Entropy

Problems from Exercise

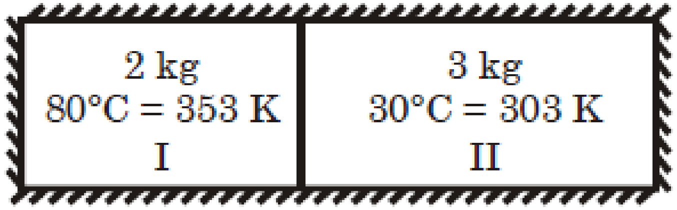
Q7.3 Two kg of water at 80°C are mixed adiabatically with 3 kg of water at 30°C in a constant pressure process of 1 atmosphere. Find the increase in the entropy of the total mass of water due to the mixing process (c_p of water = 4.187 kJ/kg K).

(**Ans.** 0.0576 kJ/K)

Solution: If final temperature of mixing is T_f then

$$2 \times c_P \ (353 - T_f)$$

$$= 3 \times c_P \ (T_f - 303)$$



or $T_f = 323$ K

$$(\Delta S)_{\text{system}} = (\Delta S)_{\text{I}} + (\Delta S)_{\text{II}}$$

$$= \int_{353}^{323} m_1 c_P \frac{dT}{T} + \int_{303}^{323} m_1 c_P \frac{dT}{T}$$

$$= 2 \times 4.187 \ln\left(\frac{323}{353}\right) + 3 \times 4.187 \times \ln\frac{323}{303}$$

$$= 0.05915 \text{ kJ/K}$$

Q7.5 A heat engine receives reversibly 420 kJ/cycle of heat from a source at 327°C, and rejects heat reversibly to a sink at 27°C. There are no other heat transfers. For each of the three hypothetical amounts of heat rejected, in (a), (b), and (c) below, compute the cyclic integral of $\text{đ}Q/T$. from these results show which case is irreversible, which reversible, and which impossible:

(a) 210 kJ/cycle rejected
(b) 105 kJ/cycle rejected
(c) 315 kJ/cycle rejected

(Ans. (a) Reversible, (b) Impossible, (c) Irreversible)

Solution: (a)
$$\oint \frac{dQ}{T} = \frac{+420}{(327+273)} - \frac{210}{(27+273)} = 0$$

∴ Cycle is Reversible, Possible

(b)
$$\oint \frac{dQ}{T} = +\frac{420}{600} - \frac{105}{300} = 0.35$$

∴ Cycle is Impossible

(c)
$$\oint \frac{dQ}{T} = +\frac{420}{600} - \frac{315}{300} = -0.35$$

∴ Cycle is irreversible but possible.

Q7.7 **Water is heated at a constant pressure of 0.7 MPa. The boiling point is 164.97°C. The initial temperature of water is 0°C. The latent heat of evaporation is 2066.3 kJ/kg. Find the increase of entropy of water, if the final state is steam**

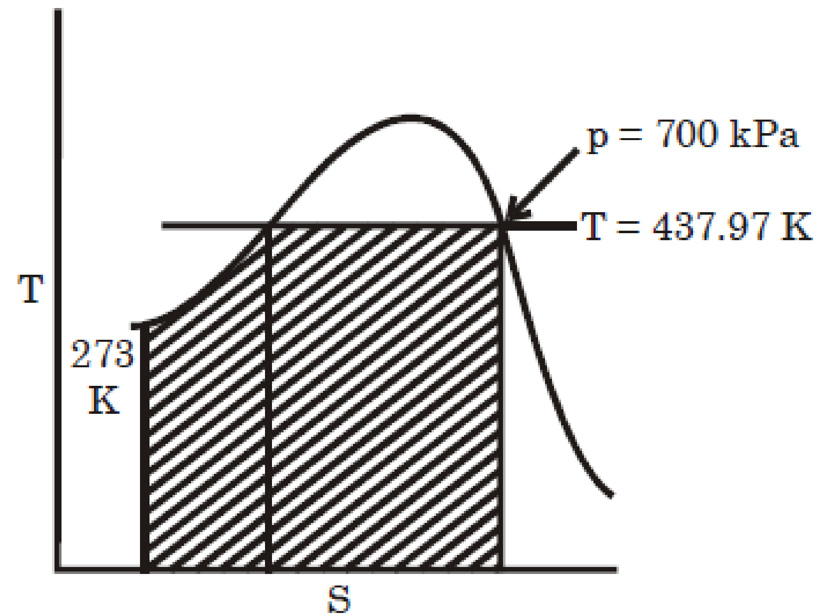
(Ans. 6.6967 kJ/kg K)

