Exclusive electroproduction revisited: treating kinematical effects

A.V. Belitsky[a], D. Müller[b]

[a]*Department of Physics, Arizona State University*
Tempe, AZ 85287-1504, USA

[b]*Institut für Theoretische Physik II, Ruhr-Universität Bochum*
D-44780 Bochum, Germany

Abstract

Generalized parton distributions of the nucleon are accessed via exclusive leptoproduction of the real photon. While earlier analytical considerations of phenomenological observables were restricted to twist-three accuracy, i.e., taking into account only terms suppressed by a single power of the hard scale, in the present study we revisit this differential cross section within the helicity formalism and restore power-suppressed effects stemming from the process kinematics exactly. We restrict ourselves to the phenomenologically important case of lepton scattering off a longitudinally polarized nucleon, where the photon flips its helicity at most by one unit.

1 Electroproduction observables

Unravelling nucleon's structure from generalized parton distributions (GPDs) [1] requires their measurements in exclusive leptoproduction experiments. Recent years had witnessed ground-breaking efforts which put the underlying theoretical framework on a firm basis with accuracy of approximation involved being under control (see, e.g., [2]). While to date reliable modeling of partonic correlations encoded in GPDs is far from being mature enough, theoretical analyses of experimental observables are not constrained by any complications of principle and rather awaited the time when experiments reached competing precision.

The cross section for exclusive electroproduction of photons, being the cleanest probe of GPDs, was computed analytically already for some time to twist-three accuracy [3], i.e., keeping terms suppressed at most by one power of the hard scale and neglecting everything else. While this approximation is robust for kinematical regimes with moderately hard virtualities of the exchanged photon at large energies, it was shown to overestimate available data at low momentum transfer in the valence quark region, i.e., for moderate values of the Bjorken variable. This calls for the restoration of contributions ignored previously on the basis of their parametric suppression. In a more recent investigation [4], we demonstrated that the deviation between the data and theoretical estimates could be reconciled by calculating kinematical corrections in hard scale exactly while ignoring dynamical high-twist contributions altogether. The latter assumptions can be motivated by the expected hierarchy of low-energy scales associated with hadronic matrix elements of high-twist operators which are smaller than other soft kinematical scales in the problem, like the hadron mass or the t-channel momentum transfer. This phenomenon exhibits itself as precocious scaling in conventional deep-inelastic scattering.

While our earlier analysis was preformed for the (pseudo) scalar target [4], presently we will generalize this consideration to the case of a spin one-half hadron. The main focus of our consideration is the differential cross section for scattering of the electron/positron $\ell = e^{\mp}$ off the nucleon N with the emission of the real photon in the final state, $\ell(k)N(p_1) \to \ell(k')N(p_2)\gamma(q_2)$,

$$d\sigma = \frac{\alpha^3 x_{\rm B} y^2}{8\,\pi\,\mathcal{Q}^4\sqrt{1+\epsilon^2}} \left|\frac{\mathcal{T}}{e^3}\right|^2 dx_{\rm B} d\mathcal{Q}^2 d|t| d\phi \,. \qquad (1.1)$$

The phase space of the process is parameterized by the Bjorken variable $x_{\rm B} = \mathcal{Q}^2/(2p_1 \cdot q_1)$ determined in terms of the momentum $q_1 = k - k'$ carried by the virtual photon of mass $\mathcal{Q}^2 = -q_1^2$, the square of the t-channel momentum $t \equiv \Delta^2$ with $\Delta = p_2 - p_1$ and the lepton energy loss $y = p_1 \cdot q_1 / p_1 \cdot k$. The azimuthal angle ϕ of the recoiled nucleon is defined in the rest frame of the target with the z-axis directed counter-along the photon three-momentum $\boldsymbol{q}_1$. While the theoretical analysis of the microscopic physics is cleanest when one formally takes the limit $\mathcal{Q} \to \infty$, realistic experiments are done in a few GeV region where the effects from kinematical parameters suppressed by $\mathcal{Q}$,

$$\epsilon \equiv 2 x_{\rm B} \frac{M}{\mathcal{Q}} \,, \qquad \frac{t}{\mathcal{Q}^2} \,, \qquad (1.2)$$

may be significant.

The electroproduction amplitude $\mathcal{T}$ is a linear superposition of the Bethe-Heitler and deeply virtual Compton scattering (DVCS) amplitudes. In the former process, the real photon is emitted from the lepton which then scatters off the target nucleons via the transition matrix element of the electromagnetic quark current J_μ, parameterized in terms of the Pauli and Dirac form factors

$F_1 = F_1(t)$ and $F_2 = F_2(t)$,

$$J_\mu = \langle p_2 | j_\mu(0) | p_1 \rangle = \bar{u}_2 \left(\gamma_\mu F_1 + i \sigma_{\mu\nu} \frac{\Delta^\nu}{2M} F_2 \right) u_1 \, , \tag{1.3}$$

with the nucleon bispinors $u_i = u(p_i)$ normalized conventionally as $\bar{u}u = 2M$. The DVCS amplitude

$$T_{\mu\nu} = \frac{i}{e^2} \int d^4 z \, \mathrm{e}^{\frac{i}{2}(q_1+q_2)\cdot z} \langle p_2 | T\{ j_\mu(z/2) j_\nu(-z/2) \} | p_1 \rangle \, , \tag{1.4}$$

encodes the partonic structure of the nucleon and is the object of interest. In the square of the scattering amplitude

$$\mathcal{T}^2 = |\mathcal{T}^{\mathrm{BH}}|^2 + |\mathcal{T}^{\mathrm{DVCS}}|^2 + \mathcal{I} \, , \qquad \mathcal{I} = \mathcal{T}^{\mathrm{DVCS}} (\mathcal{T}^{\mathrm{BH}})^* + (\mathcal{T}^{\mathrm{DVCS}})^* \mathcal{T}^{\mathrm{BH}} \, , \tag{1.5}$$

the Bethe-Heitler contribution $|\mathcal{T}^{\mathrm{BH}}|^2$ is merely an undesirable contamination which was computed exactly already in Ref. [3] and can be subtracted from the cross section making use of the available vast data on the nucleon electromagnetic form factors measured at facilities around the world. The main observables for extraction of GPDs emerge from the remaining two contributions involving $\mathcal{T}^{\mathrm{DVCS}}$, the square of the DVCS amplitude and the interference term $\mathcal{I}$.

In analogy to the hadronic electromagnetic current (1.3), decomposed in terms of the Dirac bilinears accompanied by the form factors, we parameterize the DVCS amplitude as follows

$$T_{\mu\nu} = -\mathcal{P}_{\mu\sigma} g_{\sigma\tau} \mathcal{P}_{\tau\nu} \frac{q \cdot V_1}{p \cdot q} + \left(\mathcal{P}_{\mu\sigma} p_\sigma \mathcal{P}_{\rho\nu} + \mathcal{P}_{\mu\rho} p_\sigma \mathcal{P}_{\sigma\nu} \right) \frac{V_{2\,\rho}}{p \cdot q} - \mathcal{P}_{\mu\sigma} i \varepsilon_{\sigma\tau q \rho} \mathcal{P}_{\tau\nu} \frac{A_{1\,\rho}}{p \cdot q} \, , \tag{1.6}$$

where we have kept all dynamical contributions up to twist-three accuracy and, at the same time, kinematically restored the electromagnetic gauge invariance exactly. Note, however, that the so-called gluon transversity contribution, inducing the photon helicity-flip amplitude by two units at leading twist level but suppressed by a power of α_s, is not included here. The average four-momenta which enter this equation are $p = p_1 + p_2$ and $q = \frac{1}{2}(q_1 + q_2)$. The parametrization (1.6) is similar to the one used in deep-inelastic scattering. Indeed, the twist-two part of the generalized functions V_1 and A_1 corresponds to the conventional F_1 and g_1 structure functions. The current conservation is ensured by means of the projection operator

$$\mathcal{P}_{\mu\nu} = g_{\mu\nu} - \frac{q_{1\mu} q_{2\nu}}{q_1 \cdot q_2} \, , \tag{1.7}$$

whose particular form is driven by the explicit calculation of the Compton amplitude via the operator product expansion to twist-three accuracy [5, 6], (see also Refs. [7, 8] for spinless targets). The $V_{2\,\rho}$ structure is not independent and is expressed in terms of the other two vector functions $V_{1\,\rho}$ and $A_{1\,\rho}$,

$$V_{2\,\rho} = \xi \left(V_{1\,\rho} - \frac{p_\rho}{2} \frac{q \cdot V_1}{p \cdot q} \right) + \frac{i}{2} \frac{\varepsilon_{\rho\sigma\Delta q}}{p \cdot q} A_{1\,\sigma} \, , \tag{1.8}$$

where $\xi = -q^2/p \cdot q$. The amplitudes V_1 and A_1 depend on the scaling variable x_{B}, the momentum transfer Δ^2, and the hard momentum of the probe $\mathcal{Q}^2$, however, in order to simplify our notations, we will drop this dependence when it is not essential for the presentation. Their general decomposition in a complete basis of Compton form factors (CFFs) reads

$$
\begin{aligned}
V_{1\,\rho} &= \frac{1}{p\cdot q}\bar{u}_2\left(\not{q}\left[p_\rho \mathcal{H} + \Delta_{\perp\rho}\mathcal{H}^3_+\right] + i\sigma_{\mu\nu}\frac{q_\mu\Delta_\nu}{2M}\left[p_\rho\mathcal{E} + \Delta_{\perp\rho}\mathcal{E}^3_+\right] + \widetilde{\Delta}_{\perp\rho}\left[\not{q}\widetilde{\mathcal{H}}^3_- + \frac{q\cdot\Delta}{2M}\widetilde{\mathcal{E}}^3_-\right]\gamma_5\right)u_1\,,\\
A_{1\,\rho} &= \frac{1}{p\cdot q}\bar{u}_2\left(\not{q}\gamma_5\left[p_\rho\widetilde{\mathcal{H}} + \Delta_{\perp\rho}\widetilde{\mathcal{H}}^3_+\right] + \frac{q\cdot\Delta}{2M}\gamma_5\left[p_\rho\widetilde{\mathcal{E}} + \Delta_{\perp\rho}\widetilde{\mathcal{E}}^3_+\right] + \widetilde{\Delta}_{\perp\rho}\left[\not{q}\mathcal{H}^3_- + i\sigma_{\mu\nu}\frac{q_\mu\Delta_\nu}{2M}\mathcal{E}^3_-\right]\right)u_1\,,
\end{aligned}
$$

again to twist-three accuracy. Here the CFFs given by convolutions of perturbatively calculable coefficient functions and a set of twist-two and -three GPDs (see Ref. [3] for details). In the above equations we use the following notations for the transverse components of the t-channel momentum

$$
\Delta^\perp_\rho \equiv \Delta_\rho - \frac{\Delta\cdot q}{p\cdot q}p_\rho \quad\text{and}\quad \widetilde{\Delta}^\perp_\rho \equiv \frac{i\,\varepsilon_{\rho\Delta pq}}{p\cdot q}
$$

and where $\Delta\cdot q/p\cdot q \approx -\xi$ in DVCS kinematics.

2 Helicity amplitudes

While in the BKM consideration [3], one is restricted to the twist-three approximation for dynamical as well as kinematical effects, in the current analysis the latter will be restored exactly since they account for the bulk of power-suppressed corrections provided that there is a hierarchy of hadronic scales associated with higher-twist operator matrix elements, such that, e.g., ϵ^2tw-2 $\gg \frac{1}{Q^2}$tw-4. An analysis of twist-four effects and higher is intrinsically involved due to complications and ambiguities in the choice of operator bases. On the other hand, the incorporation of kinematical power-suppressed effects is straightforward. In order to achieve this in the most efficient manner we separate power corrections that arise from the leptonic and hadronic parts by evaluating photon helicity amplitudes utilizing the polarization vectors for the incoming and outgoing photons in the target rest frame. In addition to being a concise calculation scheme, it has an advantage of localizing the azimuthal angle dependence in the lepton helicity amplitudes for the choice of the reference frame with the z-axis counter-aligned with the incoming photon three-momentum. It also allows for a straightforward reduction to the harmonic expansion introduced in Refs. [3, 9].

We define the hadronic helicity amplitudes as

$$
\mathcal{T}^{\rm DVCS}_{ac}(\phi) = (-1)^{a-1}\varepsilon_2^{\mu *}(c)T_{\mu\nu}\varepsilon_1^\nu(a)\,, \tag{2.1}
$$

where the overall phase $(-1)^{a-1}$ accounts for the signature factor in the completeness relation for the photon polarization vectors. These are constrained by the parity conservation and, as a consequence, we have six independent functions,

$$
\begin{aligned}
\mathcal{T}^{\rm DVCS}_{--}(\mathcal{F}) &= \mathcal{T}^{\rm DVCS}_{++}(\mathcal{F})\Big|_{\mathcal{F}^{P=\pm1}\to\pm\mathcal{F}^{P=\pm1}}\,,\\
\mathcal{T}^{\rm DVCS}_{0-}(\mathcal{F}) &= \mathcal{T}^{\rm DVCS}_{0+}(\mathcal{F})\Big|_{\mathcal{F}^{P=\pm1}\to\pm\mathcal{F}^{P=\pm1}}\,,\\
\mathcal{T}^{\rm DVCS}_{-+}(\mathcal{F}) &= \mathcal{T}^{\rm DVCS}_{+-}(\mathcal{F})\Big|_{\mathcal{F}^{P=\pm1}\to\pm\mathcal{F}^{P=\pm1}}\,.
\end{aligned} \tag{2.2}
$$

Substitution of the explicit parametrization for the Compton amplitude (1.6) yields dynamical twist-three approximation for the helicity amplitudes

$$\mathcal{T}^{\rm DVCS}_{aa} = \frac{1}{p\cdot q}\left[q\cdot V(\mathcal{F}) - a\, q\cdot A(\mathcal{F})\right] + \mathcal{O}(\mathcal{Q}^{-2})\,, \tag{2.3}$$

$$\mathcal{T}^{\rm DVCS}_{0a} = \frac{\sqrt{2}\widetilde{K}}{\mathcal{Q}(2-x_{\rm B})}\frac{1}{p\cdot q}\left[q\cdot V(\mathcal{F}_{\rm eff}) - a\, q\cdot A(\mathcal{F}_{\rm eff})\right] + \mathcal{O}(\mathcal{Q}^{-3}, \alpha_s\mathcal{Q}^{-1})\,, \tag{2.4}$$

where $a=\pm 1$ labels the helicity states of the final photon, $\mathcal{F}_{\rm eff}$ denotes the effective twist-three contribution in the notation of Ref. [3], see Eqs. (84)[1]–(87) there, and

$$\widetilde{K} = \sqrt{t_{\rm min}-t}\sqrt{(1-x_{\rm B})\sqrt{1+\epsilon^2} + \frac{(t_{\rm min}-t)\,(\epsilon^2+4(1-x_{\rm B})x_{\rm B})}{4\mathcal{Q}^2}}\,. \tag{2.5}$$

Note that the helicity flip amplitude $\mathcal{T}^{\rm DVCS}_{-+}$ arises from twist-two gluon transversity, formally suppressed by α_s, and higher twist contributions. Both of them will not be considered here. In the following two sections we address the square of the DVCS amplitude and the interference term in turn.

