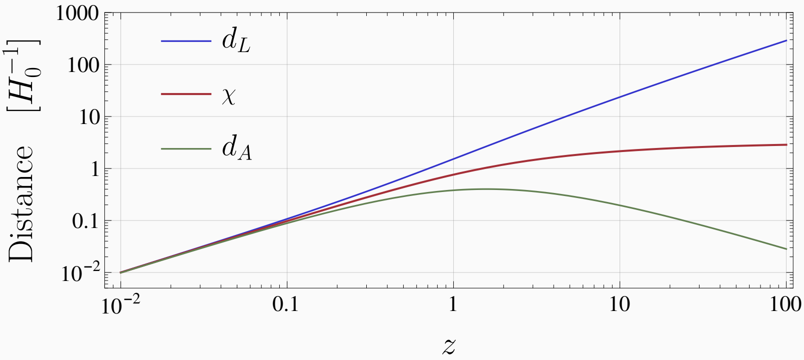
such the total comoving distance traveled by light emitted by a source at time t, when the scale factor was equal to $a = a(t)$, up to today is

$$\chi(t) = \int_t^{t_0} \frac{dt'}{a(t')} = \int_0^z \frac{dz'}{H(z')}$$

Conformal time: $$\eta(t) = \int_0^t \frac{dt'}{a(t')} = \eta_0 - \chi(t)$$

Summary of distances

$$\chi(z) = \frac{c}{H_0}\int_0^z \frac{dz'}{H(z')/H_0}, \quad d_L(z) = (1+z)\chi(z), \quad d_A(z) = \frac{1}{1+z}\chi(z)$$



At small redshift, the three distances coincide: $d = H_0^{-1} z$, (or $z = H_0 d$)

Longitudinal Doppler effect: $\lambda_0/\lambda_S = \sqrt{(1+v)/(1-v)} \approx 1 + v$, then $v = H_0 d$

Cosmological principle

Background: The universe is homogeneous and isotropic

$$X(\boldsymbol{x},t) = X(\boldsymbol{x}+\boldsymbol{b},t), \qquad X(\boldsymbol{x},t) = X(\mathbf{R}\,\boldsymbol{x},t)$$

+ perturbations: The universe is *statistically* homogeneous and isotropic

$$\begin{aligned}\langle X(\boldsymbol{x}_1,t)Y(\boldsymbol{x}_2,t)\cdots Z(\boldsymbol{x}_n,t)\rangle &= \langle X(\boldsymbol{x}_1+\boldsymbol{b})Y(\boldsymbol{x}_2+\boldsymbol{b})\cdots Z(\boldsymbol{x}_n+\boldsymbol{b})\rangle \\ &= \langle X(\mathbf{R}\,\boldsymbol{x}_1)Y(\mathbf{R}\,\boldsymbol{x}_2)\cdots Z(\mathbf{R}\,\boldsymbol{x}_n)\rangle\end{aligned}$$

Density fluctuations

$$\rho(\boldsymbol{x}, t) = \bar{\rho}(t)(1 + \delta(\boldsymbol{x}, t))$$

Since $\quad \bar{\rho}(t) = \langle \rho(\boldsymbol{x}, t) \rangle, \quad$ then $\quad \langle \delta(\boldsymbol{x}, t) \rangle = 0.$

2-point correlation function (2PCF): $\qquad \xi(\boldsymbol{x}_1, \boldsymbol{x}_2) = \langle \delta(\boldsymbol{x}_1, t)\delta(\boldsymbol{x}_2, t) \rangle$

$$\xi(\boldsymbol{x}_1, \boldsymbol{x}_2) \quad \underset{\text{homogeneity}}{\longrightarrow} \quad \xi(\boldsymbol{x}_2 - \boldsymbol{x}_1) \quad \underset{\text{isotropy}}{\longrightarrow} \quad \xi(r = |\boldsymbol{x}_2 - \boldsymbol{x}_1|)$$

Fourier Transform conventions

$$f(\boldsymbol{k}) = \int d^3x\, e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} f(\boldsymbol{x}), \qquad f(\boldsymbol{x}) = \int \frac{d^3k}{(2\pi)^3}\, e^{i\boldsymbol{k}\cdot\boldsymbol{x}} f(\boldsymbol{k})$$

