



Mohammad Ali Jinnah University

Chartered by Government of Sindh - Recognized by HEC

Assignment 3

Name: Muhamad Fahad

Id: FA19-BSSE-0014

Subject: Linear Algebra (Fall 2020)

Section: AM

Teacher: Dr. Asmat Ara

Date: Monday, December 21, 2020

Muhammad Ishaq
F019-0355-0014

A # 3

Date: _____

Q3 find the characteristic equation of the following
(a) matrix

$$\begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$$

Soln

$$A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$$

$$[\lambda I - A] = 0$$

$$\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix} = 0$$

$$\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix} = 0$$

$$\begin{bmatrix} \lambda - 3 & 0 \\ 8 & \lambda + 1 \end{bmatrix} = 0$$

$$(\lambda - 3)(\lambda + 1) - (0) = 0$$

$$\lambda^2 + \lambda - 3\lambda - 3 = 0$$

$$\lambda^2 - 2\lambda - 3 = 0$$

Characteristic equation

✓

MIGHTY PAPER PRODUCT

$$b) \begin{bmatrix} 10 & -9 \\ 4 & -2 \end{bmatrix}$$

soln

$$[\lambda I - A] = 0$$

$$-\lambda \begin{bmatrix} 10 & -9 \\ 4 & -2 \end{bmatrix} + \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} \lambda - 10 & -9 \\ 4 & \lambda + 2 \end{bmatrix} = 0$$

$$\begin{bmatrix} \lambda - 10 & +9 \\ -4 & \lambda + 2 \end{bmatrix} = 0$$

$$(\lambda - 10)(\lambda + 2) - (+9 \times -4) = 0$$

$$\lambda^2 + 2\lambda - 10\lambda - 20 + 36 = 0$$

$$\lambda^2 + (-8\lambda) + (16) = 0$$

$$\lambda^2 - 8\lambda + 16 = 0$$

Ans

$$c) \begin{bmatrix} 0 & 3 \\ 4 & 0 \end{bmatrix}$$

Soln

$$[\lambda I - A] = 0$$

$$\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 3 \\ 4 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 3 \\ 4 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \lambda & -3 \\ -4 & \lambda \end{bmatrix}$$

$$\lambda^2 - (-4\lambda - 3)$$

$$\lambda^2 - 12$$

eq of characteristic.

$$d) \begin{bmatrix} -2 & -7 \\ 1 & 2 \end{bmatrix}$$

Sol:-

$$[\lambda I - A]$$

$$A = \begin{bmatrix} -2 & -7 \\ 1 & 2 \end{bmatrix}$$

$$\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -7 \\ 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} -2 & -7 \\ 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} \lambda+2 & 7 \\ -1 & \lambda+2 \end{bmatrix}$$

$$((\lambda+2)(\lambda+2)) - (-7)$$

$$\lambda^2 + 4\lambda + 7$$

$$\lambda^2 + 3$$

Ans

$$e) \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Sol:-

$$[\lambda I - A] = 0$$

$$\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

$$\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0$$

$$\lambda^2 = 0$$

$$(f) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[\lambda I - A]$$

$$\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \lambda - 1 & 0 \\ 0 & \lambda - 1 \end{bmatrix} = 0$$

$$(\lambda - 1)(\lambda - 1) - 0 = 0$$

$$\lambda^2 - \lambda - \lambda + 1 = 0$$

$$\lambda^2 - 2\lambda + 1 = 0$$

Ans.

$$Q_6) \begin{pmatrix} A \end{pmatrix} \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

Sol:-

$$[\lambda I - A]$$

$$(\lambda) \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \lambda - 4 & 0 & -1 \\ 2 & \lambda - 1 & 0 \\ 2 & 0 & \lambda - 1 \end{bmatrix} = 0$$

$$(\lambda - 4) \begin{vmatrix} \lambda - 1 & 0 \\ 0 & \lambda - 1 \end{vmatrix} - 0 + (-1) \begin{vmatrix} 2 & \lambda - 1 \\ 2 & 0 \end{vmatrix} = 0$$

$$\begin{aligned} &(\lambda - 4) \{ (\lambda - 1)(\lambda - 1) \} - 0 - 1 \{ 0 - (2\lambda - 2) \} = 0 \\ &(\lambda - 4) (\lambda^2 - 2\lambda + 1) - 1(-2\lambda + 2) = 0 \\ &\lambda^3 - 2\lambda^2 - 4\lambda^2 + 8\lambda + \lambda - 4 + 2\lambda - 2 = 0 \\ &\lambda^3 - 6\lambda^2 + 11\lambda - 6 \end{aligned}$$

n..