2.1 Squared DVCS term

Using the completeness relations for the photon polarization vectors, we can rewrite the square of the DVCS amplitude

$$|\mathcal{T}^{\rm DVCS}|^2 = \frac{1}{\mathcal{Q}^2}\sum_{a=-,0,+}\sum_{b=-,0,+}\mathcal{L}_{ab}(\lambda,\phi)\mathcal{W}_{ab}\,, \tag{2.6}$$

in terms of the hadronic,

$$\mathcal{W}_{ab} = \mathcal{T}^{\rm DVCS}_{a+}\left(\mathcal{T}^{\rm DVCS}_{b+}\right)^* + \mathcal{T}^{\rm DVCS}_{a-}\left(\mathcal{T}^{\rm DVCS}_{b-}\right)^*\,, \tag{2.7}$$

and leptonic,

$$\mathcal{L}_{ab}(\lambda,\phi) = \varepsilon_1^{\mu *}(a)\mathcal{L}_{\mu\nu}(\lambda)\varepsilon_1^{\nu}(b)\,, \tag{2.8}$$

amplitudes, labeled by the helicity states of the initial photon. The latter can be calculated exactly with the result already presented in Ref. [4]

$$\mathcal{L}_{++}(\lambda) = \frac{1}{y^2(1+\epsilon^2)}\left(2-2y+y^2+\frac{\epsilon^2}{2}y^2\right) - \frac{2-y}{\sqrt{1+\epsilon^2}y}\lambda\,, \tag{2.9}$$

$$\mathcal{L}_{00} = \frac{4}{y^2(1+\epsilon^2)}\left(1-y-\frac{\epsilon^2}{4}y^2\right)\,, \tag{2.10}$$

$$\mathcal{L}_{0+}(\lambda,\phi) = \frac{2-y-\lambda y\sqrt{1+\epsilon^2}}{y^2(1+\epsilon^2)}\sqrt{2}\sqrt{1-y-\frac{\epsilon^2}{4}y^2}\,e^{-i\phi}\,, \tag{2.11}$$

$$\mathcal{L}_{-+}(\phi) = \frac{2}{y^2(1+\epsilon^2)}\left(1-y-\frac{\epsilon^2}{4}y^2\right)e^{-i2\phi}\,, \tag{2.12}$$

[1]We like to thank M. Diehl for pointing out that the general relation (84) is not applicable for the CFF $\mathcal{E}_{\rm eff}$. For this specific case we refer the reader to our original work [6].

where the remaining amplitudes are related to the above ones by parity and time-reversal invariance,

$$\begin{aligned}
\mathcal{L}_{0-}(\lambda,\phi) &= \mathcal{L}_{0+}(-\lambda,-\phi)\,, & \mathcal{L}_{\pm,0}(\lambda,\phi) &= \mathcal{L}_{0,\pm}(-\lambda,\phi)\,,\\
\mathcal{L}_{--}(\lambda) &= \mathcal{L}_{++}(-\lambda)\,, & \mathcal{L}_{-+}(\phi) &= \mathcal{L}_{+-}(-\phi)\,.
\end{aligned} \tag{2.13}$$

More explicitly, neglecting transverse photon helicity-flip contributions, one finds for the squared DVCS amplitude

$$\begin{aligned}
\mathcal{Q}^2|\mathcal{T}^{\rm DVCS}|^2 &= \mathcal{L}_{++}(\lambda)\mathcal{T}^{\rm DVCS}_{++}\left(\mathcal{T}^{\rm DVCS}_{++}\right)^* + \mathcal{L}_{++}(-\lambda)\mathcal{T}^{\rm DVCS}_{--}\left(\mathcal{T}^{\rm DVCS}_{--}\right)^*\\
&+ \mathcal{L}_{00}\left[\mathcal{T}^{\rm DVCS}_{0+}\left(\mathcal{T}^{\rm DVCS}_{0+}\right)^* + \mathcal{T}^{\rm DVCS}_{0-}\left(\mathcal{T}^{\rm DVCS}_{0-}\right)^*\right]\\
&+ \mathcal{L}_{0+}(\lambda,\phi)\mathcal{T}^{\rm DVCS}_{0+}\left(\mathcal{T}^{\rm DVCS}_{++}\right)^* + \mathcal{L}_{0+}(-\lambda,-\phi)\mathcal{T}^{\rm DVCS}_{0-}\left(\mathcal{T}^{\rm DVCS}_{--}\right)^*\\
&+ \mathcal{L}_{0+}(\lambda,-\phi)\mathcal{T}^{\rm DVCS}_{++}\left(\mathcal{T}^{\rm DVCS}_{0+}\right)^* + \mathcal{L}_{0+}(-\lambda,\phi)\mathcal{T}^{\rm DVCS}_{--}\left(\mathcal{T}^{\rm DVCS}_{0-}\right)^*\,.
\end{aligned} \tag{2.14}$$

These findings immediately allow one to get the Fourier coefficients in the refined approximation. In addition to the overall prefactors

$$\frac{1}{1+\epsilon^2} \qquad \text{and} \qquad \frac{\lambda}{\sqrt{1+\epsilon^2}}\,, \tag{2.15}$$

accompanying the lepton helicity independent and dependent parts of the amplitude, respectively, one find that the following substitutions in the refined approximation for the lepton-photon "splitting kernels",

$$\begin{aligned}
2-2y+y^2 &\to 2-2y+y^2+\tfrac{1}{2}\epsilon^2y^2\,,\\
1-y &\to 1-y-\tfrac{1}{4}\epsilon^2y^2\,.
\end{aligned} \tag{2.16}$$

From here, we can read off the kinematically improved DVCS harmonics in the decomposition

$$|\mathcal{T}^{\rm DVCS}|^2 = \frac{e^6}{y^2\mathcal{Q}^2}\left\{c_0^{\rm DVCS} + \sum_{n=1}^{2}\left[c_n^{\rm DVCS}\cos(n\phi) + s_n^{\rm DVCS}\sin(n\phi)\right]\right\}\,, \tag{2.17}$$

with

$$c^{\rm DVCS}_{0,\rm unp} = 2\frac{2-2y+y^2+\frac{\epsilon^2}{2}y^2}{1+\epsilon^2}\mathcal{C}^{\rm DVCS}_{\rm unp}(\mathcal{F},\mathcal{F}^*) + \frac{16K^2}{(2-x_{\rm B})^2(1+\epsilon^2)}\mathcal{C}^{\rm DVCS}_{\rm unp}(\mathcal{F}_{\rm eff},\mathcal{F}^*_{\rm eff})\,, \tag{2.18}$$

$$\left\{ {c^{\rm DVCS}_{1,\rm unp} \atop s^{\rm DVCS}_{1,\rm unp}} \right\} = \frac{8K}{(2-x_{\rm B})(1+\epsilon^2)}\left\{ {(2-y) \atop -\lambda y\sqrt{1+\epsilon^2}} \right\}\left\{ {\Re{\rm e} \atop \Im{\rm m}} \right\}\mathcal{C}^{\rm DVCS}_{\rm unp}\left(\mathcal{F}_{\rm eff},\mathcal{F}^*\right) \tag{2.19}$$

for an unpolarized target and

$$c^{\rm DVCS}_{0,\rm LP} = \frac{2\lambda\Lambda y(2-y)}{\sqrt{1+\epsilon^2}}\mathcal{C}^{\rm DVCS}_{\rm LP}(\mathcal{F},\mathcal{F}^*)\,, \tag{2.20}$$

$$\left\{ {c^{\rm DVCS}_{1,\rm LP} \atop s^{\rm DVCS}_{1,\rm LP}} \right\} = -\frac{8\Lambda K}{(2-x_{\rm B})(1+\epsilon^2)}\left\{ {-\lambda y\sqrt{1+\epsilon^2} \atop (2-y)} \right\}\left\{ {\Re{\rm e} \atop \Im{\rm m}} \right\}\mathcal{C}^{\rm DVCS}_{\rm LP}\left(\mathcal{F}_{\rm eff},\mathcal{F}^*\right) \tag{2.21}$$

for the longitudinal polarized part, proportional to the polarization Λ. As in Ref. [3], we use the shorthand

$$K = \sqrt{1 - y + \frac{\epsilon^2}{4} y^2} \frac{\widetilde{K}}{\mathcal{Q}}.$$

We emphasis that the squared twist-three contribution in Eq. (2.18) is a $1/\mathcal{Q}^2$ suppressed contribution and that the transversity contribution $\mathcal{F}_T$ is set to zero.

To evaluate the bilinear combinations $\mathcal{C}^{\rm DVCS}$ of CFFs, we rely on the approximations (2.3)–(2.4). By means of Eq. (2.14), we find the following result for the unpolarized

$$\begin{aligned} \mathcal{C}^{\rm DVCS}_{\rm unp} = & \frac{\mathcal{Q}^2(\mathcal{Q}^2 + x_{\rm B} t)}{\left((2 - x_{\rm B})\mathcal{Q}^2 + x_{\rm B} t\right)^2} \Bigg\{ 4(1 - x_{\rm B}) \mathcal{H}\mathcal{H}^* + 4 \left(1 - x_{\rm B} + \frac{2\mathcal{Q}^2 + t}{\mathcal{Q}^2 + x_{\rm B} t} \frac{\epsilon^2}{4}\right) \widetilde{\mathcal{H}}\widetilde{\mathcal{H}}^* \\ & - \frac{x_{\rm B}^2 (\mathcal{Q}^2 + t)^2}{\mathcal{Q}^2 (\mathcal{Q}^2 + x_{\rm B} t)} \left(\mathcal{H}\mathcal{E}^* + \mathcal{E}\mathcal{H}^*\right) - \frac{x_{\rm B}^2 \mathcal{Q}^2}{\mathcal{Q}^2 + x_{\rm B} t} \left(\widetilde{\mathcal{H}}\widetilde{\mathcal{E}}^* + \widetilde{\mathcal{E}}\widetilde{\mathcal{H}}^*\right) \\ & - \left(\frac{x_{\rm B}^2 \left(\mathcal{Q}^2 + t\right)^2}{\mathcal{Q}^2 (\mathcal{Q}^2 + x_{\rm B} t)} + \frac{\left((2 - x_{\rm B})\mathcal{Q}^2 + x_{\rm B} t\right)^2}{\mathcal{Q}^2 (\mathcal{Q}^2 + x_{\rm B} t)} \frac{t}{4M^2}\right) \mathcal{E}\mathcal{E}^* - \frac{x_{\rm B}^2 \mathcal{Q}^2}{\mathcal{Q}^2 + x_{\rm B} t} \frac{t}{4M^2} \widetilde{\mathcal{E}}\widetilde{\mathcal{E}}^* \Bigg\} \end{aligned} \tag{2.22}$$

and longitudinally polarized combinations of CFFs

$$\begin{aligned} \mathcal{C}^{\rm DVCS}_{\rm LP} = & \frac{\mathcal{Q}^2(\mathcal{Q}^2 + x_{\rm B} t)}{\sqrt{1+\epsilon^2}\left((2 - x_{\rm B})\mathcal{Q}^2 + x_{\rm B} t\right)^2} \Bigg\{ 4\left(1 - x_{\rm B} + \frac{(3 - 2x_{\rm B})\mathcal{Q}^2 + t}{\mathcal{Q}^2 + x_{\rm B} t} \frac{\epsilon^2}{4}\right) \left(\mathcal{H}\widetilde{\mathcal{H}}^* + \widetilde{\mathcal{H}}\mathcal{H}^*\right) \\ & - \frac{\mathcal{Q}^2 - x_{\rm B}(1 - 2x_{\rm B})t}{\mathcal{Q}^2 + x_{\rm B} t} x_{\rm B}^2 \left(\mathcal{H}\widetilde{\mathcal{E}}^* + \widetilde{\mathcal{E}}\mathcal{H}^* + \widetilde{\mathcal{H}}\mathcal{E}^* + \mathcal{E}\widetilde{\mathcal{H}}^*\right) \\ & - \frac{4(1 - x_{\rm B})\left(\mathcal{Q}^2 + x_{\rm B} t\right) t + \left(\mathcal{Q}^2 + t\right)^2 \epsilon^2}{2\mathcal{Q}^2\left(\mathcal{Q}^2 + x_{\rm B} t\right)} x_{\rm B} \left(\widetilde{\mathcal{H}}\mathcal{E}^* + \mathcal{E}\widetilde{\mathcal{H}}^*\right) \\ & - \frac{(2 - x_{\rm B})\mathcal{Q}^2 + x_{\rm B} t}{\mathcal{Q}^2 + x_{\rm B} t} \left(\frac{x_{\rm B}^2 \left(\mathcal{Q}^2 + t\right)^2}{2\mathcal{Q}^2\left((2 - x_{\rm B})\mathcal{Q}^2 + x_{\rm B} t\right)} + \frac{t}{4M^2}\right) x_{\rm B} \left(\mathcal{E}\widetilde{\mathcal{E}}^* + \widetilde{\mathcal{E}}\mathcal{E}^*\right) \Bigg\}, \end{aligned} \tag{2.23}$$

respectively. The uncertainties from remaining kinematical and dynamical higher twist contributions are included in the bilinear combinations $\mathcal{C}^{\rm DVCS}$ of CFFs. As shown in Ref. [13] for a (pseudo) scalar target, i.e., setting

$$\frac{q \cdot V(\mathcal{F})}{q \cdot p} = \mathcal{H}\,, \quad \frac{q \cdot V(\mathcal{F}_{\rm eff})}{q \cdot p} = \mathcal{H}_{\rm eff}\,, \quad q \cdot A = 0\,,$$

in Eqs. (2.3)–(2.4), different parameterizations of the DVCS amplitude result only in small numerical deviations even at rather low energy and photon virtualities. Finally, neglecting $1/\mathcal{Q}^2$ power suppressed terms in the presented findings for the squared DVCS amplitude leads to those of Ref. [3].

