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SP19-BSSE-0018

BM

HW-06

Operations Research

Q1 (a)

As we know the the change occurs in available resources so find FR.

$$\bar{b} = B^{-1}b$$

$$\bar{b} = \begin{bmatrix} 1 & 2 & -8 \\ 0 & 2 & -4 \\ 0 & -1/2 & 3/2 \end{bmatrix} \begin{bmatrix} 48 \\ 30 \\ 8 \end{bmatrix} = \begin{bmatrix} 44 \\ 28 \\ -3 \end{bmatrix}$$

$$Z = C_B B^{-1} b$$

$$\cancel{= \begin{bmatrix} 0 & 20 & 60 \end{bmatrix} \begin{bmatrix} 44 \\ 30 \\ 8 \end{bmatrix}} =$$

$$Z = \begin{bmatrix} 0 & 20 & 60 \end{bmatrix} \begin{bmatrix} 44 \\ 28 \\ -3 \end{bmatrix} = 380$$

BV	x_1	x_2	x_3	S_1	S_2	S_3	RHS
Z	0	5	0	0	10	10	380
S_1	0	-2	0	1	2	-8	44
x_3	0	-2	1	0	2	-4	28
x_1	1	5/4	0	0	-1/2	3/2	-3

Applying dual simplex method, x_1 is leaving and S_2 is Entering.

BV	x_1	x_2	x_3	S_1	S_2	S_3	RHS
Z	20	30	0	0	0	40	320
S_1	~~3~~ 4	3	0	1	0	-2	32
x_3	~~3~~ 4	3	1	0	0	2	16
S_2	-2	-5/2	0	0	1	-3	6

The optimal answer/solution after changing available resources.

Answer

Q1 (b)

$$\text{Max } Z = 60x_1 + 33x_2 + 20x_3$$

Compute reduced cost of x_2

$$\bar{C_2} = C_B B^{-1} a_2 - C_2$$

$$= \begin{bmatrix} 0 & 20 & 60 \end{bmatrix} \begin{bmatrix} -2 \\ -2 \\ 5/4 \end{bmatrix} - 33$$

$$\boxed{\bar{C_2} = 2}$$

BV	x_1	x_2	x_3	S_1	S_2	S_3	RHS
Z	0	2	0	0	10	10	280
S_1	0	-2	0	1	2	-8	~~4~~ 24
x_3	0	-2	1	0	2	-4	~~8~~ 8
x_1	1	5/4	0	0	-1/2	3/2	2

Q 1 (c)

Max $Z = 60x_1 + 45x_2 + 30x_3$

Finding reduced cost of all Non-basic variables

$$\bar{C_j} = C_B B^{-1} a_j - C_j$$

$$\bar{C_2} = C_B B^{-1} a_2 - C_2$$

$$\bar{C_2} = \begin{bmatrix} 0 & 30 & 60 \end{bmatrix} \begin{bmatrix} -2 \\ -2 \\ 5/4 \end{bmatrix} - 45 = -10$$

$$\bar{C_{S_2}} = C_B B^{-1} a_{S_2} - C_{S_2}$$

$$\bar{C_{S_2}} = \begin{bmatrix} 0 & 30 & 60 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ -1/2 \end{bmatrix} - 0 = \cancel{10}\, 30$$

$$\bar{C_{S_3}} = C_B B^{-1} a_{S_3} - C_{S_3}$$

$$\bar{C_{S_3}} = \begin{bmatrix} 0 & 30 & 60 \end{bmatrix} \begin{bmatrix} -8 \\ -4 \\ 3/2 \end{bmatrix} - 0 = 30$$

$\rightarrow Z = C_B B^{-1} b$

$$Z = \begin{bmatrix} 0 & 30 & 60 \end{bmatrix} \begin{bmatrix} 24 \\ 8 \\ 2 \end{bmatrix}$$

$Z = 360$

BV	x_1	x_2	x_3	S_1	S_2	S_3 ↓	RHS	MR
Z	0	-10	0	0	30	-30	~~280~~ 360	
S_1	0	-2	0	1	2	-8	24	—
x_3	0	-2	1	0	2	-4	8	—
← x_1	1	5/4	0	0	-1/2	3/2	2	4/3

Applying simplex method, S_3 Entering and x_1 is leaving.

