1^{st} law of Thermodynamics

Problems from Exercise

Q4.2 In a cyclic process, heat transfers are + 14.7 kJ, – 25.2 kJ, – 3.56 kJ and + 31.5 kJ. What is the net work for this cyclic process?

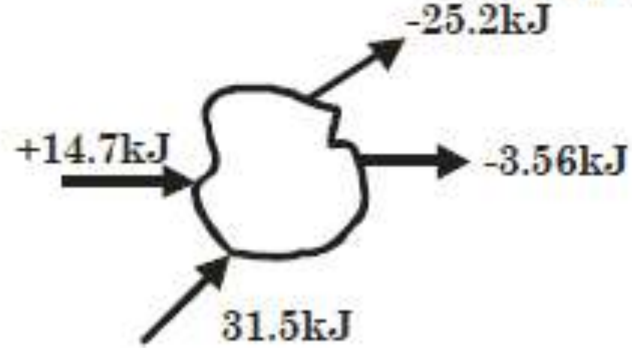
(Ans. 17.34 kJ)

Solution :

$$\sum Q = (14.7 + 31.5 - 25.2 - 3.56)\ \text{kJ}$$
$$= 17.44\ \text{kJ}$$

From first law of thermodynamics (for a cyclic process)

$$\sum Q = \sum W$$
$$\therefore \sum W = 17.44\ \text{kJ}$$



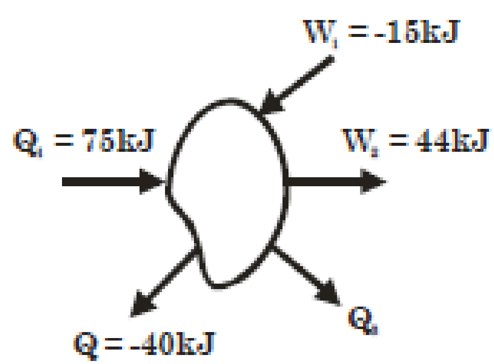
Q4.4 During one cycle the working fluid in an engine engages in two work interactions: 15 kJ to the fluid and 44 kJ from the fluid, and three heat interactions, two of which are known: 75 kJ to the fluid and 40 kJ from the fluid. Evaluate the magnitude and direction of the third heat transfer.

(Ans. – 6 kJ)

Solution: From first law of thermodynamics

$$\sum đQ = \sum đW$$
$$\therefore \quad Q_1 + Q_2 + Q_3 = W_1 + W_2$$
$$\text{or} \quad 75 - 40 + Q_3 = -15 + 44$$
$$Q_3 = -6\text{kJ}$$

i.e. 6kJ from the system



Q4.5 A domestic refrigerator is loaded with food and the door closed. During a certain period the machine consumes 1 kWh of energy and the internal energy of the system drops by 5000 kJ. Find the net heat transfer for the system.

(Ans. – 8.6 MJ)

Solution: $Q = \Delta E + W$

$$Q_{2-1} = (E_2 - E_1) + W_{2-1}$$

$$= -5000\text{kJ} + \frac{-1000 \times 3600}{1000}\text{kJ}$$

$$= -8.6\text{MJ}$$

Q4.6 1.5 kg of liquid having a constant specific heat of 2.5 kJ/kg K is stirred in a well-insulated chamber causing the temperature to rise by 15°C. Find ΔE and W for the process.

(Ans. $\Delta E = 56.25$ kJ, $W = -56.25$ kJ)

Solution: Heat added to the system $= 1.5 \times 2.5 \times 15\text{kJ}$

$= 56.25$ kJ

$\therefore$ ΔE rise $= 56.25$kJ

As it is insulated then $\cancel{d}Q = 0$

$\therefore$ $\Delta Q = \Delta E + W$

or $0 = 56.25 + W$

or $W = -56.25$ kJ

Q4.7 **The same liquid as in Problem 4.6 is stirred in a conducting chamber. During the process 1.7 kJ of heat are transferred from the liquid to the surroundings, while the temperature of the liquid is rising to 15°C. Find ΔE and W for the process.**