2.2 Interference term

Let us now treat the interference term in a manner completely analogous to the consideration of the squared DVCS amplitude given above. Inserting the completeness condition for the initial and final photon polarization states, one finds $\mathcal{I}$ as a linear superposition

$$\mathcal{I} = \frac{\pm e^6}{t\, \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \sum_{a=-,0,+} \sum_{b=-,+} \sum_{S'} \left\{ \mathcal{L}^{\rho}_{ab}(\lambda, \phi) \mathcal{T}_{ab} J^{\dagger}_{\rho} + \left(\mathcal{L}^{\rho}_{ab}(\lambda, \phi) \mathcal{T}_{ab} J^{\dagger}_{\rho}\right)^* \right\}, \tag{2.24}$$

of products of hadronic and leptonic helicity amplitudes. The former were defined earlier in Eqs. (2.3)–(2.4) while the latter read

$$\mathcal{L}^{\rho}_{ab}(\lambda,\phi) = \varepsilon_1^{\mu *}(a) L_{\mu}{}^{\rho}{}_{\nu} \varepsilon_2^{\nu}(b) \, . \tag{2.25}$$

Summation over the final nucleon polarization states yields the following result for the building blocks of the hadronic amplitudes (2.3)–(2.4)

$$\sum_{S'} \frac{q\cdot V}{p\cdot q} J^{\dagger}_{\rho} = p_\rho \left[\mathcal{C}^{\mathcal{I}}_{\rm unp} - \mathcal{C}^{\mathcal{I},A}_{\rm unp}\right](\mathcal{F}) + 2q_\rho \frac{t}{\mathcal{Q}^2}\mathcal{C}^{\mathcal{I},V}_{\rm unp}(\mathcal{F}) + \frac{2\Lambda}{\sqrt{1+\epsilon^2}} \frac{i\varepsilon_{pq\Delta\rho}}{\mathcal{Q}^2} \mathcal{C}^{\mathcal{I},V}_{\rm LP}(\mathcal{F}) \, , \tag{2.26}$$

$$\sum_{S'} \frac{q\cdot A}{p\cdot q} J^{\dagger}_{\rho} = \frac{\Lambda p_\rho}{\sqrt{1+\epsilon^2}} \left[\mathcal{C}^{\mathcal{I}}_{\rm LP} - \mathcal{C}^{\mathcal{I},V}_{\rm LP}\right](\mathcal{F}) + 2q_\rho \frac{\Lambda}{\sqrt{1+\epsilon^2}} \frac{t}{\mathcal{Q}^2} \mathcal{C}^{\mathcal{I},A}_{\rm LP}(\mathcal{F}) + 2\frac{i\varepsilon_{pq\Delta\rho}}{\mathcal{Q}^2} \mathcal{C}^{\mathcal{I},A}_{\rm unp}(\mathcal{F}) \, . \tag{2.27}$$

Here, to match the notation of Ref. [3], we introduced the following combination of CFFs

$$\mathcal{C}^{\mathcal{I}}_{\rm unp}(\mathcal{F}) = F_1\mathcal{H} - \frac{t}{4M^2}F_2\mathcal{E} + \frac{x_{\rm B}}{2-x_{\rm B}+x_{\rm B}\frac{t}{\mathcal{Q}^2}}(F_1+F_2)\widetilde{\mathcal{H}} \, , \tag{2.28}$$

$$\mathcal{C}^{\mathcal{I},V}_{\rm unp}(\mathcal{F}) = \frac{x_{\rm B}}{2-x_{\rm B}+x_{\rm B}\frac{t}{\mathcal{Q}^2}}(F_1+F_2)(\mathcal{H}+\mathcal{E}) \, , \tag{2.29}$$

$$\mathcal{C}^{\mathcal{I},A}_{\rm unp}(\mathcal{F}) = \frac{x_{\rm B}}{2-x_{\rm B}+x_{\rm B}\frac{t}{\mathcal{Q}^2}}(F_1+F_2)\widetilde{\mathcal{H}} \, , \tag{2.30}$$

$$\begin{aligned}\mathcal{C}^{\mathcal{I}}_{\rm LP}(\mathcal{F}) = &\frac{x_{\rm B}}{2-x_{\rm B}+x_{\rm B}\frac{t}{\mathcal{Q}^2}}(F_1+F_2)\left[\mathcal{H} + \frac{x_{\rm B}}{2}\left(1-\frac{t}{\mathcal{Q}^2}\right)\mathcal{E}\right] \\ &+ \left[1 + \frac{M^2}{\mathcal{Q}^2}\frac{x_{\rm B}^2}{2-x_{\rm B}+x_{\rm B}\frac{t}{\mathcal{Q}^2}}\left(3+\frac{t}{\mathcal{Q}^2}\right)\right]F_1\widetilde{\mathcal{H}} - \frac{t}{\mathcal{Q}^2}\frac{2x_{\rm B}(1-2x_{\rm B})}{2-x_{\rm B}+x_{\rm B}\frac{t}{\mathcal{Q}^2}}F_2\widetilde{\mathcal{H}} \\ &- \frac{x_{\rm B}}{2-x_{\rm B}+x_{\rm B}\frac{t}{\mathcal{Q}^2}}\left[\frac{x_{\rm B}}{2}\left(1-\frac{t}{\mathcal{Q}^2}\right)F_1 + \frac{t}{4M^2}F_2\right]\widetilde{\mathcal{E}} \, ,\end{aligned} \tag{2.31}$$

$$\mathcal{C}^{\mathcal{I},V}_{\rm LP}(\mathcal{F}) = \frac{x_{\rm B}}{2-x_{\rm B}+x_{\rm B}\frac{t}{\mathcal{Q}^2}}(F_1+F_2)\left[\mathcal{H} + \frac{x_{\rm B}}{2}\left(1-\frac{t}{\mathcal{Q}^2}\right)\mathcal{E}\right] , \tag{2.32}$$

$$\mathcal{C}^{\mathcal{I},A}_{\rm LP}(\mathcal{F}) = \frac{x_{\rm B}}{2-x_{\rm B}+x_{\rm B}\frac{t}{\mathcal{Q}^2}}(F_1+F_2)\left[\widetilde{\mathcal{H}} + 2x_{\rm B}\frac{M^2}{\mathcal{Q}^2}\widetilde{\mathcal{H}} + \frac{x_{\rm B}}{2}\widetilde{\mathcal{E}}\right] . \tag{2.33}$$

Note that the ambiguity in the parameterization of hadronic helicity amplitudes (2.3)–(2.4) is also exhibited in the q_ρ-structure of Eqs. (2.26)–(2.27), which are kinematically suppressed by $t/\mathcal{Q}^2$. Such terms appear in the azimuthal angle independent part of the interference term at "twist-three" level, yielding the addenda

$$\Delta\mathcal{C}^{\mathcal{I}}_{\rm unp}(\mathcal{F}) = -\lim_{\mathcal{Q}\to\infty}\left[\frac{x_{\rm B}}{2-x_{\rm B}}\mathcal{C}^{\mathcal{I},V}_{\rm unp} + \mathcal{C}^{\mathcal{I},A}_{\rm unp}\right](\mathcal{F}) \, , \quad \Delta\mathcal{C}^{\mathcal{I}}_{\rm LP}(\mathcal{F}) = -\lim_{\mathcal{Q}\to\infty}\left[\mathcal{C}^{\mathcal{I},V}_{\rm LP} + \frac{x_{\rm B}}{2-x_{\rm B}}\mathcal{C}^{\mathcal{I},A}_{\rm LP}\right](\mathcal{F}) \, .$$

As a cross check, neglecting the power suppressed contributions yields the CFF and FF combinations that appear in $\mathcal{C}^{\mathcal{I}}(\mathcal{F})$ and $\Delta\mathcal{C}^{\mathcal{I}}(\mathcal{F})$ of Ref. [3].

Now we turn to the leptonic helicity amplitudes which contain the entire azimuthal angular dependence of the interference term. Their contraction with the hadronic amplitude with respect to the Lorentz indices introduces the Fourier harmonics in the definition (2.24) of the interference term yields

$$\mathcal{I} = \frac{\pm e^6}{x_{\rm B} y^3 t \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)}\left\{c_0^{\mathcal{I}} + \sum_{n=1}^{3}\left[c_n^{\mathcal{I}}\cos(n\phi) + s_n^{\mathcal{I}}\sin(n\phi)\right]\right\} , \tag{2.34}$$

with kinematically power-suppressed contributions exactly accounted for,

$$c_n^{\mathcal{I}} = C_{++}(n)\,\Re\mathrm{e}\,\mathcal{C}_{++}^{\mathcal{I}}\left(n|\mathcal{F}\right) + C_{0+}(n)\,\Re\mathrm{e}\,\mathcal{C}_{0+}^{\mathcal{I}}\left(n|\mathcal{F}_{\mathrm{eff}}\right) + C_{-+}(n)\,\Re\mathrm{e}\,\mathcal{C}_{-+}^{\mathcal{I}}\left(n|\mathcal{F}_T\right) ,$$
$$s_n^{\mathcal{I}} = S_{++}(n)\,\Im\mathrm{m}\,\mathcal{S}_{++}^{\mathcal{I}}\left(n|\mathcal{F}\right) + S_{0+}(n)\,\Im\mathrm{m}\,\mathcal{S}_{0+}^{\mathcal{I}}\left(n|\mathcal{F}_{\mathrm{eff}}\right) + S_{-+}(n)\,\Im\mathrm{m}\,\mathcal{S}_{-+}^{\mathcal{I}}\left(n|\mathcal{F}_T\right) . \tag{2.35}$$

The above coefficients are defined in terms of the photon helicity-conserving

$$\mathcal{C}_{++}^{\mathcal{I}}\left(n|\mathcal{F}\right) = \mathcal{C}^{\mathcal{I}}\left(\mathcal{F}\right) + \frac{C_{++}^{V}(n)}{C_{++}(n)}\mathcal{C}^{\mathcal{I},V}\left(\mathcal{F}\right) + \frac{C_{++}^{A}(n)}{C_{++}(n)}\mathcal{C}^{\mathcal{I},A}\left(\mathcal{F}\right)$$
$$\mathcal{S}_{++}^{\mathcal{I}}\left(n|\mathcal{F}\right) = \mathcal{C}^{\mathcal{I}}\left(\mathcal{F}\right) + \frac{S_{++}^{V}(n)}{S_{++}(n)}\mathcal{C}^{\mathcal{I},V}\left(\mathcal{F}\right) + \frac{S_{++}^{A}(n)}{S_{++}(n)}\mathcal{C}^{\mathcal{I},A}\left(\mathcal{F}\right) \tag{2.36}$$

and helicity-changing amplitudes

$$\mathcal{C}_{0+}^{\mathcal{I}}\left(n|\mathcal{F}_{\mathrm{eff}}\right) = \frac{\sqrt{2}}{2-x_{\mathrm{B}}}\,\frac{\widetilde{K}}{\mathcal{Q}}\left[\mathcal{C}^{\mathcal{I}}\left(\mathcal{F}_{\mathrm{eff}}\right) + \frac{C_{0+}^{V}(n)}{C_{0+}(n)}\mathcal{C}^{\mathcal{I},V}\left(\mathcal{F}_{\mathrm{eff}}\right) + \frac{C_{0+}^{A}(n)}{C_{0+}(n)}\mathcal{C}^{\mathcal{I},A}\left(\mathcal{F}_{\mathrm{eff}}\right)\right]$$
$$\mathcal{S}_{0+}^{\mathcal{I}}\left(n|\mathcal{F}_{\mathrm{eff}}\right) = \frac{\sqrt{2}}{2-x_{\mathrm{B}}}\,\frac{\widetilde{K}}{\mathcal{Q}}\left[\mathcal{C}^{\mathcal{I}}\left(\mathcal{F}_{\mathrm{eff}}\right) + \frac{S_{0+}^{V}(n)}{S_{0+}(n)}\mathcal{C}^{\mathcal{I},V}\left(\mathcal{F}_{\mathrm{eff}}\right) + \frac{S_{0+}^{A}(n)}{S_{0+}(n)}\mathcal{C}^{\mathcal{I},A}\left(\mathcal{F}_{\mathrm{eff}}\right)\right] , \tag{2.37}$$

respectively. For an unpolarized target the coefficients $C_{ab}(n)$ and $S_{ab}(n)$ were already known from the study of a (pseudo) scalar target [13]. The complete set of coefficients $C_{ab}^{\mathcal{I}}(n)$ and $S_{ab}^{\mathcal{I}}(n)$ is given in Appendix A.

3 Discussion and conclusions

Let us shortly summarize our framework. To separate leptonic and hadronic contributions, we defined helicity amplitudes in a specific reference frame that is commonly used to confront experimental measurements and theoretical predictions. Within this convention, the leptonic part was calculated exactly. As far as the hadronic part is concerned, a few comments are in order.

- To evaluate the hadronic part, we employed the parametrization (2.3)–(2.4). The $1/\mathcal{Q}^2$-suppressed terms in both the bilinear (2.22)–(2.23) and linear combinations (2.29)–(2.33) of CFFs mainly arise from the exact treatment of the hadronic states, including parameterization of the polarization vector.

- There are intrinsic twist-four uncertainties in the above definitions, induced by the parameterization of the light cone projection, i.e., $(n \cdot V)$ and $(n \cdot A)$ in terms of the four-vectors defining the process kinematics and by missing pieces in the DVCS tensor that are needed for the restoration of the electromagnetic current conservation, see discussion in Ref. [13].

- Assuming that there is a hierarchy of hadronic scales, associated with higher-twist operator matrix elements, we mainly kept power suppressed twist-two contributions while neglecting genuine dynamical twist-four effects, i.e., $\frac{t}{Q^2}$tw-2 $\gg \frac{1}{Q^2}$tw-4.

