COMP 417 – Fall 2019

Midterm #1 – Oct. 15, 2019

80 minutes to complete (2:35 – 3:55)
Closed book and notes.
No phones or calculators required.
This test has 20 questions and 10 pages.

Write the answers to the (16) short answer questions on the answer grid on page 9 of this exam (after first entering your name and McGill ID both on this page and that page).

For the (4) long answer questions, you will answer directly on the pages of this exam, in the boxes indicated.

Each short answer question is worth 1 and each long answer question is worth 4. Therefore, the short and long answer sections have equal weight (consider using this for time management).

First Name:

Last Name:

McGill ID:

Short Answer Questions

Answer each question on the last page of this test for marking

1. Suppose a robot has a 1-D state space {x, any positive real number} and moves following the equation $x(t+1) = 3*x(t)*x(t)+ u(t)$. Command u is limited by the motor strength, so that $|u| < 10$. Note that the passive dynamics are repulsive (you move away from the origin quadratically if the command is zero). This robot is fully actuated. True or false?

2. Suppose a robot is an automated corkscrew. Its state is represented by the x, y and z coordinates of its point while moving through the cork (constrained to fall on the typical "corkscrew" shaped path). Assume that the robot's motor is strong enough to turn the corkscrew in either direction (into and out of the cork) at any moment. This robot is holonomic. True or false?

3. The inverse kinematics of a differential drive robot could involve counting how many wheel rotations of the left and right wheels are required to move the robot's center to a goal (x,y,theta) location. True or false?

4. "The number of steps from the start to the current node" is an admissible heuristic that could be used for A* planning. True or false?

5. Quadtrees must always be balanced (that is every right sub-tree and left sub-tree have the same height). True or false?

6. Visibility graphs require nodes in the middle of edges. True or false?

For the following 3 questions, match the best planner to the problem and requirements stated. You can only use each planner once:

A) RRT
B) A* on a discretized grid
C) Bug #2 algorithm

7. An automated lawn mower must reach its charging station. It has no full map of the lawn it's mowing, only proximity sensors to determine what is directly in front and beside. Choose A, B, or C.

8. A self-driving car that operates on the 2D plane: {x,y,theta}. The user will only be happy if they are taken on the shortest possible path through the city. Choose A, B, or C.

9. A disaster response robot arm has 10 joints, each which can rotate about 2 different axes, for a 20-dimensional state-space. It must find some way to pass a survivor some medicine without any of the joints impacting the world, but we do not care about the shortest path. Choose A, B, or C.

10. A globe (sphere painted with a map of Earth) is an example of a topological map. True or false?

11. The RRT algorithm, given infinitely many iterations to sample points and build its tree, will eventually converge to the shortest possible path. True or false?

For the following 3 questions, assume that a robot's belief in its current state, $\boldsymbol{x}$, is modeled as a normal distribution (Gaussian), over x, y, and theta elements of the state-space. This means the likelihood of it being at any state is:

$$p(\boldsymbol{x}) \propto e^{-(x-\mu)^T\Sigma^{-1}(x-\mu)},$$

for mean vector $\mu = \begin{bmatrix} \mu_x \\ \mu_y \\ \mu_\theta \end{bmatrix}$ and covariance matrix $\Sigma = \begin{bmatrix} Var(x) & Cov(x,y) & Cov(x,\theta) \\ Cov(x,y) & Var(y) & Cov(y,\theta) \\ Cov(x,\theta) & Cov(y,\theta) & Var(\theta) \end{bmatrix}$.

12. Suppose our mean for x is 2, our mean for y is 0 and our mean for theta is 1. Is this sufficient information to specify Cov(x,y)? Write its value if you know it, "ambiguous" if you cannot tell from the provided info and "impossible" if the provided info is self-contradictory.

13. Starting fresh (as in, without the items "supposed" in previous questions), suppose that x, y, and theta are all independent of one another, and that each state dimension has variance 3. Is this sufficient information to specify all 9 entries of the covariance matrix, Σ? Write its value if you know it, "ambiguous" if you cannot tell from the provided info and "impossible" if the provided info is self-contradictory.

14. Starting fresh (as in, without the items "supposed" in previous questions), suppose that:
    a) as x increases, y decreases at the same rate;
    b) as theta increases, both x and y increase at the same rate; and
    c) each state variable variance 10.

    Is this sufficient information to specify the whole covariance matrix, Σ? Write its value if you know it, "ambiguous" if you cannot tell from the provided info and "impossible" if the provided info is self-contradictory.

15. Suppose the "innovation" (z-h(x)), in the Kalman filter, is zero. This will cause the ***mean value*** of the belief to also become zero when applying the measurement update. True or false?

16. Suppose the motion model is imprecise, that is $x_t = x_{t-1} + u_{t-1} + \omega$ with $\omega \sim \mathcal{N}(0, {\sigma_u}^2)$ has a large value of σ_u. This will cause the Kalman filter propagation step to leave the ***mean value*** of the belief unchanged from the previous belief. True or false?

