Rock Paper Scissors and Evolutionary Game Theory

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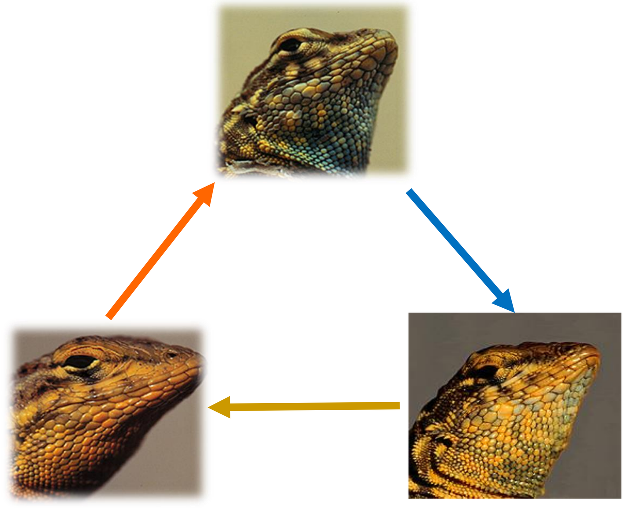
Introduction

In Rock Paper Scissors (RPS), three different "species" compete, but no single species has a dominating strategy. In evolutionary game theory, replicator equations model population densities over time. When a mutation is introduced, they are called "replicator-mutator" equations. Using the replicator-mutator equation in [1] we have shown how population density of three species change.



[3]

Rock Paper Scissors and the Payoff Matrix

In game theory, a payoff matrix represents all interactions between two agents. Table 1 represents a simple zero sum RPS game. Zero sum means that the matrix follows the zero-sum property where all rows and columns sum to zero.

	X_1 (ROCK)	X_2 (PAPER)	X_3 (SCISSOR)
X_1 (R)	0	-1	1
X_2 (P)	1	0	-1
X_3 (S)	-1	1	0

Table 1

In order to generalize this model to non zero-sum games, -1 is replaced with parameter ϵ. This is shown in Table 2. Table 2 has the zero-sum property if $\epsilon = 0$.

	X_1	X_2	X_3
X_1	0	$-(\epsilon+1)$	1
X_2	1	0	$-(\epsilon+1)$
X_3	$-(\epsilon+1)$	1	0

Table 2

Payoff (continued)

When we multiply the Payoff Matrix P given by the right table entries by the vector $\mathbf{x}$ we get the following fitness functions. We substitute $x_3 = 1 - x_1 - x_2$ in order to describe the system in 2 variables:

$$f_{x_1} = 1 - x_1 - (\epsilon+2)x_2$$

$$f_{x_2} = (\epsilon+2)x_1 + (\epsilon+1)(x_2 - 1)$$

$$f_{x_3} = -(\epsilon+1)x_1 + x_2$$

The Replicator-Mutator Equation for Global Mutations:

$$\frac{dx_1}{dt} = x_1(f_{x_1} - \varphi) + \mu(-2x_1 + x_2 + x_3)$$

$$\frac{dx_2}{dt} = x_2(f_{x_2} - \varphi) + \mu(-2x_2 + x_1 + x_3)$$

- The vector $\mathbf{x}$ refers to population densities of each individual element.
- $(f_{x_1} - \varphi)$ refers the relative fitness of an element's population aka the Replicator part of the equation.
  - $\varphi = x_1(f_{x_1}) + x_2(f_{x_2}) + x_3(f_{x_3})$ this refers to the weighted average of all fitness functions.
- $(-2x_{1or2} + x_{2or1} + x_3)$ refers to the change in the population aka the Mutator part of the equation.
  - μ refers to the rate which the population changes.

Converting to Phase Diagram

Eliminating x_3 removes the graphical intuition. The following matrix multiplication using a transformation matrix. preserves this graphical intuition. The closer a point on the phase portrait is to each of the triangle's vertices, the higher the population density of that element.

