



Mohammad Ali Jinnah University

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Assignment 1

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Id: FA19-BSSE-0014

Subject: Linear Algebra (Fall 2020)

Section: AM

Teacher: Dr. Asmat Ara

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Attempt all questions.

Q.1 (a) Evaluate the matrix

$$\begin{bmatrix} 2 & -1 & -1 & 4 \\ 3 & 4 & -2 & 11 \\ 3 & -2 & 4 & 11 \end{bmatrix} \text{ into Reduced Echelon form.}$$

Solution:

utions:

$$\begin{bmatrix} 2 & -1 & -1 & 4 \\ 3 & 4 & -2 & 11 \\ 3 & -2 & 4 & 11 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} & 2 \\ 3 & 4 & -2 & 11 \\ 3 & -2 & 4 & 11 \end{bmatrix} \quad \frac{1}{2} R_1$$

$$\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} & 2 \\ 0 & \frac{5}{2} & -\frac{3}{2} & 5 \\ 0 & -\frac{1}{2} & \frac{5}{2} & 5 \end{bmatrix} \quad \begin{array}{l} -3R_1 + R_2 \\ -3R_1 + R_3 \end{array}$$

$$\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} & 2 \\ 0 & 1 & -\frac{3}{2} & \frac{10}{5} \\ 0 & -\frac{1}{2} & \frac{5}{2} & 5 \end{bmatrix} \quad \frac{2}{11} R_2$$

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} & \frac{27}{11} \\ 0 & 1 & -\frac{3}{11} & \frac{10}{11} \\ 0 & 0 & \frac{6}{11} & \frac{6}{11} \end{bmatrix} \quad \begin{array}{l} \frac{1}{2} R_2 + R_1 \\ \frac{2}{6} R_2 + R_3 \end{array}$$

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} & \frac{27}{11} \\ 0 & 1 & -\frac{1}{11} & \frac{10}{11} \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad \frac{1}{60} R_3$$

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad \begin{array}{l} -\frac{2}{6} R_3 + R_1 \\ \frac{1}{11} R_3 + R_2 \end{array}$$

R = E = F
Ans

(b) Evaluate the matrix

$$\begin{bmatrix} 1 & -1 & 2 & 0 \\ 4 & 1 & 2 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix} \text{ into Echelon form.}$$

Solution:

$$\Rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 4 & 1 & 2 & 1 \\ 1 & 1 & 0 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 5 & -6 & 1 \\ 0 & 2 & -2 & -1 \end{bmatrix} \begin{array}{l} -4R_1 + R_1 \\ -R_1 + R_3 \end{array}$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & -6/5 & 1/5 \\ 0 & 2 & -2 & -1 \end{bmatrix} \begin{array}{l} 1/5 R_2 \\ -4 \times -1 = 4 + 1 = 5 \end{array}$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & -6/5 & 1/5 \\ 0 & 0 & 7/5 & -7/5 \end{bmatrix} -2R_2 + R_3$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & -6/5 & 1/5 \\ 0 & 0 & 1 & -1 \end{bmatrix} 5/7 R_3$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & -6/5 & 1/5 \\ 0 & 0 & 1 & -1 \end{bmatrix} R \cdot E \cdot F$$

Ans



Q.2 Solve the following system by Gaussian Jordan method.

$$5x_1 + \quad + 4x_3 + 2x_4 = 3$$

$$x_1 - x_2 + 2x_3 + x_4 = 1$$

$$4x_1 + x_2 + 2x_3 = 1$$

$$x_1 + x_2 + x_3 + x_4 = 0$$

Solution:

Solve

$$\begin{bmatrix} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 5 & 0 & 4 & 2 & 3 \end{bmatrix} \quad R_{21}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 1 \\ 0 & -3 & -2 & 4 & 1 \\ 0 & -5 & -1 & -3 & 3 \end{bmatrix} \quad \begin{array}{l} -R_1 + R_2 \\ -4R_1 + R_3 \\ -5R_1 + R_4 \end{array}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & -3 & -2 & -4 & 1 \\ 0 & -5 & -1 & -3 & 3 \end{bmatrix} \quad -\frac{1}{2}R_2$$

