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SP19-BSSE-0018
BM
HW5

QNO 1

$$\text{Max } Z = 7x_1 + 12x_2 + 4x_3$$

$$x_1 + 2x_2 + x_3 \leq 10$$
$$2x_1 - x_2 + 3x_3 \leq 8$$
$$x_1, x_2, x_3 \geq 0$$

(a)

Standard Form:-

$$\text{Max } Z = 7x_1 + 12x_2 + 4x_3$$

(Y_1) $\quad x_1 + 2x_2 + x_3 + S_1 = 10$

(Y_2) $\quad 2x_1 - x_2 + 3x_3 + S_2 = 8$

$$x_1, x_2, x_3, S_1, S_2 \geq 0$$

Convert into dual Form:-

$$\text{Min } W = 10Y_1 + 8Y_2$$

$$Y_1 + 2Y_2 \geq 7$$
$$2Y_1 - Y_2 \geq 12$$
$$Y_1 + 3Y_2 \geq 4$$

$$Y_1, Y_2 \geq 0$$

Answer

(b)

$$B^{-1} = \begin{bmatrix} 2/5 & -1/5 \\ 1/5 & 2/5 \end{bmatrix}$$

$$X_B = [x_2 \quad x_1]^T$$

$$C_B = [12 \quad 7]$$

BV	x_1	x_2	x_3	S_1	S_2	RHS
Z	0	0				
x_2	0	1		2/5	-1/5	
x_1	1	0		1/5	2/5	

For optimal values of basic variables

$\because \bar{b} = B^{-1} b$

$$\bar{b} = \begin{bmatrix} 2/5 & -1/5 \\ 1/5 & 2/5 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \end{bmatrix} = \begin{bmatrix} 12/5 \\ 26/5 \end{bmatrix}$$

For optimal values of objective function

$\because Z = B^{-1} b \, C_B$

$$Z = [12 \quad 7] \begin{bmatrix} 12/5 \\ 26/5 \end{bmatrix} = 326/5 \qquad \{\because \bar{b} = B^{-1} b\}$$

For optimal coefficients of variable in constraints

$\because \bar{a}_3 = B^{-1} a_3$

$$\bar{a}_3 = \begin{bmatrix} 2/5 & -1/5 \\ 1/5 & 2/5 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -1/5 \\ 7/5 \end{bmatrix}$$

For optimal coefficients of variables in obj. function

$$\because \bar{C}_j = C_B B^{-1} a_j - C_j$$

$$\bar{C}_3 = C_B B^{-1} a_3 - C_3 \Rightarrow \begin{bmatrix} 12 & 7 \end{bmatrix} \begin{bmatrix} -1/5 \\ 7/5 \end{bmatrix} - 4$$

$$\boxed{\bar{C}_3 = \frac{37}{5} - 4 \Rightarrow 17/5}$$

$$\because \bar{C}_{S_1} = C_B B^{-1} a_{S_1} - C_{S_1} \Rightarrow \begin{bmatrix} 12 & 7 \end{bmatrix} \begin{bmatrix} 2/5 \\ 1/5 \end{bmatrix} - 0$$

$$\boxed{\bar{C}_{S_1} = 31/5}$$

$$\bar{C}_{S_2} = C_B B^{-1} a_{S_2} - C_{S_2} \Rightarrow \begin{bmatrix} 12 & 7 \end{bmatrix} \begin{bmatrix} -1/5 \\ 2/5 \end{bmatrix} - 0$$

$$\boxed{\bar{C}_{S_2} = 2/5}$$

The optimal Tableau is;

BV	x_1	x_2	x_3	S_1	S_2	RHS
Z	0	0	17/5	31/5	2/5	326/5
x_2	0	1	-1/5	2/5	-1/5	12/5
x_1	1	0	7/5	1/5	2/5	26/5

Answer

(c)

$$Y : C_B B^{-1}$$

$$\because B^{-1} = \begin{bmatrix} 2/5 & -1/5 \\ 1/5 & 2/5 \end{bmatrix}$$

$$\because C_B = \begin{bmatrix} 12 & 7 \end{bmatrix}$$

$$Y : \begin{bmatrix} 12 & 7 \end{bmatrix} \begin{bmatrix} 2/5 & -1/5 \\ 1/5 & 2/5 \end{bmatrix}$$

$$\boxed{Y : \begin{bmatrix} 31/5 & 2/5 \end{bmatrix}}$$ Dual variables values

$$\because w = 10(31/5) + 8(2/5)$$

$$\boxed{w = 326/5}$$ optimal value of the dual.

