Redshift and Age of the Universe

Shubhayan
Dept.of.Math, Jadavpur University

November 14 , 2023

Redshift in Astrophysics

- Redshift is a fundamental concept in astrophysics, observed in the spectra of distant celestial objects.
- It results from the Doppler effect, where the wavelength of light is stretched as the source moves away from an observer.
- The redshift (z) is defined as the fractional change in wavelength and can be expressed as:

$$z = \frac{\lambda_{\text{obs}} - \lambda_{\text{emit}}}{\lambda_{\text{emit}}} = \frac{\Delta\lambda}{\lambda_{\text{emit}}} \tag{1}$$

Here, λ_{obs} is the observed wavelength, λ_{emit} is the wavelength when the light was emitted, and $\Delta\lambda$ is the change in wavelength.

On this presentation

The relationship between redshift and the age of the Universe is rooted in the expansion of space. Utilizing Hubble's Law and the redshift-distance relationship, scientists can estimate the rate of expansion and, consequently, the age of our Universe.
The mathematical foundation for redshift involves concepts from general relativity and cosmology. The redshift (z) is related to the scale factor (a) of the Universe through the equation $z = \frac{a_0}{a} - 1$, where a_0 is the scale factor at the present time. The relationship between redshift, recessional velocity, and Hubble's constant (H_0) is given by $v = H_0 d$, where v is the recessional velocity and d is the distance to the object.

Doppler Effect

- The Doppler effect explains the shift in frequency or wavelength of a wave in relation to an observer moving relative to its source.
- In the case of redshift, as celestial objects move away, the observed wavelength is longer, shifting towards the red end of the spectrum.
- The Doppler shift ($\Delta\lambda$) is given by:

$$\Delta\lambda = \frac{v}{c}\lambda_{\text{emit}} \tag{2}$$

where v is the recessional velocity of the object, c is the speed of light, and λ_{emit} is the emitted wavelength.

History of Redshift Discovery

- The redshift phenomenon was first observed by Vesto Melvin Slipher in the early 20th century.
- Edwin Hubble later built upon this work and demonstrated a correlation between redshift and distance, establishing the expanding universe theory.
- The discovery of the cosmic microwave background radiation by Arno Penzias and Robert Wilson further supported the Big Bang theory.

Hubble's Law

- Hubble's law is a fundamental relation in cosmology, connecting the redshift (z) of a distant object to its recessional velocity (v) and the Hubble constant (H_0):

$$v = H_0 \cdot d \tag{3}$$

- This law implies that galaxies are moving away from us, and the velocity is proportional to their distance.
- The relation between redshift and distance is given by:

$$z = \frac{v}{c} = \frac{H_0 \cdot d}{c} \tag{4}$$

Hubble Constant and Expansion Rate

- The Hubble constant (H_0) represents the current rate of expansion of the universe.
- Measured in km/s/Mpc, it quantifies how fast galaxies are moving away from us per unit distance.
- The expansion rate of the universe is given by:

$$H(t) = \frac{\dot{a}(t)}{a(t)} \tag{5}$$

where $a(t)$ is the scale factor of the universe, and $\dot{a}(t)$ is its time derivative.

Cosmological Redshift

- Cosmological redshift (z_{cosmo}) is related to the scale factor (a) of the universe:

$$1 + z_{cosmo} = \frac{a_{now}}{a_{emit}} \tag{6}$$

- This equation describes the expansion of the universe over time, where a_{now} is the current scale factor and a_{emit} is the scale factor when the light was emitted.

Expansion Factor and Scale Factor

- The expansion factor ($a(t)$) represents the relative size of the universe at cosmic time t:

$$a(t) = \frac{R(t)}{R_0} \tag{7}$$

- $R(t)$ is the scale factor at time t, and R_0 is the present scale factor.

Redshift and Expansion Factor

- The redshift is related to the expansion factor by:

$$1 + z = \frac{a_{\text{now}}}{a_{\text{emit}}} \tag{8}$$

- a_{now} is the current expansion factor, and a_{emit} is the expansion factor when the light was emitted.

Cosmic Time and Age of the Universe

- Cosmic time (t) is related to the age of the universe through the integral:

$$t = \int_0^{a_{\text{now}}} \frac{da}{aH(a)} \tag{9}$$

- The age of the universe is given by this integral, where $H(a)$ is the Hubble parameter as a function of the scale factor a.

Derivation of Age of the Universe as a Function of Redshift

The age of the universe (t) is given by the integral of the scale factor (a) with respect to cosmic time (dt):

$$t = \int_0^t dt = \int_{a_{\text{emit}}}^{a_{\text{now}}} \frac{da}{aH(a)} \tag{10}$$

where a_{emit} is the scale factor at the time the light was emitted, and a_{now} is the current scale factor.

Using the relationship between redshift (z) and the scale factor (a):

$$1 + z = \frac{a_{\text{now}}}{a_{\text{emit}}} \tag{11}$$

we can express a_{emit} in terms of a_{now} and z:

$$a_{\text{emit}} = \frac{a_{\text{now}}}{1+z} \tag{12}$$

Substitute this into the integral:

$$t = \int_{\frac{a_{\text{now}}}{1+z}}^{a_{\text{now}}} \frac{da}{aH(a)} \tag{13}$$

Redshift and Age of the Universe

- Expressing the age in terms of redshift involves a relation between cosmic time and redshift:

$$t(z) = \frac{1}{H_0} \int_z^{\infty} \frac{dz'}{(1+z')\sqrt{\Omega_M (1+z')^3 + \Omega_\Lambda}} \tag{16}$$

- Ω_M and Ω_Λ are the density parameters for matter and dark energy, respectively.

Understanding the Equations

- z represents the redshift, providing a measure of how much the universe has expanded since the light was emitted.
- Ω_M and Ω_Λ are the present-day density parameters for matter and dark energy, affecting the expansion dynamics.
- The integral involves a detailed calculation of the expansion history, considering the contributions of matter and dark energy to the cosmic evolution.

Significance and Summary

- Redshift is a crucial tool in astronomy for understanding the dynamics and large-scale structure of the universe.
- The observation of redshift supports the idea of an expanding universe, leading to the formulation of the Big Bang theory.
- In summary, redshift provides valuable information about the motion and evolution of celestial objects, contributing significantly to our understanding of the cosmos.

Research on Redshift

- Ongoing research involves using redshift data to study the large-scale structure of the universe and the distribution of galaxies.
- Advanced surveys, such as the Sloan Digital Sky Survey (SDSS), provide detailed redshift maps for millions of celestial objects.
- Researchers use statistical methods and cosmological simulations to understand the evolution and clustering of galaxies over cosmic time.

Beyond Redshift: Dark Energy

- Redshift observations have led to the discovery of dark energy, a mysterious force driving the accelerated expansion of the universe.
- Current cosmological models incorporate dark energy to explain observations of distant supernovae and cosmic microwave background radiation.

Conclusion

- Redshift is a versatile tool, not only for understanding the past but also for probing the future and mysteries of the cosmos.
- Continued advancements in technology and observational techniques promise exciting discoveries in the field of astrophysics.