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$$\left[\begin{array}{ccccc|l} 1 & 0 & \frac{3}{2} & 1 & \frac{1}{2} & \\ 0 & 1 & -\frac{1}{2} & 0 & -\frac{1}{2} & 3R_2 + R_3 \\ 0 & 0 & -\frac{3}{2} & -4 & -\frac{1}{2} & 5R_2 + R_4 \\ 0 & 0 & -\frac{3}{2} & -3 & \frac{1}{2} & -R_2 + R_1 \end{array} \right]$$

$$\left[\begin{array}{ccccc|l} 1 & 0 & \frac{3}{2} & 1 & \frac{2}{7} & \\ 0 & 1 & -\frac{1}{2} & 0 & -\frac{3}{7} & \frac{2}{7} R_3 \\ 0 & 0 & 1 & \frac{8}{7} & \frac{1}{7} & \\ 0 & 0 & \frac{3}{2} & -3 & \frac{1}{2} & \end{array} \right]$$

$$\left[\begin{array}{ccccc|l} 1 & 0 & 0 & -\frac{5}{7} & \frac{2}{7} & \frac{3}{2} R_3 + R_4 \\ 0 & 1 & 0 & \frac{4}{7} & -\frac{3}{7} & \frac{1}{2} R_3 + R_4 \\ 0 & 0 & 1 & \frac{8}{7} & \frac{1}{7} & -\frac{3}{2} R_3 + R_4 \\ 0 & 0 & 0 & 1 & 1 & \end{array} \right]$$

$$\left[\begin{array}{ccccc|l} 1 & 0 & 0 & 0 & 1 & -\frac{8}{7} R_4 + R_3 \\ 0 & 1 & 0 & 0 & -1 & -\frac{4}{7} R_4 + R_3 \\ 0 & 0 & 1 & 0 & -1 & \frac{5}{7} R_4 + R_3 \\ 0 & 0 & 0 & 1 & 1 & \end{array} \right]$$

$$\left. \begin{array}{l} x_1 = 1 \\ x_2 = -1 \\ x_3 = -1 \\ x_4 = 1 \end{array} \right\} \text{Unique Solution}$$

Consistency Criteria

$$\text{Rank}(AD) = \text{Rank } A = \text{No of Unknown}$$

$$4 = 4 = 4$$

Unique solution



Q.3 Solve by elementary row operations.

$$4x + 5y = 2$$

$$11x + y + 2z = 3$$

$$x + 5y + 2z = 1$$

Solution:

Solution: (by Gauss Elimination method)

$$\begin{bmatrix} 4 & 5 & 0 & 2 \\ 11 & 1 & 2 & 3 \\ 1 & 5 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 5 & 2 & 1 \\ 11 & 1 & 2 & 3 \\ 4 & 5 & 0 & 2 \end{bmatrix} \quad R_{13}$$

$$\begin{bmatrix} 1 & 5 & 2 & 1 \\ 0 & -54 & -20 & -8 \\ 0 & -15 & -8 & -2 \end{bmatrix} \quad \begin{array}{l} -11R_1 + R_2 \\ -4R_1 + R_3 \end{array}$$

$$\begin{bmatrix} 1 & 5 & 2 & 1 \\ 0 & 1 & \frac{10}{27} & \frac{4}{27} \\ 0 & -15 & -8 & 2 \end{bmatrix} \quad -\frac{1}{54}R_2$$

$$\begin{bmatrix} 1 & 5 & 2 & 1 \\ 0 & 1 & \frac{10}{27} & \frac{4}{27} \\ 0 & 0 & -\frac{22}{9} & \frac{2}{9} \end{bmatrix} \quad 15R_1 + R_3$$

$$\begin{bmatrix} 1 & 5 & 2 & 1 \\ 0 & 1 & \frac{10}{27} & \frac{4}{27} \\ 0 & 0 & 1 & \frac{1}{14} \end{bmatrix} \quad -\frac{9}{22}R_3$$

$$\begin{aligned} x_1 + 5x_2 + 2x_3 &= 1 & \text{--- (i)} \\ + x_2 + \frac{10}{27}x_3 &= \frac{4}{27} & \text{--- (ii)} \\ x_3 &= -\frac{1}{11} & \text{--- (iii)} \end{aligned}$$