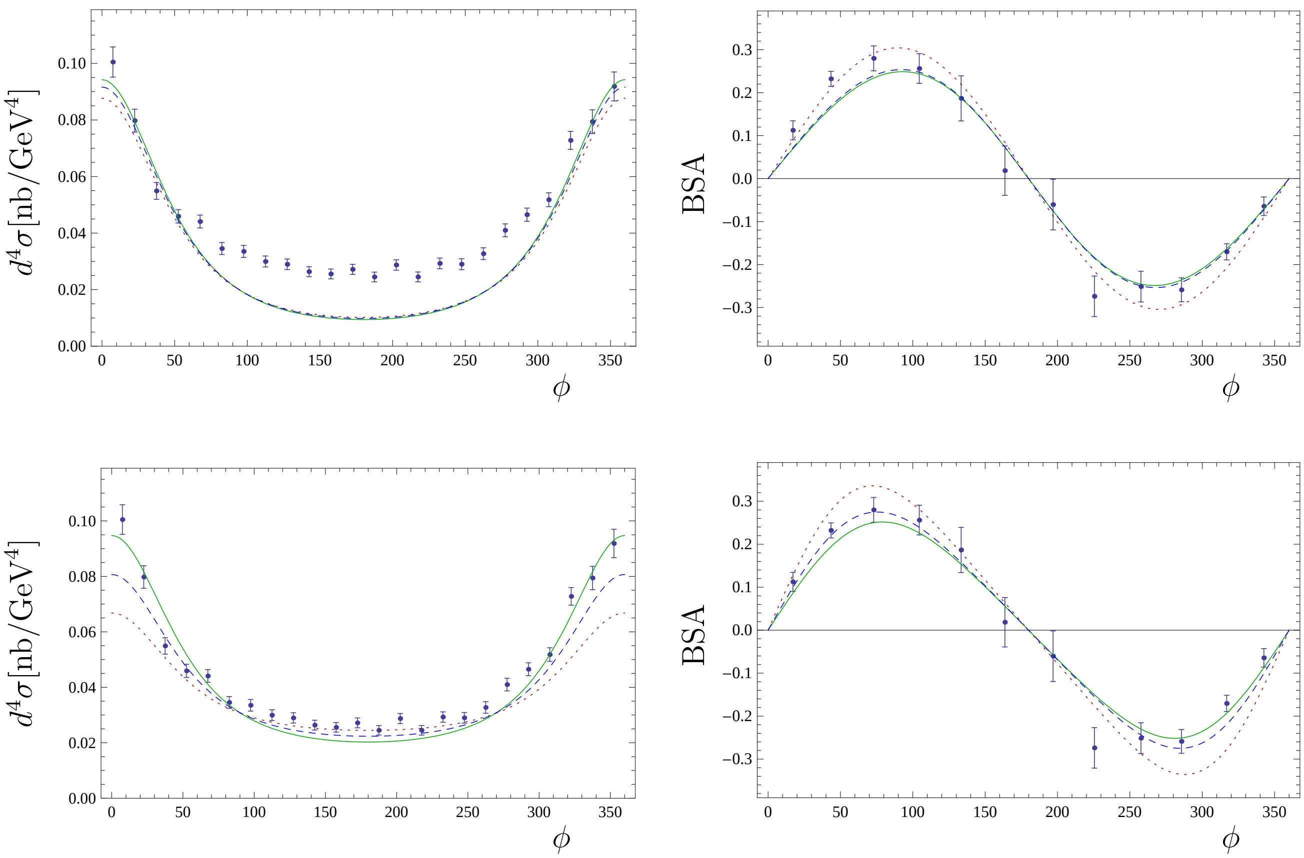

Figure 1: GPD predictions, resulting from two fits of Ref. [10] as described in the main body of the paper, for the BKM approximation (red dotted) to the exact (green solid) and "hot fix" (blue dashed) for the four-fold cross section (1.1) evaluated for lepton beam of positive helicity $\lambda = 1$ in the left panels and the beam-spin asymmetry (3.2) in the right panels.

Let us now explore the magnitude of power suppressed effects we have accounted for in this work compared to the approximate treatment of the older analysis in Ref. [3]. Before we present our predictions, let us introduce a simplified treatment of exact kinematical correction to the BKM formalism, which can be regarded as an improvement of the BKM analysis for an unpolarized target. This consists of replacing the BKM coefficients, entering the angular dependence of the cross section, with exact ones from the spin-zero case and ignoring at the same time all other induced harmonics for the same hadronic helicity amplitudes. Moreover, the BKM expressions for the hadronic $\mathcal{C}$-coefficients are taken. We will dub this scheme as "hot fix". It consists of substitutions

$$c_n^{\mathcal{I}}|_{\mathrm{BKM}} \to C_{++}(n)\,\Re\mathrm{e}\,\mathcal{C}^{\mathcal{I}}_{++}\,(n|\mathcal{F})\ , \qquad s_n^{\mathcal{I}}|_{\mathrm{BKM}} \to S_{++}(n)\,\Im\mathrm{m}\,\mathcal{S}^{\mathcal{I}}_{++}\,(n|\mathcal{F})\ , \tag{3.1}$$

with expression on the right-hand sides given in Appendix A. It provides a very accurate description of experimental observables in favorable situations. To demonstrate our point, we evaluated the four-fold cross section (1.1) and the beam-spin asymmetry defined as

$$\mathrm{BSA} = \frac{d^4\sigma(\lambda = +1) - d^4\sigma(\lambda = -1)}{d^4\sigma(\lambda = +1) - d^4\sigma(\lambda = -1)}\,, \tag{3.2}$$

where power suppressed corrections in the hadronic sector are still neglected. For illustration, we show in Figure 1 predictions for approximation schemes advocated in this paper for two dispersive approach fits [10] to the available experimental data, where twist-three and gluon transversity CFFs were projected out. (Thus, the observables, shown in the figure, are not the ones used in the fit, rather they contain an admixture of higher harmonics.) In the left panels, we display the unpolarized cross section measurement of Jefferson Laboratory's Hall A [11] for the kinematics

$$E = 5.75\ \mathrm{GeV}\,, \quad \mathcal{Q}^2 = 2.3\ \mathrm{GeV}^2\,, \quad t = -0.36\ \mathrm{GeV}^2\,, \quad x_{\mathrm{B}} = 0.36\,,$$

while on the right ones a beam spin measurement of the CLAS collaboration [12] is shown for

$$E = 5.77\ \mathrm{GeV}\,, \quad \mathcal{Q}^2 = 1.95\ \mathrm{GeV}^2\,, \quad t = -0.28\ \mathrm{GeV}^2\,, \quad x_{\mathrm{B}} = 0.25\,.$$

In the upper panels, the predictions are given for a fit that excluded the Hall A data [11] and assumed the dominance of the unpolarized GPD H in the DVCS amplitude. It is clearly demonstrated that (with present understanding of GPD magnitude) such a hypothesis is in conflict with the data. In the lower panels, the unpolarized cross section measurements of Hall A were included, with the fit performed to the ratio of $\cos(1 \cdot \phi)$ and $\cos(0 \cdot \phi)$ harmonics of the weighted cross section, see Eq. (103) of Ref. [3], rather than to the cross section itself. To describe the data, one required a large real part in the DVCS amplitude which was *effectively* obtained from an abnormally large contribution of the GPD $\widetilde{H}$. As we demonstrate in the figure, in the former case, the difference between the BKM (dotted) and exact (solid) results, while more accidentally of order of a few percent in the cross section (except for the end-point regions), reaches $20 - 25\%$ in the BSA asymmetry (right panel). However, the deviations of the "hot fix" (dashed) from the exact treatment is vanishingly small. Confronting the two fits, done with different dynamical assumptions about contributing GPDs, exhibits first of all larger effects of power suppressed correction in the differential cross section in the lower compared to upper panels, and second, demonstrates significant differences between the "hot fix" (dashed) and exact (solid) results. In other words, in a fitting procedure relying on cross section formulas with exact treatment of kinematical effects rather than the ones based on a hot fix, one anticipates that the magnitude of $\widetilde{H}$ becomes smaller.

The improvement on the BKM approximation scheme that we advocated in this paper demonstrate the necessity to incorporate power-suppressed corrections stemming from the kinematical effects in the leptonic part of the electroproduction scattering amplitudes. The results for gluon transversity and transversal polarized target will be presented somewhere else. The next set of problems of paramount importance is to develop a calculational scheme for analysis of dynamical higher twist correlation functions contributing to the DVCS amplitude, echoing formalism developed before for deep-inelastic scattering [14, 15] as well as target mass and momentum transfer corrections, extending earlier result beyond the leading twist order [16, 17].

A Fourier harmonics of the leptonic tensor

Let us present explicit expressions for the Fourier coefficients entering the leptonic part of the interference term (2.24).

A.1 Unpolarized target

The angular coefficients $C_{ab}^{\rm unp}(n)$ and $S_{ab}^{\rm unp}(n)$ for unpolarized target are given by the expressions $C_{ab}(n)$ and $\lambda S_{ab}(n)$ for scalar target [13], while the results for $C_{ab}^{{\rm unp},V}(n), C_{ab}^{{\rm unp},A}(n), S_{ab}^{{\rm unp},V}(n)$ and $S_{ab}^{{\rm unp},A}(n)$ are new. The third odd harmonics vanishes, i.e.,

$$S_{ab}^{\rm unp}(n=3) = S_{ab}^{{\rm unp},V}(n=3) = S_{ab}^{{\rm unp},A}(n=3) = 0\,,$$

and will be not listed.

Conserved photon-helicity coefficients:

$$\begin{aligned}
C_{++}^{\rm unp}(n=0) &= -\frac{4(2-y)\left(1+\sqrt{1+\epsilon^2}\right)}{(1+\epsilon^2)^2}\left\{\frac{\widetilde{K}^2}{\mathcal{Q}^2}\frac{(2-y)^2}{\sqrt{1+\epsilon^2}}\right. \\
&\quad \left. + \frac{t}{\mathcal{Q}^2}\left(1-y-\frac{\epsilon^2}{4}y^2\right)(2-x_{\rm B})\left(1+\frac{2x_{\rm B}\left(2-x_{\rm B}+\frac{\sqrt{1+\epsilon^2}-1}{2}+\frac{\epsilon^2}{2x_{\rm B}}\right)\frac{t}{\mathcal{Q}^2}+\epsilon^2}{(2-x_{\rm B})(1+\sqrt{1+\epsilon^2})}\right)\right\}, \\
C_{++}^{{\rm unp},V}(n=0) &= \frac{8(2-y)}{(1+\epsilon^2)^2}\frac{x_{\rm B}t}{\mathcal{Q}^2}\left\{\frac{(2-y)^2\widetilde{K}^2}{\sqrt{1+\epsilon^2}\mathcal{Q}^2}+\left(1-y-\frac{\epsilon^2}{4}y^2\right)\frac{1+\sqrt{1+\epsilon^2}}{2}\right. \\
&\quad \left.\times\left(1+\frac{t}{\mathcal{Q}^2}\right)\left(1+\frac{\sqrt{1+\epsilon^2}-1+2x_{\rm B}}{1+\sqrt{1+\epsilon^2}}\frac{t}{\mathcal{Q}^2}\right)\right\}, \\
C_{++}^{{\rm unp},A}(n=0) &= \frac{8(2-y)}{(1+\epsilon^2)^2}\frac{t}{\mathcal{Q}^2}\left\{\frac{(2-y)^2\widetilde{K}^2}{\sqrt{1+\epsilon^2}\mathcal{Q}^2}\frac{1+\sqrt{1+\epsilon^2}-2x_{\rm B}}{2}+\left(1-y-\frac{\epsilon^2}{4}y^2\right)\left[\frac{1+\sqrt{1+\epsilon^2}}{2}\right.\right. \\
&\quad \left.\left.\times\left(1+\sqrt{1+\epsilon^2}-x_{\rm B}+\left(\sqrt{1+\epsilon^2}-1+x_{\rm B}\frac{3+\sqrt{1+\epsilon^2}-2x_{\rm B}}{1+\sqrt{1+\epsilon^2}}\right)\frac{t}{\mathcal{Q}^2}\right)-\frac{2\widetilde{K}^2}{\mathcal{Q}^2}\right]\right\},
\end{aligned} \tag{A.1}$$

$$\begin{aligned}
C_{++}^{\rm unp}(n=1) &= \frac{-16K\left(1-y-\frac{\epsilon^2}{4}y^2\right)}{(1+\epsilon^2)^{5/2}}\left\{\left(1+(1-x_{\rm B})\frac{\sqrt{\epsilon^2+1}-1}{2x_{\rm B}}+\frac{\epsilon^2}{4x_{\rm B}}\right)\frac{x_{\rm B}t}{\mathcal{Q}^2}-\frac{3\epsilon^2}{4}\right\} \\
&\quad -4K\left(2-2y+y^2+\frac{\epsilon^2}{2}y^2\right)\frac{1+\sqrt{1+\epsilon^2}-\epsilon^2}{(1+\epsilon^2)^{5/2}}\left\{1-(1-3x_{\rm B})\frac{t}{\mathcal{Q}^2}\right. \\
&\quad \left. +\frac{1-\sqrt{1+\epsilon^2}+3\epsilon^2}{1+\sqrt{1+\epsilon^2}-\epsilon^2}\frac{x_{\rm B}t}{\mathcal{Q}^2}\right\}, \\
C_{++}^{{\rm unp},V}(n=1) &= \frac{16K}{(1+\epsilon^2)^{5/2}}\frac{x_{\rm B}t}{\mathcal{Q}^2}\left\{(2-y)^2\left(1-(1-2x_{\rm B})\frac{t}{\mathcal{Q}^2}\right)+\left(1-y-\frac{\epsilon^2}{4}y^2\right)\right. \\
&\quad \left.\times\frac{1+\sqrt{1+\epsilon^2}-2x_{\rm B}}{2}\frac{t'}{\mathcal{Q}^2}\right\}, \\
C_{++}^{{\rm unp},A}(n=1) &= \frac{-16K}{(1+\epsilon^2)^2}\frac{t}{\mathcal{Q}^2}\left\{\left(1-y-\frac{\epsilon^2}{4}y^2\right)\left(1-(1-2x_{\rm B})\frac{t}{\mathcal{Q}^2}+\frac{4x_{\rm B}(1-x_{\rm B})+\epsilon^2}{4\sqrt{1+\epsilon^2}}\frac{t'}{\mathcal{Q}^2}\right)\right. \\
&\quad \left. -(2-y)^2\left(1-\frac{x_{\rm B}}{2}+\frac{1+\sqrt{1+\epsilon^2}-2x_{\rm B}}{4}\left(1-\frac{t}{\mathcal{Q}^2}\right)+\frac{4x_{\rm B}(1-x_{\rm B})+\epsilon^2}{2\sqrt{1+\epsilon^2}}\frac{t'}{\mathcal{Q}^2}\right)\right\},
\end{aligned}$$

$$C^{\rm unp}_{++}(n=2) = \frac{8(2-y)\left(1-y-\frac{\epsilon^2}{4}y^2\right)}{(1+\epsilon^2)^2}\Bigg\{\frac{2\epsilon^2}{\sqrt{1+\epsilon^2}(1+\sqrt{1+\epsilon^2})}\frac{\widetilde{K}^2}{\mathcal{Q}^2} + \frac{x_{\rm B} t\, t'}{\mathcal{Q}^4}\left(1-x_{\rm B}-\frac{\sqrt{1+\epsilon^2}-1}{2}+\frac{\epsilon^2}{2x_{\rm B}}\right)\Bigg\},$$

$$C^{{\rm unp},V}_{++}(n=2) = \frac{8(2-y)\left(1-y-\frac{\epsilon^2}{4}y^2\right)}{(1+\epsilon^2)^2}\frac{x_{\rm B} t}{\mathcal{Q}^2}\Bigg\{\frac{4\widetilde{K}^2}{\sqrt{1+\epsilon^2}\mathcal{Q}^2} + \frac{1+\sqrt{1+\epsilon^2}-2x_{\rm B}}{2}\left(1+\frac{t}{\mathcal{Q}^2}\right)\frac{t'}{\mathcal{Q}^2}\Bigg\},$$

