2^{nd} law of thermodynamics

Problems from Exercise

Q6.1 An inventor claims to have developed an engine that takes in 105 MJ at a temperature of 400 K, rejects 42 MJ at a temperature of 200 K, and delivers 15 kWh of mechanical work. Would you advise investing money to put this engine in the market?

(Ans. No)

Solution: Maximum thermal efficiency of his engine possible

$$\eta_{max} = 1 - \frac{200}{400} = 50\%$$

$\therefore$ That engine and deliver output = $\eta \times$ input

$$= 0.5 \times 105 \text{ MJ}$$
$$= 52.5 \text{ MJ} = 14.58 \text{ kWh}$$

As he claims that his engine can deliver more work than ideally possible so I would not advise to investing money.

Q6.3 Using an engine of 30% thermal efficiency to drive a refrigerator having a COP of 5, what is the heat input into the engine for each MJ removed from the cold body by the refrigerator?

If this system is used as a heat pump, how many MJ of heat would be available for heating for each MJ of heat input to the engine?

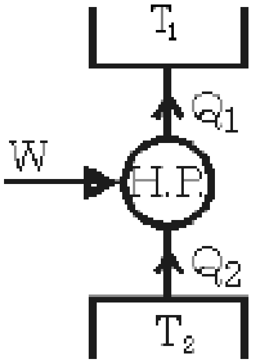
Solution: COP of the Ref. is 5

So for each MJ removed from the cold body we need work

$$= \frac{1\,MJ}{5} = 200\text{ kJ}$$

For 200 kJ work output of heat engine hair $\eta = 30\%$

We have to supply heat $= \dfrac{200\text{ kJ}}{0.3} = 666.67\text{ kJ}$



Now

COP of H.P. = COP of Ref. + 1

$= 5 + 1 = 6$

Heat input to the H.E. = 1 MJ

$\therefore$ Work output (W) $= 1 \times 0.3$ MJ $= 300$ kJ

That will be the input to H.P.

$$\therefore (\text{COP})_{H.P} = \frac{Q_1}{W}$$

$$\therefore \quad Q_1 = (\text{COP})_{H.P.} \times W = 6 \times 300\text{ kJ} = 1.8\text{ MJ}$$

Q6.6 A heat pump working on the Carnot cycle takes in heat from a reservoir at 5°C and delivers heat to a reservoir at 60°C. The heat pump is driven by a reversible heat engine which takes in heat from a reservoir at 840°C and rejects heat to a reservoir at 60°C. The reversible heat engine also drives a machine that absorbs 30 kW. If the heat pump extracts 17 kJ/s from the 5°C reservoir, determine

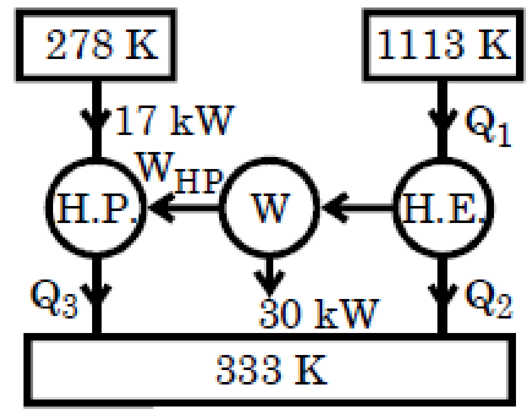
(a) The rate of heat supply from the 840°C source

(b) The rate of heat rejection to the 60°C sink.

(Ans. (a) 47.61 kW; (b) 34.61 kW)

Solution:

COP of H.P.