Put x_3 in eq (ii)

$$\begin{aligned} x_2 &= \frac{4}{27} - \frac{10}{27}x_3 \\ x_2 &= \frac{4}{27} - \frac{10}{27} \left(-\frac{1}{11} \right) \end{aligned}$$

$$x_2 = \frac{4}{27} + \frac{10}{297}$$

$$x_2 = \frac{44 + 10}{297} \Rightarrow \frac{54}{297}$$

$$x_2 = \frac{2}{11} \text{ --- (ii)}$$

Put eq (ii) & (iii) in eq (i)

$$\begin{aligned} x_1 &= 1 - 5x_2 + 2x_3 \\ x_1 &= 1 - 5\left(\frac{2}{11}\right) + 2\left(-\frac{1}{11}\right) \\ x_1 &= 1 - \frac{10}{11} - \frac{2}{11} \\ x_1 &= \frac{11 - 10 - 2}{11} \Rightarrow \frac{3}{11} \end{aligned}$$

$$x_1 = \frac{3}{11}, x_2 = \frac{2}{11}, x_3 = -\frac{1}{11} = \text{Unique Solution}$$

Consistency Criteria:

$$\text{Rank}(AD) = \text{Rank}(A) + 1 = \text{No of Unknowns}$$

$$3 = 3 = 3$$

Unique Solution:



Q.4 Solve by elementary row operations. (by Gauss Elimination)

$$2x_1 + x_2 + 5x_3 = 4$$

$$3x_1 - 2x_2 + 2x_3 = 2$$

$$5x_1 - 8x_2 - 4x_3 = 1$$

Solution:

Handwritten solution for the system of linear equations:

$$\begin{bmatrix} 2 & 1 & 5 & 4 \\ 3 & -2 & 2 & 2 \\ 5 & -8 & -4 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1/2 & 5/2 & 2 \\ 3 & -2 & 2 & 2 \\ 5 & -8 & -4 & 1 \end{bmatrix} \begin{array}{l} \\ \times R_1 \\ \end{array}$$

$$\begin{bmatrix} 1 & 1/2 & 5/2 & 2 \\ 0 & 3/2 & -1/2 & -4 \\ 0 & 3/2 & 3/2 & -9 \end{bmatrix} \begin{array}{l} \\ -3R_1 + R_2 \\ -5R_1 + R_3 \end{array}$$

$$\begin{bmatrix} 1 & 1/2 & 5/2 & 2 \\ 0 & 1 & 1/2 & 8/3 \\ 0 & 3/2 & 3/2 & -9 \end{bmatrix} \begin{array}{l} \\ \\ 3/2 R_2 \end{array}$$

$$\begin{bmatrix} 1 & 1/2 & 5/2 & 2 \\ 0 & 1 & 1/2 & 8/3 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

$x_1 + 1/2 x_2 + 5/2 x_3 = 2$
 $x_2 + 1/2 x_3 = 8/3$

No Solution:

Consistency Criterion (No solution)

$$\text{Rank}(AD) \neq \text{Rank}(A)$$
$$3 \neq 2$$

No solution.

Q.5 Solve by elementary row operations. (by Gauss Elimination)

$$2x_1 + \quad + x_3 = 1$$

$$2x_1 + 4x_2 - x_3 = -2$$

$$x_1 - 8x_2 - 3x_3 = 2$$

Solution:

$$\begin{bmatrix} 2 & 0 & 1 & 1 \\ 2 & 4 & -1 & -2 \\ 1 & -8 & -3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -8 & -3 & 2 \\ 2 & 4 & -1 & -2 \\ 2 & 0 & 1 & 1 \end{bmatrix} \quad R_{13}$$

$$\begin{bmatrix} 1 & -8 & -3 & 2 \\ 0 & 20 & 5 & -6 \\ 0 & 16 & 7 & -3 \end{bmatrix} \quad \begin{array}{l} -2R_1 + R_2 \\ -2R_1 + R_3 \end{array}$$

$$\begin{bmatrix} 1 & -8 & -3 & 2 \\ 0 & 1 & 1/4 & -3/10 \\ 0 & 0 & 7 & -3 \end{bmatrix} \quad \frac{1}{20} R_2$$

