Active SLAM Utility Function Exploiting Path Entropy

Muhammad Farhan Ahmed, Vincent Fremont and Isabelle Fantoni

CNRS, LS2N, Ecole Centrale De Nantes (ECN), France

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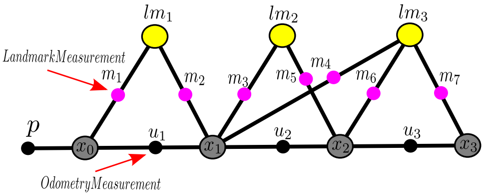
Outline

Simultaneous Localization And Mapping (SLAM)

1. Robot localizes itself and simultaneously maps the environment while navigating through it.
2. Localization is a problem of estimating the pose of the robot with respect to the map, while mapping makes up the reconstruction of the environment.
3. Modern SLAM approaches adopt a graphical approach. Where each node represents the robot or landmark pose and each edge represents a pose to pose or pose to landmark measurement measurement.
4. The objective of the SLAM problem is to find the optimal state vector x^* which minimizes the measurement error



$$\mathbf{x}_i = \begin{pmatrix} x_i \\ y_i \\ \theta_i \end{pmatrix} \quad \mathbf{x}_l = \begin{pmatrix} x_l \\ y_l \end{pmatrix}$$

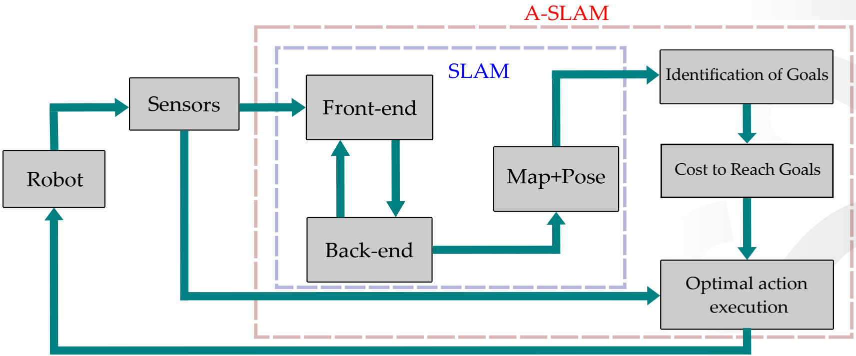
$$\mathbf{e}_i(\mathbf{x}) = \mathbf{Z}_i - f_i(\mathbf{x})$$
$$e_i(\mathbf{x}) = \mathbf{e}_i^T(\mathbf{x})\Omega_i\mathbf{e}_i(\mathbf{x})$$

$$\begin{aligned} \mathbf{x}^* &= \arg\min_x \textstyle\sum_i e_i(\mathbf{x}) \\ &= \arg\min_x \textstyle\sum_i \mathbf{e}_i^T(\mathbf{x})\Omega_i\mathbf{e}_i\mathbf{x} \end{aligned}$$

What is the Active SLAM (A-SLAM) problem?

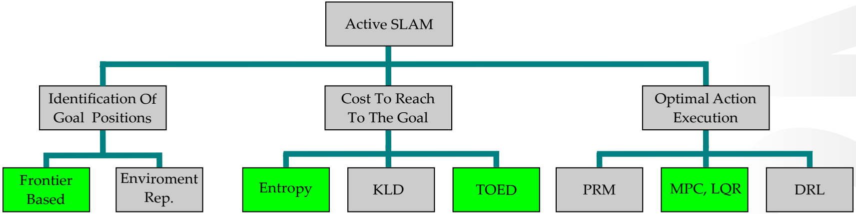
1. A-SLAM deals with designing robot trajectories towards the goal locations subject to minimizing the uncertainty in its map localization.
2. The aim is to perform autonomous navigation and exploration of the environment without an external controller or human effort.