$$= \frac{333}{333-278} = 6.05454$$

$$Q_3 = W_{H.P.} + 17$$

$$\therefore \quad \frac{W_{H.P.} + 17}{W_{H.P.}} = 6.05454$$

$$\therefore \quad \frac{17}{W_{H.P.}} = 5.05454$$

$$\therefore \quad W_{H.P.} = \frac{17}{5.05454} = 3.36 \text{ kW}$$

$\therefore$ Work output of the Heat engine

$$W_{H.E.} = 30 + 3.36 = 33.36 \text{ kW}$$

$$\eta \text{ of the H.E.} = 1 - \frac{333}{1113} = 0.7$$

(a) $\therefore \quad \dfrac{W}{Q_1} = 0.7$

$$\therefore \quad Q_1 = \frac{W}{0.7} = 47.61 \text{ kW}$$

(b) Rate of heat rejection to the 333 K

(i) From H.E. = $Q_1 - W = 47.61 - 33.36 = 14.25$ kW

(ii) For H.P. = $17 + 3.36 = 20.36$ kW

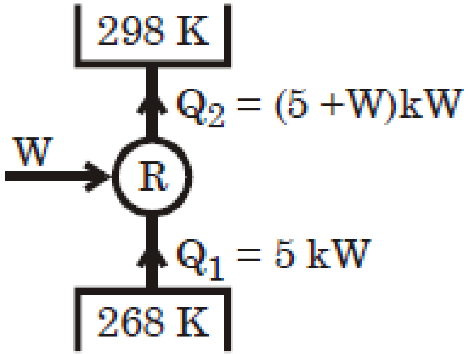
$\therefore$ Total = 34.61 kW

Q6.7 **A refrigeration plant for a food store operates as a reversed Carnot heat engine cycle. The store is to be maintained at a temperature of – 5°C and the heat transfer from the store to the cycle is at the rate of 5 kW. If heat is transferred from the cycle to the atmosphere at a temperature of 25°C, calculate the power required to drive the plant.**

(Ans. 0.56 kW)

Solution: $$(\text{COP})_R = \frac{268}{298 - 268} = 8.933$$

$$= \frac{5 \text{ kW}}{W}$$



$$\therefore \quad W = \frac{5}{8.933} \text{ kW} = 0.56 \text{ kW}$$

Q6.8 A heat engine is used to drive a heat pump. The heat transfers from the heat engine and from the heat pump are used to heat the water circulating through the radiators of a building. The efficiency of the heat engine is 27% and the COP of the heat pump is 4. Evaluate the ratio of the heat transfer to the circulating water to the heat transfer to the heat engine.

(Ans. 1.81)

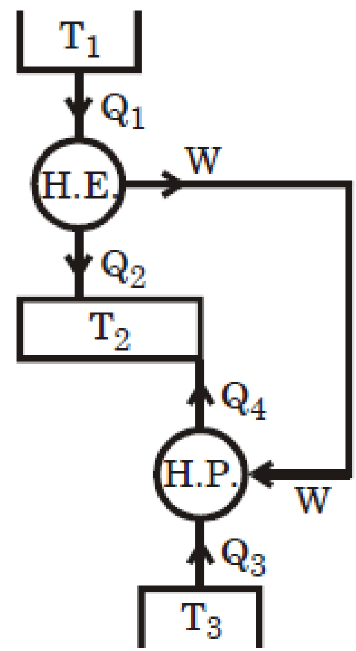
Solution: For H.E.

$$1 - \frac{Q_2}{Q_1} = 0.27$$

$$\frac{Q_2}{Q_1} = 0.73$$

$$Q_2 = 0.73\ Q_1$$

$$W = Q_1 - Q_2 = 0.27\ Q_1$$



For H.P.

$$\frac{Q_4}{W} = 4$$

$$\therefore \quad Q_4 = 4W = 1.08\ Q_1$$

$$\therefore \quad Q_2 + Q_4 = (0.73 + 1.08)\ Q_1 = 1.81\ Q_1$$

$$\therefore \quad \frac{\text{Heat transfer to the circulating water}}{\text{Heat for to the Heat Engine}}$$

$$= \frac{1.81\ Q_1}{Q_1} = 1.81$$

Q6.10 Two reversible heat engines *A* and *B* are arranged in series, A rejecting heat directly to *B*. Engine *A* receives 200 kJ at a temperature of 421°C from a hot source, while engine *B* is in communication with a cold sink at a temperature of 4.4°C. If the work output of *A* is twice that of *B*, find

