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Section: Am

You have to show rough work (dry run) in detail to get points, otherwise ZERO.

Q1: Show, using the definition of the big-Oh, that the function $f(n) = 0.5n + 255$ is $O(n)$

O^1 $f_n = 0.5n + 255$ is $O(n)$

$f_n \leq C(g_n)$.

$$0.5(n) + 255 \leq C(g_n)$$

$$0.5(n) + 255 \leq C(n)$$

$n=2$

$C=2$

$$0.5(2) + 255 \leq 2 \times 2 \\ 256 \leq 4$$

$$0.5(10) + 255 \leq 10 \times 10 \\ 260 \leq 100$$

$n=10$

$C=10$

$$0.5(20) + 255 \leq 20 \times 20 \\ 265 \leq 400$$

$n=20$

$C=20$

$O(n)$

Q2: Using the big-Oh notation, the worst-case running time of the following algorithm:

Let A be a given array of n integers.

```
for i <- 0 to n-1 do
    for j <- 0 to (i*i)-1 do
        Let A[j mod n] <- j.
    end for
end for
```

Q2

$$\begin{aligned} & (n-1) \times (n^2-1) \\ & (n-1) \times (n^2-1) \\ & (n-1) \times (n^2-1) \\ & n^3 - n - n^2 + 1 \\ & O(n^3) \end{aligned}$$

Q3: Describe the worst case running time of the following (a, b and c) codes in “big-Oh” notation in terms of the variable n.

a)

```
void f1(int n) {  
    for(int i=0; i < n; i++) {  
        for(int j=0; j < n; j++) {  
            for(int k=0; k < n; k++) {  
                for(int m=0; m < n; m++) {  
                    System.out.println("!");  
                } } } }
```

}

b)

```
int f2(int n) {  
    if (n < 10) {  
        System.out.println("!");  
        return n+3;  
    } else {  
        return f4(n-1) + f4(n-1);  
    }  
}  
  
int f3(int n) {  
    if (n < 10) {  
        System.out.println("!");  
        return n+3;  
    } else {  
        return f5(n-1) + 1;  
    }  
}
```

Q3

a)

$$m \times n \times n \times n \\ n^4 \\ O(n^4)$$

$$\begin{array}{ccc} m & \xrightarrow{n^4} & m \times n \\ 16 & \xrightarrow{4} & 4 \end{array} \quad \begin{array}{l} n=2 \\ \hookrightarrow \end{array}$$

b)

$$\begin{array}{r} n \\ \cancel{n} \\ \cancel{n} \\ \cancel{n} \\ O(n) \end{array} \begin{array}{r} +3 \\ +3 \\ +3 \end{array}$$

$$\begin{array}{c} n \\ 2 \\ 2 \\ 2 \end{array} \geq c(g(n)) \quad \begin{array}{c} n \times n \\ 4 \end{array}$$

b)

$$n \times n + 3$$

$$O(n^4) \quad \begin{array}{r} n^2 \\ n^2 \end{array} + 3$$

$$\begin{array}{r} \geq \\ 2^2 \\ 2^2 \\ 4 \end{array} \geq n \times n \quad \begin{array}{r} n \times n \\ 2 \times 2 \\ 4 \end{array}$$

Q2

c)

$$n \times n + 3$$
$$\Theta(n^2) \cancel{+ 3}$$

$$n^2 \geq n \times n$$
$$3^2 \geq 3 \times 3$$
$$9 \geq 9$$
$$\Theta(n^2)$$

Q4: Suppose you have a queue Q, which contains two elements in the following order: 5 4.

a) Write down the resulting sequence (in order), after you do the following operations (in the order presented):

Q.insert(3);

Q.insert(8);

Q.remove();

Q.remove();

Q.remove();

Q.insert(9);

(b) Suppose you have the same sequence as in (a) at the beginning. Write a possible ordering of statements to bring it to the following state:

1 2 3 4 5

(c) Now, write a program to verify the above two subparts

**Attach screenshots in word file and submit code as separate file

a)

[5, 4]

[5, 4, 3, 8]

[4, 3, 8]

[, 4, 3, 8]

[, , 3, 8]

[, , , 8]

[, , { 8 }, 8 }, 9]

output [8, 89]

b)

[5, 4]

[5, 4, 3, 2, 1]

Q, remove()

Q, remove()

Q, insert(1)

Q, insert(2)

Q, insert(3)

Q, insert(5)

Q, insert(4)

Q5: What will be output of the following programs

The array `a` contains the following 3 elements:

`a = [a | b | c]`

For each of the following two program fragments indicate what they print:

```
n = a.size();
Stack s = Stack();
for (i=0; i<n; i++) {
    s.push(a[i]);
    for (j=0; j<i; j++)
        s.push(a[j]);
}
while (!s.empty()) {
    System.out.println(s.pop());
}
```

```
n = a.size();
Queue s = Queue();
for (i=0; i<n; i++) {
    s.enqueue(a[i]);
    for (j=0; j<i; j++)
        s.enqueue(a[j]);
}
while (!s.empty()) {
    System.out.println(s.dequeue());
}
```

Q5

stack
a = [a, b, c].

n = a.size(); 3

stack s = stack();

[a, b / a, ab, a, c]
[a, b / a, c, a, b].
[a, b / a, c, a, b].
(pop) (pop) (pop) (pop) (pop) (pop)

[]

Q6

queue
[a, b, a, c, ab]

[a, b, a, c, ab].

(dequeue) (dequeue) (dequeue) (dequeue) (dequeue) (dequeue)

[].

***Q6: Design and analyze an algorithm that for a given positive integer n counts the number of different ways to write n as a sum of 1, 2, 3, and 4. For examples, if n = 4, the output should be 8, as**

$$4 = 4,$$

$$4 = 1 + 3,$$

$$4 = 1 + 1 + 2,$$

$$4 = 2 + 2,$$

$$4 = 1 + 1 + 1 + 1,$$

$$4 = 1 + 2 + 1,$$

$$4 = 2 + 1 + 1,$$

$$4 = 3 + 1.$$

Note that $3 + 1$ and $1 + 3$ are counted as different ways of writing 4.

Q7: Write the code to satisfy the requirement, output should be as depicted in figure (attach screenshots of the program output in word file and submit code as separate file)

Quicksort is another recursive sorting algorithm, discovered by Tony Hoare in 1959 and first published in 1961. In this algorithm, the hard work is splitting the array into smaller subarrays *before* recursion, so that merging the sorted subarrays is trivial.

1. Choose a *pivot* element from the array.
2. Partition the array into three subarrays containing the elements smaller than the pivot, the pivot element itself, and the elements larger than the pivot.
3. Recursively quicksort the first and last subarrays.

| | | | | | | | | | | | | | |
|------------------------|---|---|---|---|---|---|---|---|----------|---|---|----------|---|
| Input: | S | O | R | T | I | N | G | E | X | A | M | P | L |
| Choose a pivot: | S | O | R | T | I | N | G | E | X | A | M | P | L |
| Partition: | A | G | O | E | I | N | L | M | P | T | X | S | R |
| Recurse Left: | A | E | G | I | L | M | N | O | P | T | X | S | R |
| Recurse Right: | A | E | G | I | L | M | N | O | P | R | S | T | X |

A quicksort example.

Q8. Evaluate and write the result for the following given expression, take values for the operands (show detailed work/dry run)

- i) abc*+de*f+g*+
- ii) -+a*/bc^def

Convert the given expression to Postfix and Prefix

- iii) (((A + B) * C) - ((D + E) / F))

Q8

$$a=1, b=2, c=3$$

i) $abc * + de * f + g * +$

$$(a + (b \times c)) + (f \times (d * e))$$

$$((a + (b \times c)) + (g * (f + (d * e))))$$

$$((1 + (2 \times 3)) + (4 \times (5 + (6 \times 7))))$$

$$((1 + 6)) + (4 \times (5 + (6 \times 7))))$$

$$((7) + (4 \times (5 + 42)))$$

$\frac{+}{7} \quad 188$

195

ii) $- + a * / bc ^ \wedge def.$

$$fed ^ \wedge cb / * a + -$$

$$((f * (e / (d ^ \wedge c))))$$

$$q(d \wedge F)$$

$$a=1, b=2, c=3$$

$$(c/b) * (d^e)$$

$$d=4, e=5, F=6$$

$$((F - (a + (c/b)) * (d^e)))$$

$$((F - (a + (c/b))) * (d^e))$$

$$((6 - (1 + (3/2))) * (4^5))$$

$$((6 - (1 + (1))) * (1024))$$

$$((6 - 2)) * (1024))$$

$$(4 \times 1024)$$

4096

iii)

$$((A + B) * C) - ((D + E) / F))$$

prostix

$$AB + C * DE + F / -$$

((F /
prefix.

$$-/F + ED * C + BA$$

Q9: Here is an array of ten integers:

