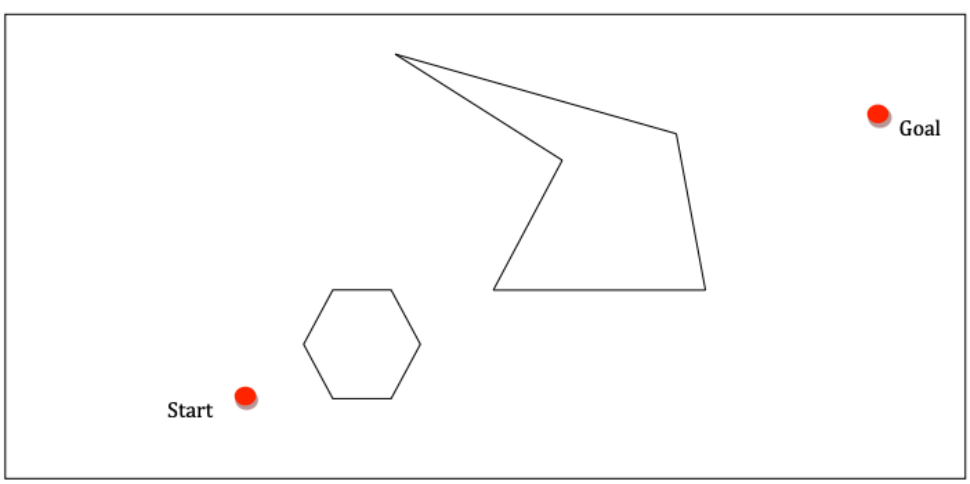
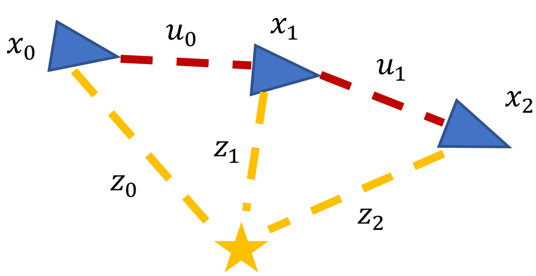
COMP 417 – Fall 2019

Practice Midterm #1:

1. Starting from our objective with Bayesian filtering, $Bel(x_t) = p(x_t|z_{1:t}, u_{1:t})$, simplify the equation until it can be expressed as an expression that contains the motion and measurement likelihoods: $p(x_t|x_{t-1}, u_{t-1})$ and $p(z_t|x_t)$. Show each step aligned along the left of the page and explain on the right which assumptions or rules have been used. Consider the following:
    a. Bayes rule
    b. Conditional independence
    c. Each measurement is independent
    d. The state at time t is independent of current and future commands, $u_t$onwards.
2. Give an example of a source of error (inaccuracy) in each of the following, and explain how that might be accounted for by probabilistic modeling:
    a. A GPS receiver and corresponding $p(z_t|x_t)$
    b. A self-driving car's wheel odometry and $p(x_t|x_{t-1}, u_{t-1})$
    c. An architect's creation of a floorplan, and $p(m_{ij} = 1)$
3. Define forward kinematics and inverse kinematics.
4. For a differential drive robot, where the wheels are distance d apart and the wheel velocities are V_r and V_l (right and left): give the expression for the vehicles linear velocity and turning rate.
5. For the map below, draw a possible solution given by each one of the listed planning algorithms:
    a. Visibility graph (draw the graph and the plan selected)
    b. Rapidly exploring random tree (draw the planning tree and the plan selected)
    c. Dijkstra's method (just the plan)
    d. A* planning (just the plan)
    e. The Bug #2 algorithm (the path the robot ends up following)



6. In the box below, draw an environment that shows the Bug #1 algorithm is not complete. Ensure to include the start, goal and obstacles (shaded regions) and also draw with a dotted line the path the robot ends up following.
7. Explain the difference between a topological map and a metric map. Explain how each could be used to represent Montreal's metro system.
8. Explain how the parameters that optimize the least squares solution compare to the maximum likelihood parameters, assuming a Gaussian distribution on the data.
9. Suppose we have three random variables a, b and c. Each variable on its own follows a zero-mean Gaussian distribution with variance 1, 2 and 3 respectively. Variables a and b are independent, while the covariance of a with c is 4 and b with c is 5. Write the mean vector and covariance matrix of the 3D Gaussian that represents the distribution of these 3 variables.
10. A colleague tells you that, because of the values above, we can conclude that "c causes a and b". Do you agree or disagree and why?
11. Using the following Graph SLAM diagram, write out the expression we will try to maximize to find the most likely robot poses over time and map points:



12. Write out at least 3 assumptions that we made in order to derive the Kalman filter.
13. Explain why applying A* planning to a robot whose state space has 23 dimensions is infeasible. If you're able to, state the complexity of performing each step in the planning process (try to get at least to the one that has exponential complexity in dimension).
14. Explain how the variance of our sensor impacts a Kalman filter measurement update. Use symbols and math if you are able but ensure to also explain the effect clearly in plain English.