$$\left[\begin{array}{cccc|c} 1 & -8 & -3 & 2 & \\ 0 & 1 & \frac{1}{4} & -\frac{3}{10} & \\ 0 & 0 & 3 & \frac{9}{5} & \end{array} \right] \quad -16R_2 + R_3$$

$$\left[\begin{array}{cccc|c} 1 & -8 & -3 & 2 & \\ 0 & 1 & \frac{1}{4} & -\frac{3}{10} & \\ 0 & 0 & 1 & \frac{3}{5} & \end{array} \right] \quad \frac{1}{3} R_3$$

$$\begin{aligned} x_1 - 8x_2 + 3x_3 &= 2 & \text{--- (i)} \\ x_2 + \frac{1}{4}x_3 &= -\frac{3}{10} & \text{--- (ii)} \\ x_3 &= \frac{3}{5} & \text{--- (iii)} \end{aligned}$$

Now eq (ii)

$$x_2 = -\frac{3}{10} - \frac{1}{4}x_3 \quad \because \text{Put } x_3 \text{ value}$$

$$x_2 = -\frac{3}{10} - \frac{1}{4}\left(\frac{3}{5}\right)$$

$$x_2 = -\frac{3}{10} - \frac{3}{20}$$

$$x_2 = \frac{-6 - 3}{20}$$

$$\boxed{x_2 = -\frac{9}{20}}$$

Now eq (i):

$$\begin{aligned} x_1 &= 2 + 8x_2 + 3x_3 \\ x_1 &= 2 + 8\left(\frac{-9}{20}\right) + 3\left(\frac{3}{5}\right) \end{aligned}$$

$$\text{Unique sol } \left\{ \begin{aligned} x_1 &= 2 - \frac{8}{5} + \frac{9}{5} \\ x_1 &= \frac{1}{5}, x_2 = -\frac{9}{20}, x_3 = \frac{3}{5} \end{aligned} \right\}$$

Consistency Criteria:-

$$\begin{aligned} \text{Rank}(A \cdot D) &= \text{Rank}(A) = \text{No of Unknown} \\ 3 &= 3 = 3 \end{aligned}$$

Unique Solution



Q.6 Solve by elementary row operations. (by Gauss Jordan)

$$6x_1 - 6x_2 + 6x_3 = 6$$

$$2x_1 - 4x_2 - 6x_3 = 12$$

$$10x_1 - 5x_2 + 5x_3 = 30$$

Solution:

Solution:

$$\begin{bmatrix} 6 & -6 & 6 & 6 \\ 2 & -4 & -6 & 12 \\ 10 & -5 & 5 & 30 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 1 \\ 2 & -4 & -6 & 12 \\ 10 & -5 & 5 & 30 \end{bmatrix} \begin{array}{l} \\ \frac{1}{2}R_1 \\ \end{array}$$

$$\begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & -2 & -8 & 10 \\ 0 & 5 & -5 & 20 \end{bmatrix} \begin{array}{l} \\ -2R_1 + R_2 \\ -10R_1 + R_3 \end{array}$$

$$\begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 1 & -4 & -5 \\ 0 & 5 & -5 & 20 \end{bmatrix} \begin{array}{l} \\ \\ -\frac{1}{2}R_2 \end{array}$$

$$\begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 1 & -4 & -5 \\ 0 & 0 & -25 & 45 \end{bmatrix} \begin{array}{l} \\ -5R_2 + R_3 \\ R_3 + R_2 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 5 & -4 \\ 0 & 1 & -4 & -5 \\ 0 & 0 & 1 & -9/5 \end{bmatrix} \begin{array}{l} \\ -\frac{1}{5}R_3 \\ \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & -9/5 \end{bmatrix} \begin{array}{l} \\ -4R_3 + R_2 \\ -5R_3 + R_1 \end{array}$$

$$\left. \begin{array}{l} x_1 = -4 \quad - (i) \\ x_2 = -5 \quad - (ii) \\ x_3 = -9/5 \quad - (iii) \end{array} \right\} \text{Unique Solution}$$

Consistency Criteria -

$$\begin{array}{ccccccc} \text{Rank (AD)} & = & \text{Rank (A)} & = & \text{No of Un Know} \\ 3 & = & 3 & = & 3 \\ & & & & \text{Unique Solution.} \end{array}$$

