COMP 417 – Fall 2019

Practice Midterm #2:

1. Applying Bayes' rule with a Gaussian prior and Gaussian measurement likelihood always results in a Gaussian posterior. True or false?
2. The Extended Kalman filter (EKF) is an optimal estimator, true or false?
3. Would you prefer to use the EKF on problems where the true state distribution is unimodal (single peak) or multi-modal (more than one peak)? Why?
4. Explain why a particle filter could track a state distribution with two distinct peaks. List elements of the algorithm where you would need to take care to ensure this works well.
5. You want to use a potential field controller to navigate safely in a room made up of 3 square box obstacles with centers at (2,3), (5,6) and (7,9), all with side-length=1, towards a goal at (10,10).
    a. Write an expression for the potential field at every point (x,y) in the 2D state-space. Don't forget to normalize your force vector.
    b. Write out the pseudo-code for the navigation controller that could run live on this robot to ensure it reaches the goal.
6. How would the accuracy of a robot's EKF interact with the settings of a potential field controller for the same robot? Are these two factors completely unrelated? Explain why or why not.
7. You are given an omnidirectional robot that modes on a surface with friction. The continuous dynamics of the robot are given by the following equation, which extends the double integrator dynamics: $m\ddot{p} = u - \alpha\dot{p}$, where the 2D vector p denotes the position (x,y) of the robot. The 2D vector u denotes the controls (ux,uy). m is the mass and alpha is the coefficient of friction. Formulate this control problem as an LQR problem. Specifically, define the state x, discretize the system and derive linear dynamics that follow $x_{t+1} = Ax_t + Bu_t$, as well as a cost function $g(x_t, u_t) = x_t^T Q x_t + u_t^T R u_t$ that will stabilize the point robot around the zero state and zero control. Write down your choice of matrices A, B, Q, R. Hint: use the fact that $\dot{p} = v$ to make the system linear.
8. Adding two rotation matrices always results in another rotation matrix, true or false?
9. Is the cost function $g(x, u) = x^T \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} x{+}u^T u$ valid for an LQR controller? Justify your answer.
10. The purpose of the derivative term in the PID is to predict the value of the error in the near future. True or false?
11. Is the following a valid rotation matrix?

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

12. A potential field that is the weighted sum of a single repulsive term (due to a single obstacle) and a single attractive term (due to a single destination), can never have a local minimum other than at the goal. True or false?
13. The matrices A and B in the linear dynamics model need to be positive definite if we want to apply LQR. True or false?
14. Consider the problem in which a robot is trying to determine if a door is open or not by taking a series of noisy measurements. Assuem the door is static over time. Let *m* be the binary random variable that denotes whether the door is open. The robot's initial belief, before any measurements are obtained, is p(m=open)=0.5. The robot's sensor takes binary measurements z of whether the door is open, according to the following sensor model:
    P(z=open|m=open) = 0.8
    P(z=open|m=close) = 0.4
    This does not depend on time or the robot's position. Suppose that the first measurement is z1=closed and the second measurement is z2=closed.
    a) What is the robot's belief about whether the door is open given the first measurement? Show your work.
    b) What is the robot's belief about whether the door is open given the first two measurements? You may assume tha thte measurments are independent given the state of the door, *m*.