Figure 1: SLAM and A-SLAM Pipeline



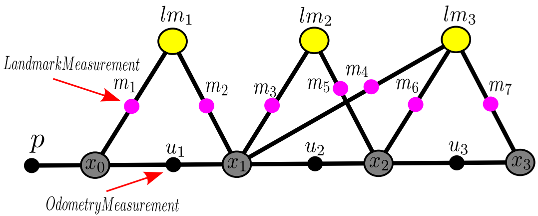
A-SLAM Components

Figure 2: A-SLAM Submodules



- Frontiers
  - Frontier is the border between known and unknown map locations
- Entropy
  - Entropy measures the amount of uncertainty associated with a random variable or random quantity.
- Theory of Optimal Experimental Design (TOED)
  - Provides scalar mapping of the covariance matrix.
- MPC, LQR
  - formulate the robot path planning problem as an Optimal Control Problem (OCP).

Brief Graph Theory



- $\mathcal{G} = (\mathbb{V}, \mathbb{E})$ with $|\mathbb{V}| = n$ and $|\mathbb{E}| = m$
- The Degree Matrix $\mathbb{D}$ is a diagonal matrix $\mathbb{D}_{i,j} = \begin{cases} deg(v_i), & \text{if } i = j \\ 0, & \text{Otherwise} \end{cases}$
- The Adjacency Matrix $\mathbb{A}$ is given as $\mathbb{A}_{i,j} = \begin{cases} 1, & \text{if } i, j \in \mathbb{E} \\ 0, & \text{Otherwise} \end{cases}$
- The Laplacian Matrix is an nxn and is given as $\mathcal{L} = \mathbb{D} - \mathbb{A}$
- The Incidence Matrix $\mathcal{Q}$ is an nxm and is given $\mathcal{Q}_{i,j} = \begin{cases} 1, & \text{if vertex } v_i \text{ is incident with edge } e_j \\ 0, & \text{Otherwise} \end{cases}$

Proposed approach

- Motivated by [1] and starting from AGS[2]
- Ray Tracing using Bresenham's line algorithm.
- Path Entropy
- Distance

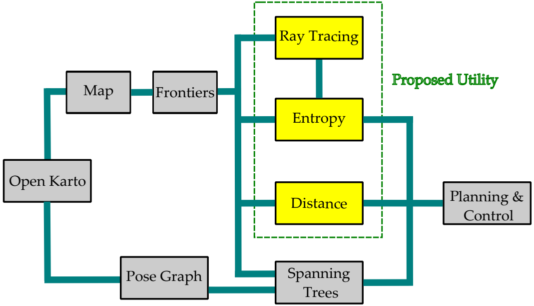


Figure 3: Proposed Approach Single Robot

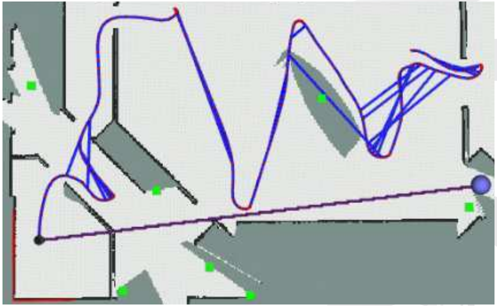


Figure 4: Proposed utility function ROS implementation

[1] Khosoussi, K. et al Reliable Graphs for SLAM. The International Journal of Robotics Research 2019

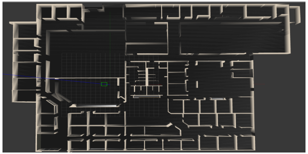
[2] Placed, J.A. et al"Fast Autonomous Robotic Exploration Using the Underlying Graph Structure", IROS, pp. 6672?6679, 2021

