Work and Heat

Problems from Exercise

Q3.4 A mass of 1.5 kg of air is compressed in a quasi-static process from 0.1 MPa to 0.7 MPa for which *pv* = constant. The initial density of air is 1.16 kg/m³. Find the work done by the piston to compress the air.

(Ans. 251.62 kJ)

Solution: For quasi-static process

Work done $= \int p\,dV$ [given $pV = C$

$$= p_1V_1 \int_{V_1}^{V_2} \frac{dV}{V} \qquad \therefore \quad p_1V_1 = pV = p_2V_2 = C$$

$$= p_1V_1 \ln\left(\frac{V_2}{V_1}\right) \qquad \therefore \quad p = \frac{p_1V_1}{V}$$

$$= p_1V_1 \ln\left(\frac{p_1}{p_2}\right) \qquad \therefore \quad \frac{p_1}{p_2} = \frac{V_2}{V_1}$$

$$= 0.1 \times 1.2931 \times \ln\left|\frac{0.1}{0.7}\right| \text{MJ} \qquad \text{given } p_1 = 0.1 \text{ MPa}$$

$$= 251.63 \text{ kJ} \qquad V_1 = \frac{m_1}{\rho_1} = \frac{1.5}{1.16} \text{ m}^3$$

$$p_2 = 0.7 \text{ MPa}$$

Q3.5 A mass of gas is compressed in a quasi-static process from 80 kPa, 0.1 m^3 to 0.4 MPa, 0.03 m^3. Assuming that the pressure and volume are related by pv^n = constant, find the work done by the gas system.

(Ans. –11.83 kJ)

Solution: Given initial pressure $(p_1) = 80\text{kPa}$

$$\text{Initial volume}(V_1) = 0.1\text{ m}^3$$

$$\text{Final pressure }(p_2) = 0.4\text{ MPa} = 400\text{ kPa}$$

$$\text{Final volume}(V_2) = 0.03\text{ m}^3$$

$$\text{As p-V relation } pV^n = C$$

$$\therefore \quad p_1V_1^n = p_2V_2^n$$

taking $\log_e$ both side

$$\ln p_1 + n\ln V_1 = \ln p_2 + n\ln V_2$$

$$\text{or} \quad n[\ln V_1 - \ln V_2] = \ln p_2 - \ln p_1$$

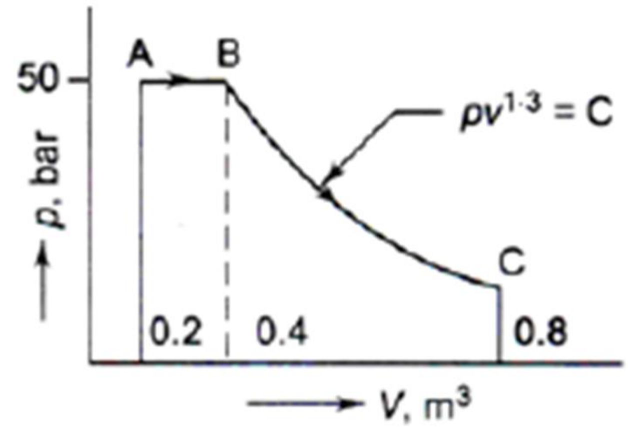
$$\text{or} \quad n\ln\left(\frac{V_1}{V_2}\right) = \ln\left(\frac{p_2}{p_1}\right)$$

$$\text{or} \quad n = \frac{\ln\left({p_2}/{p_1}\right)}{\ln\left({V_1}/{V_2}\right)} = \frac{\ln\left(\frac{400}{80}\right)}{\ln\left(\frac{0.1}{0.03}\right)} = \frac{1.60944}{1.20397} \approx 1.3367 \approx 1.34$$

$$\therefore \quad \text{Work done}(W) = \frac{p_1V_1 - p_2V_2}{n-1}$$

$$= \frac{80\times 0.1 - 400\times 0.03}{1.34-1} = -11.764\text{ kJ}$$

Q3.13 **Determine the total work done by a gas system following an expansion process as shown in Figure.**

