Mohammad Ali Jinnah University

Chartered by Government of Sindh - Recognized by HEC

Assignment 1

Name: Muhamad Fahad

Id: FA19-BSSE-0014

Subject: Linear Algebra (Fall 2020)

Section: AM

Teacher: Dr. Asmat Ara

Date: Wednesday, October 28, 2020

Attempt all questions.

Q.1 (a) Evaluate the matrix

$$\begin{bmatrix} 2 & -1 & -1 & 4 \\ 3 & 4 & -2 & 11 \\ 3 & -2 & 4 & 11 \end{bmatrix}$$ into Reduced Echelon form.

Solution:

Solution:-

$$\Rightarrow \begin{bmatrix} 2 & -1 & -1 & 4 \\ 3 & 4 & -2 & 11 \\ 3 & -2 & 4 & 11 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} & 2 \\ 3 & 4 & -2 & 11 \\ 3 & -2 & 4 & 11 \end{bmatrix} \quad \tfrac{1}{2}R_1$$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} & 2 \\ 0 & \frac{11}{2} & -\frac{1}{2} & 5 \\ 0 & -\frac{1}{2} & \frac{11}{2} & 5 \end{bmatrix} \quad \begin{matrix} -3R_1 + R_2 \\ -3R_1 + R_3 \end{matrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} & 2 \\ 0 & 1 & -\frac{1}{11} & \frac{10}{11} \\ 0 & -\frac{1}{2} & \frac{11}{2} & 5 \end{bmatrix} \quad \begin{matrix} \tfrac{1}{2}R_2 + R_1 \\ \tfrac{1}{2}R_1 + R_2 \end{matrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{6}{11} & \frac{17}{11} \\ 0 & 1 & -\frac{1}{11} & \frac{10}{11} \\ 0 & 0 & \frac{60}{11} & \frac{60}{11} \end{bmatrix} \quad \tfrac{11}{60}R_3$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{6}{11} & \frac{17}{11} \\ 0 & 1 & -\frac{1}{11} & \frac{10}{11} \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad \begin{matrix} \tfrac{1}{11}R_3 + R_2 \\ \tfrac{6}{11}R_3 + R_1 \end{matrix}$$

Date

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad R.E.f$$

Ans

(b) Evaluate the matrix

$$\begin{bmatrix} 1 & -1 & 2 & 0 \\ 4 & 1 & 2 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix} \text{ into Echelon form.}$$

Solution:

$$\Rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 4 & 1 & 2 & 1 \\ 1 & 1 & 0 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 5 & -6 & 1 \\ 0 & 2 & -2 & -1 \end{bmatrix} \begin{matrix} -4R_1 + R_2 \\ -R_1 + R_3 \end{matrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & -6/5 & 1/5 \\ 0 & 2 & -2 & -1 \end{bmatrix} \frac{1}{5}R_2$$

$-4 \times -1 = 4 + 1$
$= 5$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & -6/5 & 1/5 \\ 0 & 0 & 7/5 & -7/5 \end{bmatrix} -2R_2 + R_3$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & -6/5 & 1/5 \\ 0 & 0 & 1 & -1 \end{bmatrix} \frac{5}{7}R_3$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & -6/5 & 1/5 \\ 0 & 0 & 1 & -1 \end{bmatrix} \quad R.E.F$$

Ans

Q.2 Solve the following system by Gaussian Jordan method.

$$5x_1 + \quad +4x_3 + 2x_4 = 3$$

$$x_1 - x_2 + 2x_3 + x_4 = 1$$

$$4x_1 + x_2 + 2x_3 \quad = 1$$

$$x_1 + x_2 + x_3 + x_4 = 0$$

<u>**Solution:**</u>

Soln

$$\begin{bmatrix} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 5 & 0 & 4 & 2 & 3 \end{bmatrix} \quad R_{14}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 1 \\ 0 & -3 & -2 & 4 & 1 \\ 0 & -5 & -1 & -3 & 3 \end{bmatrix} \quad \begin{matrix} -R_1 + R_2 \\ -4R_1 + R_3 \\ -5R_1 + R_4 \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1/2 & 0 & -1/2 \\ 0 & -3 & -2 & -4 & 1 \\ 0 & -5 & -1 & -3 & 3 \end{bmatrix} \quad -1/2\, R_2$$