$$\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

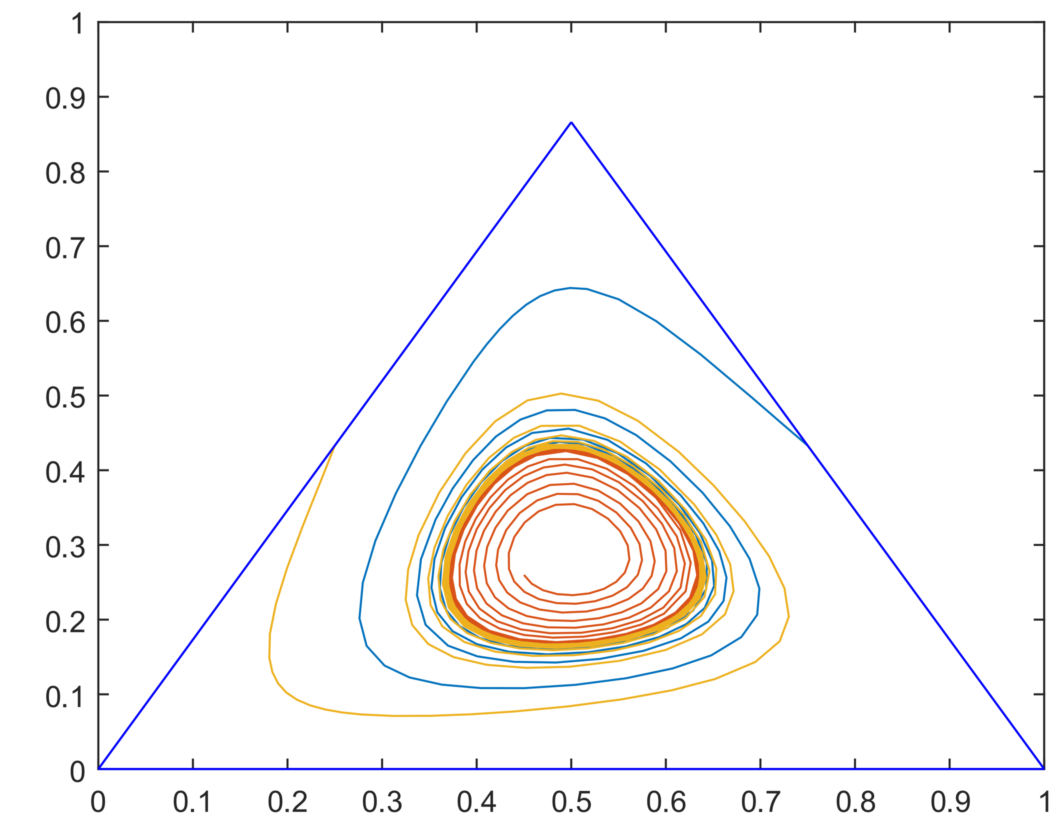


Figure 1: Phase Portrait with stable limit cycle. Conditions where $\epsilon = 4$, and $\mu = .2$. All initial conditions eventually end in a stable limit cycle.

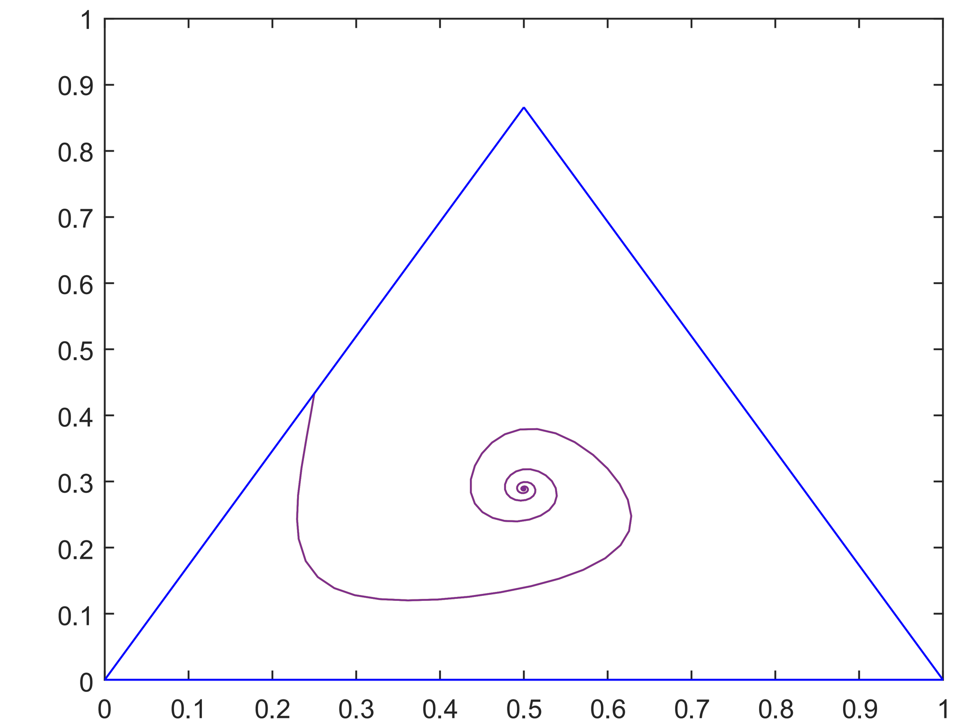


Figure 2: Phase Portrait with stable interior point. Conditions where $\epsilon = 8$, and $\mu = .6$. All initial conditions converge to x(1) = 1/3, x(2) =1/3, x(3)=1/3

Applications in Biology

- In nature, the common side blotched lizard demonstrates RPS mating strategies. The realistic scenario has only the yellow mutating into blue. Solving a simplified form (for only one mutation) of the system of differential equations in MATLAB and then applying the transformation to the resulting points, we have shown in Figure 3 how the population densities of the lizards change.