(Ans. $\Delta E = 54.55$ kJ, $W = 56.25$ kJ)

Solution: As temperature rise is same so internal energy is same

$$\Delta E = 56.25 \text{ kJ}$$

As heat is transferred form the system so we have to give more work = 1.7 kJ to the system

So $$W = -56.25 - 1.7 \text{ kJ}$$
$$= -57.95 \text{ kJ}$$

Q4.8 The properties of a certain fluid are related as follows:

$$u = 196 + 0.718t$$
$$pv = 0.287\ (t + 273)$$

Where u is the specific internal energy (kJ/kg), t is in °C, p is pressure (kN/m²), and v is specific volume (m³/kg). For this fluid, find c_v and c_p.

(Ans. 0.718, 1.005 kJ/kg K)

Solution:

$$C_p = \left(\frac{\partial h}{\partial T}\right)_p$$
$$= \left[\frac{\partial(u + pV)}{\partial T}\right]_p$$
$$= \left[\frac{\partial\{196 + 0.718t + 0.287(t + 273)\}}{\partial T}\right]_p$$
$$= \left[\frac{0 + 0.718\ \partial t + 0.287\ \partial t + 0}{\partial T}\right]_p$$
$$= \left[1.005\frac{\partial t}{\partial T}\right]_p \qquad \begin{bmatrix} T = t + 273 \\ \therefore \partial T = \partial t \end{bmatrix}$$
$$= 1.005\ \text{kJ / kg} - \text{K}$$

$$c_v = \left(\frac{\partial u}{\partial T}\right)_v$$
$$= \left[\frac{\partial(196 + 0.718t)}{\partial T}\right]_v$$
$$= \left[0 + 0.718\frac{\partial t}{\partial T}\right]_v$$
$$= 0.718\ \text{kJ / kg} - \text{K}$$

Q4.9 A system composed of 2 kg of the above fluid expands in a frictionless piston and cylinder machine from an initial state of 1 MPa, 100°C to a final temperature of 30°C. If there is no heat transfer, find the net work for the process.

(Ans. 100.52 kJ)

Solution: Heat transfer is not there so

$$Q = \Delta E + W$$

$$\begin{aligned} W &= -\Delta E \\ &= -\Delta U \\ &= -\int_1^2 C_v dT \\ &= -0.718(T_2 - T_1) \\ &= -0.718(100 - 30) \\ &= -50.26 \text{ kJ/kg} \end{aligned}$$

$\therefore$ Total work (W) = 2 × (-50.26) = -100.52 kJ

Q 4.10 If all the work in the expansion of Problem 4.9 is done on the moving piston, show that the equation representing the path of the expansion in the *pv*-plane is given by $pv^{1.4}$ = constant.

Solution: Let the process is pV^n = constant.

Then

$$\text{Work done} = \frac{p_1V_1 - p_2V_2}{n-1} \qquad [\because pV = mRT]$$

$$= \frac{mRT_1 - mRT_2}{n-1} \qquad \left[\begin{aligned} R &= (c_p - c_v) \\ &= 1.005 - 0.718 \\ &= 0.287 \text{ kJ/kg} - K \end{aligned}\right]$$

$$= \frac{mR}{n-1}(T_1 - T_2)$$

or $$= \frac{2 \times 0.287 \times (100 - 30)}{n-1} = 100.52$$

or $$n - 1 = 0.39972$$

or $$n = 1.39972 \approx 1.4$$

Q4.13 A mass of 8 kg gas expands within a flexible container so that the p–v relationship is of the from $pv^{1.2}$ = constant. The initial pressure is 1000 kPa and the initial volume is 1 m³. The final pressure is 5 kPa. If specific internal energy of the gas decreases by 40 kJ/kg, find the heat transfer in magnitude and direction.