Hence

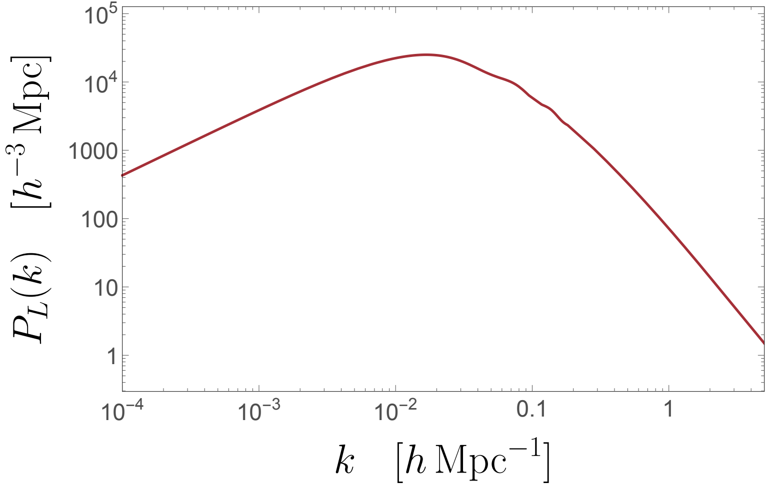
$$(2\pi)^3 \delta_{\rm D}(\boldsymbol{k}+\boldsymbol{k}') = \int d^3x\, e^{i(\boldsymbol{k}+\boldsymbol{k}')\cdot\boldsymbol{x}}$$

For a real field $f(\boldsymbol{x})$

$$f(\boldsymbol{k}) = f^*(-\boldsymbol{k})$$

Matter power spectrum

$$\langle \delta(\boldsymbol{k})\delta(\boldsymbol{k}')\rangle = (2\pi)^3\delta_{\mathrm{D}}(\boldsymbol{k}+\boldsymbol{k}')P_L(k)$$

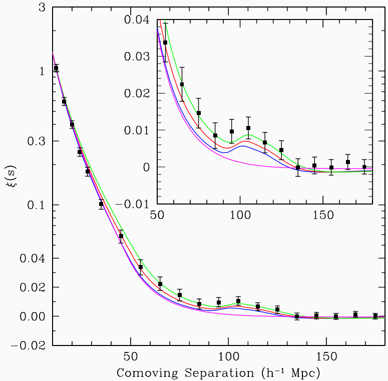


The (2-point) matter correlation function

SUBMITTED TO *The Astrophysical Journal* 12/31/2004
Preprint typeset using LaTeX style emulateapj v. 11/12/01

DETECTION OF THE BARYON ACOUSTIC PEAK IN THE LARGE-SCALE CORRELATION FUNCTION OF SDSS LUMINOUS RED GALAXIES

DANIEL J. EISENSTEIN[1,2], IDIT ZEHAVI[1], DAVID W. HOGG[3], ROMAN SCOCCIMARRO[3], MICHAEL R.



The correlation function is the Fourier Transform of the power spectrum

$$\xi(r) = \int \frac{d^3k}{(2\pi)^3} e^{i\boldsymbol{k}\cdot\boldsymbol{r}} P(k)$$

Because $P(\boldsymbol{k}) = P(k)$, the angular integral can be performed analytically

$$\xi(r) = \frac{1}{2\pi^2} \int_0^\infty dk\, k^2 P(k) j_0(kr)$$

with

$$j_0(x) = \frac{\sin x}{x}$$

the spherical Bessel function of degree zero.

This is a Hankel transformation. Due to the oscillatory behaviour of j_0, is computational expensive to integrate it correctly.

Use FFTLog methods: Hamilton (2000), arxiv:9905191 [astro-ph].

Numerical Issues

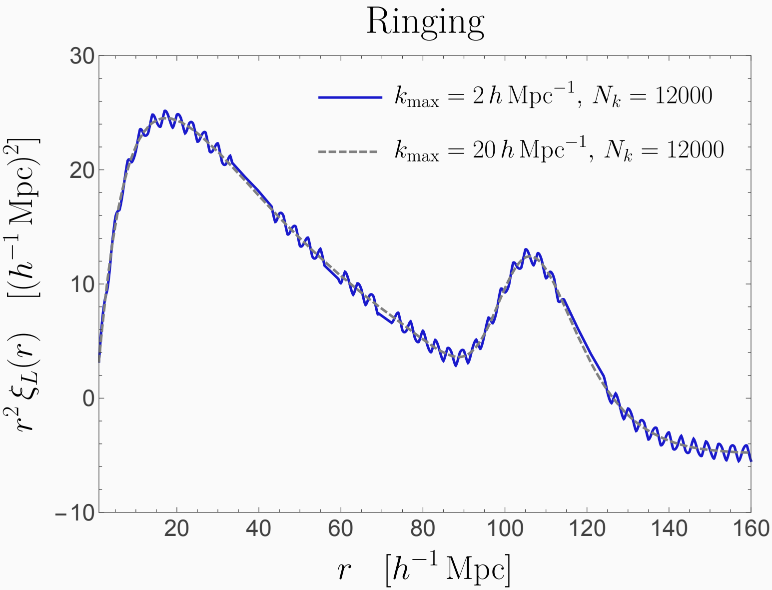
$$\xi(r) = \frac{1}{2\pi^2}\int_0^{\infty} dk\, k^2 P(k) j_0(kr)$$
$$= \frac{1}{2\pi^2}\int_{k_{\min}}^{k_{\max}} dk\, k^2 P(k) j_0(kr)$$

$$\xi(r) = \sum_{i=1}^{N_k} \frac{k_i^3}{2\pi^2} P(k_i) j_0(k_i r) \Delta(\log k_i)$$