Date

$$\begin{bmatrix} 1 & 0 & 3/2 & 1 & 1/2 \\ 0 & 1 & -1/2 & 0 & -1/2 \\ 0 & 0 & -3/2 & -4 & -1/2 \\ 0 & 0 & -3/2 & -3 & 1/2 \end{bmatrix} \begin{matrix} \\ 3R_2 + R_3 \\ 5R_2 + R_4 \\ -R_2 + R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 3/2 & 1 & 2/7 \\ 0 & 1 & -1/2 & 0 & -3/7 \\ 0 & 0 & 1 & 8/7 & 1/7 \\ 0 & 0 & 7/2 & -3 & 1/2 \end{bmatrix} \quad 2/7\, R_3$$

$$\begin{bmatrix} 1 & 0 & 0 & -5/7 & 2/7 \\ 0 & 1 & 0 & 4/7 & -3/7 \\ 0 & 0 & 1 & 8/7 & 1/7 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{matrix} 7/2\, R_3 + R_4 \\ 1/2\, R_3 + R_4 \\ -3/2\, R_3 + R_4 \\ \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{matrix} -8/7\, R_4 + R_3 \\ -4/7\, R_4 + R_3 \\ 5/7\, R_4 + R_3 \\ \end{matrix}$$

$$\left.\begin{matrix} x_1 = 1 \\ x_2 = -1 \\ x_3 = -1 \\ x_4 = 1 \end{matrix}\right\} \text{Unique Solution}$$

Consistency Criteria

$$\text{Rank}(AD) = \text{Rank}A = \text{No of Unknown}$$

$$4 = 4 = 4$$

Unique solution

Q.3 Solve by elementary row operations.

$$4x + 5y = 2$$

$$11x + y + 2z = 3$$

$$x + 5y + 2z = 1$$

Solution:

Solution:- (by Gauss Elimination method)

$$\begin{bmatrix} 4 & 5 & 0 & 2 \\ 11 & 1 & 2 & 3 \\ 1 & 5 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 5 & 2 & 1 \\ 11 & 1 & 2 & 3 \\ 4 & 5 & 0 & 2 \end{bmatrix} R_{13}$$

$$\begin{bmatrix} 1 & 5 & 2 & 1 \\ 0 & -54 & -20 & -8 \\ 0 & -15 & -8 & -2 \end{bmatrix} \begin{matrix} -11R_1 + R_2 \\ \\ -4R_1 + R_3 \end{matrix}$$

$$\begin{bmatrix} 1 & 5 & 2 & 1 \\ 0 & 1 & 10/27 & 4/27 \\ 0 & -15 & -8 & 2 \end{bmatrix} -1/54\, R_2$$

$$\begin{bmatrix} 1 & 5 & 2 & 1 \\ 0 & 1 & 10/27 & 4/27 \\ 0 & 0 & -22/9 & 2/9 \end{bmatrix} 15R_2 + R_3$$

$$\begin{bmatrix} 1 & 5 & 2 & 1 \\ 0 & 1 & 10/27 & 4/27 \\ 0 & 0 & 1 & 1/14 \end{bmatrix} -9/22\, R_3$$

$x_1 + 5x_2 + 2x_3 = 1$ — (i)

$+ x_2 + \frac{10}{27}x_3 = \frac{4}{27}$ — (ii)

$x_3 = -\frac{1}{11}$ — (iii)

Put x_3 in eq (ii)

$x_2 = \frac{4}{27} - \frac{10}{27}x_3$

$x_2 = \frac{4}{27} - \frac{10}{27}\left(\frac{-1}{11}\right)$

$x_2 = \frac{4}{27} + \frac{10}{297}$

$x_2 = \frac{44 + 10}{297} \Rightarrow \frac{\cancel{54}^{2}}{\cancel{297}_{11}}$

$x_2 = \frac{2}{11}$ — (ii)

Put eq (ii) & (iii) in eq (i)

$x_1 = 1 - 5x_2 + 2x_3$

$x_1 = 1 - 5\left(\frac{2}{11}\right) + 2\left(-\frac{1}{11}\right)$

$x_1 = 1 - \frac{10}{11} + \frac{2}{11}$

$x_1 = \frac{13 - 10 + 2}{11} \Rightarrow \frac{3}{11}$

$x_1 = \frac{3}{11}$, $x_2 = \frac{2}{11}$, $x_3 = -\frac{1}{11}$ = Unique Solution

Consistency Criteria:

