Dated: 20th April 2022
Due Date: 5th May 2022 (In class)
Home Work # 5
Total Points 50

DUALITY ANALYSIS (20 points)

1. Maximize

$$z = 7x_1 + 12x_2 + 4x_3$$

Subject to

$$x_1 + 2x_2 + x_3 \leq 10$$
$$2x_1 - x_2 + 3x_3 \leq 8$$
$$x_1, x_2, x_3 \geq 0$$

For the above problem,

(a) Convert it into its dual (5 points).

(b) Given $B^{-1} = \begin{bmatrix} 2/5 & -1/5 \\ 1/5 & 2/5 \end{bmatrix}$ and $x_B = [x_2 \quad x_1]^T$, calculate the whole optimal tableau (10 points)

(c) Calculate the values of the dual variable as well as the optimal value of the dual. (5 points)

PRIMAL-DUAL CONVERSION (5 points each)

2. Convert the following problems into their dual

(a)Maximize

$$z = 66x_1 - 22x_2$$

Subject to

$$-x_1 + x_2 \leq -2$$
$$2x_1 + 3x_2 \leq 5$$
$$x_1, x_2 \geq 0$$

(b)Minimize

$$z = 6x_1 + 3x_2$$

Subject to

$$6x_1 - 3x_2 + x_3 \geq 25$$
$$3x_1 + 4x_2 + x_3 \geq 55$$
$$x_1 urs, x_2, x_3 \geq 0$$

DUAL SIMPLEX ALGORITHM (10 points each)

3. Consider the following set of constraints:

Minimize $z = 4x_1 + 8x_2 + 3x_3$

Subject to

$$x_1 + x_2 + x_3 = 7$$
$$2x_1 - 5x_2 + x_3 \geq 10$$
$$x_1, x_2, x_3 \geq 0$$

4. Consider the following LP:

Minimize $z = 2x_1 + 2x_2 + 4x_3$

Subject to

$$2x_1 + x_2 - x_3 \leq 2$$
$$3x_1 + 4x_2 + 2x_3 \geq 8$$
$$x_1, x_2, x_3 \geq 0$$