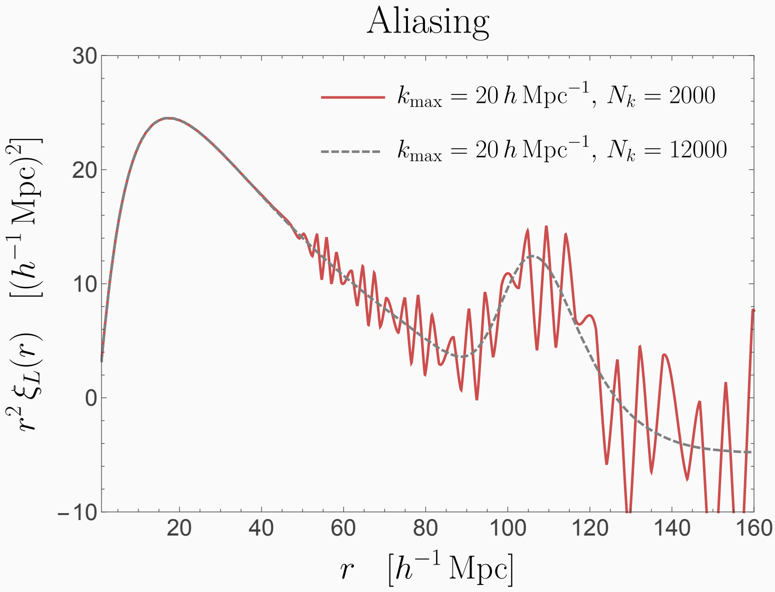
with

$$i = 1,\, 2, \cdots, N_k$$
$$k_i \in \{k_1 = k_{\min},\, k_2, \cdots, k_{N_k} = k_{\max}\}$$

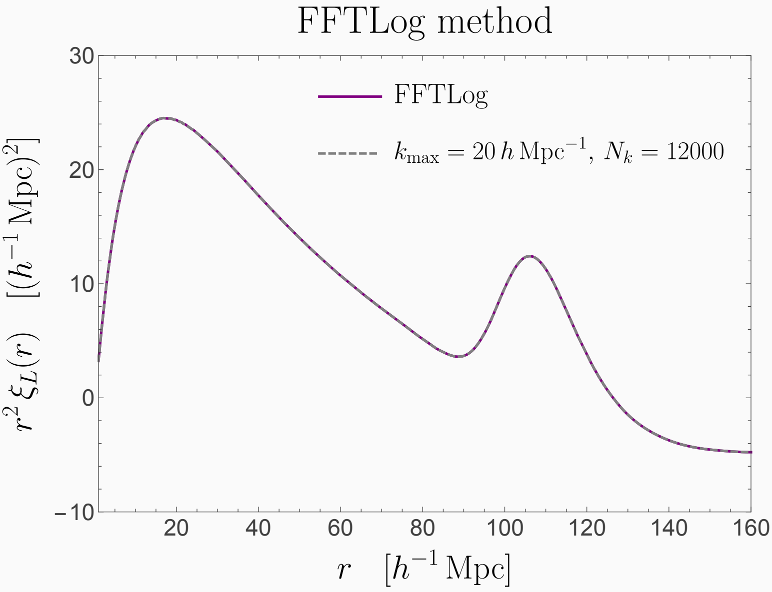
Ringing: cutting off high frequencies



Aliasing: poor sampling



FFTLog. Hamilton (2000)



Matter fluctuations linear evolution

Boltzmann eq. $\longrightarrow$ Fluid eqs. $\longrightarrow$ linearize

$$\Longrightarrow \quad \ddot{\delta}(\boldsymbol{k}, t) + 2H\dot{\delta} - \frac{3}{2}\Omega_m(a)H^2\delta = 0.$$

This equation does not depend on $\boldsymbol{k}$, hence the solution is separable

$$\delta(\boldsymbol{k}, t) = D_+(t)A(\boldsymbol{k}) + D_-(t)B(\boldsymbol{k})$$

with $D_\pm$ the solutions to the equation

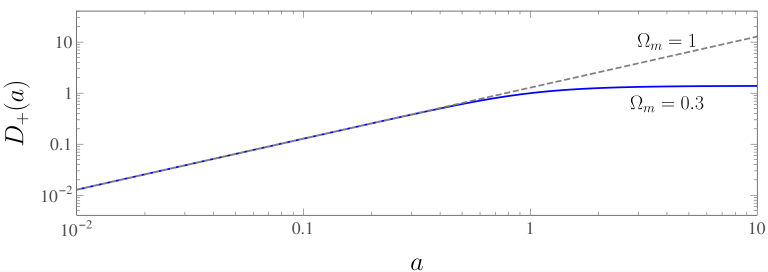
$$\left(\frac{d^2}{dt^2} + 2H\frac{d}{dt} - \frac{3}{2}\Omega_m(a)H^2\right)D(t) = 0$$