Solution: $(\Delta S)_{Water}$

$$= \int_{273}^{437.97} 1 \times 4187 \times \frac{dT}{T}$$

$$= 4.187 \ln\left(\frac{437.97}{273}\right) kJ/K$$

$$= 1.979 \text{ kJ/K}$$



$(\Delta S)_{Eva\ pour}$

$$= \frac{1 \times 2066.3}{437.97} kJ/K$$

$$= 4.7179 \text{ kJ/K}$$

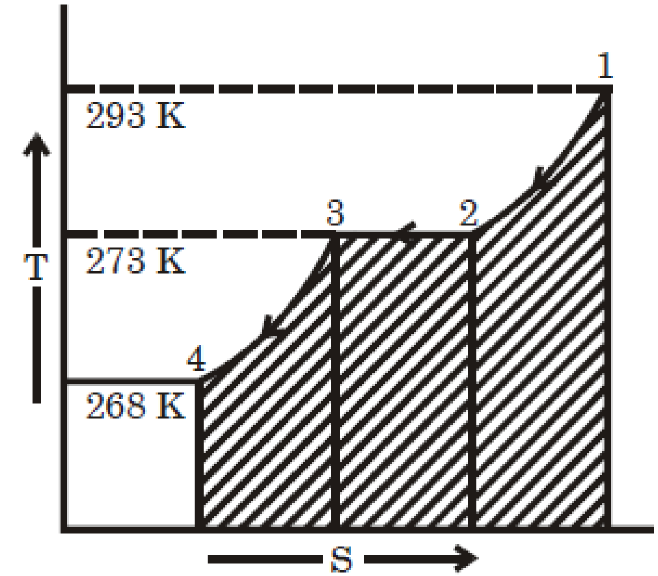
$(\Delta s)_{\text{system}}$

$$= 6.697 \text{ kJ/kg} - \text{K}$$

Q7.9 Ten grammes of water at 20°C is converted into ice at –10°C at constant atmospheric pressure. Assuming the specific heat of liquid water to remain constant at 4.2 J/gK and that of ice to be half of this value, and taking the latent heat of fusion of ice at 0°C to be 335 J/g, calculate the total entropy change of the system.

(Ans. 16.02 J/K)

Solution:



$$S_2 - S_1 = \int_{293}^{273} \frac{m\,c_p\,dT}{T}$$

$$= 0.01 \times 4.2 \times \ln\frac{273}{293}\ \text{kJ}/K$$

$$= -0.00297\ \text{kJ/K}$$

$$= -2.9694\ \text{J/K}$$

$$S_3 - S_2 = \frac{-mL}{T}$$

$$= \frac{-0.01 \times 335 \times 1000}{273}$$

$$= -12.271\ \text{J/K}$$

$$S_4 - S_3 = \int_{273}^{268} \frac{m\,c_p\,dT}{T} = 0.01 \times \left(\frac{4.2}{2}\right) \times \ln\frac{268}{273}\ \text{kJ}/K$$

$$= -0.3882\ \text{J/K}$$

$\therefore$ $S_4 - S_1 = -\ 15.63$ J/K

$\therefore$ Net Entropy change = 15.63 J/K

Q7.10 Calculate the entropy change of the universe as a result of the following processes:

(a) A copper block of 600 g mass and with C_p of 150 J/K at 100°C is placed in a lake at 8°C.

(b) The same block, at 8°C, is dropped from a height of 100 m into the lake.

(c) Two such blocks, at 100 and 0°C, are joined together.