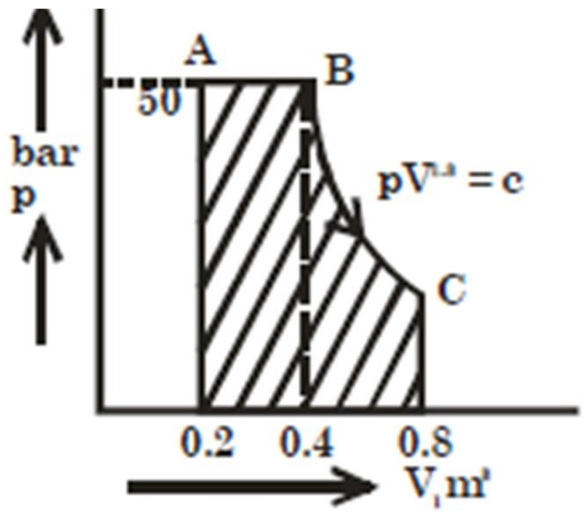


(Ans. 2.253 MJ)

Solution: Area under AB

$= (0.4 - 0.2) \times 50 \times 10^5$ J

$= 10^6$ W $=$ 1 MJ



Area under BC

$$= \frac{p_1 V_1 - p_2 V_2}{n-1}$$

$$= \frac{50\times10^5 \times 0.4 - 20.31\times10^5 \times 0.8}{1.3-1} W$$

$$= 1.251 MJ$$

Here $p_B = p_B = 50 \text{ bar} = 50\times10^5 \text{ Pa}$

$V_B = 0.4 m^3$

$V_C = 0.8 m^3$

$$p_C = \frac{p_B V_B^{1.5}}{V_C^{1.5}} = \frac{50\times10^5 \times 0.4^{1.5}}{0.8^{1.5}}$$

$$= 20.31\times10^5 \text{ Pa}$$

Total work = 2.251MJ

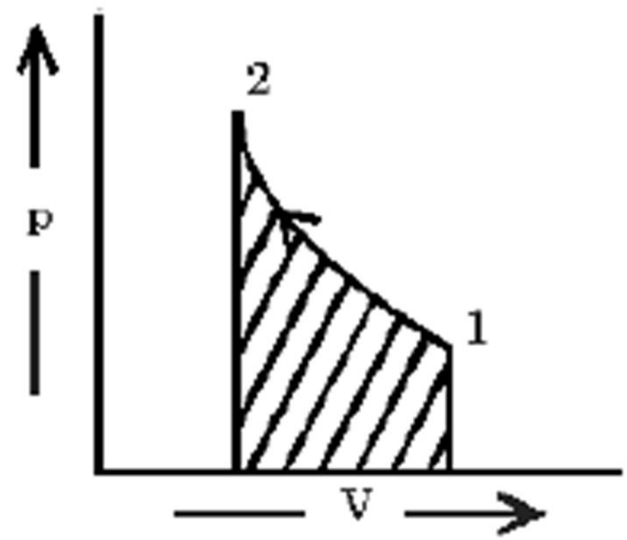
Q3.15 **If a gas of volume 6000 cm³ and at pressure of 100 kPa is compressed quasistatically according to pV^2 = constant until the volume becomes 2000 cm³, determine the final pressure and the work transfer.**

(Ans. 900 kPa, – 1.2 kJ)

Solution: Initial volume $(V_1) = 6000\ \text{cm}^3$

$= 0.006\ \text{m}^3$

Initial pressure $(p_1) = 100$ kPa



Final volume $(V_2) = 2000\ \text{cm}^3$

$= 0.002\ \text{m}^3$

If final pressure (p_2)

$$\therefore \quad p_2 = \frac{p_1 V_1^2}{V_2^2} = \frac{100 \times (0.006)^2}{(0.002)^2} = 900\ \text{kPa}$$

$$\text{work done on the system} = \frac{1}{n-1}\left[p_2 V_2 - p_1 V_1\right]$$

$$= \frac{1}{2-1}\left[900 \times 0.002 - 100 \times 0.006\right] \text{kJ}$$

$$= 1.2\ \text{kJ}$$