Q.7 Solve by elementary row operations. (by Gauss Jordan)

$$5x_1 - 2x_2 + x_3 = 0$$

$$3x_1 + 2x_2 + 7x_3 = 0$$

$$x_1 + x_2 + 3x_3 = 0$$

Solution:

Q7) Solve by elementary row operation (by Gauss Jordan)

$$\begin{aligned} 5x_1 - 2x_2 + x_3 &= 0 \\ 3x_1 + 2x_2 + 7x_3 &= 0 \\ x_1 + x_2 + 3x_3 &= 0 \end{aligned}$$

$$\begin{bmatrix} 5 & -2 & 1 & 0 \\ 3 & 2 & 7 & 0 \\ 1 & 1 & 3 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 3 & 0 \\ 3 & 2 & 7 & 0 \\ 5 & -2 & 1 & 0 \end{bmatrix} \quad R_{13}$$

$$\begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & -2 & -2 & 0 \\ 0 & -3 & -14 & 0 \end{bmatrix} \quad \begin{aligned} &-3R_1 + R_2 \\ &-5R_1 + R_3 \end{aligned}$$

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$$\left[\begin{array}{cccc} 1 & 1 & 3 & 0 \\ 0 & 1 & +2 & 0 \\ 0 & -3 & -14 & 0 \end{array} \right] \quad -R_2$$

$$\left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \quad \begin{array}{l} -R_2 + R_2 \\ 7R_2 + R_2 \end{array}$$

$$x_1 + x_3 = 0 \quad (i)$$

$$x_2 + 2x_3 = 0 \quad (ii)$$

Now eq (i)

$$x_1 = -x_3 \quad (i)$$

Let $x_3 = K$ put in eq (i)

$$x_1 = -K$$

$$x_2 = -2K$$

$$x_3 = K$$

Infinite

Consistency Criteria:

$$\text{Rank}(AD) = \text{Rank}(A) < \text{No of unknown}$$

$$2 = 2$$

$$< 3$$

Infinite Solution.

Q.8 Solve by elementary row operations. (by Gauss Jordan)

$$3x_1 + 2x_2 + 4x_3 = 7$$

$$2x_1 + x_2 + x_3 = 4$$

$$x_1 + 3x_2 + 5x_3 = 3$$

Solution:

$$\begin{bmatrix} 3 & 2 & 4 & 7 \\ 2 & 1 & 1 & 4 \\ 1 & 3 & 5 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 5 & 3 \\ 2 & 1 & 1 & 4 \\ 3 & 2 & 4 & 7 \end{bmatrix} \quad R_{13}$$

$$\begin{bmatrix} 1 & 3 & 5 & 3 \\ 0 & -5 & -9 & -2 \\ 0 & -7 & -11 & -2 \end{bmatrix} \quad \begin{array}{l} -2R_1 + R_2 \\ -3R_1 + R_3 \end{array}$$

$$\begin{bmatrix} 1 & 3 & 5 & 3 \\ 0 & 1 & 9/5 & 2/5 \\ 0 & -7 & -11 & -2 \end{bmatrix} \quad -1/5 R_2$$

$$\begin{bmatrix} 1 & 0 & -2/5 & 3/5 \\ 0 & 1 & 9/5 & 2/5 \\ 0 & 0 & 8/5 & 4/5 \end{bmatrix} \quad \begin{array}{l} -3R_2 + R_1 \\ +7R_2 + R_3 \end{array}$$

$$\begin{bmatrix} 1 & 0 & -2/5 & 3/5 \\ 0 & 1 & 9/5 & 2/5 \\ 0 & 0 & 1 & 1/2 \end{bmatrix} \quad 5/8 R_3$$

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$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix} \quad \begin{array}{l} -\frac{9}{5}R_3 + R_1 \\ \\ \frac{2}{5}R_3 + R_2 \end{array}$$

$$\left. \begin{array}{l} x_1 = 2 \\ x_2 = -\frac{1}{2} \\ x_3 = \frac{1}{2} \end{array} \right\} \text{Unique Solution}$$

Consistency Criteria:

$$\begin{array}{ccccccc} \text{Rank}(A|D) & = & \text{Rank}(A) & = & \text{No of Unknowns} \\ 3 & = & 3 & = & 3 \end{array}$$

Unique Solution