(Ans. (a) 6.69 J/K, (b) 2.095 J/K, (c) 3.64 J/K)

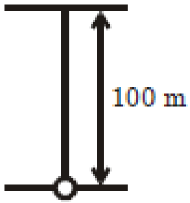
Solution:

(a)

$$(\Delta S)_{copper} = \int_{373}^{281} m c_p \frac{dT}{T}$$

$$= 150 \ln \frac{281}{373} J/K$$

$$= -42.48 \text{ J/K}$$



As unit of Cp is J/K there for

∴ It is heat capacity

i.e. $C_p = m c_p$

$$(\Delta S)_{lake} = \frac{C_p(100-8)}{281} J/K$$

$$= \frac{150(100-8)}{281} J/K = 49.11 \text{ J/K}$$

$$(\Delta S)_{univ} = (\Delta S)_{COP} + (\Delta S)_{lake} = 6.63 \text{ J/K}$$

(b) Work when it touch water = $0.600 \times 9.81 \times 100$ J = 588.6 J

As work dissipated from the copper

$$(\Delta S)_{copper} = 0$$

As the work is converted to heat and absorbed by water then

$$(\Delta S)_{lake} = \frac{W = Q}{281} = \frac{588.6}{281} J/K = 2.09466 \text{ J/K}$$

∴ $(\Delta S)_{univ} = 0 + 2.09466$ J/k = 2.09466 J/K

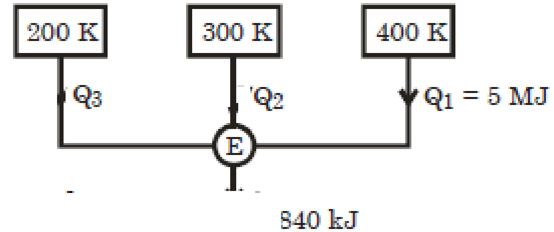
(c) Final temperature $(T_f) = \dfrac{100+0}{2} = 50°\text{ C} = 323$ K

$$(\Delta S)_I = C_p \int_{T_1}^{T_f} \frac{dT}{T} \;; \quad (\Delta S)_{II} = C_p \int_{T_2}^{T_f} \frac{dT}{T}$$

$$\therefore (\Delta S)_{system} = 150 \ln \left(\frac{T_f}{T_1}\right) + 150 \ln \left(\frac{T_f}{T_2}\right)$$

$$= 150 \left\{\ln \frac{323}{373} + \ln \frac{323}{273}\right\} J/K = 3.638 \text{ J/K}$$

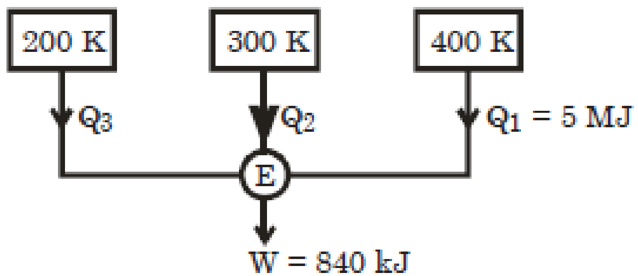
Q7.18 A reversible engine, as shown in Figure during a cycle of operations draws 5 MJ from the 400 K reservoir and does 840 kJ of work. Find the amount and direction of heat interaction with other reservoirs.



Solution: Let Q_2 and Q_3 both incoming i.e. out from the system

$\therefore$ $Q_2 \to$ +ve, $Q_3 \to$ +ve

$$(\Delta S)_{univ} = \frac{Q_3}{200} + \frac{Q_2}{300} + \frac{5000}{400} + (\Delta S)_{H.E.} + (\Delta S)_{surrounds} = 0$$



Or $$\frac{Q_3}{2} + \frac{Q_2}{3} + \frac{5000}{4} + 0 + 0 = 0$$

or $6\,Q_3 + 4\,Q_2 + 3 \times 5000 = 0$ ... (i)

$Q_3 + Q_2 + 5000 - 840 = 0$ ... (ii)

Heat balance

or $4\,Q_3 + 4\,Q_2 + 16640 = 0$ ... (iii)

$\therefore$ (i) – (iii) gives

$2\,Q_3 = +1640$

$\therefore$ $Q_3 = +820$ kJ

(Here –ve sign means heat flow opposite to our assumption)

$\therefore$ $Q_2 = -4980$ kJ