$$6) \quad (b) \quad \begin{bmatrix} 3 & 0 & -5 \\ \frac{1}{5} & -1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

$$[\lambda I - A] = 0$$

$$\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 3 & 0 & -5 \\ \frac{1}{5} & -1 & 0 \\ 1 & 1 & 2 \end{bmatrix} = 0$$

$$\begin{bmatrix} \lambda - 3 & 0 & 5 \\ -\frac{1}{5} & \lambda + 1 & 0 \\ -1 & -1 & \lambda - 2 \end{bmatrix} = 0$$

$$(\lambda - 3) \begin{vmatrix} \lambda + 1 & 0 \\ -1 & \lambda - 2 \end{vmatrix} - 0 + (5) \begin{vmatrix} -\frac{1}{5} & \lambda + 1 \\ -1 & -1 \end{vmatrix}$$

$$(\lambda - 3) \{ (\lambda + 1)(\lambda - 2) - 0 \} + 5 \{ (\frac{1}{5}) - (\lambda + 1)(-1) \}$$

$$(\lambda - 3) \{ \lambda^2 + 3\lambda + 2 \} + 5 \left(\frac{1}{5} - (\lambda + 1) \right)$$

$$(\lambda^3 + 3\lambda^2 + 3\lambda^2 + 9\lambda + 2\lambda - 6) + 5 \left(\frac{1}{5} + \lambda - 1 \right)$$

$$(\lambda^3 - 7\lambda - 6) + (5\lambda + 6)$$

$$\lambda^3 - 2\lambda = 0$$

eq of characteristic

h

$$d) \begin{bmatrix} -1 & 0 & 1 \\ -1 & 3 & 0 \\ -4 & 13 & -1 \end{bmatrix}$$

Soln.

$$[\lambda I - A]$$

$$\begin{bmatrix} \lambda+1 & 0 & -1 \\ 1 & \lambda-3 & 0 \\ 4 & -13 & \lambda+1 \end{bmatrix}$$

$$(\lambda+1) \begin{vmatrix} \lambda-3 & 0 \\ -13 & \lambda+1 \end{vmatrix} - 0 - 1 \begin{vmatrix} 1 & \lambda-3 \\ 4 & -13 \end{vmatrix}$$

$$(\lambda+1) ((\lambda-3)(\lambda+1)) - 1 (-13 - (4\lambda - 12))$$

$$(\lambda+1) (\lambda^2 - 2\lambda - 3) - 1 (-4\lambda - 1)$$

$$(\lambda^3 - 2\lambda^2 + \lambda^2 - 2\lambda - 3\lambda - 3) + 4\lambda + 1$$

$$\lambda^3 - \lambda^2 - \lambda - 2 = 0$$

eq of characteristic

Ans

MIGHTY PAPER PRODUCT

Date: _____

$$\begin{array}{l} \text{Q6} \\ \text{(c)} \end{array} \begin{bmatrix} 5 & 0 & 1 \\ 1 & 1 & 0 \\ -7 & 1 & 0 \end{bmatrix}$$

$$[\lambda I - A]$$

$$\begin{bmatrix} \lambda - 5 & 0 & -1 \\ -1 & \lambda - 1 & 0 \\ 7 & -1 & \lambda \end{bmatrix} = 0$$

$$(\lambda - 5) \begin{vmatrix} \lambda - 1 & 0 \\ -1 & \lambda \end{vmatrix} - 0 \begin{vmatrix} -0 & -1 \\ -1 & \lambda \end{vmatrix} - 1 \begin{vmatrix} -1 & \lambda - 1 \\ 7 & -1 \end{vmatrix} = 0$$

$$((\lambda - 5)(\lambda^2 - \lambda)) - 1(1 + (7\lambda - 7)) = 0$$

$$\lambda^3 - 5\lambda^2 - \lambda^2 + 5\lambda + 7\lambda + 8 = 0$$

$$\lambda^3 - 6\lambda^2$$

$$\lambda^3 - 6\lambda^2 + 12\lambda + 8 = 0$$

eq of characteristic

Ans.

