Mohammad Ali Jinnah University

Chartered by Government of Sindh - Recognized by HEC

Assignment 4

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Id: FA19-BSSE-0014

Subject: Linear Algebra (Fall 2020)

Section: AM

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Q.1 Solve the following system by Gaussian Jordan method.

Solution:

$$\begin{bmatrix} 6 & -6 & 6 & 6 \\ 2 & -4 & -6 & 12 \\ 10 & -5 & 5 & 30 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 1 \\ 2 & -4 & -6 & 12 \\ 10 & -5 & 5 & 30 \end{bmatrix} \quad \tfrac{1}{6}R_1$$

$$\begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & -2 & 8 & 10 \\ 0 & 5 & -5 & 20 \end{bmatrix} \quad \begin{matrix} -2R_1+R_2 \\ -10R_1+R_3 \end{matrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 1 & -4 & -5 \\ 0 & 5 & -5 & 20 \end{bmatrix} \quad -\tfrac{1}{2}R_2$$

$$\begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 1 & -4 & -5 \\ 0 & 0 & -25 & 45 \end{bmatrix} \quad \begin{matrix} -5R_2+R_3 \\ R_2+R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 5 & -4 \\ 0 & 1 & -4 & -5 \\ 0 & 0 & 1 & 9/5 \end{bmatrix} \quad -\tfrac{1}{25}$$

Date

$$\begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & -9/5 \end{bmatrix} \begin{matrix} \\ -4R_3 + R_2 \\ -5R_3 + R_1 \end{matrix}$$

$$\left.\begin{matrix} x_1 = -4 & —(i) \\ x_2 = -5 & —(ii) \\ x_3 = -9/5 & —(iii) \end{matrix}\right\} \text{Unique Solution}$$

Consistency Criteria:-

Rank (AB) = Rank (A) = No of UnKnow

$3 = 3 = 3$

Unique Solution.

Q.2 Show that. is an idempotent matrix.

Q2

Solution:

$$P = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 4+2-4 & 2+9-8 & -8-8+12 \\ -2+9-8 & 2+9-8 & 4+12-12 \\ -4-8+9 & 2-6+6 & -4-8+9 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$$A^{k+1} = A$$

$$P^{1+1} = P^2$$

$$P^2 = P$$

Ans.

Q.3 Find the inverse of the following matrix.

Q3

Solution:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 2 & 4 & 1 & 0 & 1 & 0 \\ 1 & 3 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 4 & -5 & -2 & 1 & 0 \\ 0 & 3 & 3 & -1 & 0 & 1 \end{array}\right] \quad \begin{array}{l} -2R_1 + R_2 \\ -R_1 + R_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & -5/4 & -1/2 & 1/4 & 0 \\ 0 & 3 & 3 & -1 & 0 & 1 \end{array}\right] \quad \tfrac{1}{4}R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & -5/4 & -1/2 & 1/4 & 0 \\ 0 & 0 & 3/4 & -1 & -3/4 & 1 \end{array}\right] \quad -3R_2 + R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 3 & -4 \\ 0 & 1 & 0 & -1/2 & -1 & 5/3 \\ 0 & 0 & 1 & -1 & -3/4 & 4/3 \end{array}\right] \quad \begin{array}{l} \tfrac{4}{3}R_3 \\ -\tfrac{4}{5}R_3 + R_2 \\ -3R_3 + R_1 \end{array}$$

$$A^{-1} = \begin{bmatrix} 1 & 3 & -4 \\ 1/3 & -1 & 5/3 \\ 2/3 & -1 & 4/3 \end{bmatrix} \quad \text{Ans}$$

Q.4 Find the basis and dimension for the solution space of the system.

Q4)
Solution:-

$x_1 - 3x_2 + x_3 = 0$
$2x_1 - 6x_2 + 2x_3 = 0$
$3x_1 - 9x_2 + 3x_3 = 0$

$$\begin{bmatrix} 1 & -3 & 1 \\ 2 & -6 & 2 \\ 3 & -9 & 3 \end{bmatrix} \quad \begin{matrix} -2R_1 + R_2 \\ -3R_1 + R_3 \end{matrix}$$

$$\begin{bmatrix} 1 & -3 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 - 3x_2 + x_3 = 0$ — (i)

Let $x_2 = s$ & $x_3 = t$ -

$x_1 = +3s - t$
$x_2 = s$
$x_3 = t$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = s \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Rank: 1
Base: 2 $\left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$
Dimension: 2(t, s)

Aas

Q.5 Determine whether the following vectors in R^4 are linearly

Q5
Solution:-

$$\overset{u_1}{(1,4,-1,3)},\ \overset{v_1}{(2,1,-3,-1)},\ \overset{w_1}{(0,2,1,5)}$$

$$k_1u_1 + k_2v_1 + k_3w_1 = 0$$

$$\begin{bmatrix} 1 & 2 & 0 \\ 4 & 1 & 2 \\ -1 & -3 & 1 \\ 3 & -1 & 5 \end{bmatrix} \quad \begin{array}{l} -4R_1+R_2 \\ +R_1+R_3 \\ -3R_1+R_4 \end{array}$$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & -7 & 2 \\ 0 & -1 & 1 \\ 0 & -7 & 5 \end{bmatrix} \quad -(R_{23})$$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & -1 \\ 0 & -7 & 2 \\ 0 & -7 & 5 \end{bmatrix} \quad \begin{array}{l} -2R_2+R_1 \\ 7R_2+R_3 \\ 7R_2+R_4 \end{array} \Rightarrow \begin{bmatrix} 1 & 0 & +2 \\ 0 & 1 & -1 \\ 0 & 0 & -5 \\ 0 & 0 & -2 \end{bmatrix} \quad \begin{array}{l} -\frac{1}{5}R_3 \\ R_3+R_2 \\ -2R_3+R_1 \\ 2R_3+R_4 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \left.\begin{array}{l} k_1 = 0 \\ k_2 = 0 \\ k_3 = 0 \end{array}\right\} \text{linear independent}$$

Ans.

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iQ.6 Reduced the matrix

Solution

$$\Rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 4 & 1 & 2 & 1 \\ 1 & 1 & 0 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 5 & -6 & 1 \\ 0 & 2 & -2 & -1 \end{bmatrix} \begin{matrix} -4R_1 + R_1 \\ -R_1 + R_3 \end{matrix}$$

$$-4 \times -1 = 4+1 = 5$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & -6/5 & 1/5 \\ 0 & 2 & -2 & -1 \end{bmatrix} \tfrac{1}{5} R_2$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & -6/5 & 1/5 \\ 0 & 0 & 7/5 & -7/5 \end{bmatrix} -2R_2 + R_3$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & -6/5 & 1/5 \\ 0 & 0 & 1 & -1 \end{bmatrix} \tfrac{5}{7} R_3$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & -6/5 & 1/5 \\ 0 & 0 & 1 & -1 \end{bmatrix} \text{R.E.F}$$

Ans

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