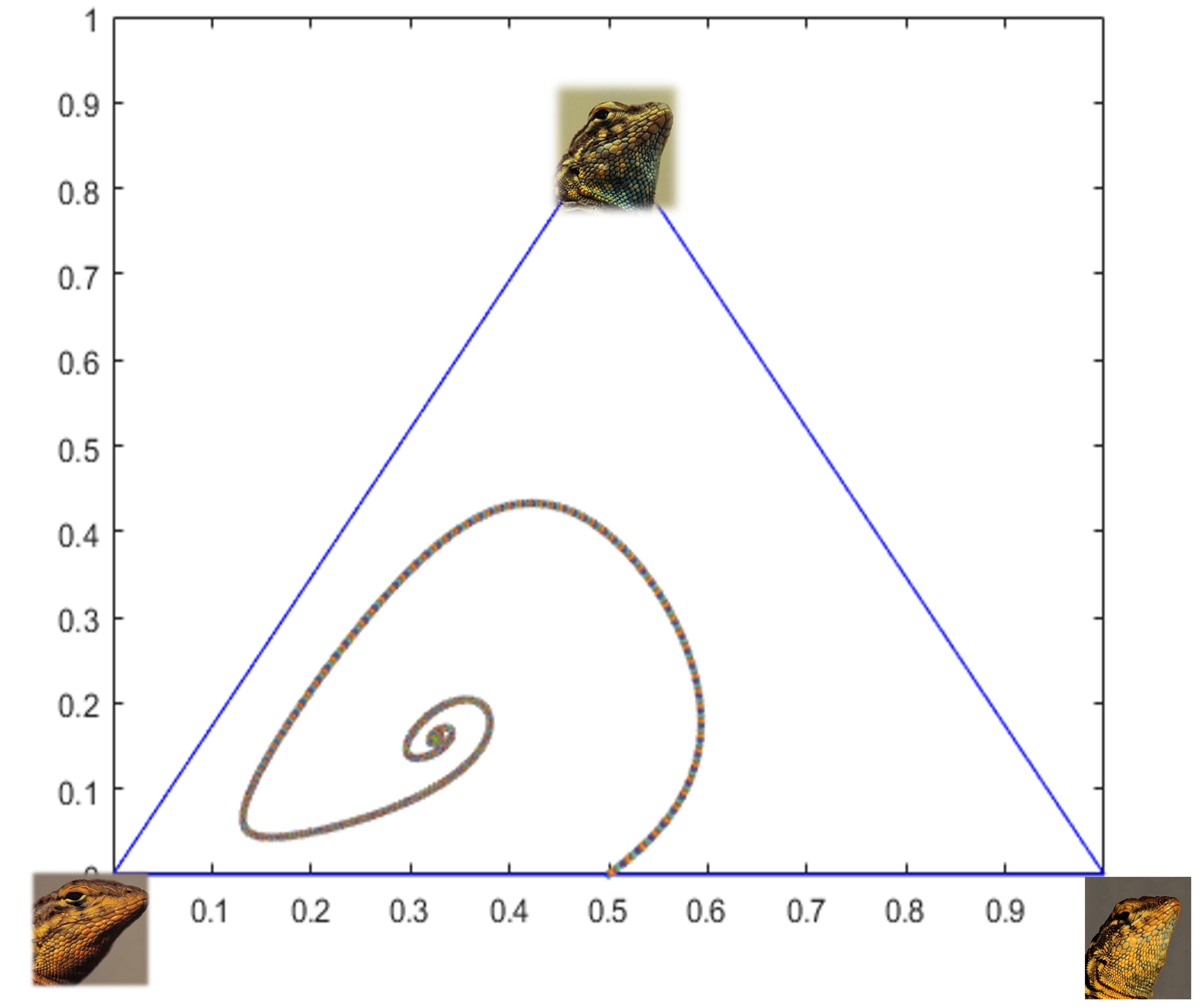


Figure 3: Phase Portrait with stable interior point when $\epsilon = 0.5$ and $\mu = 0.4$ and initial conditions $x_1 = 0.5$, and $x_3 = 0.5$.

Conclusion

As seen, the replicator-mutator equation derived from RPS can serve as a useful tool in obtaining a grasp of how competing species interact. Additionally, this equation has relevance to many areas of science including genetics, theoretical biochemistry, language evolution and population biology. Like any universal equation, replicator-mutator equation is an approximation to reality, but it also grasps many important features of the dynamics, common to a wide variety of systems.

References

[1] Toupo, Danielle F. P., and Steven H. Strogatz. "Nonlinear Dynamics of the Rock-Paper-Scissors Game with Mutations." 12 Feb. 2015, pp. 1–6., Rock-Paper-Scissors Game with Mutations.

[3] Stephens, Tim. "Cooperation between Unrelated Male Lizards Adds a New Wrinkle to Evolutionary Theory." *Blue-Throated Lizards*, UC Santa Cruz Currents Online, 23 June 2003, www1.ucsc.edu/currents/02-03/06-23/lizards.html.