Rank(AD) = Rank(A) = No of Unknowns

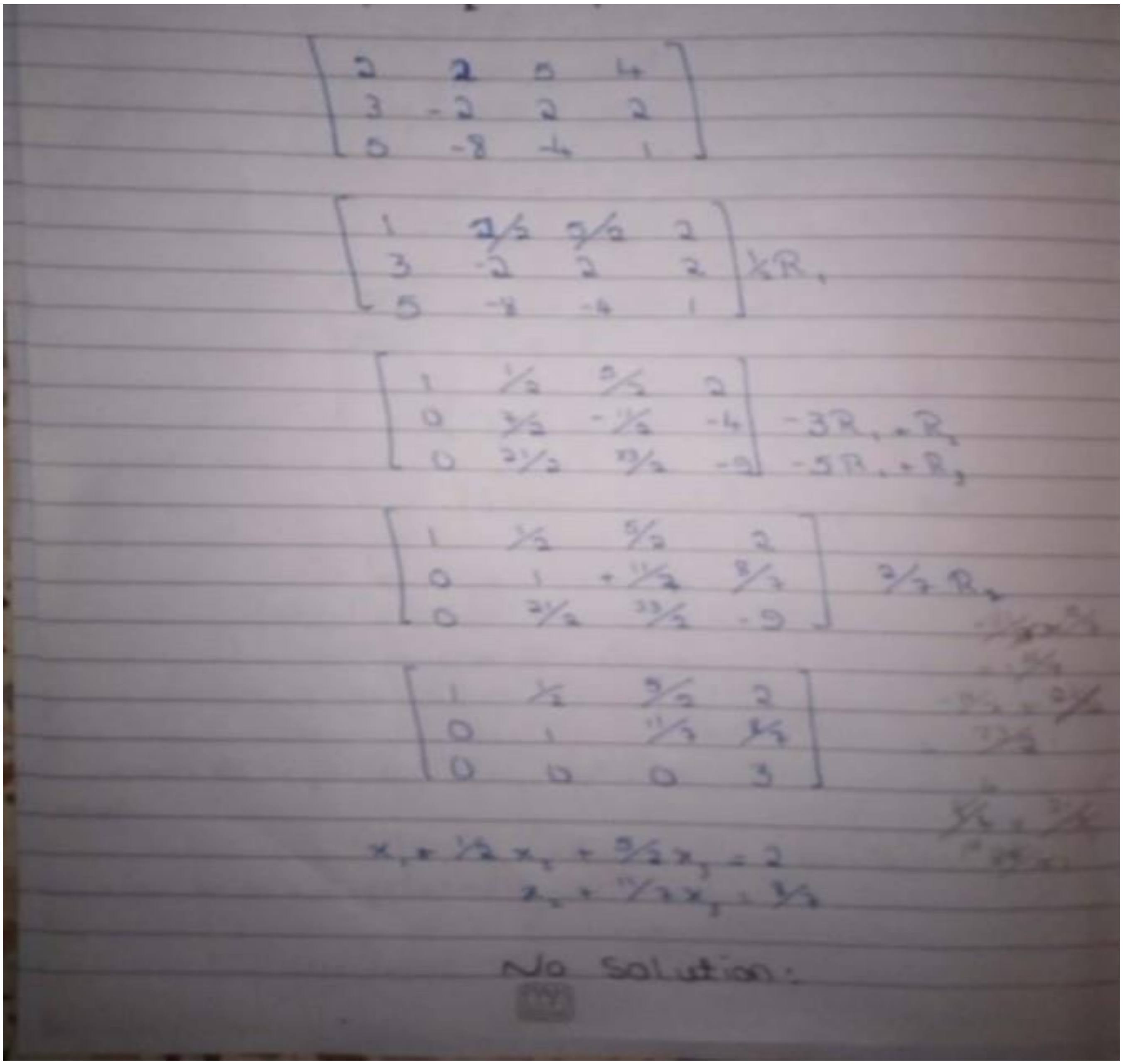
3 = 3 = 3

Unique Solution.

Q.4 Solve by elementary row operations. (by Gauss Elimination)

$$2x_1 + x_2 + 5x_3 = 4$$
$$3x_1 - 2x_2 + 2x_3 = 2$$
$$5x_1 - 8x_2 - 4x_3 = 1$$

Solution:



$$\begin{bmatrix} 2 & 1 & 5 & 4 \\ 3 & -2 & 2 & 2 \\ 5 & -8 & -4 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1/2 & 5/2 & 2 \\ 3 & -2 & 2 & 2 \\ 5 & -8 & -4 & 1 \end{bmatrix} \tfrac{1}{2}R_1$$

$$\begin{bmatrix} 1 & 1/2 & 5/2 & 2 \\ 0 & -7/2 & -11/2 & -4 \\ 0 & -21/2 & -33/2 & -9 \end{bmatrix} \begin{matrix} -3R_1+R_2 \\ -5R_1+R_3 \end{matrix}$$

$$\begin{bmatrix} 1 & 1/2 & 5/2 & 2 \\ 0 & 1 & 11/7 & 8/7 \\ 0 & -21/2 & -33/2 & -9 \end{bmatrix} -\tfrac{2}{7}R_2$$

$$\begin{bmatrix} 1 & 1/2 & 5/2 & 2 \\ 0 & 1 & 11/7 & 8/7 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

$x_1 + \tfrac{1}{2}x_2 + \tfrac{5}{2}x_3 = 2$

$x_2 + \tfrac{11}{7}x_3 = 8/7$

No Solution.