Linear growth function

The fastest growing solution is called the *linear growth function* D_+, then

$$\delta^{(1)}(\boldsymbol{k}, t) = D_+(t)\delta^{(1)}(\boldsymbol{k}, t_0)$$

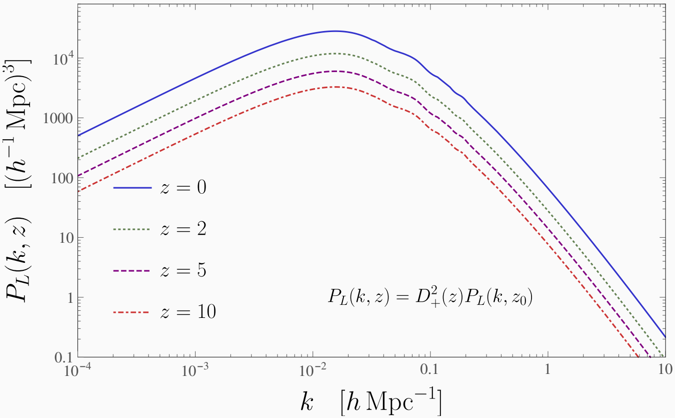
where one normalize $D_+(t_0) = 1$. Typically one chooses t_0 to be the present time. The other solution is $D_- \propto H$.

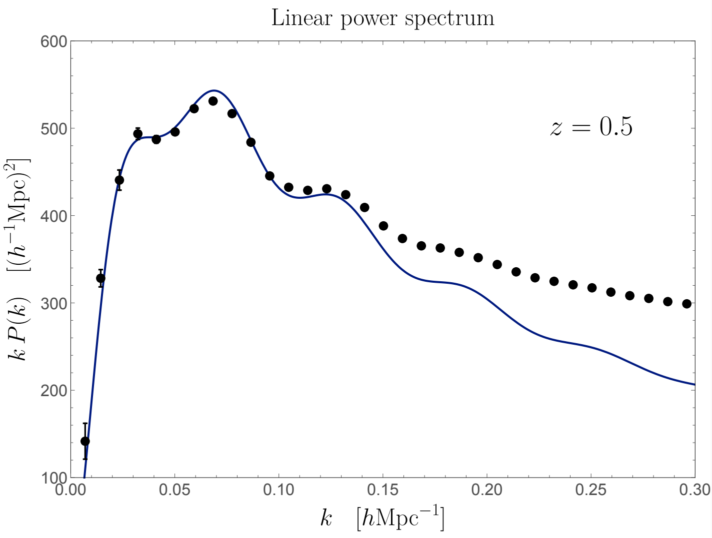


Linear power spectrum

$$\langle \delta^{(1)}(\boldsymbol{k},t)\delta^{(1)}(\boldsymbol{k}',t)\rangle = D_+^2(t)\langle \delta^{(1)}(\boldsymbol{k},t_0)\delta^{(1)}(\boldsymbol{k}',t_0)\rangle$$

$$\Longrightarrow \qquad P_L(k,t) = D_+^2(t)P_L(k,t_0)$$





Linear power spectrum
$z = 0.5$
$k\,P(k)\quad [(h^{-1}\mathrm{Mpc})^2]$
$k\quad [h\mathrm{Mpc}^{-1}]$
600
500
400
300
200
100
0.00
0.05
0.10
0.15
0.20
0.25
0.30

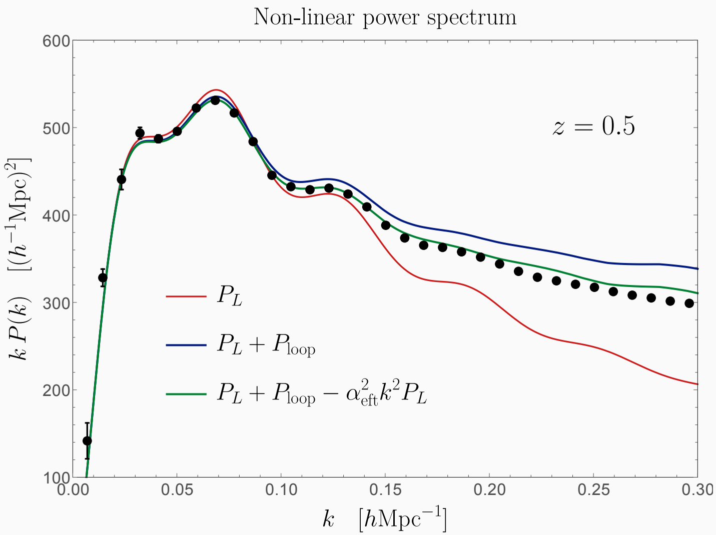
Perturbation theory

$$P^{\text{SPT/EFT}}(k) = (1 - \alpha_{\text{EFT}} k^2) P_L(k) + P_{\text{1-loop}}(k) + \cdots$$
$$P_{\text{1-loop}}(k) = P_{22}(k) + P_{13}(k)$$