Proposed Utility

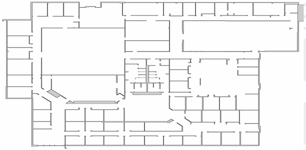
1. $F = \{f_1, f_2, ...f_N\} \in \mathbb{R}^2$, we get the occupancy values of each path as $G^n = \{m_0, m_1, ...m_L\}, \forall n \in N$
2. $m_0 ... m_L$ are the pixel occupancy values of path length L. We assign the probability value of $P_{unk} = 0.1$ for unknown pixels
3. $E^n = E^n[p(m)]_{m \in G^n} = -\sum_{m \in G^n}(P(m_{i,j})log_2(p(m_{i,j})) + P(1 - m_{i,j})log_2(1 - p(m_{i,j})), \forall m_{i,j} \in M$
4. Once the path entropy is computed it is normalized with the number of pixels within the path $K^n = \sum_{i=R_x}^{n_x} \sum_{j=R_y}^{n_y} m_{i,j}$.
5. $n = \{n_x, n_y\}$ and $R = \{R_x, R_y\}$ are the selected frontier and robot positions respectively.
6. $\gamma^n = \exp^{-(\lambda * dist(R,n))}$
7. $U_2^n = (1 - E^n/K^n) * \rho^n + \gamma^n$
8. $U_1^n = \text{Spann}(L_w^n)^3$
9. $U_{tot} = \max(U_1^n + U_2^n)$

[3] Placed, J.A. et al"Fast Autonomous Robotic Exploration Using the Underlying Graph Structure", IROS, pp. 6672?6679, 2021

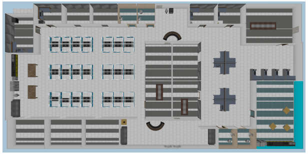
Simulation Environment



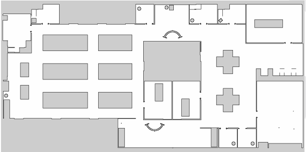
Willow Garage modified $2072m^2$



Occupancy grid map[4]



AWS Office modified $60 \times 50741m^2$



Occupancy grid map

[4] https://github.com/marinaKollmitz/gazebo.

Performance Metrics

Reliable Graphs for SLAM[5]

1. Algebraic Connectivity (A.C)
   - Algebraic connectivity is a measure of how well a graph is connected and is associated with the second smallest eigenvalue of graph Laplacian.
2. Average Degree of Graph ($\bar{d}$)
   - For a fixed number of poses, maximizing the average degree is equivalent to maximizing the number of measurements leading to a precise estimate of the log likelihood.
3. Normalized Tree connectivity ($\hat{\tau}(\mathcal{G})$)
   - The weighted number of spanning trees as a measure of graph connectivity has significant impact on the determinant of the estimation error covariance of the Maximum likelihood (ML) estimator.

$$t_w(\mathcal{G}) = det(L_w) \text{ and } \hat{\tau}_w((G)) := \begin{cases} log_{tw}(\mathcal{G}), & \text{if}(G)\text{is connected} \\ 0, & \text{Otherwise} \end{cases}$$

[5] Khosoussi, K. et al Reliable Graphs for SLAM. The International Journal of Robotics Research 2019

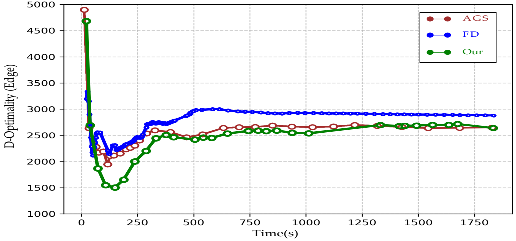
Simulation Results

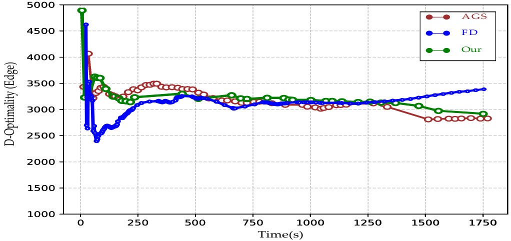
Env.	Meth.	A.C	$\bar{d}$	$\hat{\tau}(\mathcal{G})$	SSIM	RMSE
W.G	FD	0.104	**3.290**	1.016	0.05	0.70
	AGS	0.426	2.907	1.139	0.05	0.64
	Our	**0.845**	2.925	**1.205**	**0.08**	**0.60**
Office	FD	3.061	**3.179**	1.229	0.09	0.83
	AGS	5.740	2.742	1.312	0.07	0.80
	Our	**9.617**	2.612	**1.941**	**0.11**	**0.77**

Average graph connectivity and map quality comparison of 15 simulations.