Consistency Criterian (No solution)

$$\text{Rank}(AD) \neq \text{Rank}(A)$$
$$3 \neq 2$$

No solution.

Q.5 Solve by elementary row operations. (by Gauss Elimination)

$$2x_1 + \quad + x_3 = 1$$
$$2x_1 + 4x_2 - x_3 = -2$$
$$x_1 - 8x_2 - 3x_3 = 2$$

Solution:

$$\begin{bmatrix} 2 & 0 & 1 & 1 \\ 2 & 4 & -1 & -2 \\ 1 & -8 & -3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -8 & -3 & 2 \\ 2 & 4 & -1 & -2 \\ 2 & 0 & 1 & 1 \end{bmatrix} \quad R_{13}$$

$$\begin{bmatrix} 1 & -8 & -3 & 2 \\ 0 & 20 & 5 & -6 \\ 0 & 16 & 7 & -3 \end{bmatrix} \quad \begin{array}{l} -2R_1 + R_2 \\ -2R_1 + R_3 \end{array}$$

$$\begin{bmatrix} 1 & -8 & -3 & 2 \\ 0 & 1 & 1/4 & -3/10 \\ 0 & 0 & 7 & -3 \end{bmatrix} \quad \frac{1}{20}R_2$$

$$\begin{bmatrix} 1 & -8 & -3 & 2 \\ 0 & 1 & 1/4 & -3/10 \\ 0 & 0 & 3 & 9/5 \end{bmatrix} -16R_2+R_3$$

$$\begin{bmatrix} 1 & -8 & -3 & 2 \\ 0 & 1 & 1/4 & -3/10 \\ 0 & 0 & 1 & 3/5 \end{bmatrix} 1/3\, R_3$$

$$x_1 - 8x_2 - 3x_3 = 2 \quad -(i)$$
$$x_2 + 1/4\, x_3 = -3/10 \quad -(ii)$$
$$x_3 = 3/5 \quad -(iii)$$

Now eq (ii)

$$x_2 = -3/10 - 1/4\, x_3 \quad \because \text{put } x_3 \text{ value}$$
$$x_2 = -3/10 - 1/4(3/5)$$
$$x_2 = \frac{-3}{10} - \frac{3}{20}$$
$$x_2 = (-6-3)/20$$
$$\boxed{x_2 = -9/20}$$

Now eq (i):

$$x_1 = 2 + 8x_2 + 3x_3$$
$$x_1 = 2 + 8\left(\frac{-9}{20}\right) + 3\left(\frac{3}{5}\right)$$

$$x_1 = 2 - 8/5 + 9/5$$

Unique sol $\{ x_1 = 1/5,\ x_2 = -9/20,\ x_3 = 3/5 \}$

Consistency Criteria:-

Rank(A:D) = Rank(A) = No of Unknow

3 = 3 = 3

Unique Solution

Q.6 Solve by elementary row operations. (by Gauss Jordan)

$$6x_1 - 6x_2 + 6x_3 = 6$$
$$2x_1 - 4x_2 - 6x_3 = 12$$
$$10x_1 - 5x_2 + 5x_3 = 30$$

Solution:

Solution:

$$\begin{bmatrix} 6 & -6 & 6 & 6 \\ 2 & -4 & -6 & 12 \\ 10 & -5 & 5 & 30 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 1 \\ 2 & -4 & -6 & 12 \\ 10 & -5 & 5 & 30 \end{bmatrix} \tfrac{1}{6}R_1$$

$$\begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & -2 & -8 & 10 \\ 0 & 5 & -5 & 20 \end{bmatrix} \begin{matrix} -2R_1 + R_2 \\ -10R_1 + R_3 \end{matrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 1 & 4 & -5 \\ 0 & 5 & -5 & 20 \end{bmatrix} -\tfrac{1}{2}R_2$$

$$\begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 1 & -4 & -5 \\ 0 & 0 & -25 & 45 \end{bmatrix} \begin{matrix} -5R_2 + R_3 \\ R_2 + R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 5 & -4 \\ 0 & 1 & -4 & -5 \\ 0 & 0 & 1 & 9/5 \end{bmatrix} -\tfrac{1}{25}$$

Date

$$\begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & -9/5 \end{bmatrix} \quad \begin{array}{l} -4R_3 + R_2 \\ -5R_3 + R_1 \end{array}$$

$$\left.\begin{array}{ll} x_1 = -4 & \text{— (i)} \\ x_2 = -5 & \text{— (ii)} \\ x_3 = -9/5 & \text{— (iii)} \end{array}\right\} \text{Unique Solution}$$

Consistency Criteria:-

Rank (AD) = Rank (A) = No of Unknow

3 = 3 = 3

Unique Solution.