with

$$P_{22}(k) = \langle \delta^{(2)}(\boldsymbol{k}) \delta^{(2)}(\boldsymbol{k}') \rangle' = 2 \int \frac{d^3p}{(2\pi)^3} \left[F_2(\boldsymbol{p}, \boldsymbol{k} - \boldsymbol{p}) \right]^2 P_L(p) P_L(|\boldsymbol{k} - \boldsymbol{p}|)$$
$$P_{13}(k) = 2 \langle \delta^{(1)}(\boldsymbol{k}) \delta^{(3)}(\boldsymbol{k}') \rangle' = 6 P_L(k) \int \frac{d^3p}{(2\pi)^3} F_3(\boldsymbol{k}, -\boldsymbol{p}, \boldsymbol{p}) P_L(p)$$

$$P^{\text{EFT}}_{\text{1-loop}}(k,t) = \left[1 - \alpha_{\text{EFT}}(t) k^2 \right] D_+^2(t) P_L(k, t_0) + D_+^4(t) P_{\text{1-loop}}(k, t_0) + \mathcal{O}(D_+^6 P^3)$$



Non-linear power spectrum
$z = 0.5$
P_L
$P_L + P_{\mathrm{loop}}$
$P_L + P_{\mathrm{loop}} - \alpha_{\mathrm{eft}}^2 k^2 P_L$
$k\,P(k)\quad [(h^{-1}\mathrm{Mpc})^2]$
$k\quad [h\mathrm{Mpc}^{-1}]$
600
500
400
300
200
100
0.00
0.05
0.10
0.15
0.20
0.25
0.30

Bias

We observe galaxies in the sky. But galaxies do not exactly follow the matter distribution, they are biased tracers. To linear order (at large scales) the galaxy density fluctuation is

$$\delta_g(\boldsymbol{x}) = b_1 \delta(\boldsymbol{x}) \qquad \longrightarrow \qquad P_g(k) = b_1^2 P_L(k)$$

Formal theory of bias (McDonald & Roy [JCAP 08 (2009) 020])

$$\delta_g(\boldsymbol{x}) = \sum_{\mathcal{O}} b_{\mathcal{O}} \mathcal{O}(\boldsymbol{x})$$

where $\mathcal{O}$ are a set of local and non-local operators (functions) of the gravitational and velocity potentials. e.g.: $\delta, \delta^2, \cdots \in \mathcal{O}$

Angular power spectra and correlations

Photometric redshift

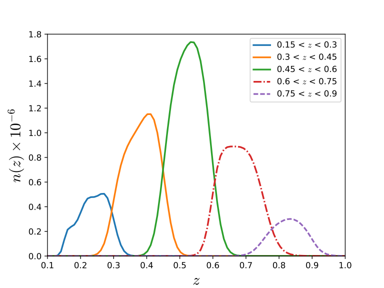
We do not always have access to accurate redshift measurements of astronomical objects. This is the case of, for example, photometric surveys. However, we do have high-quality images in several wavelength bands. These colors can be converted into rough estimates of redshift, so-called photometric redshifts, which can be used as proxies (with significant spread) for true redshifts.

More precisely, we can infer distributions of number of galaxies,

$$W_g(z) = \frac{1}{N_g}\frac{dN_g}{dz},$$

and depending on the features of each galaxy decide to which distribution it belongs.

DES 1 yr results (1708.01536)

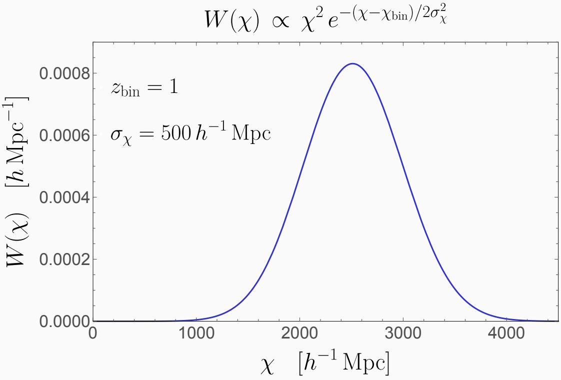


Instead of the redshift z, it is common to use the comoving radial distance $\chi(z)$