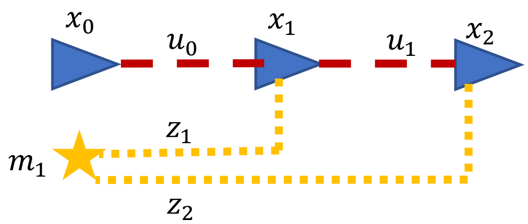
Long(er) Answer Questions

For each item below, please write your response directly below the question, in the space provided. There is intentionally no space in the answer grid for these responses.

17. Recall the Bug #1 Algorithm, which has the pseudo-code:
    a) Move in a straight-line path towards the goal, G.
    b) If the robot reaches G, we are done.
    c) If the robot hits and obstacle, then follow the right-hand rule and traverse its boundary.
    d) This continues until the wall no longer is pointing against the straight line from X towards G. Once this occurs, continue from step a.

    In the box below, draw an environment that shows the Bug #1 Algorithm is not complete. Ensure to include the start, goal and obstacles (shaded regions) and also draw with a dotted line the path the robot ends up following.

For the next 3 questions, assume a robot with a 1D state-space starts at the origin (x=0), it makes 2 motions, following controls u_0 and u_1, and takes two sensory measurements, z_1 and z_2.



18. Given that the robot's motion model is $x_t = f(x_{t-1}, u_{t-1})$ and the robot's measurement model is $z_t^i = g(x_t, m_i)$, write out a least-squares objective for the unknown variables in the SLAM problem. That is, the states x_1 and x_2 and the single map feature, m_1.

Show your work starting here and write the final answer in the box provided:

$$\begin{bmatrix} x_1 \\ x_2 \\ m_1 \end{bmatrix} =$$

19. Using the probabilistic form of the Bayes filter, write out the expression for $p(x_1|u_0, z_1)$, also known as our ***belief*** in the robot's location at time 1. Make sure your final expression is composed of only the 3 basic elements of a Bayes filter:

a) The previous belief (in this case, the belief in the robot's location at time 0), $bel(x_0)$

b) The measurement model, $p(z_t|x_t)$

c) The motion model, $p(x_t|x_{t-1}, u_{t-1})$

Show your work starting here and write the final answer in the box provided:

$p(x_1|u_0, z_1) =$

20. Now assume that all of the assumptions for the Kalman filter have been met. Given this, compute the KF starting from the mean and variance that form $bel(x_1)$ to integrate the new information provided by u_1 and z_2 and arrive at the mean and variance of $bel(x_2)$.

    The robot's motion model is $x_t = x_{t-1} + u_{t-1} + \omega$ with $\omega \sim \mathcal{N}(0, {\sigma_u}^2)$.
    The robot's measurement model is $z_t = x_t + \varepsilon$ with $\varepsilon \sim \mathcal{N}(0, {\sigma_z}^2)$.
    The previous belief, $bel(x_1)$ had mean value **a** and variance **b**.
    The commanded motion, u_1 was to move forward (positive x direction) **c** units.
    The measurement received, z_2 was **d**.
    The motion and measurement variances are ${\sigma_u}^2 =$ **e** and ${\sigma_z}^2 =$ **f**.

    The 1-D Kalman filter measurement updates that we derived to compute $bel(x_t)$ from $\overline{bel}(x_{t-1})$ are:
    $\mu = \mu_B + \frac{\sigma_B^2}{\sigma_A^2+\sigma_B^2}(\mu_A - \mu_B)$ and $\sigma^2 = \sigma_B^2 - \frac{\sigma_B^2}{\sigma_A^2+\sigma_B^2}\sigma_B^2$
    (B indexes the $\overline{bel}$ parameters while A indexes the measurement model)

    The 1-D Kalman filter propagation/prediction step that we derived to compute $\overline{bel}(x_{t-1})$ from $bel(x_{t-1})$ are:
    $\mu = \mu_C + u_{t-1}$ and $\sigma^2 = \sigma_C^2 + q^2$
    (C indexes the previous bel parameters)

Show your work here and write the final answer in the box provided at the bottom of the page. Note that your answer should be some math expression comprised of the letters **a** through **f**:

$bel(x_2) = \mathcal{N}(\mu, \sigma)$ with:
$\mu =$ and $\sigma =$

THIS PAGE INTENTIONALLY BLANK

Use for your rough work

THIS PAGE INTENTIONALLY BLANK

Short Answer Responses:

First Name:

Last Name:

McGill ID:

Q#:	Response	Grade (filled by the TAs)	Q#:	Response	Grade (filled by the TAs)
1			9		
2			10		
3			11		
4			12		
5			13		
6			14		
7			15		
8			16		

THIS PAGE INTENTIONALLY BLANK

Use for your rough work

THIS PAGE INTENTIONALLY BLANK