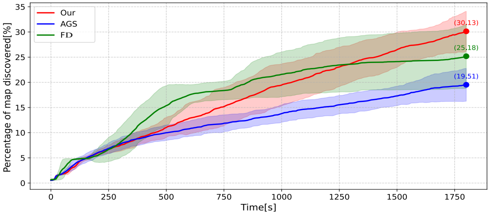
Env.	Method	D-Opti(Max&Min)	Diff	%R
W.G	FD	3700&2900	800	20
	AGS	4800&2600	2200	**45**
	Our	4700&2600	2100	44
Office	FD	4600&3400	1200	26
	AGS	4100&2700	1400	34
	Our	4900&2900	2000	**40**

Uncertainty reduction (%R) comparison

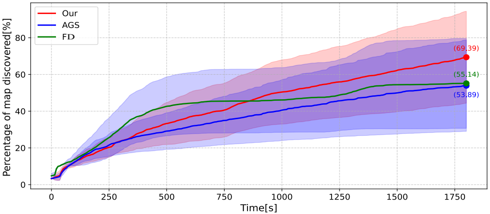




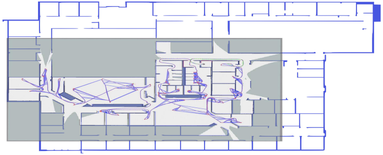
Simulation Results



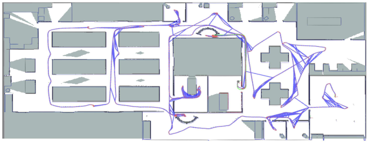
Comparison of evolution of map discovered (W.G).



Comparison of evolution of map discovered (AWS Office)

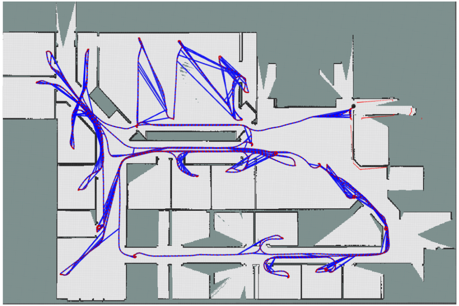


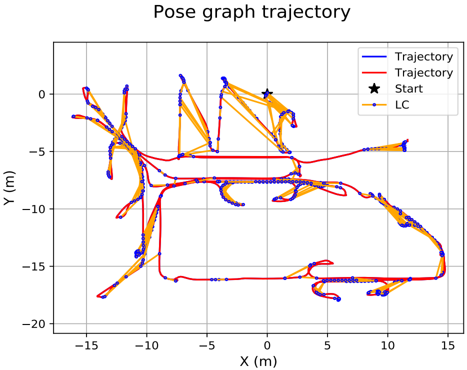
Obtained pose graphs using Our approach W.G



Obtained pose graphs using Our approach AWS Office

Simulation Results

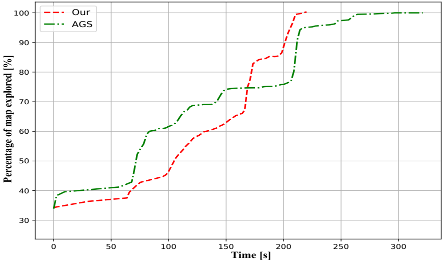




Experimental Results

RosBot 2.

Mapped environment.



Gazebo Simulation Video

Conclusion

1. We have presented a utility function that selects the most favorable frontier goal location within an occupancy grid map
2. The proposed utility function incorporates path entropy to select the frontier goal location which has the highest amount of unknown cells within its path thus maximizing the area coverage.
3. Using simulation and experimental results on publicly available environment maps we have proved the efficiency of our approach as compared to similar methods.
4. As a future prospective, we plan to incorporate our method in a multi-robot scenario utilizing efficient frontier-sharing for maximum environment exploration