

ASSIGNMENT #03

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SP19-BSSE-0018

BM

Operations Research

Q# 1 (a)

Max $Z = 2x_1 + x_2 - 3x_3 + 5x_4$

$$x_1 + x_2 + 2x_3 + 2x_4 \leq 42$$

$$2x_1 - x_2 + x_3 + 2x_4 \leq 8$$

$$4x_1 - 2x_2 + x_3 - x_4 \leq 12$$

$$x_1, x_2, x_3, x_4 \geq 0$$

St.

$$Z = 2x_1 + x_2 - 3x_3 + 5x_4$$

$$x_1 + x_2 + 2x_3 + 2x_4 + S_1 = 42$$

$$2x_1 - x_2 + x_3 + 2x_4 + S_2 = 8$$

$$4x_1 - 2x_2 + x_3 - x_4 + S_3 = 12$$

$$x_1, x_2, x_3, x_4 \geq 0$$

$$S_1, S_2, S_3 \geq 0$$

BV	x_1	x_2	x_3	x_4	S_1	S_2	S_3	R.H.S.	MR.
Z	-2	-1	3	-5	0	0	0	0	
S_1	1	1	2	2	1	0	0	42	21
$\leftarrow S_2$	2	-1	1	2	0	1	0	8	4
S_3	4	-2	1	-1	0	0	1	12	-

x_4 Entering
 S_2 leaving

$$(0, 0, 0, 0, 42, 8, 12) = (x_1, x_2, x_3, x_4, S_1, S_2, S_3)$$

BV	x_1	x_2	x_3	x_4	s_1	s_2	s_3	RHS	MR.
Z	3	$-7/2$	$11/2$	0	0	$5/2$	0	20	
$\leftarrow s_1$	-1	2	1	0	1	-1	0	34	17
x_4	1	$-1/2$	$1/2$	1	0	$1/2$	0	4	—
s_3	5	$-5/2$	$3/2$	0	0	$1/2$	1	16	—

x_2 is Entering
 s_1 is leaving

$$(0, 0, 0, 4, 34, 0, 16) = (x_1, x_2, x_3, x_4, s_1, s_2, s_3)$$

BV	x_1	x_2	x_3	x_4	s_1	s_2	s_3	RHS
Z	$5/4$	0	$29/4$	0	$7/4$	$3/4$	0	$159/2$
x_2	$-1/2$	1	$1/2$	0	$1/2$	$-1/2$	0	17
x_4	$3/4$	0	$3/4$	1	$1/4$	$1/4$	0	$25/2$
s_3	$15/4$	0	$11/4$	0	$5/4$	$-7/4$	1	$117/2$

$$(0, 17, 0, 25/2, 0, 0, 117/2) = (x_1, x_2, x_3, x_4, s_1, s_2, s_3)$$

Verification:

$$159/2 = 2(0) + 17 - 3(0) + 5(25/2)$$

$$159/2 = 17 + (25/2)$$

$$159/2 = 159/2 \quad \text{verified}$$

QNO# 1 (d)

$$\text{Min } Z = 5x_1 - 4x_2 + 6x_3 - 8x_4$$

$$x_1 + x_2 + 2x_3 + 2x_4 \leq 42$$

$$2x_1 - x_2 + x_3 + 2x_4 \leq 8$$

$$4x_1 - 2x_2 + x_3 - x_4 \leq 12$$

$$x_1, x_2, x_3, x_4 \geq 0$$

St.

Multiply by -1

$$-Z = -5x_1 + 4x_2 - 6x_3 + 8x_4$$

$$-5x_1 + 4x_2 - 6x_3 + 8x_4 + Z = 0$$

$$x_1 + x_2 + 2x_3 + 2x_4 + S_1 = 42$$

$$2x_1 - x_2 + x_3 + 2x_4 + S_2 = 8$$

$$4x_1 - 2x_2 + x_3 - x_4 + S_3 = 12$$

$$x_1, x_2, x_3, x_4 \geq 0$$

$$S_1, S_2, S_3 \geq 0$$

BV	x_1	x_2	\downarrow x_3	x_4	S_1	S_2	S_3	RHS	MR
Z	-5	4	-6	8	0	0	0	0	
S_1	1	1	2	2	1	0	0	42	21
$\leftarrow S_2$	2	-1	1	2	0	1	0	8	8
S_3	4	-2	1	-1	0	0	1	12	12

x_3 is Entering
 S_2 is leaving

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BV	x_1	x_2	x_3	x_4	s_1	s_2	s_3	RHS.	MR
Z	7	-2	0	20	0	6	0	48	
← s_1	-3	3	0	2	1	-2	0	26	$26/3$
x_3	2	-1	1	2	0	1	0	8	—
s_3	2	-1	0	-3	0	-1	1	4	—

$(0, 0, 8, 0, 26, 0, 4) = (x_1, x_2, x_3, x_4, s_1, s_2, s_3)$ x_2 is entering
 s_1 is leaving

BV	x_1	x_2	x_3	x_4	s_1	s_2	s_3	RHS
Z	5	0	0	$64/3$	$2/3$	$14/3$	0	$196/3$
x_2	-1	1	0	$2/3$	$1/3$	$-2/3$	0	$26/3$
x_3	-1	0	1	$8/3$	$1/3$	-1	0	$50/3$
s_3	-1	0	0	$-7/3$	$1/3$	-3	1	$38/3$

$(0, 26/3, 50/3, 0, 0, 0, 38/3) = (x_1, x_2, x_3, x_4, s_1, s_2, s_3)$

Answer

QNO # 2

Max $Z = 20x_1 + 15x_2$

$$15x_1 + 20x_2 \leq 120$$

$$-x_1 + x_2 \leq 1$$

$$x_1 \leq 3$$

$$x_1, x_2 \geq 0$$

et.

Max $Z = 20x_1 + 15x_2$

$$15x_1 + 20x_2 + S_1 = 120$$

$$-x_1 + x_2 + S_2 = 1$$

$$x_1 + S_3 = 3$$

$$x_1, x_2 \geq 0$$

$$S_1, S_2, S_3 \geq 0$$

↓

BV	x_1	x_2	S_1	S_2	S_3	RHS	MR.
Z	-20	-15	0	0	0	0	
S_1	15	20	1	0	0	120	8
S_2	-1	1	0	1	0	1	1
← S_3	1	0	0	0	1	3	3

x_1 is Entering
 S_3 is leaving

$$(0, 0, 120, 1, 3) = (x_1, x_2, S_1, S_2, S_3)$$

BV	x_1	x_2	s_1	s_2	s_3	RHS	MR
Z	0	-15	0	0	20	60	
$\leftarrow s_1$	0	20	1	0	-15	75	15/4
s_2	0	1	0	1	1	4	4
x_1	1	0	0	0	1	3	—

x_2 is Entering
 s_1 is leaving

$$(3, 0, 75, 4, 0) = (x_1, x_2, s_1, s_2, s_3)$$

BV	x_1	x_2	s_1	s_2	s_3	RHS
Z	0	0	15/20	0	35/4	415/4
x_2	0	1	1/20	0	-3/4	15/4
s_2	0	0	-1/20	1	7/4	1/4
x_1	1	0	0	0	1	3

$$(3, 15/4, 0, 1/4, 0) = (x_1, x_2, s_1, s_2, s_3)$$

Verification:

$$415/4 = 20(3) + 15(15/4)$$

$$415/4 = 415/4$$

verified