$$C^{{\rm unp},A}_{++}(n=2) = \frac{4(2-y)\left(1-y-\frac{\epsilon^2}{4}y^2\right)}{(1+\epsilon^2)^2}\frac{t}{\mathcal{Q}^2}\Bigg\{\frac{4(1-2x_{\rm B})\widetilde{K}^2}{\sqrt{1+\epsilon^2}\mathcal{Q}^2} - \left(3-\sqrt{1+\epsilon^2}-2x_{\rm B}+\frac{\epsilon^2}{x_{\rm B}}\right)\frac{x_{\rm B} t'}{\mathcal{Q}^2}\Bigg\},$$

$$C^{\rm unp}_{++}(n=3) = -8K\left(1-y-\frac{\epsilon^2}{4}y^2\right)\frac{\sqrt{1+\epsilon^2}-1}{(1+\epsilon^2)^{5/2}}\left\{(1-x_{\rm B})\frac{t}{\mathcal{Q}^2} + \frac{\sqrt{1+\epsilon^2}-1}{2}\left(1+\frac{t}{\mathcal{Q}^2}\right)\right\},$$

$$C^{{\rm unp},V}_{++}(n=3) = -\frac{8K\left(1-y-\frac{\epsilon^2}{4}y^2\right)}{(1+\epsilon^2)^{5/2}}\frac{x_{\rm B} t}{\mathcal{Q}^2}\left\{\sqrt{1+\epsilon^2}-1+\left(1+\sqrt{1+\epsilon^2}-2x_{\rm B}\right)\frac{t}{\mathcal{Q}^2}\right\},$$

$$C^{{\rm unp},A}_{++}(n=3) = \frac{16K\left(1-y-\frac{\epsilon^2}{4}y^2\right)}{(1+\epsilon^2)^{5/2}}\frac{t\, t'}{\mathcal{Q}^4}\left\{x_{\rm B}(1-x_{\rm B})+\frac{\epsilon^2}{4}\right\},$$

$$S^{\rm unp}_{++}(n=1) = \frac{8\lambda K(2-y)y}{1+\epsilon^2}\Bigg\{1+\frac{1-x_{\rm B}+\frac{\sqrt{1+\epsilon^2}-1}{2}}{1+\epsilon^2}\frac{t'}{\mathcal{Q}^2}\Bigg\},$$

$$S^{{\rm unp},V}_{++}(n=1) = -\frac{8\lambda K(2-y)y}{(1+\epsilon^2)^2}\frac{x_{\rm B} t}{\mathcal{Q}^2}\left\{\sqrt{1+\epsilon^2}-1+\left(1+\sqrt{1+\epsilon^2}-2x_{\rm B}\right)\frac{t}{\mathcal{Q}^2}\right\},$$

$$S^{{\rm unp},A}_{++}(n=1) = \frac{8\lambda K(2-y)y}{(1+\epsilon^2)}\frac{t}{\mathcal{Q}^2}\Bigg\{1-(1-2x_{\rm B})\frac{1+\sqrt{1+\epsilon^2}-2x_{\rm B}}{2\sqrt{1+\epsilon^2}}\frac{t'}{\mathcal{Q}^2}\Bigg\},$$

$$S^{\rm unp}_{++}(n=2) = -\frac{4\lambda\left(1-y-\frac{\epsilon^2}{4}y^2\right)y}{(1+\epsilon^2)^{3/2}}\left(1+\sqrt{1+\epsilon^2}-2x_{\rm B}\right)\frac{t'}{\mathcal{Q}^2}\Bigg\{\frac{\epsilon^2-x_{\rm B}(\sqrt{1+\epsilon^2}-1)}{1+\sqrt{\epsilon^2+1}-2x_{\rm B}} - \frac{2x_{\rm B}+\epsilon^2}{2\sqrt{1+\epsilon^2}}\frac{t'}{\mathcal{Q}^2}\Bigg\},$$

$$S^{{\rm unp},V}_{++}(n=2) = -\frac{4\lambda\left(1-y-\frac{\epsilon^2}{4}y^2\right)y}{(1+\epsilon^2)^2}\frac{x_{\rm B} t}{\mathcal{Q}^2} \times\left(1-(1-2x_{\rm B})\frac{t}{\mathcal{Q}^2}\right)\left\{\sqrt{1+\epsilon^2}-1+\left(1+\sqrt{1+\epsilon^2}-2x_{\rm B}\right)\frac{t}{\mathcal{Q}^2}\right\},$$

$$S^{{\rm unp},A}_{++}(n=2) = -\frac{8\lambda\left(1-y-\frac{\epsilon^2}{4}y^2\right)y}{(1+\epsilon^2)^2}\frac{t\, t'}{\mathcal{Q}^4} \times\left(1+\sqrt{1+\epsilon^2}-2x_{\rm B}\right)\left(1+\frac{4(1-x_{\rm B})x_{\rm B}+\epsilon^2}{4-2x_{\rm B}+3\epsilon^2}\frac{t}{\mathcal{Q}^2}\right).$$

Longitudinal-transverse coefficients:

$$C_{0+}^{\mathrm{unp}}(n=0) = \frac{12\sqrt{2}K(2-y)\sqrt{1-y-\frac{\epsilon^2}{4}y^2}}{(1+\epsilon^2)^{5/2}}\left\{\epsilon^2 + \frac{2-6x_{\mathrm{B}}-\epsilon^2}{3}\frac{t}{\mathcal{Q}^2}\right\}, \tag{A.2}$$

$$C_{0+}^{\mathrm{unp}}(n=1) = \frac{8\sqrt{2}\sqrt{1-y-\frac{\epsilon^2}{4}y^2}}{(1+\epsilon^2)^2}\left\{(2-y)^2\frac{t'}{\mathcal{Q}^2}\left(1-x_{\mathrm{B}}+\frac{(1-x_{\mathrm{B}})x_{\mathrm{B}}+\frac{\epsilon^2}{4}}{\sqrt{1+\epsilon^2}}\frac{t'}{\mathcal{Q}^2}\right)\right.$$
$$\left. + \frac{1-y-\frac{\epsilon^2}{4}y^2}{\sqrt{1+\epsilon^2}}\left(1-(1-2x_{\mathrm{B}})\frac{t}{\mathcal{Q}^2}\right)\left(\epsilon^2 - 2\left(1+\frac{\epsilon^2}{2x_{\mathrm{B}}}\right)\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}\right)\right\},$$

$$C_{0+}^{\mathrm{unp}}(n=2) = -\frac{8\sqrt{2}K(2-y)\sqrt{1-y-\frac{\epsilon^2}{4}y^2}}{(1+\epsilon^2)^{5/2}}\left(1+\frac{\epsilon^2}{2}\right)\left\{1+\frac{1+\frac{\epsilon^2}{2x_{\mathrm{B}}}}{1+\frac{\epsilon^2}{2}}\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}\right\},$$

$$S_{0+}^{\mathrm{unp}}(n=1) = \frac{8\lambda\sqrt{2}(2-y)y\sqrt{1-y-\frac{\epsilon^2}{4}y^2}}{(1+\epsilon^2)^2}\frac{\widetilde{K}^2}{\mathcal{Q}^2},$$

$$S_{0+}^{\mathrm{unp}}(n=2) = \frac{8\lambda\sqrt{2}Ky\sqrt{1-y-\frac{\epsilon^2}{4}y^2}}{(1+\epsilon^2)^2}\left(1+\frac{\epsilon^2}{2}\right)\left\{1+\frac{1+\frac{\epsilon^2}{2x_{\mathrm{B}}}}{1+\frac{\epsilon^2}{2}}\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}\right\},$$

$$C_{0+}^{\mathrm{unp},V}(n=0) = \frac{24\sqrt{2}K(2-y)\sqrt{1-y-\frac{\epsilon^2}{4}y^2}}{(1+\epsilon^2)^{5/2}}\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}\left\{1-(1-2x_{\mathrm{B}})\frac{t}{\mathcal{Q}^2}\right\},$$

$$C_{0+}^{\mathrm{unp},V}(n=1) = \frac{16\sqrt{2}\sqrt{1-y-\frac{\epsilon^2}{4}y^2}}{(1+\epsilon^2)^{5/2}}\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}\left\{\frac{\widetilde{K}^2(2-y)^2}{\mathcal{Q}^2}+\left(1-(1-2x_{\mathrm{B}})\frac{t}{\mathcal{Q}^2}\right)^2\left(1-y-\frac{\epsilon^2}{4}y^2\right)\right\},$$

$$C_{0+}^{\mathrm{unp},V}(n=2) = \frac{8\sqrt{2}K(2-y)\sqrt{1-y-\frac{\epsilon^2}{4}y^2}}{(1+\epsilon^2)^{5/2}}\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}\left(1-(1-2x_{\mathrm{B}})\frac{t}{\mathcal{Q}^2}\right),$$

$$S_{0+}^{\mathrm{unp},V}(n=1) = \frac{4\sqrt{2}\lambda y(2-y)\sqrt{1-y-\frac{\epsilon^2}{4}y^2}}{(1+\epsilon^2)^2}\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}\left\{4(1-2x_{\mathrm{B}})\frac{t}{\mathcal{Q}^2}\left(1+\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}\right)+\epsilon^2\left(1+\frac{t}{\mathcal{Q}^2}\right)^2\right\},$$

$$S_{0+}^{\mathrm{unp},V}(n=2) = -\frac{8\sqrt{2}\lambda Ky\sqrt{1-y-\frac{\epsilon^2}{4}y^2}}{(1+\epsilon^2)^2}\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}\left\{1-(1-2x_{\mathrm{B}})\frac{t}{\mathcal{Q}^2}\right\},$$

$$C_{0+}^{\mathrm{unp},A}(n=0) = \frac{4\sqrt{2}K(2-y)\sqrt{1-y-\frac{\epsilon^2}{4}y^2}}{(1+\epsilon^2)^{5/2}}\frac{t}{\mathcal{Q}^2}(8-6x_{\mathrm{B}}+5\epsilon^2)\left\{1-\frac{t}{\mathcal{Q}^2}\frac{2-12x_{\mathrm{B}}(1-x_{\mathrm{B}})-\epsilon^2}{8-6x_{\mathrm{B}}+5\epsilon^2}\right\},$$

$$C_{0+}^{\mathrm{unp},A}(n=1) = \frac{8\sqrt{2}\sqrt{1-y-\frac{\epsilon^2}{4}y^2}}{(1+\epsilon^2)^{5/2}}\frac{t}{\mathcal{Q}^2}\left\{\frac{\widetilde{K}^2}{\mathcal{Q}^2}(1-2x_{\mathrm{B}})(2-y)^2\right.$$
$$\left. + \left(1-(1-2x_{\mathrm{B}})\frac{t}{\mathcal{Q}^2}\right)\left(1-y-\frac{y^2\epsilon^2}{4}\right)\left(4-2x_{\mathrm{B}}+3\epsilon^2+\frac{t}{\mathcal{Q}^2}(4x_{\mathrm{B}}(1-x_{\mathrm{B}})+\epsilon^2)\right)\right\},$$

$$C_{0+}^{\mathrm{unp},A}(n=2) = \frac{8\sqrt{2}K(2-y)\sqrt{1-y-\frac{\epsilon^2}{4}y^2}}{(1+\epsilon^2)^2}\frac{t}{\mathcal{Q}^2}\left\{1-x_{\mathrm{B}}+\frac{t'}{2\mathcal{Q}^2}\frac{4x_{\mathrm{B}}(1-x_{\mathrm{B}})+\epsilon^2}{\sqrt{1+\epsilon^2}}\right\},$$

$$S_{0+}^{\mathrm{unp},A}(n=1) = -\frac{8\sqrt{2}\lambda y(2-y)(1-2x_{\mathrm{B}})\sqrt{1-y-\frac{\epsilon^2}{4}y^2}}{(1+\epsilon^2)^2}\frac{tK^2}{\mathcal{Q}^4},$$

$$S_{0+}^{\mathrm{unp},A}(n=2) = -\frac{2\sqrt{2}\lambda K y\sqrt{1-y-\frac{\epsilon^2}{4}y^2}}{(1+\epsilon^2)^2}\frac{t}{\mathcal{Q}^2}\left(4-4x_{\mathrm{B}}+2\epsilon^2+4\frac{t}{\mathcal{Q}^2}(4x_{\mathrm{B}}(1-x_{\mathrm{B}})+\epsilon^2)\right)\,, \tag{A.3}$$

Transverse-transverse helicity-flip coefficients:

$$\begin{aligned}
C_{-+}^{\mathrm{unp}}(n=0) &= \frac{8(2-y)}{(1+\epsilon^2)^{3/2}}\Bigg\{(2-y)^2\frac{\sqrt{1+\epsilon^2}-1}{2(1+\epsilon^2)}\frac{\widetilde{K}^2}{\mathcal{Q}^2} \\
&\quad +\frac{1-y-\frac{\epsilon^2}{4}y^2}{\sqrt{1+\epsilon^2}}\left(1-x_{\mathrm{B}}-\frac{\sqrt{1+\epsilon^2}-1}{2}+\frac{\epsilon^2}{2x_{\mathrm{B}}}\right)\frac{x_{\mathrm{B}}t\,t'}{\mathcal{Q}^4}\Bigg\}, \\
C_{-+}^{\mathrm{unp}}(n=1) &= \frac{8K}{(1+\epsilon^2)^{3/2}}\Bigg\{(2-y)^2\frac{2-\sqrt{1+\epsilon^2}}{1+\epsilon^2}\left(\frac{\sqrt{1+\epsilon^2}-1+\epsilon^2}{2\left(2-\sqrt{1+\epsilon^2}\right)}\left(1-\frac{t}{\mathcal{Q}^2}\right)-\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}\right) \\
&\quad + 2\frac{1-y-\frac{\epsilon^2}{4}y^2}{\sqrt{1+\epsilon^2}}\left(\frac{1-\sqrt{1+\epsilon^2}+\frac{\epsilon^2}{2}}{2\sqrt{1+\epsilon^2}}+\frac{t}{\mathcal{Q}^2}\left(1-\frac{3x_{\mathrm{B}}}{2}+\frac{x_{\mathrm{B}}+\frac{\epsilon^2}{2}}{2\sqrt{1+\epsilon^2}}\right)\right)\Bigg\}, \\
C_{-+}^{\mathrm{unp}}(n=2) &= 4(2-y)\left(1-y-\frac{\epsilon^2}{4}y^2\right)\frac{1+\sqrt{1+\epsilon^2}}{(1+\epsilon^2)^{5/2}}\Bigg\{(2-3x_{\mathrm{B}})\frac{t}{\mathcal{Q}^2} \\
&\quad + \left(1-2x_{\mathrm{B}}+\frac{2(1-x_{\mathrm{B}})}{1+\sqrt{1+\epsilon^2}}\right)\frac{x_{\mathrm{B}}t^2}{\mathcal{Q}^4}+\left(1+\frac{\sqrt{1+\epsilon^2}+x_{\mathrm{B}}+(1-x_{\mathrm{B}})\frac{t}{\mathcal{Q}^2}}{1+\sqrt{1+\epsilon^2}}\frac{t}{\mathcal{Q}^2}\right)\epsilon^2\Bigg\}, \\
C_{-+}^{\mathrm{unp}}(n=3) &= -8K\left(1-y-\frac{\epsilon^2}{4}y^2\right)\frac{1+\sqrt{1+\epsilon^2}+\frac{\epsilon^2}{2}}{(1+\epsilon^2)^{5/2}}\left\{1+\frac{1+\sqrt{1+\epsilon^2}+\frac{\epsilon^2}{2x_{\mathrm{B}}}}{1+\sqrt{1+\epsilon^2}+\frac{\epsilon^2}{2}}\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}\right\}, \\
S_{-+}^{\mathrm{unp}}(n=1) &= \frac{4\lambda K(2-y)y}{(1+\epsilon^2)^2}\left\{1-\sqrt{1+\epsilon^2}+2\epsilon^2-2\left(1+\frac{\sqrt{1+\epsilon^2}-1}{2x_{\mathrm{B}}}\right)\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}\right\}, \\
S_{-+}^{\mathrm{unp}}(n=2) &= 2\lambda y\left(1-y-\frac{\epsilon^2}{4}y^2\right)\frac{1+\sqrt{1+\epsilon^2}}{(1+\epsilon^2)^2}\left(\epsilon^2-2\left(1+\frac{\epsilon^2}{2x_{\mathrm{B}}}\right)\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}\right) \\
&\quad \times\left\{1+\frac{\sqrt{1+\epsilon^2}-1+2x_{\mathrm{B}}}{1+\sqrt{1+\epsilon^2}}\frac{t}{\mathcal{Q}^2}\right\}, \\
C_{-+}^{\mathrm{unp},V}(n=0) &= \frac{4(2-y)}{(1+\epsilon^2)^{5/2}}\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}\Bigg\{\frac{2\widetilde{K}^2}{\mathcal{Q}^2}\left(2-2y+y^2+\frac{y^2\epsilon^2}{2}\right) \\
&\quad - \left(1-(1-2x_{\mathrm{B}})\frac{t}{\mathcal{Q}^2}\right)\left(1-y-\frac{y^2\epsilon^2}{4}\right)\left(\sqrt{1+\epsilon^2}-1+\left(\sqrt{1+\epsilon^2}+1-2x_{\mathrm{B}}\right)\frac{t}{\mathcal{Q}^2}\right)\Bigg\}. \\
C_{-+}^{\mathrm{unp},V}(n=1) &= \frac{8K}{(1+\epsilon^2)^{5/2}}\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}\Bigg\{2\left(1-(1-2x_{\mathrm{B}})\frac{t}{\mathcal{Q}^2}\right)\left(2-2y+y^2+\frac{y^2\epsilon^2}{2}\right) \\
&\quad + \left(1-y-\frac{y^2\epsilon^2}{4}\right)\left(3-\sqrt{1+\epsilon^2}-\left(3(1-2x_{\mathrm{B}})+\sqrt{1+\epsilon^2}\right)\frac{t}{\mathcal{Q}^2}\right)\Bigg\}, \\
C_{-+}^{\mathrm{unp},V}(n=2) &= \frac{4(2-y)\left(1-y-\frac{y^2\epsilon^2}{4}\right)}{(1+\epsilon^2)^{5/2}}\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}\Bigg\{4\frac{\widetilde{K}^2}{\mathcal{Q}^2} \\
&\quad + 1+\sqrt{1+\epsilon^2}+\frac{t}{\mathcal{Q}^2}\left((1-2x_{\mathrm{B}})\left(1-2x_{\mathrm{B}}-\sqrt{1+\epsilon^2}\right)\frac{t}{\mathcal{Q}^2}-2+4x_{\mathrm{B}}+2x_{\mathrm{B}}\sqrt{1+\epsilon^2}\right)\Bigg\},
\end{aligned} \tag{A.4}$$

$$
\begin{aligned}
C^{\mathrm{unp},V}_{-+}(n=3) &= \frac{8K\left(1-y-\frac{y^2\epsilon^2}{4}\right)}{(1+\epsilon^2)^{5/2}}\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}\left(1+\sqrt{1+\epsilon^2}\right)\left\{1-\frac{t}{\mathcal{Q}^2}\frac{1-2x_{\mathrm{B}}-\sqrt{1+\epsilon^2}}{1+\sqrt{1+\epsilon^2}}\right\},\\
S^{\mathrm{unp},V}_{-+}(n=1) &= \frac{8\lambda K y(2-y)}{(1+\epsilon^2)^2}\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}\left(1+\sqrt{1+\epsilon^2}\right)\left\{1-\frac{t}{\mathcal{Q}^2}\frac{1-2x_{\mathrm{B}}-\sqrt{1+\epsilon^2}}{1+\sqrt{1+\epsilon^2}}\right\},\\
S^{\mathrm{unp},V}_{-+}(n=2) &= \frac{4\lambda y\left(1-y-\frac{y^2\epsilon^2}{4}\right)}{(1+\epsilon^2)^2}\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}\\
&\quad\times\left(1+\sqrt{1+\epsilon^2}\right)\left(1-(1-2x_{\mathrm{B}})\frac{t}{\mathcal{Q}^2}\right)\left\{1-\frac{t}{\mathcal{Q}^2}\frac{1-2x_{\mathrm{B}}-\sqrt{1+\epsilon^2}}{1+\sqrt{1+\epsilon^2}}\right\},\\
C^{\mathrm{unp},A}_{-+}(n=0) &= \frac{4(2-y)}{(1+\epsilon^2)^2}\frac{t}{\mathcal{Q}^2}\left\{\frac{t'}{\mathcal{Q}^2}\left(1-y-\frac{y^2\epsilon^2}{4}\right)\left(2x_{\mathrm{B}}^2-\epsilon^2-3x_{\mathrm{B}}+x_{\mathrm{B}}\sqrt{1+\epsilon^2}\right)\right.\\
&\quad\left.+\frac{\widetilde{K}^2}{\mathcal{Q}^2\sqrt{1+\epsilon^2}}\left(4-2x_{\mathrm{B}}(2-y)^2-4y+y^2-(1+\epsilon^2)^{3/2}\right)\right\},\\
C^{\mathrm{unp},A}_{-+}(n=1) &= \frac{4K}{(1+\epsilon^2)^{5/2}}\frac{t}{\mathcal{Q}^2}\left\{\left(2-2y+y^2+\frac{y^2\epsilon^2}{2}\right)\right.\\
&\quad\times\left(5-4x_{\mathrm{B}}+3\epsilon^2-\sqrt{1+\epsilon^2}-\frac{t}{\mathcal{Q}^2}\left(1-\epsilon^2-\sqrt{1+\epsilon^2}-2x_{\mathrm{B}}(4-4x_{\mathrm{B}}-\sqrt{1+\epsilon^2})\right)\right)\\
&\quad+\left(1-y-\frac{y^2\epsilon^2}{4}\right)\\
&\quad\left.\times\left(8+5\epsilon^2-6x_{\mathrm{B}}+2x_{\mathrm{B}}\sqrt{1+\epsilon^2}-\frac{t}{\mathcal{Q}^2}\left(2-\epsilon^2+2\sqrt{1+\epsilon^2}-4x_{\mathrm{B}}(3-3x_{\mathrm{B}}+\sqrt{1+\epsilon^2})\right)\right)\right\},\\
C^{\mathrm{unp},A}_{-+}(n=2) &= \frac{16(2-y)\left(1-y-\frac{y^2\epsilon^2}{4}\right)}{(1+\epsilon^2)^{3/2}}\frac{t}{\mathcal{Q}^2}\left\{\frac{\widetilde{K}^2}{\mathcal{Q}^2}\frac{1-2x_{\mathrm{B}}}{1+\epsilon^2}\right.\\
&\quad\left.-\frac{1-x_{\mathrm{B}}}{4x_{\mathrm{B}}(1-x_{\mathrm{B}})+\epsilon^2}\left(2x_{\mathrm{B}}^2-\epsilon^2-3x_{\mathrm{B}}-x_{\mathrm{B}}\sqrt{1+\epsilon^2}\right)-\frac{t'}{\mathcal{Q}^2}\frac{2x_{\mathrm{B}}^2-\epsilon^2-3x_{\mathrm{B}}-x_{\mathrm{B}}\sqrt{1+\epsilon^2}}{4\sqrt{1+\epsilon^2}}\right\},\\
C^{\mathrm{unp},A}_{-+}(n=3) &= \frac{16K\left(1-y-\frac{y^2\epsilon^2}{4}\right)}{(1+\epsilon^2)^2}\frac{t}{\mathcal{Q}^2}\left\{1-x_{\mathrm{B}}+\frac{t'}{\mathcal{Q}^2}\frac{4x_{\mathrm{B}}(1-x_{\mathrm{B}})+\epsilon^2}{4\sqrt{1+\epsilon^2}}\right\},\\
S^{\mathrm{unp},A}_{-+}(n=1) &= \frac{4\lambda K y(2-y)}{(1+\epsilon^2)^2}\frac{t}{\mathcal{Q}^2}\left\{3+2\epsilon^2\right.\\
&\quad\left.+\sqrt{1+\epsilon^2}-2x_{\mathrm{B}}-2x_{\mathrm{B}}\sqrt{1+\epsilon^2}-\frac{t}{\mathcal{Q}^2}(1-2x_{\mathrm{B}})\left(1-2x_{\mathrm{B}}-\sqrt{1+\epsilon^2}\right)\right\},\\
S^{\mathrm{unp},A}_{-+}(n=2) &= \frac{2\lambda\left(1-y-\frac{y^2\epsilon^2}{4}\right)}{(1+\epsilon^2)^2}\frac{t}{\mathcal{Q}^2}\left(4-2x_{\mathrm{B}}+3\epsilon^2+\frac{t}{\mathcal{Q}^2}\left(4x_{\mathrm{B}}(1-x_{\mathrm{B}})+\epsilon^2\right)\right)\\
&\quad\times\left(1+\sqrt{1+\epsilon^2}-\frac{t}{\mathcal{Q}^2}\left(1-2x_{\mathrm{B}}-\sqrt{1+\epsilon^2}\right)\right).
\end{aligned}
$$

A.2 Longitudinally polarized target

In the helicity dependent contribution of a longitudinal polarized target the third even harmonic vanishes, i.e.,

$$C^{\mathrm{LP}}_{ab}(n=3) = C^{\mathrm{LP},V}_{ab}(n=3) = C^{\mathrm{LP},A}_{ab}(n=3) = 0,$$

and will be not listed.

Conserved photon-helicity coefficients:

$$C^{\mathrm{LP}}_{++}(n=0) = -\frac{4\lambda\Lambda y\left(1+\sqrt{1+\epsilon^2}\right)}{(1+\epsilon^2)^{5/2}}\Bigg\{(2-y)^2\frac{\widetilde{K}^2}{\mathcal{Q}^2} + \left(1-y+\frac{\epsilon^2}{4}y^2\right) \times\left(\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2} - \left(1-\frac{t}{\mathcal{Q}^2}\right)\frac{\epsilon^2}{2}\right)\left(1+\frac{\sqrt{1+\epsilon^2}-1+2x_{\mathrm{B}}}{1+\sqrt{1+\epsilon^2}}\frac{t}{\mathcal{Q}^2}\right)\Bigg\}, \tag{A.5}$$

$$C^{\mathrm{LP,V}}_{++}(n=0) = \frac{4\lambda\Lambda y\left(1+\sqrt{1+\epsilon^2}\right)}{(1+\epsilon^2)^{5/2}}\frac{t}{\mathcal{Q}^2}\Bigg\{(2-y)^2\,\frac{1+\sqrt{1+\epsilon^2}-2x_{\mathrm{B}}}{1+\sqrt{1+\epsilon^2}}\,\frac{\widetilde{K}^2}{\mathcal{Q}^2} + \left(1-y-\frac{\epsilon^2}{4}y^2\right) \times\left(2-x_{\mathrm{B}}+\frac{3\epsilon^2}{2}\right)\left(1+\frac{4(1-x_{\mathrm{B}})x_{\mathrm{B}}+\epsilon^2}{4-2x_{\mathrm{B}}+3\epsilon^2}\frac{t}{\mathcal{Q}^2}\right)\left(1+\frac{\sqrt{1+\epsilon^2}-1+2x_{\mathrm{B}}}{1+\sqrt{1+\epsilon^2}}\frac{t}{\mathcal{Q}^2}\right)\Bigg\},$$

$$C^{\mathrm{LP,A}}_{++}(n=0) = \frac{4\lambda\Lambda y}{(1+\epsilon^2)^{5/2}}\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}\Bigg\{2(2-y)^2\frac{\widetilde{K}^2}{\mathcal{Q}^2} + \left(1-y-\frac{\epsilon^2}{4}y^2\right)(1+\sqrt{1+\epsilon^2})\left(1-(1-2x_{\mathrm{B}})\frac{t}{\mathcal{Q}^2}\right) \times\left(1+\frac{\sqrt{1+\epsilon^2}-1+2x_{\mathrm{B}}}{1+\sqrt{1+\epsilon^2}}\frac{t}{\mathcal{Q}^2}\right)\Bigg\},$$

$$C^{\mathrm{LP}}_{++}(n=1) = -\frac{4\lambda\Lambda K y(2-y)}{(1+\epsilon^2)^{5/2}}(1+\sqrt{1+\epsilon^2}-\epsilon^2)\left\{1-\left(1-2x_{\mathrm{B}}\frac{2+\sqrt{1+\epsilon^2}}{1+\sqrt{1+\epsilon^2}-\epsilon^2}\right)\frac{t}{\mathcal{Q}^2}\right\},$$

$$C^{\mathrm{LP,V}}_{++}(n=1) = \frac{8\lambda\Lambda K(2-y)y}{(1+\epsilon^2)^2}\left(\sqrt{1+\epsilon^2}+2(1-x_{\mathrm{B}})\right)\frac{t}{\mathcal{Q}^2}\left\{1-\frac{1+\frac{1-\epsilon^2}{\sqrt{1+\epsilon^2}}-2x_{\mathrm{B}}\left(1+\frac{4(1-x_{\mathrm{B}})}{\sqrt{1+\epsilon^2}}\right)}{2\left(\sqrt{1+\epsilon^2}+2(1-x_{\mathrm{B}})\right)}\frac{t'}{\mathcal{Q}^2}\right\},$$