(a) The intermediate temperature between *A* and *B*

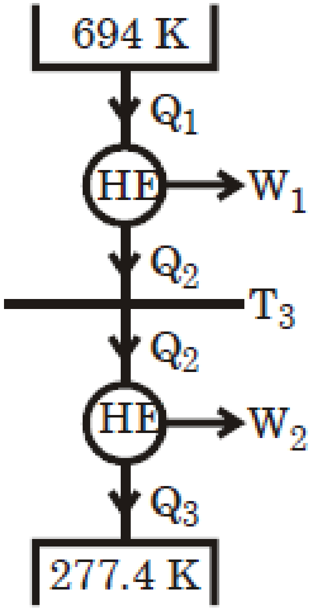
(b) The efficiency of each engine

(c) The heat rejected to the cold sink

(Ans. 143.4°C, 40% and 33.5%, 80 kJ)

Solution:

$$\frac{Q_1}{694} = \frac{Q_2}{T} = \frac{Q_1 - Q_2}{694 - T} = \frac{Q_3}{277.4} = \frac{Q_2 - Q_3}{T - 277.4}$$



Hence $Q_1 - Q_2 = 2\,W_2$

$Q_2 - Q_3 = W_2$

$$\therefore \quad \frac{2}{694 - T} = \frac{1}{T - 277.4}$$

or $\quad 2T - 277.4 \times 2 = 694 - T$

or $\quad T = 416.27\text{ K} = 143.27^\circ\text{ C}$

(b) $\quad \eta_1 = 40\%$

$$\eta_2 = 1 - \frac{277.4}{416.27} = 33.36\%$$

(c) $$Q_2 = \frac{416.27}{694} \times 200\text{ kJ} = 119.96\text{ kJ};$$

$$Q_1 = \frac{277.4}{416.27} \times 119.96 = 79.94\text{ kJ}$$

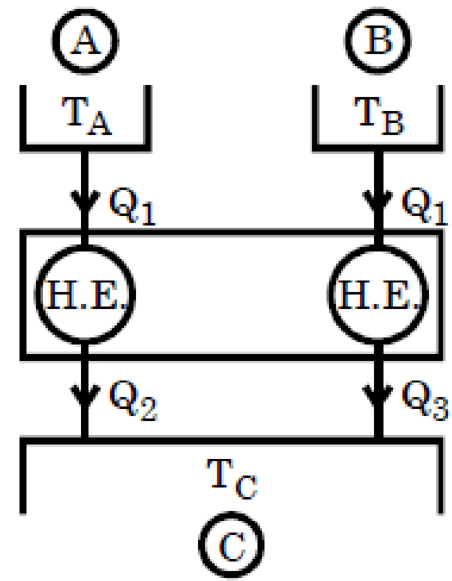
Q6.13 A reversible engine works between three thermal reservoirs, A, B and C. The engine absorbs an equal amount of heat from the thermal reservoirs A and B kept at temperatures T_A and T_B respectively, and rejects heat to the thermal reservoir C kept at temperature T_C. The efficiency of the engine is α times the efficiency of the reversible engine, which works between the two reservoirs A and C. prove that

$$\frac{T_A}{T_B} = (2\alpha - 1) + 2(1 - \alpha)\,\frac{T_A}{T_C}$$

Solution: η of H.E. between A and C

$$\eta_A = \left(1 - \frac{T_C}{T_A}\right)$$

$$\eta \text{ of our engine} = \alpha\left(1 - \frac{T_C}{T_A}\right)$$



Here $Q_2 = \frac{Q_1}{T_A} \times T_C = Q_3 = \frac{Q_1}{T_B} \times T_C$

$\therefore$ Total Heat rejection

$$(Q_2 + Q_3) = Q_1 T_C\left(\frac{1}{T_A} + \frac{1}{T_B}\right)$$

Total Heat input = $2Q_1$

$$\eta \text{ of engine} = \left[1 - \frac{Q_1 T_c\left(\frac{1}{T_A} + \frac{1}{T_B}\right)}{2Q_1}\right]$$

$$\therefore \quad \alpha - \frac{\alpha T_C}{T_A} = 1 - \frac{T_C}{2T_A} - \frac{T_C}{2T_B}$$

