Plotting a Rectangular Wave Using Fourier Series

Introduction

This document explains the mathematical concepts used in the provided Python code to plot a rectangular wave function using Fourier series.

Rectangular Wave

A rectangular wave is a type of periodic function that alternates between two values with a certain period. The Fourier series representation of a periodic function allows us to express it as a sum of sines and cosines.

Fourier Series Representation

The Fourier series of a periodic function $v(t)$ with period T is given by:

$$v(t) = a_0 + \sum_{k=1}^{\infty} (a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t))$$

where:

$$a_0 = \frac{1}{T} \int_0^T v(t)\, dt$$
$$a_k = \frac{2}{T} \int_0^T v(t) \cos(k\omega_0 t)\, dt$$
$$b_k = \frac{2}{T} \int_0^T v(t) \sin(k\omega_0 t)\, dt$$

and $\omega_0 = \frac{2\pi}{T}$ is the fundamental angular frequency.

Coefficients for Rectangular Wave

For a rectangular wave with amplitude V_m and period T, the Fourier coefficients are:

$$a_0 = \frac{V_m}{4}$$
$$a_k = \frac{V_m}{k\pi} \sin\left(\frac{k\pi}{2}\right)$$
$$b_k = \frac{V_m}{k\pi} \left(1 - \cos\left(\frac{k\pi}{2}\right)\right)$$

Sum of Harmonics

The rectangular wave can be reconstructed using the sum of its harmonics:

$$v(t) = a_0 + \sum_{k=1}^{n} \left(a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t)\right)$$

where n is the number of harmonics.

Plotting the Waveform

To plot the rectangular wave, we compute the sum of the harmonics at each time instant t over two periods ($0 \leq t < 2T$) with a step size of 0.01 seconds. The Python code calculates the Fourier series terms and sums them to generate the waveform, which is then plotted using Matplotlib.