$$C^{\mathrm{LP,A}}_{++}(n=1) = \frac{16\lambda\Lambda K(2-y)y}{(1+\epsilon^2)^{5/2}}\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}\left(1-(1-2x_{\mathrm{B}})\frac{t}{\mathcal{Q}^2}\right),$$

$$C^{\mathrm{LP}}_{++}(n=2) = -\frac{4\lambda\Lambda y\left(1-y-\frac{\epsilon^2}{4}y^2\right)}{(1+\epsilon^2)^{5/2}}\left(\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}-\left(1-\frac{t}{\mathcal{Q}^2}\right)\frac{\epsilon^2}{2}\right) \times\left\{1-\sqrt{1+\epsilon^2}-\left(1+\sqrt{1+\epsilon^2}-2x_{\mathrm{B}}\right)\frac{t}{\mathcal{Q}^2}\right\},$$

$$C^{\mathrm{LP,V}}_{++}(n=2) = -\frac{2\lambda\Lambda y\left(1-y-\frac{\epsilon^2}{4}y^2\right)}{(1+\epsilon^2)^{5/2}}\left(4-2x_{\mathrm{B}}+3\epsilon^2\right)\frac{t}{\mathcal{Q}^2}\left(1+\frac{4(1-x_{\mathrm{B}})x_{\mathrm{B}}+\epsilon^2}{4-2x_{\mathrm{B}}+3\epsilon^2}\frac{t}{\mathcal{Q}^2}\right) \times\left\{\sqrt{1+\epsilon^2}-1+\left(1+\sqrt{1+\epsilon^2}-2x_{\mathrm{B}}\right)\frac{t}{\mathcal{Q}^2}\right\},$$

$$C_{++}^{\mathrm{LP,A}}(n=2) = \frac{4\lambda\Lambda y\left(1-y-\frac{\epsilon^2}{4}y^2\right)}{\left(1+\epsilon^2\right)^{5/2}}\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}\left(1-(1-2x_{\mathrm{B}})\frac{t}{\mathcal{Q}^2}\right)$$
$$\times\left\{1-\sqrt{1+\epsilon^2}-\left(1+\sqrt{1+\epsilon^2}-2x_{\mathrm{B}}\right)\frac{t}{\mathcal{Q}^2}\right\},$$

$$S_{++}^{\mathrm{LP}}(n=1) = \frac{4\Lambda K\left(2-2y+y^2+\frac{\epsilon^2}{2}y^2\right)}{\left(1+\epsilon^2\right)^3}(1+\sqrt{1+\epsilon^2})\left\{2\sqrt{1+\epsilon^2}-1+\frac{1+\sqrt{1+\epsilon^2}-2x_{\mathrm{B}}}{1+\sqrt{1+\epsilon^2}}\frac{t}{\mathcal{Q}^2}\right\}$$
$$+\frac{8K\Lambda\left(1-y-\frac{\epsilon^2}{4}y^2\right)}{\left(1+\epsilon^2\right)^3}\left\{\frac{3\epsilon^2}{2}+\left(1-\sqrt{1+\epsilon^2}-\frac{\epsilon^2}{2}-x_{\mathrm{B}}\left(3-\sqrt{1+\epsilon^2}\right)\right)\frac{t}{\mathcal{Q}^2}\right\},$$
$$S_{++}^{\mathrm{LP,V}}(n=1) = \frac{8\Lambda K\left(2-2y+y^2+\frac{\epsilon^2}{2}y^2\right)}{\left(1+\epsilon^2\right)^2}\frac{t}{\mathcal{Q}^2}\left\{1-\frac{(1-2x_{\mathrm{B}})\left(1+\sqrt{1+\epsilon^2}-2x_{\mathrm{B}}\right)}{2(1+\epsilon^2)}\frac{t'}{\mathcal{Q}^2}\right\}$$
$$+\frac{32\Lambda K\left(1-y-\frac{\epsilon^2}{4}y^2\right)}{\left(1+\epsilon^2\right)^3}\left(1-\frac{3+\sqrt{1+\epsilon^2}}{4}x_{\mathrm{B}}+\frac{5\epsilon^2}{8}\right)\frac{t}{\mathcal{Q}^2}$$
$$\times\left\{1-\frac{1-\sqrt{1+\epsilon^2}+\frac{\epsilon^2}{2}-2x_{\mathrm{B}}\left(3(1-x_{\mathrm{B}})-\sqrt{1+\epsilon^2}\right)}{4-x_{\mathrm{B}}\left(\sqrt{1+\epsilon^2}+3\right)+\frac{5\epsilon^2}{2}}\frac{t}{\mathcal{Q}^2}\right\},$$
$$S_{++}^{\mathrm{LP,A}}(n=1) = -\frac{8\Lambda K\left(2-2y+y^2+\frac{\epsilon^2}{2}y^2\right)}{\left(1+\epsilon^2\right)^3}\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}\left\{\sqrt{1+\epsilon^2}-1+(1+\sqrt{1+\epsilon^2}-2x_{\mathrm{B}})\frac{t}{\mathcal{Q}^2}\right\}$$
$$+\frac{8\Lambda K\left(1-y-\frac{\epsilon^2}{4}y^2\right)}{\left(1+\epsilon^2\right)^3}(3+\sqrt{1+\epsilon^2})\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}\left\{1-\frac{3-\sqrt{1+\epsilon^2}-6x_{\mathrm{B}}}{3+\sqrt{1+\epsilon^2}}\frac{t}{\mathcal{Q}^2}\right\},$$

$$S_{++}^{\mathrm{LP}}(n=2) = -\frac{4\Lambda(2-y)\left(1-y-\frac{\epsilon^2}{4}y^2\right)}{\left(1+\epsilon^2\right)^{5/2}}$$
$$\times\left\{\frac{4\widetilde{K}^2}{\sqrt{1+\epsilon^2}\mathcal{Q}^2}(1+\sqrt{1+\epsilon^2}-2x_{\mathrm{B}})\left(1+\sqrt{1+\epsilon^2}+\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}\right)\frac{t'}{\mathcal{Q}^2}\right\},$$
$$S_{++}^{\mathrm{LP,V}}(n=2) = \frac{4\Lambda(2-y)\left(1-y-\frac{\epsilon^2}{4}y^2\right)}{\left(1+\epsilon^2\right)^{5/2}}\frac{t}{\mathcal{Q}^2}\left\{\frac{4(1-2x_{\mathrm{B}})\widetilde{K}^2}{\sqrt{1+\epsilon^2}\mathcal{Q}^2}-\left(3-\sqrt{1+\epsilon^2}-2x_{\mathrm{B}}+\frac{\epsilon^2}{x_{\mathrm{B}}}\right)\frac{x_{\mathrm{B}}t'}{\mathcal{Q}^2}\right\},$$
$$S_{++}^{\mathrm{LP,A}}(n=2) = \frac{4\Lambda(2-y)\left(1-y-\frac{\epsilon^2}{4}y^2\right)}{\left(1+\epsilon^2\right)^{5/2}}\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}\left\{\frac{4\widetilde{K}^2}{\mathcal{Q}^2}-\left(1+\sqrt{1+\epsilon^2}-2x_{\mathrm{B}}\right)\left(1-\frac{(1-2x_{\mathrm{B}})t}{\mathcal{Q}^2}\right)\frac{t'}{\mathcal{Q}^2}\right\},$$

$$S_{++}^{\mathrm{LP}}(n=3) = -\frac{4\Lambda K\left(1-y-\frac{\epsilon^2}{4}y^2\right)}{\left(1+\epsilon^2\right)^3}\frac{1+\sqrt{1+\epsilon^2}-2x_{\mathrm{B}}}{1+\sqrt{1+\epsilon^2}}\frac{\epsilon^2t'}{\mathcal{Q}^2},$$
$$S_{++}^{\mathrm{LP,V}}(n=3) = \frac{4\Lambda K\left(1-y-\frac{\epsilon^2}{4}y^2\right)}{\left(1+\epsilon^2\right)^3}\left(4(1-x_{\mathrm{B}})x_{\mathrm{B}}+\epsilon^2\right)\frac{t\,t'}{\mathcal{Q}^4},$$

$$S^{\mathrm{LP,A}}_{++}(n=3) = -\frac{8\Lambda K\left(1-y-\frac{\epsilon^2}{4}y^2\right)}{\left(1+\epsilon^2\right)^3}\left(1+\sqrt{1+\epsilon^2}-2x_{\mathrm{B}}\right)\frac{x_{\mathrm{B}}t\,t'}{\mathcal{Q}^4}\,.$$

<u>photon helicity-flip amplitudes by one unit:</u>

$$C^{\mathrm{LP}}_{0+}(n=0) = \frac{8\sqrt{2}\lambda\Lambda K(1-x_{\mathrm{B}})y\sqrt{1-y-\frac{y^2\epsilon^2}{4}}}{(1+\epsilon^2)^2}\frac{t}{\mathcal{Q}^2}\,,$$

$$C^{\mathrm{LP}}_{0+}(n=1) = -\frac{8\sqrt{2}\lambda\Lambda Ky(1-y)\sqrt{1-y-\frac{y^2\epsilon^2}{4}}}{(1+\epsilon^2)^2}\frac{\widetilde{K}^2}{\mathcal{Q}^2}\,,$$

$$C^{\mathrm{LP}}_{0+}(n=2) = -\frac{8\sqrt{2}\lambda\Lambda Ky\sqrt{1-y-\frac{y^2\epsilon^2}{4}}}{(1+\epsilon^2)^2}\left(1+\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}\right)\,,$$

$$S^{\mathrm{LP}}_{0+}(n=1) = \frac{8\sqrt{2}\Lambda\sqrt{1-y-\frac{y^2\epsilon^2}{4}}}{(1+\epsilon^2)^{5/2}}\left\{\frac{\widetilde{K}^2}{\mathcal{Q}^2}(2-y)^2\right.$$
$$\left.+\left(1+\frac{t}{\mathcal{Q}^2}\right)\left(1-y-\frac{y^2\epsilon^2}{4}\right)\left(2\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}-\left(1-\frac{t}{\mathcal{Q}^2}\right)\epsilon^2\right)\right\}\,,$$

$$S^{\mathrm{LP}}_{0+}(n=2) = \frac{8\sqrt{2}\Lambda K(2-y)\sqrt{1-y-\frac{y^2\epsilon^2}{4}}}{(1+\epsilon^2)^{5/2}}\left(1+\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}\right)\,,$$

$$C^{\mathrm{LP},V}_{0+}(n=0) = \frac{8\sqrt{2}\lambda\Lambda Ky\sqrt{1-y-\frac{y^2\epsilon^2}{4}}}{(1+\epsilon^2)^2}\frac{t}{\mathcal{Q}^2}\left(x_{\mathrm{B}}-\frac{t}{\mathcal{Q}^2}(1-2x_{\mathrm{B}})\right)\,,$$

$$C^{\mathrm{LP},V}_{0+}(n=1) = \frac{8\sqrt{2}\lambda\Lambda y(2-y)\sqrt{1-y-\frac{y^2\epsilon^2}{4}}}{(1+\epsilon^2)^2}\frac{t\widetilde{K}^2}{\mathcal{Q}^4}\,,$$

$$C^{\mathrm{LP},V}_{0+}(n=2) = \frac{8\sqrt{2}\lambda\Lambda Ky(1-x_{\mathrm{B}})\sqrt{1-y-\frac{y^2\epsilon^2}{4}}}{(1+\epsilon^2)^2}\frac{t}{\mathcal{Q}^2}\,,$$

$$S^{\mathrm{LP},V}_{0+}(n=1) = -\frac{8\sqrt{2}\Lambda\sqrt{1-y-\frac{y^2\epsilon^2}{4}}}{(1+\epsilon^2)^{5/2}}\frac{t}{\mathcal{Q}^2}\left\{\frac{\widetilde{K}^2}{\mathcal{Q}^2}(2-y)^2\right.$$
$$\left.+\left(1+\frac{t}{\mathcal{Q}^2}\right)\left(1-y-\frac{y^2\epsilon^2}{4}\right)\left(4-2x_{\mathrm{B}}+3\epsilon^2+\frac{t}{\mathcal{Q}^2}(4x_{\mathrm{B}}(1-x_{\mathrm{B}})+\epsilon^2)\right)\right\}\,,$$

$$S^{\mathrm{LP},V}_{0+}(n=2) = -\frac{8\sqrt{2}\Lambda K(2-y)(1-x_{\mathrm{B}})\sqrt{1-y-\frac{y^2\epsilon^2}{4}}}{(1+\epsilon^2)^{5/2}}\frac{t}{\mathcal{Q}^2}\,,$$

$$C^{\mathrm{LP},A}_{0+}(n=0) = -\frac{8\sqrt{2}\lambda\Lambda Ky\sqrt{1-y-\frac{y^2\epsilon^2}{4}}}{(1+\epsilon^2)^2}\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}\left(1+\frac{t}{\mathcal{Q}^2}\right)\,,$$

$$C^{\mathrm{LP},A}_{0+}(n=2) = \frac{8\sqrt{2}\lambda\Lambda Ky\sqrt{1-y-\frac{y^2\epsilon^2}{4}}}{(1+\epsilon^2)^2}\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}\left(1+\frac{t}{\mathcal{Q}^2}\right)\,,$$

$$S^{\mathrm{LP},A}_{0+}(n=1) = -\frac{16\sqrt{2}\Lambda\left(1-y-\frac{y^2\epsilon^2}{4}\right)^{3/2}}{(1+\epsilon^2)^{5/2}}\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}\left(1+\frac{t}{\mathcal{Q}^2}\right)\left(1-(1-2x_{\mathrm{B}})\frac{t}{\mathcal{Q}^2}\right)\,,$$

$$S^{\mathrm{LP},A}_{0+}(n=2) = -\frac{8\sqrt{2}\Lambda K(2-y)\sqrt{1-y-\frac{y^2\epsilon^2}{4}}}{(1+\epsilon^2)^{5/2}}\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}\left(1+\frac{t}{\mathcal{Q}^2}\right)\,.$$