Multiply both side by T_A and divide by T_C

or $$\alpha\frac{T_A}{T_C} - \alpha = \frac{T_A}{T_C} - \frac{1}{2} - \frac{1}{2}\frac{T_A}{T_B}$$

or $$\frac{T_A}{T_B} = (2\alpha - 1) + 2(1 - \alpha)\frac{T_A}{T_C} \quad \text{Proved}$$

Q6.15 Two Carnot engines A and B are connected in series between two thermal reservoirs maintained at 1000 K and 100 K respectively. Engine A receives 1680 kJ of heat from the high-temperature reservoir and rejects heat to the Carnot engine B. Engine B takes in heat rejected by engine A and rejects heat to the low-temperature reservoir. If engines A and B have equal thermal efficiencies, determine

(a) The heat rejected by engine B

(b) The temperature at which heat is rejected by engine, A

(c) The work done during the process by engines, A and B respectively.

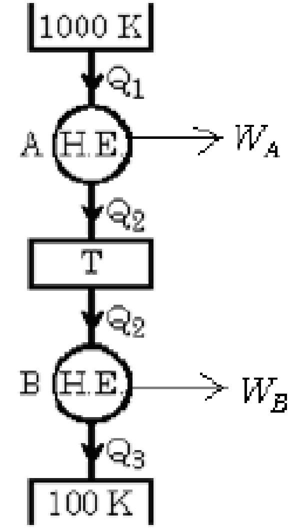
If engines A and B deliver equal work, determine

(d) The amount of heat taken in by engine B

(e) The efficiencies of engines A and B

(Ans. (a) 168 kJ, (b) 316.2 K, (c) 1148.7, 363.3 kJ, (d) 924 kJ, (e) 45%, 81.8%)

Solution: As their efficiency is same so



$$\eta_A = \eta_B$$

$$\text{or } 1 - \frac{T}{1000} = 1 - \frac{100}{T}$$

(b) $T = \sqrt{1000 \times 100} = 316.3\text{K}$

$$Q_2 = \frac{Q_1}{1000} \times T = \frac{1680 \times 316.3}{1000}$$

$$= 531.26 \text{ kJ}$$

(a) $Q_3 = \dfrac{Q_2}{316.3} \times 100 = \dfrac{531.26 \times 100}{316.3}$

$= 168$ kJ as (a)

(c) $W_A = Q_1 - Q_2 = (1880 - 531.26)$ kJ

$= 1148.74$ kJ

$W_B = (531.26 - 168)$ kJ

$= 363.26$ kJ

(d) If the equal work then $T = \dfrac{100 + 1000}{2} = 550$ K

$$\therefore Q_2 = \frac{Q_1}{1000} \times T = \frac{1680 \times 550}{1000} = 924 \text{ kJ}$$

(e) $\eta_A = 1 - \dfrac{550}{1000} = 0.45$

$\eta_B = 1 - \dfrac{100}{550} = 0.8182$

Home work

Q6.22 A heat engine operating between two reservoirs at 1000 K and 300 K is used to drive a heat pump which extracts heat from the reservoir at 300 K at a rate twice that at which the engine rejects heat to it. If the efficiency of the engine is 40% of the maximum possible and the COP of the heat pump is 50% of the maximum possible, what is the temperature of the reservoir to which the heat pump rejects heat? What is the rate of heat rejection from the heat pump if the rate of heat supply to the engine is 50 kW?

(Ans. 326.5 K, 86 kW)

Q6.23 A reversible power cycle is used to drive a reversible heat pump cycle. The power cycle takes in Q_1 heat units at T_1 and rejects Q_2 at T_2. The heat pump abstracts Q_4 from the sink at T_4 and discharges Q_3 at T_3. Develop an expression for the ratio Q_4/Q_1 in terms of the four temperatures.

$$\left(\text{Ans. } \frac{Q_4}{Q_1} = \frac{T_4(T_1 - T_2)}{T_1(T_3 - T_4)}\right)$$