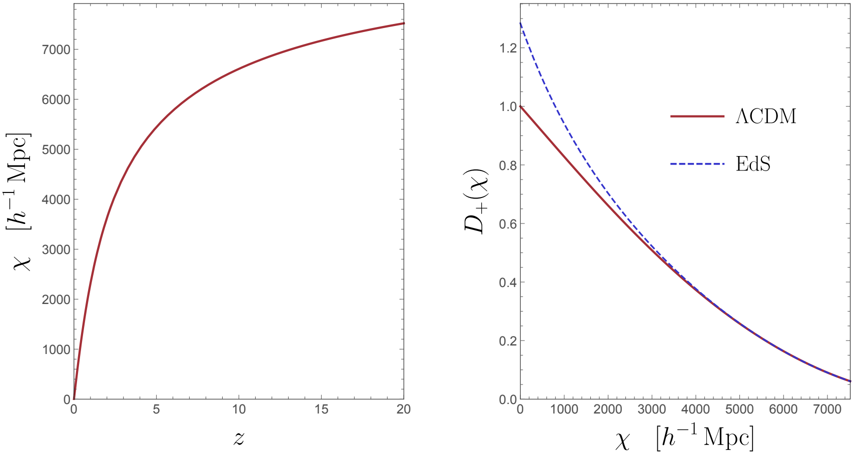
$$\chi(a) = \int_a^1 \frac{da'}{a'^2 H(a')}, \qquad \chi(z) = \int_0^z \frac{dz'}{H(z')}$$

And the distribution of galaxies becomes $W_g(\chi) = W_g(z(\chi))\dfrac{dz}{d\chi}$

normalized to unity: $\int_0^\infty W_g(\chi) d\chi = 1$.



Linear Growth function

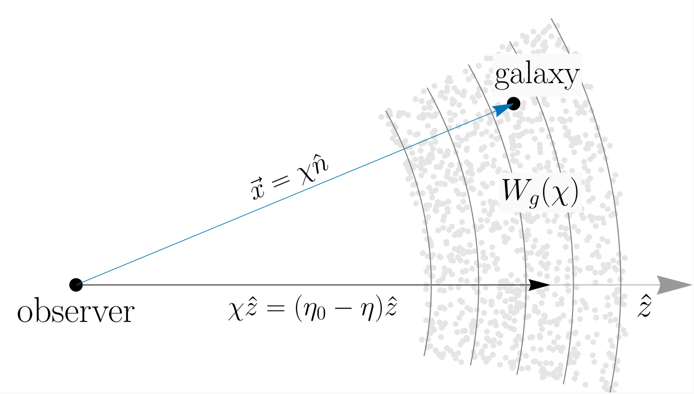


Coordinates

An event in space-time can be written as $\mathbf{p} = (\boldsymbol{x}, \eta)$, with $\boldsymbol{x}$ the comoving coordinates with the expansion and η the conformal time.

The conformal time at χ is $\eta(\chi) = \eta_0 - \chi$. And $\boldsymbol{x} = \hat{\boldsymbol{n}}\chi$ with $\hat{\boldsymbol{n}}$ the angular direction

Then, we can write $\mathbf{p} = (\hat{\boldsymbol{n}}\chi, \eta_0 - \chi)$: two numbers for $\hat{\boldsymbol{n}}$, and one for χ.



projected density

Instead of measuring the 3-dim galaxy density field, we measure its 2-dim projection on the sky,

$$\Delta_g(\hat{\boldsymbol{n}}) = \int_0^\infty d\chi\, W(\chi)\delta_g(\hat{\boldsymbol{n}}\chi, \eta = \eta_0 - \chi).$$

Now, we transform the 3D fluctuation to Fourier space:

$$\begin{aligned}\Delta_g(\hat{\boldsymbol{n}}) =& \int_0^\infty d\chi\, W(\chi) \int \frac{d^3k}{(2\pi)^3} e^{i\boldsymbol{k}\cdot\hat{\boldsymbol{n}}\chi}\delta_g(k,\chi) \\ =& 4\pi \int \frac{d^3k}{(2\pi)^3} \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell} i^\ell Y_{\ell m}(\hat{\boldsymbol{n}}) Y^*_{\ell m}(\hat{\boldsymbol{k}}) \int_0^\infty d\chi\, W(\chi) j_\ell(k\chi)\delta_g(k,\chi)\end{aligned}$$

where we used the plane wave expansion

$$e^{i\boldsymbol{k}\cdot\boldsymbol{x}} = 4\pi \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell} i^\ell j_\ell(kx) Y_{\ell m}(\hat{\boldsymbol{x}}) Y^*_{\ell m}(\hat{\boldsymbol{k}})$$

Decomposition in spherical harmonics

We can expand in spherical harmonics

$$\Delta_g(\hat{\boldsymbol{n}}) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \Delta_{g,\ell m} Y_{\ell m}(\hat{\boldsymbol{n}})$$

with

$$\Delta_{g,\ell m} = 4\pi \int \frac{d^3k}{(2\pi)^3} i^\ell Y^*_{\ell m}(\hat{\boldsymbol{k}}) \int_0^\infty d\chi\, W(\chi) j_\ell(k\chi) \delta_g(k,\chi).$$

The $\Delta_{g,\ell m}$ are the analogs to the $a_{\ell m}$ in the CMB anisotropies. The angular power spectrum of galaxy counts on the sky is then proportional to the expectation value of $|\Delta_{g,\ell m}|^2$.