Q.7 Solve by elementary row operations. (by Gauss Jordan)

$$5x_1 \; - 2\; x_2 \; + x_3 = 0$$

$$3x_1 + 2\,x_2 + 7x_3 = 0$$

$$x_1 + \; x_2 + 3x_3 \; = 0$$

Solution:

Q7) Solve by elementary row Operation (by Gauss Jordan)

$$5x_1 - 2x_2 + x_3 = 0$$
$$3x_1 + 2x_2 + 7x_3 = 0$$
$$x_1 + x_2 + 3x_3 = 0$$

$$\begin{bmatrix} 5 & -2 & 1 & 0 \\ 3 & 2 & 7 & 0 \\ 1 & 1 & 3 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 3 & 0 \\ 3 & 2 & 7 & 0 \\ 5 & -2 & 1 & 0 \end{bmatrix} R_{13}$$

$$\begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & -1 & -2 & 0 \\ 0 & -3 & -14 & 0 \end{bmatrix} \begin{matrix} \\ -3R_1 + R_2 \\ -5R_1 + R_3 \end{matrix}$$

Date

$$\begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & 1 & +2 & 0 \\ 0 & -7 & -14 & 0 \end{bmatrix} \quad -R_2$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} -R_2+R_1 \\ \\ 7R_2+R_3 \end{matrix}$$

$x_1 + x_3 = 0 \quad -(i)$

$x_2 + 2x_3 = 0 \quad -(ii)$

Now eq (i)

$x_1 = -x_3 \quad -(i)$

Let $x_3 = K$ put in eq (i)

$x_1 = -K$

$x_2 = -2K$ Infinite

$x_3 = K$

Consistency Criteria:-

Rank(AD) = Rank(A) < No of unknow

2 = 2 < 3

Infinit Solution.

Q.8 Solve by elementary row operations. (by Gauss Jordan)

$$3x_1 + 2x_2 + 4x_3 = 7$$

$$2x_1 + x_2 + x_3 = 4$$

$$x_1 + 3x_2 + 5x_3 = 3$$

Solution:

$$\begin{bmatrix} 3 & 2 & 4 & 7 \\ 2 & 1 & 1 & 4 \\ 1 & 3 & 5 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 5 & 3 \\ 2 & 1 & 1 & 4 \\ 3 & 2 & 4 & 7 \end{bmatrix} \quad R_{13}$$

$$\begin{bmatrix} 1 & 3 & 5 & 3 \\ 0 & -5 & -9 & -2 \\ 0 & -7 & -11 & -2 \end{bmatrix} \quad \begin{matrix} -2R_1 + R_2 \\ -3R_1 + R_3 \end{matrix}$$

$$\begin{bmatrix} 1 & 3 & 5 & 3 \\ 0 & 1 & 9/5 & 2/5 \\ 0 & -7 & 11 & -2 \end{bmatrix} \quad -1/5\, R_2$$

$$\begin{bmatrix} 1 & 0 & -2/5 & 9/5 \\ 0 & 1 & 9/5 & 2/5 \\ 0 & 0 & 8/5 & 4/5 \end{bmatrix} \quad \begin{matrix} -3R_2 + R_1 \\ \\ +7R_2 + R_3 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & -2/5 & 9/5 \\ 0 & 1 & 9/5 & 2/5 \\ 0 & 0 & 1 & 1/2 \end{bmatrix} \quad 5/8\, R_3$$

Date

$$\left[\begin{array}{cccc} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{array}\right] \begin{array}{l} -\frac{9}{5}R_3 + R_1 \\ \\ \frac{2}{5}R_3 + R_2 \end{array}$$

$$\left.\begin{array}{l} x_1 = 2 \\ x_2 = -\frac{1}{2} \\ x_3 = \frac{1}{2} \end{array}\right\} \text{Unique Solution}$$

Consistency Criteria:

$$\text{Rank}(AB) = \text{Rank}(A) = \text{No of Unknow}$$

$$3 = 3 = 3$$

Unique Solution