BV	x_1	x_2	x_3	S_1	S_2	S_3	RHS
Z	20	15	0	0	20	0	400
S_1	16/3	14/3	0	1	-2/3	0	104/3
x_3	8/3	4/3	1	0	2/3	0	40/3
S_3	2/3	5/6	0	0	-1/3	1	4/3

Answer

Q1 (d)

s.t. $x_1 + x_2 + x_3 \le 12$

$x_1 + x_2 + x_3 + S_4 = 12$

BV	x_1	x_2	x_3	S_1	S_2	S_3	S_4	RHS
Z	0	5	0	0	10	10	0	280
S_1	0	-2	0	1	2	-8	0	24
x_3	0	-2	1	0	2	-4	0	8
x_1	1	5/4	0	0	-1/2	3/2	0	2
S_4	1	1	1	0	0	0	1	12

BV	x_1	x_2	x_3	S_1	S_2	S_3	S_4	RHS
Z	0	5	0	0	10	10	0	280
S_1	0	-2	0	1	2	-8	0	24
x_3	0	-2	1	0	2	-4	0	8
x_1	1	5/4	0	0	-1/2	3/2	0	2
S_4	0	7/4	0	0	-3/2	5/2	1	2

Answer

Q NO # 1 (e)

Addition of new variable, such as x_4.

$$\text{Max } Z = 60x_1 + 30x_2 + 20x_3 + 15x_4$$

$$8x_1 + 6x_2 + x_3 + x_4 \leq 48$$
$$4x_1 + 2x_2 + 3/2\, x_3 + x_4 \leq 20$$
$$2x_1 + 3/2\, x_2 + 1/2\, x_3 + x_4 \leq 8$$

$$x_1, x_2, x_3, x_4 \geq 0$$

S.t.

$$8x_1 + 6x_2 + x_3 + x_4 + S_1 = 48$$
$$4x_1 + 2x_2 + \frac{3}{2}x_3 + x_4 + S_2 = 20$$
$$2x_1 + \frac{3}{2}x_2 + \frac{1}{2}x_3 + x_4 + S_3 = 8$$

$$x_1, x_2, x_3, x_4 \geq 0$$
$$S_1, S_2, S_3 \geq 0$$

BV	x_1	x_2	x_3	x_4	S_1	S_2	S_3	RHS
Z	0	5	0	25	0	10	10	280
S_1	0	-2	0	-5	1	2	-8	24
x_3	0	-2	1	-2	0	2	-4	8
x_1	1	5/4	0	1	0	-1/2	3/2	2

Answer

QNO# 1 (F)

$\because \ \bar{C}_2 = C_B B^{-1} a_2 - (c_2 + \Delta)$

$$C_B B^{-1} a_2 - (c_2 + \Delta) \geq 0$$

$$\begin{bmatrix} 0 & 20 & 60 \end{bmatrix} \begin{bmatrix} 1 & 2 & -8 \\ 0 & 2 & -4 \\ 0 & -1/2 & 3/2 \end{bmatrix} \begin{bmatrix} 6 \\ 2 \\ 3/2 \end{bmatrix} - (30 + \Delta) \geq 0$$

$$\begin{bmatrix} 0 & 20 & 60 \end{bmatrix} \begin{bmatrix} -2 \\ -2 \\ 5/4 \end{bmatrix} - (30 + \Delta) \geq 0$$

~~[0 20 60~~

$$(0 \times -2) + (20 \times -2) + (5/4 \times 60) - (30 + \Delta) \geq 0$$

$$35 - 30 + \Delta \geq 0$$

$$5 - \Delta \geq 0$$

$$\boxed{\Delta \geq 5}$$

if the change greathan equals to 5, we are in optimal state. otherwise we are not in optimal state.

QNO#01 (g)

$$\because \bar{b} = B^{-1} b' \qquad \because b' = \begin{bmatrix} b_1 \\ 20 \\ 8 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & -8 \\ 0 & 2 & -4 \\ 0 & -1/2 & 3/2 \end{bmatrix} \begin{bmatrix} b_1 \\ 20 \\ 8 \end{bmatrix} \geq 0$$

$$\begin{bmatrix} b_1 + 40 - 64 \\ 0 + 4 - 32 \\ -10 + 12 \end{bmatrix} \geq 0$$

$$b_1 - 24 \geq 0$$

$$\boxed{b_1 \geq 24}$$

if the supply of lumber is greaterthan equals to 24, we are in optimal state otherwise we are not in optimal state.