Angular power spectrum of galaxies

Let us evaluate the angular power spectrum

$$\langle \Delta_{g,\ell m} \Delta^*_{g,\ell' m'} \rangle = (4\pi)^2 i^{\ell-\ell'} \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3k'}{(2\pi)^3} Y^*_{\ell m}(\hat{\boldsymbol{k}}) Y_{\ell' m'}(\hat{\boldsymbol{k}}') \\ \times \int_0^\infty d\chi\, W(\chi) j_\ell(k\chi) \int_0^\infty d\chi'\, W(\chi') j_{\ell'}(k'\chi') \, \langle \delta_g(\boldsymbol{k},\chi) \delta^*_g(\boldsymbol{k}',\chi') \rangle$$

The ensemble average of overdensities gives Dirac delta function $\delta_{\mathrm{D}}(\boldsymbol{k}-\boldsymbol{k}')$ (notice $\delta^*_g(\boldsymbol{k}') = \delta_g(-\boldsymbol{k})$) which sets $\boldsymbol{k} = \boldsymbol{k}'$. Hence we can use the orhonormality of spherical harmonics,

$$\int d\Omega\, Y^*_{\ell m}(\hat{\boldsymbol{n}}) Y_{\ell' m'}(\hat{\boldsymbol{n}}) = \delta_{\ell\ell'} \delta_{mm'},$$

to reduce the above expression.

We obtain

$$\langle \Delta_{g,\ell m} \Delta^*_{g,\ell' m'} \rangle = C_g(\ell) \delta_{\ell\ell'} \delta_{mm'}$$

with

$$C_g(\ell) = \frac{2}{\pi} \int_0^\infty d\chi \, W(\chi) \int_0^\infty d\chi' \, W(\chi') \int_0^\infty dk \, k^2 j_\ell(k\chi) j_\ell(k\chi') P_g(\boldsymbol{k}; \chi, \chi')$$

Notice:

- To compute the angular power spectrum we need to integrate the two overdensities at all “times” χ. And then the galaxy standard power spectrum is evaluated at two different times.
- The power spectrum evaluated at two different times $P_g(\boldsymbol{k}; \chi, \chi')$ is anisotropic because it depends on the value of two overdensities evaluated at two different times.

  Indeed, using the obvious transverse symmetry $\boldsymbol{k} = (k_\parallel, \boldsymbol{k}_\perp^{2\mathrm{D}})$ or $\boldsymbol{k} = (k\mu_k, \boldsymbol{k}_\perp^{2\mathrm{D}})$, where μ_k is the cosine angle of $\boldsymbol{k}$ and the line of sight.

Limber approximation

Consider the integral representation of the Dirac delta function:

$$\int_0^\infty dk\, k^2 j_\ell(k\chi) j_\ell(k\chi') = \frac{\pi}{2\chi^2}\delta_{\rm D}(\chi - \chi')$$

We have almost the above integral, but with $P(k)$ in the integrand. But for large ℓ one obtains

$$\int dk\, k^2 j_\ell(k\chi) j_\ell(k\chi') f(k) = \frac{\pi}{2\chi^2}\delta_{\rm D}(\chi - \chi') f(\ell/\chi)$$

which can be obtained by substituting

$$j_\ell(x) \approx \sqrt{\frac{\pi}{2\ell}}\,\delta_{\rm D}(\ell + \frac{1}{2} - x) \qquad \text{for large } \ell$$

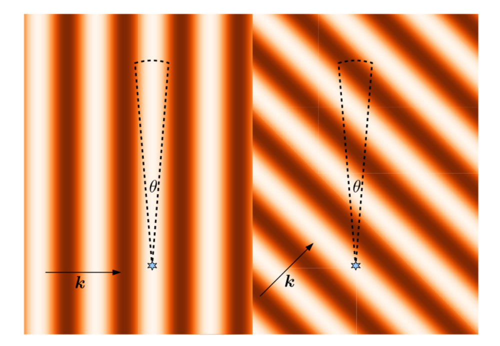
One obtains the angular power spectrum under the Limber approximation

$$C_g(\ell) = \int_0^\infty \frac{d\chi}{\chi^2} W^2(\chi) P_g\left(k = \frac{\ell + 1/2}{\chi}, \mu_k = 0; \chi\right)$$

valid for large ℓ. Typically for $\ell > 20$.