Photon helicity flip amplitudes by two units:

$$
\begin{aligned}
C^{\mathrm{LP}}_{-+}(n=0) = {}& \frac{4\lambda\Lambda y}{(1+\epsilon^2)^{5/2}}\left\{\frac{\widetilde{K}^2}{\mathcal{Q}^2}(2-y)^2\left(1-\sqrt{1+\epsilon^2}\right)\right. \\
&\left. + \frac{1}{2}\left(1-y-\frac{y^2\epsilon^2}{4}\right)\left(2\frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}-\left(1-\frac{t}{\mathcal{Q}^2}\right)\epsilon^2\right)\left(1-\sqrt{1+\epsilon^2}-\frac{t}{\mathcal{Q}^2}\left(1-2x_{\mathrm{B}}+\sqrt{1+\epsilon^2}\right)\right)\right\},
\end{aligned}
$$

$$
C^{\mathrm{LP}}_{-+}(n=1) = \frac{4\lambda\Lambda K y(2-y)}{(1+\epsilon^2)^{5/2}}\left\{1-\epsilon^2-\sqrt{1+\epsilon^2}-\frac{t}{\mathcal{Q}^2}\left(1-\epsilon^2-\sqrt{1+\epsilon^2}-2x_{\mathrm{B}}\left(2-\sqrt{1+\epsilon^2}\right)\right)\right\},
$$

$$
\begin{aligned}
C^{\mathrm{LP}}_{-+}(n=2) = {}& -\frac{2\lambda\Lambda y\left(1-y-\frac{y^2\epsilon^2}{4}\right)}{(1+\epsilon^2)^{5/2}}\left\{\epsilon^2\left(1+\sqrt{1+\epsilon^2}\right)\right. \\
&\left. - 2\frac{t}{\mathcal{Q}^2}\left((1-x_{\mathrm{B}})\epsilon^2+x_{\mathrm{B}}\left(1+\sqrt{1+\epsilon^2}\right)\right)+\frac{t^2}{\mathcal{Q}^4}(2x_{\mathrm{B}}+\epsilon^2)\left(1-2x_{\mathrm{B}}-\sqrt{1+\epsilon^2}\right)\right\},
\end{aligned}
$$

$$
\begin{aligned}
S^{\mathrm{LP}}_{-+}(n=1) = {}& -\frac{4\Lambda K}{(1+\epsilon^2)^3}\left\{(2-y)^2\left(1+2\epsilon^2-\sqrt{1+\epsilon^2}+\frac{t}{\mathcal{Q}^2}\left(1-2x_{\mathrm{B}}-\sqrt{1+\epsilon^2}\right)\right)\right. \\
&\left. - \left(1-y-\frac{y^2\epsilon^2}{4}\right)\left(2+\epsilon^2-2\sqrt{1+\epsilon^2}+\frac{t}{\mathcal{Q}^2}\left(\epsilon^2-4\sqrt{1+\epsilon^2}+2x_{\mathrm{B}}(1+\sqrt{1+\epsilon^2})\right)\right)\right\},
\end{aligned}
$$

$$
\begin{aligned}
S^{\mathrm{LP}}_{-+}(n=2) = {}& -\frac{4\Lambda(2-y)\left(1-y-\frac{y^2\epsilon^2}{4}\right)}{(1+\epsilon^2)^3}\left\{\frac{t}{\mathcal{Q}^2}\left(2+2\sqrt{1+\epsilon^2}+\epsilon^2\sqrt{1+\epsilon^2}-x_{\mathrm{B}}\left(3-\epsilon^2+3\sqrt{1+\epsilon^2}\right)\right)\right. \\
&\left. + \frac{t^2}{\mathcal{Q}^4}\left(\epsilon^2-2x_{\mathrm{B}}^2(2+\sqrt{1+\epsilon^2})+x_{\mathrm{B}}(3-\epsilon^2+\sqrt{1+\epsilon^2})\right)+\epsilon^2\left(1+\sqrt{1+\epsilon^2}\right)\right\},
\end{aligned}
$$

$$
S^{\mathrm{LP}}_{-+}(n=3) = \frac{4\Lambda K\left(1-y-\frac{y^2\epsilon^2}{4}\right)}{(1+\epsilon^2)^3}\left\{2+\epsilon^2+2\sqrt{1+\epsilon^2}+\frac{t}{\mathcal{Q}^2}\left(\epsilon^2+2x_{\mathrm{B}}(1+\sqrt{1+\epsilon^2})\right)\right\},
$$

$$
\begin{aligned}
C^{\mathrm{LP}}_{-+}(n=0) = {}& \frac{2\lambda\Lambda y}{(1+\epsilon^2)^{5/2}}\frac{t}{\mathcal{Q}^2}\left\{(4-2x_{\mathrm{B}}+3\epsilon^2)\left(1-y-\frac{y^2\epsilon^2}{4}\right)\left(1+\frac{t}{\mathcal{Q}^2}\frac{4x_{\mathrm{B}}(1-x_{\mathrm{B}})+\epsilon^2}{4-2x_{\mathrm{B}}+3\epsilon^2}\right)\right. \\
&\left. \times\left(\sqrt{1+\epsilon^2}-1+\frac{t}{\mathcal{Q}^2}\left(1-2x_{\mathrm{B}}+\sqrt{1+\epsilon^2}\right)\right)+2(2-y)^2(\sqrt{1+\epsilon^2}-1+2x_{\mathrm{B}})\frac{\widetilde{K}^2}{\mathcal{Q}^2}\right\},
\end{aligned}
$$

$$
\begin{aligned}
C^{\mathrm{LP}}_{-+}(n=1) = {}& -\frac{4\lambda\Lambda y(2-y)}{(1+\epsilon^2)^{5/2}}\frac{t}{\mathcal{Q}^2}\left\{5-4x_{\mathrm{B}}+3\epsilon^2-\sqrt{1+\epsilon^2}\right. \\
&\left. - \frac{t}{\mathcal{Q}^2}\left(1-\epsilon^2-\sqrt{1+\epsilon^2}-2x_{\mathrm{B}}(4-4x_{\mathrm{B}}-\sqrt{1+\epsilon^2})\right)\right\},
\end{aligned}
$$

$$
\begin{aligned}
C^{\mathrm{LP}}_{-+}(n=2) = {}& -\frac{2\lambda\Lambda y\left(1-y-\frac{y^2\epsilon^2}{4}\right)}{(1+\epsilon^2)^{5/2}}\frac{t}{\mathcal{Q}^2}\left(4-2x_{\mathrm{B}}+3\epsilon^2+\frac{t}{\mathcal{Q}^2}(4x_{\mathrm{B}}(1-x_{\mathrm{B}})+\epsilon^2)\right) \\
&\times\left(1+\sqrt{1+\epsilon^2}-\frac{t}{\mathcal{Q}^2}(1-\sqrt{1+\epsilon^2}-2x_{\mathrm{B}})\right),
\end{aligned}
$$

$$
\begin{aligned}
S^{\mathrm{LP},V}_{-+}(n=1) = {}& -\frac{4\Lambda K}{(1+\epsilon^2)^3}\frac{t}{\mathcal{Q}^2}\left\{\left(2-2y+y^2+\frac{y^2\epsilon^2}{2}\right)\right. \\
&\times\left(3+2\epsilon^2+\sqrt{1+\epsilon^2}-2x_{\mathrm{B}}(1+\sqrt{1+\epsilon^2})-\frac{t}{\mathcal{Q}^2}(1-2x_{\mathrm{B}})(1-2x_{\mathrm{B}}-\sqrt{1+\epsilon^2})\right)
\end{aligned}
$$

$$+\left(1-y-\frac{y^2\epsilon^2}{4}\right)\left(8+5\epsilon^2-2x_{\rm B}(3-\sqrt{1+\epsilon^2})\right.$$
$$\left.\left.-\frac{t}{\mathcal{Q}^2}\left(2-\epsilon^2+2\sqrt{1+\epsilon^2}-12x_{\rm B}(1-x_{\rm B})-4x_{\rm B}\sqrt{1+\epsilon^2}\right)\right)\right\},$$
$$S^{{\rm LP},V}_{-+}(n=2)=-\frac{4\Lambda(2-y)\sqrt{1-y-\frac{y^2\epsilon^2}{4}}}{(1+\epsilon^2)^{5/2}}\frac{t}{\mathcal{Q}^2}\left\{(2-x_{\rm B})(1+\sqrt{1+\epsilon^2})\right.$$
$$\left.+\epsilon^2+\frac{4\widetilde{K}^2(1-2x_{\rm B})}{\mathcal{Q}^2\sqrt{1+\epsilon^2}}+\frac{t}{\mathcal{Q}^2}\left(\epsilon^2+x_{\rm B}(3-2x_{\rm B}+\sqrt{1+\epsilon^2})\right)\right\},$$
$$S^{{\rm LP},V}_{-+}(n=3)=-\frac{4\Lambda K\left(1-y-\frac{y^2\epsilon^2}{4}\right)}{(1+\epsilon^2)^{5/2}}\frac{t}{\mathcal{Q}^2}\left\{4-4x_{\rm B}+\frac{t'}{\mathcal{Q}^2}\frac{4x_{\rm B}(1-x_{\rm B})+\epsilon^2}{\sqrt{1+\epsilon^2}}\right\},$$
$$C^{{\rm LP},A}_{-+}(n=0)=\frac{4\lambda\Lambda x_{\rm B}y}{(1+\epsilon^2)^{5/2}}\frac{t}{\mathcal{Q}^2}\left\{2(2-y)^2\left((1-x_{\rm B})\frac{t}{\mathcal{Q}^2}\left(1+\frac{x_{\rm B}t}{\mathcal{Q}^2}\right)+\left(1+\frac{t}{\mathcal{Q}^2}\right)^2\frac{\epsilon^2}{4}\right)\right.$$
$$\left.-\left(1-y-\frac{y^2\epsilon^2}{4}\right)\left(1-(1-2x_{\rm B})\frac{t}{\mathcal{Q}^2}\right)\left(1-\sqrt{1+\epsilon^2}-\frac{t}{\mathcal{Q}^2}(1+\sqrt{1+\epsilon^2}-2x_{\rm B})\right)\right\},$$
$$C^{{\rm LP},A}_{-+}(n=1)=-\frac{16\lambda\Lambda x_{\rm B}y(2-y)}{(1+\epsilon^2)^{5/2}}\frac{t}{\mathcal{Q}^2}\left(1-(1-2x_{\rm B})\frac{t}{\mathcal{Q}^2}\right),$$
$$C^{{\rm LP},A}_{-+}(n=2)=-\frac{4\lambda\Lambda x_{\rm B}y\left(1-y-\frac{y^2\epsilon^2}{4}\right)}{(1+\epsilon^2)^{5/2}}$$
$$\times\frac{t}{\mathcal{Q}^2}\left(1-(1-2x_{\rm B})\frac{t}{\mathcal{Q}^2}\right)\left\{1+\sqrt{1+\epsilon^2}-\frac{t}{\mathcal{Q}^2}\left(1-\sqrt{1+\epsilon^2}-2x_{\rm B}\right)\right\},$$
$$S^{{\rm LP},A}_{-+}(n=1)=-\frac{8\Lambda K\left(2-2y+y^2+\frac{y^2\epsilon^2}{2}\right)}{(1+\epsilon^2)^3}(1+\sqrt{1+\epsilon^2})\frac{x_{\rm B}t}{\mathcal{Q}^2}\left(1-\frac{t}{\mathcal{Q}^2}\frac{1-\sqrt{1+\epsilon^2}-2x_{\rm B}}{1+\sqrt{1+\epsilon^2}}\right)$$
$$-\frac{8\Lambda K\left(1-y+\frac{y^2\epsilon^2}{4}\right)}{(1+\epsilon^2)^3}\frac{x_{\rm B}t}{\mathcal{Q}^2}\left\{3-\sqrt{1+\epsilon^2}-\frac{t}{\mathcal{Q}^2}\left(3+\sqrt{1+\epsilon^2}-6x_{\rm B}\right)\right\},$$
$$S^{{\rm LP},A}_{-+}(n=2)=-\frac{4\Lambda(2-y)\left(1-y-\frac{y^2\epsilon^2}{4}\right)}{(1+\epsilon^2)^3}\frac{x_{\rm B}t}{\mathcal{Q}^2}\left\{1+4\frac{\widetilde{K}^2}{\mathcal{Q}^2}\right.$$
$$\left.+\sqrt{1+\epsilon^2}-2\frac{t}{\mathcal{Q}^2}\left(1-2x_{\rm B}-x_{\rm B}\sqrt{1+\epsilon^2}\right)-\frac{t}{\mathcal{Q}^2}(1-2x_{\rm B})\left(1-2x_{\rm B}-\sqrt{1+\epsilon^2}\right)\right\},$$
$$S^{{\rm LP},A}_{-+}(n=3)=-\frac{8\Lambda K\left(1-y-\frac{y^2\epsilon^2}{4}\right)}{(1+\epsilon^2)^3}\frac{x_{\rm B}t}{\mathcal{Q}^2}\left\{1+\sqrt{1+\epsilon^2}-\frac{t}{\mathcal{Q}^2}\left(1-2x_{\rm B}-\sqrt{1+\epsilon^2}\right)\right\}. \tag{A.6}$$

References

[1] D. Müller, D. Robaschik, B. Geyer, F.-M. Dittes, J. Horejsi, Fortschr. Phys. 42 (1994) 101;
X. Ji, Phys. Rev. D 55 (1997) 7114;
A.V. Radyushkin, Phys. Rev. D 56 (1997) 5524.

[2] K. Goeke, M.V. Polyakov, M. Vanderhaeghen, Prog. Part. Nucl. Phys. 47 (2001) 401;
M. Diehl, Phys. Rept. 388 (2003) 41;

A.V. Belitsky, A.V. Radyushkin, Phys. Rept. 418 (2005) 1.

[3] A.V. Belitsky, D. Müller, A. Kirchner, Nucl. Phys. B 629 (2002) 323.

[4] A.V. Belitsky, D. Müller, Phys. Rev. D 79 (2009) 014017.

[5] M. Penttinen, M.V. Polyakov, A.G. Shuvaev, M. Strikman, Phys. Lett. B 491 (2000) 96.

[6] A.V. Belitsky, D. Müller, Nucl. Phys. B 589 (2000) 611.

[7] I.V. Anikin, B. Pire, O.V. Teryaev, Phys. Rev. D 62 (2000) 071501.

[8] A.V. Radyushkin, C. Weiss, Phys. Lett. B 493 (2000) 332; Phys. Rev. D 63 (2001) 114012.

[9] M. Diehl, T. Gousset, B. Pire, J.P. Ralston, Phys. Lett. B 411 (1997) 193.

[10] K. Kumericki, D. Müller, *Deeply virtual Compton scattering at small* $x_{\rm B}$ *and the access to the GPD H*, arXiv:0904.0458 [hep-ph].

[11] C. Munoz Camacho et al., Phys. Rev. Lett. 97 (2006) 262002.

[12] F.X. Girod et al., Phys. Rev. Lett. 100 (2008) 162002.

[13] A.V. Belitsky, D. Mueller, Phys. Rev. D 79 (2009) 014017.

[14] R.K. Ellis, W. Furmanski, R. Petronzio, Nucl. Phys. B 212 (1983) 29.

[15] I.I. Balitsky, V.M. Braun, Nucl. Phys. B 311 (1989) 541.

[16] A.V. Belitsky, D. Müller, Phys. Lett. B 507 (2001) 173.

[17] B. Geyer, D. Robaschik, J. Eilers, Nucl. Phys. B 704 (2005) 279.