Date: _____

$$f) \begin{bmatrix} 5 & 6 & 2 \\ 0 & -1 & -8 \\ 1 & 0 & -2 \end{bmatrix}$$

Sol:-

$$A = \begin{bmatrix} 5 & 6 & 2 \\ 0 & -1 & -8 \\ 1 & 0 & -2 \end{bmatrix}$$

$$[\lambda I - A]$$

$$\begin{bmatrix} \lambda - 5 & -6 & -2 \\ 0 & \lambda + 1 & 8 \\ -1 & 0 & \lambda + 2 \end{bmatrix} = 0$$

$$(\lambda - 5) \left\{ \begin{vmatrix} \lambda + 1 & 8 \\ 0 & \lambda + 2 \end{vmatrix} \right\} - (-6) \left\{ \begin{vmatrix} 0 & 8 \\ -1 & \lambda + 2 \end{vmatrix} \right\} - 2 \left\{ \begin{vmatrix} 0 & \lambda + 1 \\ -1 & 0 \end{vmatrix} \right\}$$

$$(\lambda - 5) ((\lambda + 1)(\lambda + 2)) - 6(+8) - 2(-(-\lambda - 1))$$

$$(\lambda - 5)(\lambda^2 + 3\lambda + 2) + 48 - 2(\lambda + 1) = 0$$

$$\lambda^3 - 5\lambda^2 + 3\lambda^2 + 2\lambda - 15\lambda - 10\lambda + 48 - 2\lambda - 2 = 0$$

$$\lambda^3 - 2\lambda^2 - 15\lambda + 36 = 0$$

eq of characteristic.

MIGHTY PAPER PRODUCT

Date: _____

Q9
(a)
$$\begin{bmatrix} 0 & 0 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Soln

$$\begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 0 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \lambda & 0 & -2 & 0 \\ -1 & \lambda & -1 & 0 \\ 0 & -1 & \lambda+2 & 0 \\ 0 & 0 & 0 & \lambda-1 \end{bmatrix} = 0$$

$$(\lambda-1) \left\{ \begin{vmatrix} \lambda & 0 & -2 \\ -1 & \lambda & -1 \\ 0 & -1 & \lambda+2 \end{vmatrix} \right\} = 0$$

$$(\lambda-1) \{ (\lambda(\lambda)(\lambda+2) - 1) - 0 + (-2(1-0)) \}$$

$$(\lambda-1) \{ \lambda(\lambda^2+2\lambda)-1 \} - 2 \}$$

$$(\lambda-1) \{ \lambda^3 + 2\lambda^2 - \lambda - 2 \}$$

MIGHTY PAPER PRODUCT

Date: _____

$$\lambda^4 + \lambda^3 - 3\lambda^2 - \lambda + 2 = 0$$

Ans

$$(b) \begin{bmatrix} 10 & -9 & 0 & 0 \\ 4 & -2 & 0 & 0 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$[\lambda I - A]$$

$$\lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - A$$

$$\begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 10 & -9 & 0 & 0 \\ 4 & -2 & 0 & 0 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} \lambda - 10 & 9 & 0 & 0 \\ -4 & \lambda + 2 & 0 & 0 \\ 0 & 0 & \lambda + 2 & 2 \\ 0 & 0 & -1 & \lambda - 2 \end{bmatrix} = 0$$

$$(\lambda + 2) R_2 + R_3$$

MINOR CROSS PRODUCT

Date: _____

$$\begin{vmatrix} \lambda - 10 & 9 & 0 & 0 \\ -4 & \lambda + 2 & 0 & 0 \\ 0 & 0 & 0 & \lambda^2 + 3 \\ 0 & 0 & -1 & \lambda - 2 \end{vmatrix} = 0$$

$$-1 \left\{ \begin{vmatrix} \lambda - 10 & 9 & 0 \\ -4 & \lambda + 2 & 0 \\ 0 & 0 & \lambda^2 + 3 \end{vmatrix} \right\} = 0$$

$$-1 \{ (\lambda^2 - 3)(\lambda^2 + 2\lambda - 10\lambda - 20 + 36) \} = 0$$

$$-1 (\lambda^2 - 3)(\lambda^2 - 8\lambda + 16) = 0$$

$$\lambda^4 - 8\lambda^3 + 19\lambda^2 - 24\lambda + 48 = 0$$

Ans