- We see that $\chi = \chi'$ in the Limber approximation, hence only the power spectrum at equal-time densities in the integral is important (at large ℓ).
- It means also that the k modes involved do not have a line-of-sight component, since that would mean different distances of different points along the perturbation, i.e. $\chi \neq \chi'$.
- So, $\boldsymbol{k}$ has to be transverse to the line of sight: $\mu_k = 0$. The longitudinal components cancel out!

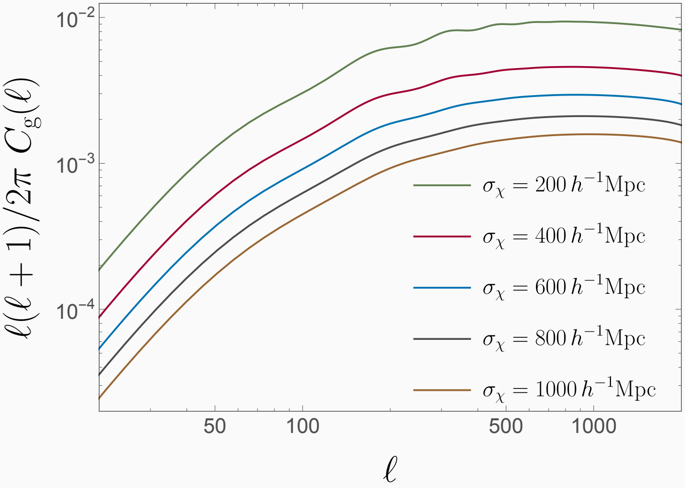
Dodelson & Schmidt figure 11.8



Focusing on small scales corresponds to looking at small angles, $\theta \sim 1/\ell$. *Right panel:* Modes with longitudinal wavenumber $\mu_k k > \chi^{-1}$ (or $\mu_k > \theta$) don't provide angular correlations because of cancelations along the line of sight. *Left panel:* Only modes with $\mu_k k \lesssim \chi^{-1}$ (or $\mu_k < \theta$) lead to angular correlations. Since χ is typically large, compared to k^{-1}, this corresponds to setting $\chi = \chi'$.

Angular galaxy power spectrum

$$C_{\rm g}(\ell) = \int d\chi \frac{W^2(\chi)}{\chi^2} P_{\rm g}\big(\ell/\chi, \eta_0 - \chi\big)$$

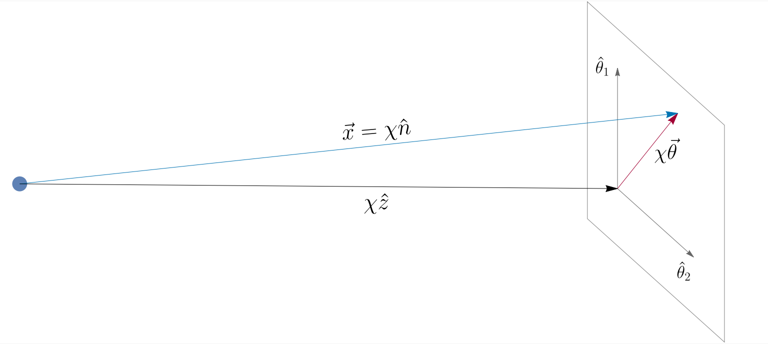


Coordinates in the plane parallel approximation

We choose a basis vector on the plane, $\{\hat{\theta}_1, \hat{\theta}_2\}$. Hence the 3-dim spatial position is given by

$$\boldsymbol{x} = \chi(z)(\theta_1, \theta_2, 1),$$

($\hat{\theta}_1 \times \hat{\theta}_2 = \hat{z}$), while the position on the plane is $\boldsymbol{\theta} = (\theta_1, \theta_2)$

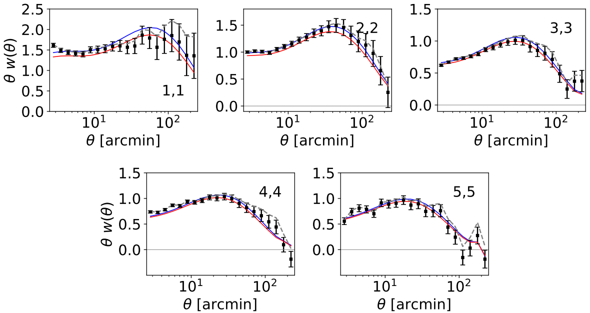


Under the plane parallel approximation $\hat{\boldsymbol{n}} = \hat{\boldsymbol{z}} + \hat{\boldsymbol{\theta}}$.

Angular correlation function

The angular correlation function is the Fourier transform of the angular power spectrum:

$$\xi_g(\theta) = \int \frac{d^2\ell}{(2\pi)^2} e^{i\boldsymbol{\ell}\cdot\boldsymbol{\theta}} C_g(\ell) = \int_0^\infty d\ell\, \ell C_g(\ell) J_0(\ell\theta)$$



DES Year 1 Results: galaxy-galaxy auto-correlation function for different photometric redshifts [1708.01536]