(Ans. + 2615 kJ)

Solution:

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{V_1}{V_2}\right)^{n-1}$$

$$\therefore \quad \frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^{n}$$

$$\text{or} \quad \frac{V_2}{V_1} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}}$$

$$\text{or} \quad V_2 = V_1 \times \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}}$$

$$= 1 \times \left(\frac{1000}{5}\right)^{\frac{1}{1.2}} = 82.7\,\text{m}^3$$

$$\therefore \quad W = \frac{p_1V_1 - p_2V_2}{n-1}$$

$$= \frac{1000 \times 1 - 5 \times 82.7}{1.2 - 1} = 2932.5\,\text{kJ}$$

$$\Delta E = -8 \times 40 = -320\ \text{kJ}$$

$$\therefore \quad Q = \Delta E + W = -320 + 2932.5 = 2612.5\,\text{kJ}$$

Q4.14 A gas of mass 1.5 kg undergoes a quasi-static expansion which follows a relationship $p = a + bV$, where a and b are constants. The initial and final pressures are 1000 kPa and 200 kPa respectively and the corresponding volumes are 0.20 m³ and 1.20 m³. The specific internal energy of the gas is given by the relation

$$u = 1.5\,pv - 85 \text{ kJ/kg}$$

Where p is the kPa and v is in m³/kg. Calculate the net heat transfer and the maximum internal energy of the gas attained during expansion.

(Ans. 660 kJ, 503.3 kJ)

Solution:

$$1000 = a + b \times 0.2 \quad \text{....(i)}$$

$$200 = a + b \times 1.2 \quad \text{...(ii)}$$

(ii) – (i) gives

$$-800 = b$$

$$\therefore \quad a = 1000 + 2 \times 800 = 1160$$

$$\therefore \quad p = 1160 - 800V$$

$$\therefore \quad W = \int_{v_1}^{v_2} p\,dV$$

$$= \int_{0.2}^{1.2} (1160 - 800V)\,dV$$

$$= \left[1160V - 400V^2\right]_{0.2}^{1.2}$$

$$= 1160 \times (1.2 - 0.2) - 400\left(1.2^2 - .2^2\right)\text{kJ}$$

$$= 1160 - 560\text{kJ} = 600\text{kJ}$$

$$u_1 = 1.5 \times 1000 \times \frac{0.2}{1.5} - 85 = 215 kJ/kg$$

$$u_2 = 1.5 \times 200 \times \frac{1.2}{1.5} - 85 = 155 kJ/kg$$

$$\therefore \ \Delta u = u_2 - u_1 = (275 - 215) = 40 kJ/kg$$

$$\therefore \Delta U = m\Delta u = 40 \times 1.5 = 60 kJ$$

$$\therefore \quad Q = \Delta U + W = 60 + 600 = 660 kJ$$

$$\Rightarrow \quad u = 1.5pv - 85 kJ/kg$$

$$= 1.5\left(\frac{1160 - 800v}{1.5}\right)v - 85 kJ/kg$$

$$= 1160v - 800v^2 - 85 kJ/kg$$

$$\frac{\partial u}{\partial v} = 1160 - 1600v$$

for maximum u, $\quad \frac{\partial u}{\partial v} = 0 \therefore \ v = \frac{1160}{1600} = 0.725$

$$\therefore \qquad u_{max.} = 1160 \times 0.725 - 800 \times (0.725)^2 - 85 kJ/kg$$

$$= 335.5 kJ/kg$$

$$U_{max} = 1.5 u_{max} = 503.25 kJ$$

Home work

4.19 A certain gas of mass 4 kg is contained within a piston cylinder assembly. The gas undergoes a process for which $pV^{1.5}$ = constant. The initial state is given by 3 bar, 0.1 m^3. The change in internal energy of the gas in the process is $u_2 - u_1 = -4.6$ kJ/kg. Find the net heat transfer for the process when the final volume is 0.2 m^3. Neglect the changes in KE and PE.

Ans. – 0.8 kJ

4.22 Air is contained in a rigid well-insulated tank with a volume of 0.2 m^3. The tank is fitted with a paddle wheel which transfers energy to the air at a constant rate of 4 W for 20 min. The initial density of the air is 1.2 kg/m^3. If no changes in KE or PE occur, determine (a) the specific volume at the final state, (b) the change in specific internal energy of the air.

Ans. (a) 0.833 m^3/kg, (b) 20 kJ/kg