5 3 8 9 1 7 0 2 6 4

- a) Write the output of this array after the FIRST iteration of the selection sort.
- b) Then, write the output of this array after FIRST iteration of the loop in an insertion sort.
- c) Finally, sort the array using bubble sort.

q)

$[5, 3, 8, 9, 1, 7, 0, 2, 6, 4]$.

a)

index
↑
↓

$[0, 3, 8, 9, 1, 7, 5, 2, 6, 4]$

b)

$[5, 3, 8, 9, 1, 7, 0, 2, 6, 4]$.

key = 3

j = 0

$\{5, 3\} \Rightarrow [3, 5]$

$[3, 5, 8, 9, 1, 7, 0, 2, 6, 4]$.

$[0, 3, 8, 9, 1, 7, 5, 2, 6, 4]$

key = 3

T = 0

$0 > 03$

$[0, 3, 8, 9, 1, 7, 5, 2, 6, 4]$

c) 6

$$[8, 3, 7, 9, 1, 7, 0, 2, 6, 4]$$

inner

$$\left[\frac{5}{7}, \frac{3}{5}, 8, 9, 1, 7, 0, 2, 6, 4 \right]$$

$$\left[\frac{3}{7}, \frac{5}{5}, \frac{8}{7}, \frac{9}{9}, \frac{1}{1}, 7, 0, 2, 6, 4 \right]$$

$$\left[\frac{1}{1}, \frac{5}{5}, 8, 9, 3, \frac{7}{7}, \frac{0}{9}, 2, 6, 4 \right]$$

inner

$$\left[\frac{0}{1}, \frac{5}{1}, 8, 9, 3, 7, 1, 2, 6, 4 \right]$$

$$\left[0, \frac{5}{1}, \frac{8}{1}, 9, 3, 7, 1, 2, 6, 4 \right]$$

$$\left[0, \frac{5}{1}, 8, \frac{9}{1}, 3, 7, 1, 2, 6, 4 \right]$$

$$\left[0, \frac{5}{1}, 8, \frac{9}{1}, \frac{3}{9}, 7, 1, 2, 6, 4 \right]$$

$$\left[0, \frac{5}{1}, 8, 9, 5, \frac{7}{7}, 1, 2, 6, 4 \right]$$

$$\left[0, \frac{5}{1}, 8, 9, 5, 7, \frac{1}{1}, 2, 6, 4 \right]$$

$$\left[0, 1, 8, 9, 5, 7, 3, 2, 6, 4 \right]$$

$[0, 1, \underline{8}, 9, 8, 7, 3, 2, 6, 4]$

$[0, 1, 2, \underline{9}, 8, 7, 3, 6, 4]$

$[0, 1, 2, 3, \underline{9}, 7, 5, 6, 4]$

$[0, 1, 2, 3, 4, \underline{9}, 7, 5, 6]$

$[0, 1, 2, 3, 4, 5, \underline{9}, 7, 6]$

$[0, 1, 2, 3, 4, 5, 6, \underline{9}, 7, 9]$

$[0, 1, 2, 3, 4, 5, 6, 7, \underline{9}]$

Q10: Do as directed (Choose sorting algorithm wisely)

- a) There are 30 Students in the class, whose result of data structures has been submitted to the administrative office for announcements. For the ease of students, the administrator has to make three columns for roll number, name and marks. Write a program for the above given scenario to show the result to the administrator in increasing order.
- b) 10 students are standing in a ground in an unsorted manner. The PT instructor start arrange them according to their height such that he pick the smaller one first and place him in the starting position, then pick the second one who is slightly taller than the first on and place him behind the first student, then pick next student who is slightly taller than the second student and place him behind him and continue till all are placed in their appropriate positions.
- c) A clerk at a shipping company is charged with the task of rearranging a number of large crates in order of the time they are to be shipped out. Thus, the cost of compares is very low (just look at the labels) relative to the cost of exchanges (move the crates). The warehouse is nearly full: there is extra space sufficient to hold any one of the crates, but not two.
- d) Suppose an array of 8 elements,

| | | | | | | | | |
|----|----|---|----|----|----|----|----|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 42 | 24 | 6 | 17 | 30 | 13 | 90 | 47 | 2 |

Which sorting algorithm (bubble, selection, insertion) produce the following result after seven pass's

| | | | | | | | | |
|---|----|----|----|----|----|----|----|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 2 | 42 | 24 | 17 | 30 | 13 | 90 | 47 | 6 |

Q10).

a) bubble sort:

b) Selection sort

c) insertion sort

d) insertion sort