Answer

QNO#2

(a)

$$\text{Max } Z = 66x_1 - 22x_2$$

$$-x_1 + x_2 \leq -2$$

$$2x_1 + 3x_2 \leq 5$$

$$x_1, x_2 \geq 0$$

Standard form:-

$$\text{Max } Z = 66x_1 - 22x_2$$

$$(Y_1) \quad x_1 - x_2 - S_1 = 2$$

$$(Y_2) \quad 2x_1 + 3x_2 + S_2 = 5$$

$$x_1, x_2 \geq 0$$

Convert into dual form:-

$$\text{Min } w = 2Y_1 + 5Y_2$$

$$Y_1 + 2Y_2 \geq 66$$

$$-Y_1 + 3Y_2 \geq -22$$

$$Y_1, Y_2 \geq 0$$

(b)

Min $Z = 6x_1 + 3x_2$

$$6x_1 - 3x_2 + x_3 \geq 25$$

$$3x_1 + 4x_2 + x_3 \geq 55$$

$$x_1 \text{ urs}, \; x_2, x_3 \geq 0$$

Standard form:-

$\times \; \cancel{\text{Min } Z = 6x_1 + 3x_2}$

$\cancel{6x_1 - 3x_2 + x_3} \; \times$

Min $Z = 6x_1^- - 6x_1^+ + 3x_2$

$$6x_1^- - 6x_1^+ - 3x_2 + x_3 - S_1 = 25$$

$$3x_1^- - 3x_1^+ + 4x_2 + x_3 - S_2 = 55$$

$$x_1^-, x_1^+, x_2, x_3 \geq 0$$

Convert into dual form:-

Max $w = Y_1$

QNO # 3

$$\text{Min } z = 4x_1 + 8x_2 + 3x_3$$

$$x_1 + x_2 + x_3 = 7$$
$$2x_1 - 5x_2 + x_3 \geq 10$$
$$x_1, x_2, x_3 \geq 0$$

Dual Simplex Algorithm:-

$$\text{Min } z = 4x_1 + 8x_2 + 3x_3$$

$$x_1 + x_2 + x_3 \leq 7$$
$$-x_1 - x_2 - x_3 \leq -7$$
$$-2x_1 + 5x_2 - x_3 \leq -10$$
$$x_1, x_2, x_3 \geq 0$$

Standard form:

$$x_1 + x_2 + x_3 + S_1 = 7$$
$$-x_1 - x_2 - x_3 + S_2 = -7$$
$$-2x_1 + 5x_2 - x_3 + S_3 = -10$$
$$x_1, x_2, x_3 \geq 0$$
$$S_1, S_2, S_3 \geq 0$$

BV	x_1 ↓	x_2	x_3	S_1	S_2	S_3	RHS
Z	-4	-8	-3	0	0	0	0
S_1	1	1	1	1	0	0	7
S_2	-1	-1	-1	0	1	0	-7
S_3	-2	5	-1	0	0	1	-10 ←

$\left|\frac{C_j}{a_{ij}}\right|, a_{ij} \leq 0$

S_3 is leaving

x_1 is Entering

$\left|\frac{-4}{-2}\right| = 2 \quad , \left|\frac{-3}{-1}\right| = 3$

BV	x_1	x_2	x_3 ↓	S_1	S_2	S_3	RHS	
Z	0	-18	-1	0	0	-2	20	
S_1	0	7/2	1/2	1	0	1/2	2	
S_2	0	-7/2	-1/2	0	1	-1/2	-2	←
x_1	1	-5/2	1/2	0	0	-1/2	5	

$\left|\frac{-18}{-7/2}\right| = \frac{36}{7}$, $\left|\frac{-1}{-1/2}\right| = 2$

S_2 is leaving
x_3 is Entering

BV	x_1	x_2	x_3	S_1	S_2	S_3	RHS
Z	0	-11	0	0	-2	-1	24
S_1	0	0	0	1	1	0	0
x_3	0	7	1	0	-2	1	4
x_1	1	-6	0	0	1	-1	3

$Z = 24$

$(3, 0, 4, 0, 0, 0)$ Answer

$\because Z = 4x_1 + 8x_2 + 3x_3$

$24 = 4(3) + 8(0) + 3(4)$

$24 = 24$ verified

QNO # 4

$$\text{Min } Z = 2x_1 + 2x_2 + 4x_3$$

$$2x_1 + x_2 - x_3 \leq 2$$
$$3x_1 + 4x_2 + 2x_3 \geq 8$$
$$x_1, x_2, x_3 \geq 0$$

Dual Simplex Algorithm:-

$$\text{Min } Z = 2x_1 + 2x_2 + 4x_3$$

$$2x_1 + x_2 - x_3 \leq 2$$
$$-3x_1 - 4x_2 - 2x_3 \leq -8$$
$$x_1, x_2, x_3 \geq 0$$

Standard form:-

$$2x_1 + x_2 - x_3 + S_1 = 2$$
$$-3x_1 - 4x_2 - 2x_3 + S_2 = -8$$
$$x_1, x_2, x_3 \geq 0$$
$$S_1, S_2 \geq 0$$

BV	x_1	x_2 ↓	x_3	S_1	S_2	RHS
Z	-2	-2	-4	0	0	0
S_1	2	1	-1	1	0	2
S_2	-3	-4	-2	0	1	-8 ←

$$\left|\frac{-2}{-3}\right| = \frac{2}{3}, \quad \left|\frac{-2}{-4}\right| = \frac{1}{2}, \quad \left|\frac{-4}{-2}\right| = 2$$

S_2 is leaving

x_2 is Entering

BV	x_1	x_2	x_3	S_1	S_2	RHS
Z	$-1/2$	0	-3	0	$-1/2$	4
S_1	$5/4$	0	$-3/2$	1	$1/4$	0
x_2	$3/4$	1	$1/2$	0	$-1/4$	2

$Z = 4$

$(0, 2, 0, 0, 0)$

$\because Z = 2x_1 + 2x_2 + 4x_3$

$4 = 2(2) + 0 + 0$

$4 = 4$ verified

Answer